

## A selected survey of umbral calculus

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# A Selected Survey of Umbral Calculus \*

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Dedicated to the memory our friend and colleague Gian-Carlo Rota (1932-1999)

## Abstract

We survey the mathematical literature on umbral calculus (otherwise known as the calculus of finite differences) from its roots in the 19th century (and earlier) as a set of “magic rules” for lowering and raising indices, through its rebirth in the 1970’s as Rota’s school set it on a firm logical foundation using operator methods, to the current state of the art with numerous generalizations and applications. The survey itself is complemented by a fairly complete bibliography (over 500 references) which we expect to update regularly.

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\*More up to date information may be available in the unofficial hypertext version of this survey at <http://www.tm.tue.nl/vakgr/ppk/bucchianico/hypersurvey/index.html> .

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## Contents

<b>1</b>	<b>What is the Umbral Calculus?</b>	<b>2</b>
<b>2</b>	<b>History</b>	<b>3</b>
<b>3</b>	<b>Applications of the umbral calculus</b>	<b>5</b>
3.1	Lagrange inversion . . . . .	5
3.2	Symmetric functions . . . . .	5
3.3	Combinatorial counting and recurrences . . . . .	5
3.4	Graph theory . . . . .	6
3.5	Coalgebras . . . . .	6
3.6	Statistics . . . . .	6
3.7	Probability theory . . . . .	6
3.8	Topology . . . . .	7
3.9	Analysis . . . . .	7
3.10	Physics . . . . .	7
3.11	Invariant theory . . . . .	8
<b>4</b>	<b>Generalizations and variants of the umbral calculus</b>	<b>8</b>
<b>5</b>	<b>Further information</b>	<b>9</b>

## 1 What is the Umbral Calculus?

The theory of binomial enumeration is variously called the calculus of finite differences or the umbral calculus. This theory studies the analogies between various sequences of polynomials  $p_n$  and the powers sequence  $x^n$ . The subscript  $n$  in  $p_n$  was thought of as the shadow ( “umbra” means “shadow” in Latin, whence the name umbral calculus) of the superscript  $n$  in  $x^n$ , and many parallels were discovered between such sequences.

Take the example of the lower factorial polynomials  $(x)_n = x(x - 1) \cdots (x - n + 1)$ . Just as  $x^n$  counts the number of functions from an  $n$ -element set to an  $x$ -element set,  $(x)_n$  counts the number of injections. Just as the derivative maps  $x^n$  to  $nx^{n-1}$ , the forward difference operator maps  $(x)_n$  to  $n(x)_{n-1}$ . Just as also polynomials can be expressed in terms of  $x^n$  via Taylor’s theorem

$$f(x + a) = \sum_{n=0}^{\infty} \frac{a^n D^n f(x)}{n!},$$

Newton's theorem allows similar expressions for  $(x)_n$

$$f(x+a) = \sum_{n=0}^{\infty} (a)_n \frac{\Delta^n f(x)}{n!}$$

where  $\Delta f(x) = f(x+1) - f(x)$ . Just as  $(x+y)^n$  is expanded using the binomial theorem

$$(x+a)^n = \sum_{k=0}^{\infty} \binom{n}{k} a^k x^{n-k},$$

$(x+y)_n$  expands by Vandermonde's identity

$$(x+a)_n = \sum_{k=0}^{\infty} \binom{n}{k} (a)_k (x)_{n-k}.$$

And so on. [289, 267]

This theory is quite classical with its roots in the works of Barrow and Newton — expressed in the belief the some polynomial sequences such as  $(x)_n$  really were just like the powers of  $x$ . Nevertheless, many doubts arose as to the correctness of such informal reasoning, despite various (see e.g., [46]) attempts to set it on an axiomatic base.

The contribution of Rota's school was to first set umbral calculus on a firm logical foundation by using operator methods [289, 389]. That being done, sequences of polynomials of binomial type

$$p_n(x+y) = \sum_{k=0}^n \binom{n}{k} p_k(x) p_{n-k}(y) \quad (1)$$

could be for once studied systematically rather than as a collection of isolated yet philosophically similar sequences. The sister sequence of divided powers  $q_n(x) = p_n(x)/n!$  then obeys the identity

$$q_n(x+y) = \sum_{k=0}^n q_k(x) q_{n-k}(y). \quad (2)$$

Given any species of combinatorial structures (or quasi-species), let  $p_n(x)$  be the number of functions from an  $n$ -element set to an  $x$ -element set enriched by this species. A function is enriched by associating a (weighted) structure with each of its fibers. All sequences of binomial type arise in this manner, and conversely, all such sequence are of binomial type.

## 2 History

As mentioned in the introduction, the history of the umbral calculus goes back to the 17th century. The rise of the umbral calculus, however, takes place in the second half of the 19th century with the work of such mathematicians as Sylvester (who invented the name), Cayley and Blissard (see e.g., [44]). Although widely used, the umbral calculus was nothing more than a set of "magic" rules of lowering and raising indices (see e.g., [188]). These rules worked well in practice, but lacked a proper foundation. Let us consider an example of such a "magic rule". The Bernoulli numbers  $B_n$  are defined by the generating function

$$\sum_{n=0}^{\infty} B_n \frac{x^n}{n!} = \frac{x}{e^x - 1}. \quad (3)$$

The magic trick used in the 19th century Umbral Calculus is to write

$$\sum_{n=0}^{\infty} B_n \frac{x^n}{n!} \simeq \sum_{n=0}^{\infty} B^n \frac{x^n}{n!} = e^{Bx}, \quad (4)$$

where we use the  $\simeq$  symbol to stress the purely formal character of this manipulation. A trivial standard algebraic manipulation then yields

$$e^{(B+1)x} - e^{Bx} \simeq x, \quad (5)$$

from which we deduce by equating coefficients of  $\frac{x^n}{n!}$  that

$$(B+1)^n - B^n \simeq \delta_{1n}, \quad (6)$$

where  $\delta_{1n}$  denotes the Kronecker delta. If we now expand (6) using the Binomial Theorem and change the superscripts back to subscripts, we obtain the following relation for the Bernoulli numbers:

$$\sum_{k=0}^{n-1} \binom{n}{k} B_k = \delta_{1n}, \quad (7)$$

which can be shown to be true (a standard direct proof is possible by considering the reciprocal power series  $(e^x - 1)/x$ ).

Early attempts to put the umbral manipulation on an axiomatic basis (see [46]) were unsuccessful. Although the mathematical world remained sceptical of the umbral calculus, it was used extensively (e.g., in Riordan's highly respected book on combinatorics).

A second line in the history of the umbral calculus in the form that we know today, is the theory of Sheffer polynomials. The history of Sheffer polynomials goes back to 1880 when Appell studied sequences of polynomials  $(p_n)_n$  satisfying  $p'_n = n p_{n-1}$  (see [20]). These sequences are nowadays called Appell polynomials. Although this class was widely studied (see the bibliography in [133] which is included in the bibliography of this survey), it was not until 1939 that Sheffer noticed the similarities with which the introduction of this survey starts. These similarities led him to extend the class of Appell polynomials which he called polynomials of type zero (see [415]), but which nowadays are called Sheffer polynomials. This class already appeared in [277]. Although Sheffer uses operators to study his polynomials, his theory is mainly based on formal power series. In 1941 the Danish actuary Steffensen also published a theory of Sheffer polynomials based on formal power series [444]. Steffensen uses the name poweroids for Sheffer polynomials (see also [423, 422, 444, 445, 446, 443]). However, these theories were not adequate as they do not provide sufficient computational tools (expansion formulas etc.).

A third line in the history of the umbral calculus is the theory of abstract linear operators. This line goes back to the work of Pincherle starting in the 1890's (i.e., in the beginnings of functional analysis). His early work is laid down in the monumental monograph [336]. Pincherle went surprisingly far considering the state of functional analysis in those days, but his work lacked explicit examples. The same applies to papers by others in this field (see e.g., [129, 133, 489]).

A prelude to the merging of these three lines can be seen in [389], in which operators methods are used to free umbral calculus from its mystery. In [289] the ideas from [389] are extended to give a beautiful theory combining enriched functions, umbral methods and operator methods. However, only the subclass of polynomials of binomial type are treated in [289]. The extension to Sheffer polynomials is accomplished in [392]. The latter paper is much more geared towards special functions, while the former paper is a combinatorial paper.

The papers [289] and [392] were soon followed by papers that reacted directly on the new umbral calculus. E.g., Fillmore and Williamson showed that with equal ease the Rota umbral

calculus could be situated in abstract vector spaces instead of the vector space of polynomials [162], Zeilberger noticed connections with Fourier analysis [505] and Garsia translated the operator methods of Rota back into formal power series [170].

We conclude this section with mentioning the remarkable papers [98, 146, 393, 394, 395] in which the authors manage to make sense of the 19th century umbral calculus (thereby fulfilling Bell's dream [44]; cf. [355]). A related paper is [132], where a related technique is used to give proofs of results like inclusion-exclusion and Bonferroni inequalities.

### 3 Applications of the umbral calculus

We now indicate papers that apply the umbral calculus to various fields.

#### 3.1 Lagrange inversion

An important property of (any extension of) the umbral calculus is that it has its own generalization of Lagrange's inversion formula (as follows from the closed forms for basic polynomials [289, Theorem 4], in particular the Transfer Formula). Thus we find many papers in which new forms of the Lagrange's inversion formula is derived using umbral calculus [24, 37, 125, 199, 213, 214, 239, 302, 441, 475, 485].

#### 3.2 Symmetric functions

In [259], the umbral calculus is generalized to symmetric functions. When counting enriched functions (functions, injections, reluctant functions, dispositions, etc.) from  $N$  to  $X$ , we can assign a weight to each function according to its fiber structure.  $w(f) = \prod_{i \in N} f(i) = \prod_{x \in X} x^{|f^{-1}(x)|}$ . The total number of such functions is a symmetric function  $p_n(X)$  of degree  $n$  where  $n = |N|$ . The elementary and complete symmetric functions are (up to a multiple of  $n!$ ) good examples of such sequences. They obey their own sort of binomial theorem

$$p_n(X \cup Y) = \sum_{k=0}^n \binom{n}{k} p_k(X) p_{n-k}(Y).$$

The generating functions of  $p_n(X)$  are directly related to that of their underlying species. By specializing all  $x$  variables to 1, we return to the study to polynomials.

Nevertheless, such a sequence  $p_n(X)$  is not a basis for the vector space of symmetric functions. Furthermore, the  $p_n(X)$  may not even be algebraically independent. These problems were solved in [279, 280] where an umbral calculus of *full* sequences of symmetric functions  $p_\lambda(X)$  indexed by an integer partition  $\lambda$  is presented.

#### 3.3 Combinatorial counting and recurrences

Another rich field of application is linear recurrences and lattice path counting. Here we should first of all mention the work of Niederhausen [292, 293, 294, 296, 300, 301, 308, 309, 310]. The starting idea of the work of Niederhausen is the fact that if  $Q$  is a delta operator, then  $E^a Q$  is also a delta operator, and hence has a basic sequence. The relations between the basic sequences of

these operators enables him to upgrade the binomial identity for basic sequences to a general Abel-like identity for Sheffer sequences. For a nice introduction to this we refer to the survey papers [308, 309]. In spite of their titles, the papers [492, 493] are more directed to the general theory of Umbral Calculus, then to specific applications in lattice path counting. A different approach to lattice path counting is taken in the papers [364, 365, 387]. In these papers a functional approach is taken in the spirit of [388, 381] rather than an operator approach. Finally, an approach based on umbrae (see end of section 2) can be found in [456]).

As stated in the introduction, umbral calculus is strongly related with the Joyal theory of species (see e.g., [114, 399]).

General papers on counting combinatorial objects include [189, 241, 240, 369, 454].

### 3.4 Graph theory

The chromatic polynomial of a graph can be studied in a fruitful way using a variant of the Umbral Calculus. This is done by Ray and co-workers, see [257, 356, 363, 361]. A generalization of the chromatic polynomial to so-called partition sets can be found in [256].

### 3.5 Coalgebras

Coalgebraic aspects of umbral calculus are treated in [100, 102, 147, 220, 231, 257, 287, 290, 354, 360]. E.g., umbral operators are exactly coalgebra automorphisms of the usual Hopf algebra of polynomials.

### 3.6 Statistics

Non-parametric statistics (or distribution-free statistics) has a highly combinatorial flavor. In particular, lattice path counting techniques are often used. It is therefore not surprising that the main applications of umbral calculus to statistics are of a combinatorial nature [291, 295, 297, 301].

However, there are also applications to parametric statistics. Di Bucchianico and Loeb link natural exponential families with Sheffer polynomials ([144]). They show that the variance function of a natural exponential family determines the delta operator of the associated Sheffer sequence. As a side result, they find all orthogonal Sheffer polynomials.

Another application concerns statistics for parameters in power series distributions ([108, 246]).

### 3.7 Probability theory

As suggested in [392, p. 752], there is a connection between polynomials of binomial type and compound Poisson processes. Two different approaches can be found in [97, 440]. A connection of polynomials of binomial type with renewal sequences can be found in [438]. Probabilistic aspects of Lagrange inversion and polynomials of binomial type can be found in [441]. Various probabilistic representations of Sheffer polynomials can be found in [137]. Many of the above results can also be found in the book [138].

The following papers do not actually use umbral calculus, but provide interesting information on probabilistic aspects of generalized Appell polynomials [180, 181, 182, 270].

### 3.8 Topology

Applications of the umbral calculus to algebraic topology can be found in the work of Ray [354, 355, 358, 357, 359]. An application of umbral calculus to (co)homology can be found in [253, 252].

### 3.9 Analysis

Cholewinski developed a version of the umbral calculus for studying differential equations of Bessel type and related topics in [120]; see also [121].

A connection between approximation operators and polynomials of binomial type can be found in [210]. Further papers in this direction are [216, 283, 455]. Padé approximants are treated in [499].

Orthogonal polynomials play an important role in analysis. It is therefore important to know whether polynomials are orthogonal. The classification of orthogonal Sheffer polynomials was first found by Meixner [277]; it has been reproved many times (see e.g., [144, 169, 229, 387, 415]). Orthogonal Sheffer polynomials on the unit circle have been characterized by Kholodov [229].

General papers on orthogonal polynomials and umbral calculus are [175, 229, 353].

Hypergeometric and related functions are dealt with in an umbral calculus way in [467, 469, 503].

There are different ways of implementing  $q$ -analysis in terms of umbral calculus. The first  $q$ -umbral calculus can be found in [16]. Other  $q$ -umbral calculi can be found in [11, 14, 124, 126, 125, 205, 233, 370, 382]. Comparisons between different  $q$ -umbral calculi can be found in [10, 382]. Various basic hypergeometric (i.e.,  $q$ -hypergeometric) identities are derived in [110]. A  $q$ -Saalschütz identity is derived in an umbral way in [425].

Constructing umbral calculi based on the operator  $f(x) \mapsto \frac{f(x)-f(y)}{x-y}$  yields a powerful way to study interpolation theory [196, 376, 422, 476, 477, 478, 479].

Banach algebras are used by Di Bucchianico [135] to study the convergence properties of the generating function of polynomials of binomial type and by Grabiner [186, 187] to extend the umbral calculus to certain classes of entire functions.

Applications of umbral calculus to numerical analysis can be found in several papers of Wimp [498, 501, 502, 500].

Umbral calculus is a powerful tool for dealing with recurrences. Recurrences play an important role in the theory of filter banks in signal processing. An umbral calculus approach based on recursive matrices can be found in [33, 34]. A related theory is the theory of wavelets. An umbral approach to the refinement equations for wavelets can be found in [420].

### 3.10 Physics

An application of umbral calculus to the physics of gases can be found in [497].

Biedenharn and his co-workers use umbral techniques in group theory and quantum mechanics [51, 50].

Gzyl found connections between umbral calculus, the Hamiltonian approach in physics and quantum mechanics [190, 191, 192]. Closely related to this topic is the work by Feinsilver (see in particular [155, 154, 158]).

Morikawa developed an Umbral Calculus for differential polynomials in infinitely many variables with applications to statistical physics ([285]).



### 3.11 Invariant theory

There are some papers that link invariant theory (either classical or modern forms like supersymmetric algebras) with Umbral Calculus ([66, 107]).

## 4 Generalizations and variants of the umbral calculus

The umbral calculus of [392] is restricted to the class of Sheffer polynomials. It was therefore natural to extend the umbral calculus to larger classes of polynomials. Viskov first extended the umbral calculus to so-called generalized Appell polynomials (or Boas-Buck polynomials) [483] and then went on to generalize this to arbitrary polynomials [484]. The extension to generalized Appell polynomials makes it possible to apply umbral calculus to  $q$ -analysis (see section 3.9) or important classes of orthogonal polynomials like the Jacobi polynomials [382]. Roman remarks [381] that Ward back in 1936 attempted to construct an umbral calculus for generalized Appell polynomials [489]. Other interesting papers in this direction are [87, 140, 274].

An extension of the umbral calculus to certain classes of entire function can be found in [186, 187].

Another extension of the umbral calculus is to allow several variables [23, 73, 172, 216, 281, 323, 367, 373, 427, 490, 491]. However, all these extensions suffer from the same drawback, viz. they are basis dependent. A first version of a basis-free umbral calculus in finite and infinite dimensions was obtained by Di Bucchianico, Loeb and Rota ([145]).

Roman [377, 378, 380, 381] developed a version to the umbral calculus for inverse formal power series of negative degree. Most theorems of umbral calculus have their analog in this context. In particular, any shift-invariant operator of degree one (delta operator) has a special sequence associated with it satisfying a type of binomial theorem. Nevertheless, despite its philosophical connections, this theory remained completely distinct from Rota's theory treating polynomials.

Later, in [267], a theory was discovered which generalized simultaneously Roman and Rota's umbral calculi by embedding them in a logarithmic algebra containing both positive and negative powers of  $x$ , and logarithms. A subsequent generalization [258, 260] extends this algebra to a field which includes not only  $x$  and  $\log(x)$  but also the iterated logarithms, all of whom may be raised to any real power. Sequences of polynomials  $p_n(x)$  are then replaced with sequence of asymptotic series  $p_a^\alpha$  where the degree  $a$  is a real and the level  $\alpha$  is a sequence of reals. Rota's theory is the restriction to level  $\alpha = (0)$ , and degree  $a \in \mathbf{N}$ . Roman's theory is the restriction to level  $\alpha = (1)$  and degree  $a \in \mathbf{Z}^-$ . Thus, the difficulty in uniting Roman and Rota's theories was essentially that they lay on different levels of some larger yet unknown algebra. Other papers in this direction are [230, 304, 384, 385].

Rota's operator approach to the calculus of finite difference can be thought of as a systematic study of shift-invariant operators on the algebra of polynomials. The expansion theorem [289, Theorem 2] states that all shift-invariant operators can be written as formal power series in the derivative  $D$ . If  $\theta : \mathbf{C}[x] \rightarrow \mathbf{C}[x]$  is a shift-invariant operator, then

$$\theta = \sum_{k=0}^{\infty} a_k D^k / k!$$

where  $a_n = [\theta x^n]_{x=0}$ .

However, a generalization of this by Kurbanov and Maksimov [245] to arbitrary linear operators has received surprisingly little attention. Any linear operator  $\theta : \mathbf{C}[x] \rightarrow \mathbf{C}[x]$  can be

expanded as a formal power series in  $X$  and  $D$  where  $X$  is the operator of multiplication by  $x$ . More generally, let  $B$  be any linear operator which reduces the degree of nonzero polynomials by one. (By convention,  $\deg(0) = -1$ .) Thus,  $B$  might be not only the derivative or any delta operator, but also the  $q$ -derivative, the divided difference operator, etc. Then  $\theta$  can be expanded in terms of  $x$  and  $B$ :

$$\theta = \sum_{k=0}^{\infty} f_n(X)B^n.$$

A detailed study of this kind of expansion and its sister expansion  $\theta = \sum_{k=0}^{\infty} f_n(B)X^n$  can be found in [142].

Extensions of umbral calculus to symmetric functions have already been mentioned in section 3.2..

Another interesting extension is the divided difference umbral calculus, which is useful for interpolation theory (see Section 3.9).

Extensions of the umbral calculus to the case where the base field is not of characteristic zero ([470, 480, 481]).

Finally, we mention that there also interesting variants of the umbral calculus. An important variant is the umbral calculus that appears by restricting the polynomials to integers. This theory is developed by Barnabei and co-authors ([26, 27, 28]) and is called the theory of recursive matrices. There are applications in signal processing ([33, 34]) and to inversion of combinatorial sums ([127]).

## 5 Further information

A software package for performing calculations in the umbral calculus is available ([60, 61]).

The bibliography of this survey is based on searches in the Mathematical Reviews and on the bibliographies in [392, 388] which have not yet been included completely. The bibliography contains papers on Umbral Calculus and related topics such as Sheffer polynomials.

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