BACHELOR

Density Gradients and Stratification in Estuaries

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Density Gradients and Stratification in Estuaries

Bachelor Thesis

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Abstract

Estuaries, where the rivers and sea meet, are of interest because of both ecological and economical reasons. In these areas, it is important to predict the flow field which is highly influenced by the buoyancy impact from the river. In an estuary, the difference in densities between the water from the river and the sea generates a horizontal density gradient that leads to a pressure gradient. This pressure gradient drives a flow in which the salt water from the sea flows towards the river at the bottom of the estuary, and the fresh water from the river flows towards the sea at the surface. This is called the *estuarine circulation* in which the water column becomes stratified instead of mixed.

This project investigates the possibility of decomposing the density into a fraction due to a horizontal gradient and a fraction due to a vertical gradient (i.e. stratification). Decomposing the density in a horizontal and a vertical fraction could lead to new research opportunities, for example simulations requiering horizontal boundary conditions. We also would like to verify whether a horizontal density gradient that is initially imposed to be linear, conserves its linearity with the onset of stratification.

The estuary is modelled as a two-dimensional problem that is solved using numerical simulations without turbulence modelling. The simulations showed that the horizontal density gradient between the river and the sea does not depend on the horizontal coordinate, but only on the depth and the time. The approximation of decomposing the density seems to hold for depths very close to the fresh and salt water interface and far away from the interface.

Furthermore, the time evolution of the horizontal density gradient had a different behaviour depending on its place in the water column. Three layers could be identified. Close to the fresh and salt water interface, the gradient increases linearly over time, until it reaches a value that only depends on the depth. Far from the interface and far from the walls, the gradient decreases to zero rapidly. Close to the walls, the gradient decreases to zero after a long time because of boundary effects that oppose this process. After the gradient reaches this value, it remains unchanged for a long time until diffusive effects become dominant over the stratification.

Furthermore, the stratification seems to be uniform over the horizontal, and there seems to be a linear relationship between the horizontal density gradient and the stratification. However, more research about this relation is needed.

Contents

Chapter 1

Introduction

Estuaries are the areas where the fresh water from the river meets the salt water from the sea or the ocean, and they are very important for both ecological and economical reasons. To maintain the estuarine area as safe and as efficient as possible, it is crucial to know about the hydrodynamics of an estuary.

In estuaries, the density gradient between the sea and the river leads to a pressure gradient that drives a flow, where the salt water flows to the river and the fresh water flows to the sea. This flow results in a stratified flow, where the fresh water flows at the top and the salt water at the bottom of the estuary, which is called an *estuarine circulation.* Figure 1.1 shows a schematic representation about this process.

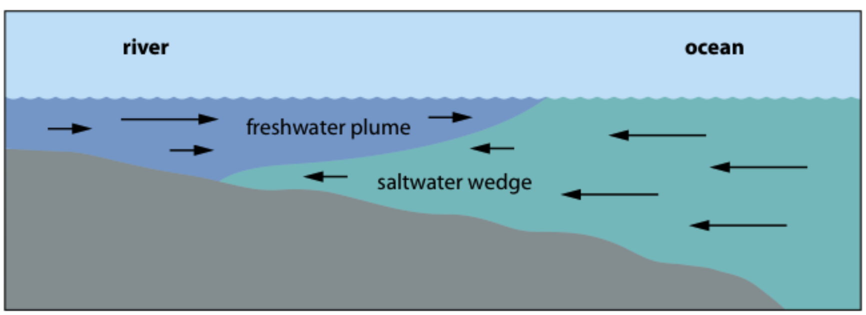

Figure 1.1: Circulation of fresh and salt water between a river and an ocean. The fresh water flows through the top of the estuary and the salt water flows through the bottom. *Figure from Oceanographic Phenomena by Norman* [1].

This project is inspired by the experiments done by Simpson and Linden [2] . Under particular assumptions, the estuary can be reduced to a simplified model, which is then solved using two-dimensional (2D) numerical simulations without turbulence modelling. These simulations are done using the software COMSOL, and then post processed by MATLAB.

The aim of this project is to study the evolution of the density field in an idealized case and test a simple model for this evolution. If this model holds, it could provide new research opportunities.

In the theory chapter, the model and the variables of the system with the relevant equations will be explained. In the computational model, the numerical setup of the model is presented with the initial and boundary conditions imposed on the system. Also the simulation done by COMSOL and the post processing by MATLAB is explained in this chapter. After that, the results chapter presents the relevant data obtained from the simulations, from which we try to verify our model. At the end, the conclusion presents the important information gathered from this project with some ideas for future research.

Chapter 2

Theory

2.1 Problem definition

To save computational time and for simplicity, the estuary is modeled as a 2D channel. We only consider the direction from the river to the sea (the x-direction) and the vertical direction (the z-direction). Figure 2.1 presents a sketch of this channel.

The tides and the Coriolis force are neglected in this project, which means the third dimension has no influence on the stratification. In a later stage, the third dimension could be implemented into the model to make it more realistic, but that is beyond the scope of this project.

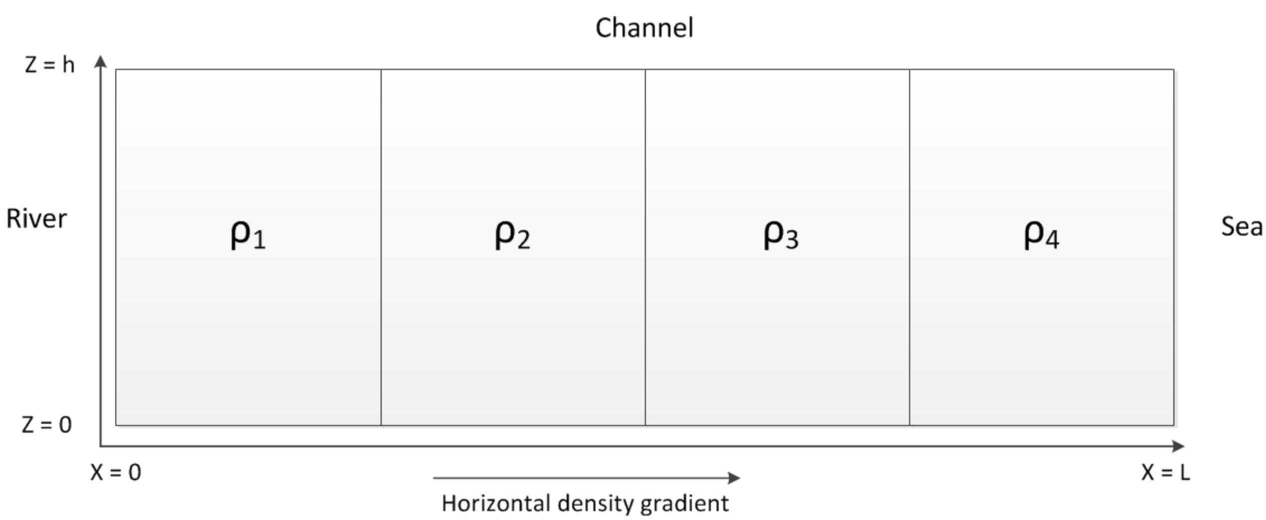

Figure 2.1: Sketch of the the channel that represents the two-dimensional estuary. Due to the horizontal density gradient, $\rho_4 > \rho_3 > \rho_2 > \rho_1$.

To investigate the process of stratification, we start with an estuary that is not yet stratified. Initially, there is only a horizontal density gradient between the river and the sea, and no dependence on the vertical. Here we consider a channel of height h and length L. At one side of the channel, the seawater has density ρ_+ and on the other side, the river water has density ρ_-. Between the river and the sea, there is a linear horizontal density gradient. The horizontal density gradient leads to a pressure gradient, that consists of a baroclinic and a barotropic part. When these two parts are combined, the total pressure gradient generates a flow where the fresh water flows at the top of the estuary and the salt water at the bottom. A sketch of this process is presented in figure 2.2.

Under the influence of gravity, this flow starts to stratify over time. The result is a circulation of salt and fresh water which is the estuarine circulation.

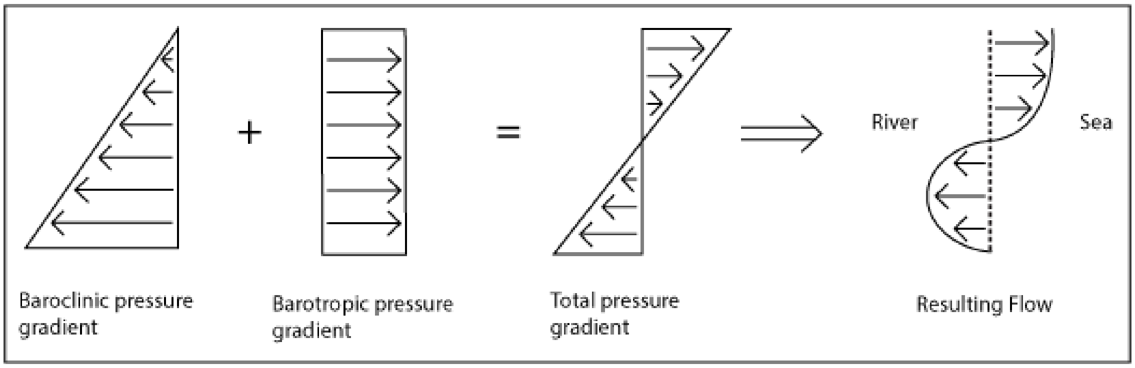

Figure 2.2: Sketch of the pressure gradient that generates the flow in the estuary.

The main question of this project is if the density ρ can be decomposed into a fraction due to a horizontal density gradient and a fraction due to a vertical density gradient, and whether this horizontal part increases linearly over x.

More precisely, we would like to write the density in the form of:

$$\rho = \rho_0 + f(\rho_s)\, x + \rho_s. \tag{2.1}$$

Here ρ is the total density, ρ_s is the stratification that represents the vertical fraction and f is a function that depends on the stratification. According to this approximation, the density should only have a linear increase in the horizontal direction but is also influenced by the stratification. With the results in this project, we will test whether this approximation is valid and what the error is.

2.2 Basic equations

The system is governed by a set of parameters that define the whole system, they are given in table 2.1.

Table 2.1: Relevant parameters of the model.

Variable	**Symbol**	**SI Unit**
Length channel	L	m
Height channel	h	m
Viscosity	ν	m^2 s^{-1}
Background density	$\rho_0 = (\rho_+ + \rho_-)/2$	kg m^{-3}
Density difference	$\Delta\rho = \rho_+ - \rho_-$	kg m^{-3}
Diffusion coefficient	D	m^{2}s^{-1}
Gravity acceleration	g	m s^{-2}

The equations used in this project are the Navier-Stokes equations and the transport equation. Using the Boussinesq approximation [4], the equations can be reduced to:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = \frac{1}{\rho_0}\frac{\partial P}{\partial x} + \nu\frac{\partial^2 u}{\partial x^2} + \nu\frac{\partial^2 u}{\partial z^2}, \tag{2.2}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z} = \frac{1}{\rho_0}\frac{\partial P}{\partial z} + \nu\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial z^2} - \frac{\rho}{\rho_0}g, \tag{2.3}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho w)}{\partial z} = D\frac{\partial^2 \rho}{\partial x^2} + D\frac{\partial^2 \rho}{\partial z^2}, \tag{2.4}$$

where u and w are the velocities in the x-direction and z-direction respectively, P is the pressure, ρ the density, ν the kinematic viscosity and D the diffusion coefficient.

2.3 Dimensionless equations

Often it is more useful to write the basic equations in dimensionless form. To do this, we need the dimensionless variables presented in table 2.2.

Table 2.2: Variables in dimensionless form. The primed variables are the dimensionless variables.

Variable	**Dimensionless form**
x	hx'
z	hz'
u	Uu'
w	Uw'
t	$h/U\ t'$
ρ	$\rho_0\rho'$
P	$\rho_0 U^2 P'$

The typical velocity scale U is defined as:

$$U = \sqrt{\frac{\Delta\rho}{\rho_0}\frac{gh^2}{L}}. \tag{2.5}$$

Using table 2.2, the basic equations can be rewritten:

$$\frac{\partial u'}{\partial t'} + u'\frac{\partial u'}{\partial x'} + w'\frac{\partial u'}{\partial z'} = \frac{\partial P'}{\partial x'} + \frac{\nu}{Uh}\frac{\partial^2 u'}{\partial x'^2} + \frac{\nu}{Uh}\frac{\partial^2 u'}{\partial z'^2}, \tag{2.6}$$

$$\frac{\partial w'}{\partial t'} + u'\frac{\partial w'}{\partial x'} + w'\frac{\partial w'}{\partial z'} = \frac{\partial P'}{\partial z'} + \frac{\nu}{Uh}\frac{\partial^2 w'}{\partial x'^2} + \frac{\nu}{Uh}\frac{\partial^2 w'}{\partial z'^2} - \frac{\rho_0}{\Delta\rho}\frac{L}{h}\rho', \tag{2.7}$$

$$\frac{\partial \rho'}{\partial t'} + u'\frac{\partial \rho'}{\partial x'} + w'\frac{\partial \rho'}{\partial z'} = \frac{D}{Uh}(\frac{\partial^2 \rho'}{\partial x'^2} + \frac{\partial^2 \rho'}{\partial z'^2}). \tag{2.8}$$

According to the Buckingham-π theorem, there are four dimensionless parameters relevant for this problem. These parameters are the Reynolds number (Re), the Atwood number (A), the Schmidt number (Sc) and the aspect ratio between the height and length of the channel (h/L). These dimensionless parameters and their definitions are presented in table 2.3.

Table 2.3: Definition dimensionless parameters.

Parameter	**Definition**
Reynolds	Uh/ν
Atwood	$\Delta\rho/\rho_0$
Schmidt	ν/D
Aspect ratio	h/L

Using these dimensionless parameters, the equations can be written as:

$$\frac{\partial u'}{\partial t'} + u'\frac{\partial u'}{\partial x'} + w'\frac{\partial u'}{\partial z'} = \frac{\partial P'}{\partial x'} + \frac{1}{\mathrm{Re}}\frac{\partial^2 u'}{\partial x'^2} + \frac{1}{\mathrm{Re}}\frac{\partial^2 u'}{\partial z'^2}, \tag{2.9}$$

$$\frac{\partial w'}{\partial t'} + u'\frac{\partial w'}{\partial x'} + w'\frac{\partial w'}{\partial z'} = \frac{\partial P'}{\partial z'} + \frac{1}{\mathrm{Re}}\frac{\partial^2 w'}{\partial x'^2} + \frac{1}{\mathrm{Re}}\frac{\partial^2 w'}{\partial z'^2} - \frac{1}{\mathrm{A}}g, \tag{2.10}$$

$$\frac{\partial \rho'}{\partial t'} + u'\frac{\partial \rho'}{\partial x'} + w'\frac{\partial \rho'}{\partial z'} = \frac{1}{Sc}\frac{1}{Re}\left(\frac{\partial^2 \rho'}{\partial x'^2} + \frac{\partial^2 \rho'}{\partial z'^2}\right). \tag{2.11}$$

These equations will be solved with the numerical simulations.

Chapter 3

Computational Model

3.1 Numerical setup

In this project, the estuary is modeled as a 2-dimensional channel that connects two containers. The left container represents the river and the right container represents the sea. The density in the channel depends on the coordinates and varies in time, similar to an estuary. The containers are chosen to be large compared to the channel in order not to be influenced by the developing stratification. The numerical setup is presented in figure 3.1.

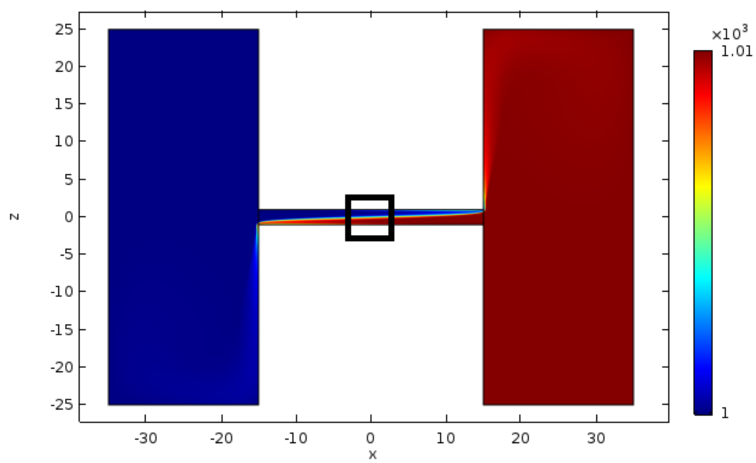

Figure 3.1: Model of two containers filled with fresh water on the left and salt water on the right. The containers are connected with a channel in which the initial density profile is imposed. The small box in the middle is the part where the results will be computed.

In this figure, it becomes clear why this model is chosen. Since the only acting force is gravity, the water in the channel will stratify naturally. This gives us a simple but effective model to investigate the stratification induced by the estuarine circulation.

This project focuses only on a small part of the channel: the area from $x = -1$ to $x = 1$ and from $z = -1$ to $z = 1$ (the small box in figure 3.1), far away from the inlets so they do not influence the velocity and density fields in the center of the channel.

3.2 Initial and boundary conditions

All the walls in the model have a no slip boundary condition so the system becomes symmetric with respect to $x = 0$ and $z = 0$, which simplifies the problem. Note that in a real estuarine flow, the top of the channel should have a free surface.

Initially there is no circulation and the density distribution in the channel is independent of the vertical. The density increases linearly in x from the river side of the channel until it reaches the density at the sea side of the channel (as shown in figure 3.2).

The low density container has a density $\rho_- = 1000$ and the high density container has a density $\rho_+ = 1010$. The channel has a height of $h = 2$ and a lenght of $L = 30$. This means that the initial density gradient has a value of $\partial\rho/\partial x = (\rho_+ - \rho_-)/L = 0.333$. The dimensionless numbers are: $Re = 200$, $A = 0.01$, $Sc = 600$ and the aspect ratio is $h/L = 0.067$.

When a simulation is started in COMSOL with these initial conditions, the velocities, the pressure and the density are computed in triangular grid points at each time step. The channel has a high refinement and the containers have a low refinement. This use of locally increasing and decreasing of refinement saves a lot of computation time without any loss in relevant information.

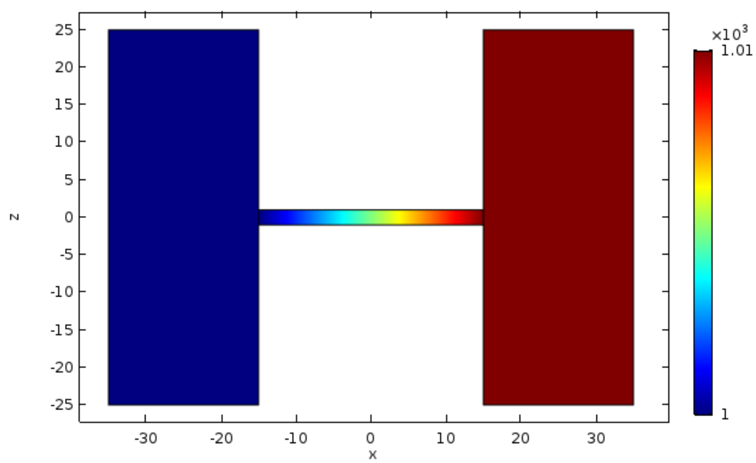

Figure 3.2: Initial distribution of density in the numerical domain.

3.3 Post processing

The triangular grid from COMSOL does have some downsides. The grid points are not structered, which makes it hard to compute gradients in the channel. To solve this problem, the data generated in COMSOL is processed with MATLAB. The information from the triangular grid in COMSOL is interpolated into a structured rectangular grid in MATLAB.

Chapter 4

Results

We start by looking at the whole channel before we focus on the center. The four graphs in figure 4.1 show how the whole channel stratifies and how the fresh water flows at the top of the channel and the salt water flows at the bottom. After $t = 1500$, there are hardly any changes over time and a steady state can be assumed.

For the purposes of this project, we are only interested in the center part of the channel as explained before. Figure 4.2 presents the density in the center of the channel at different times. This gives a better understanding of how the density is distributed far from the edges of the channel.

After $t = 1500$, the system appears to have reached a steady state as well. However, diffusion effects might still change the system slightly, but these changes are slow. The density has a small dependence on x after reaching this state.

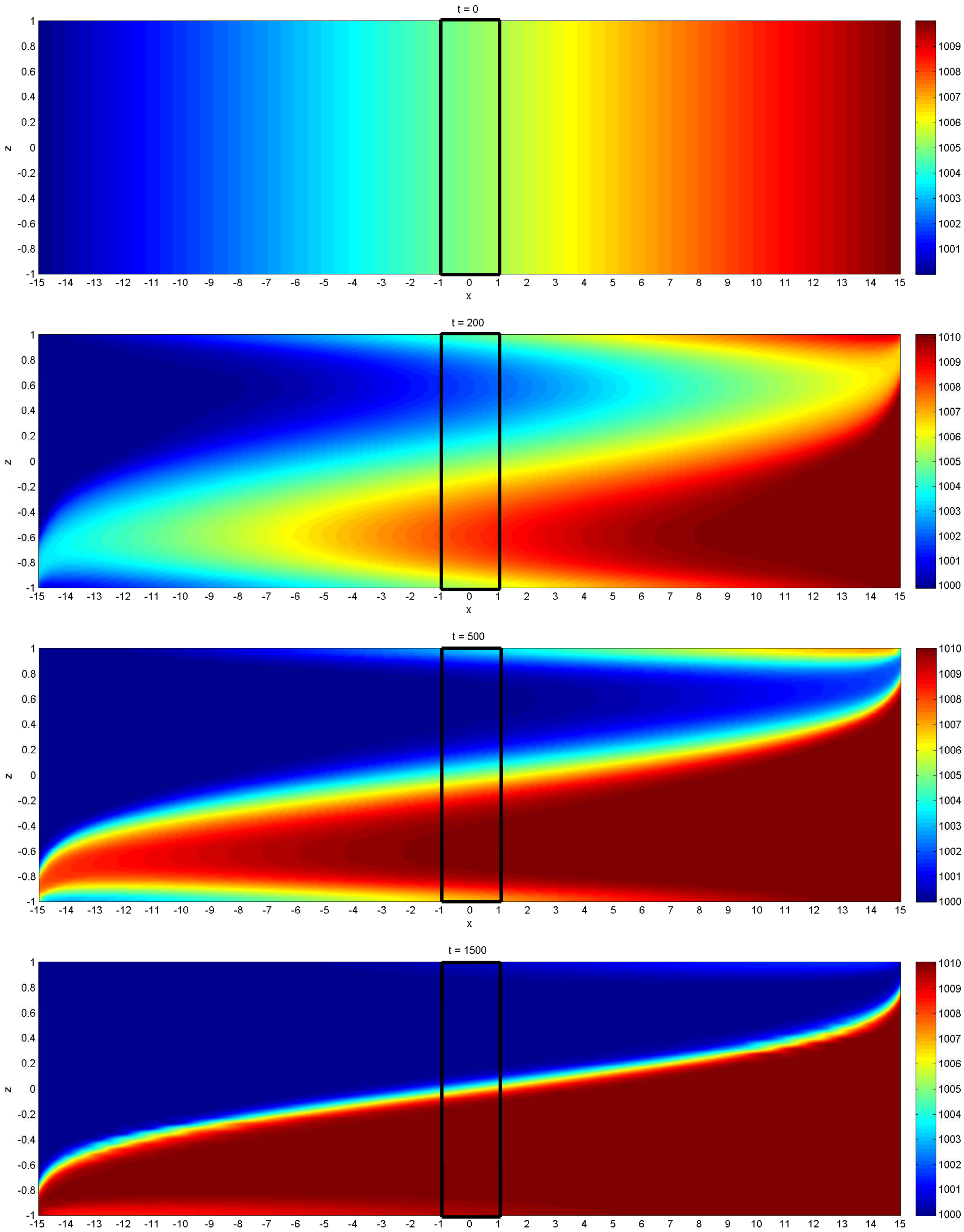

Figure 4.1: Plot of the density in the whole channel at different times. The rectangle in the middle represents the center of the channel.

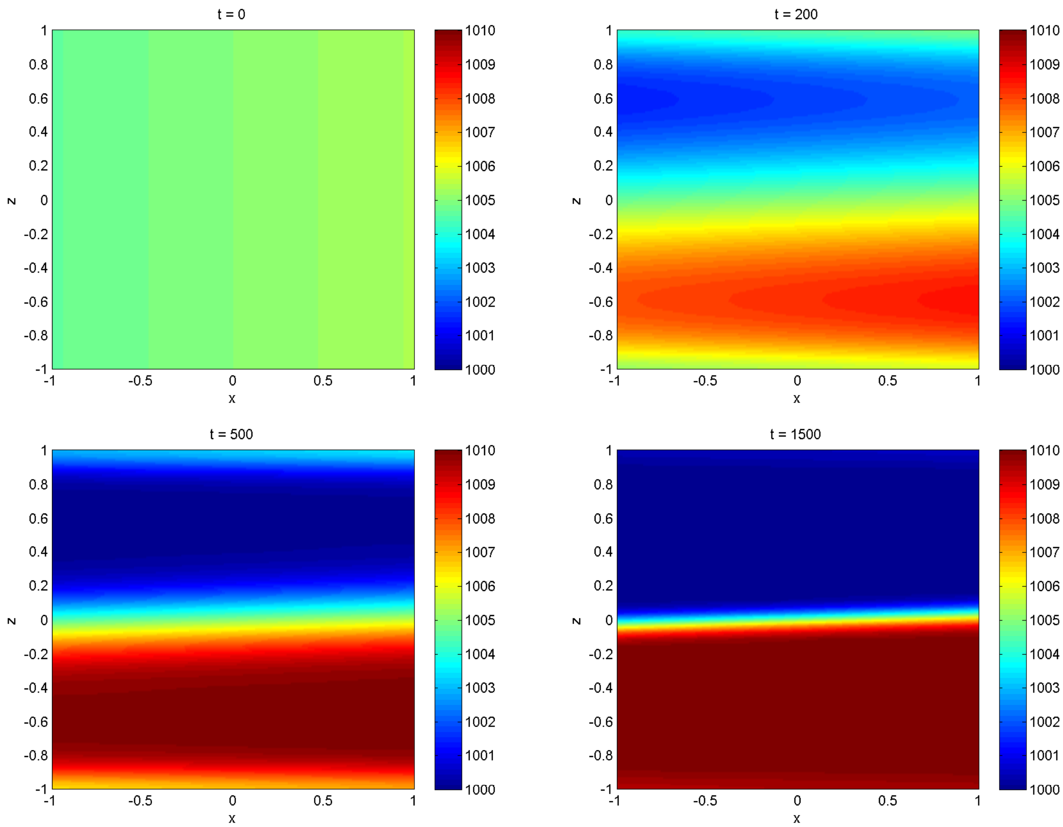

Figure 4.2: Plot of the density in the center of the channel at different times. These plots are the rectangles from figure 4.1.

4.1 Density gradient

An important aspect in this project is the evolution of the initial horizontal density gradient in the channel. In order to find the horizontal density gradient, we plot the density as a function of x, along lines where $z = 0$, see figure 4.3.

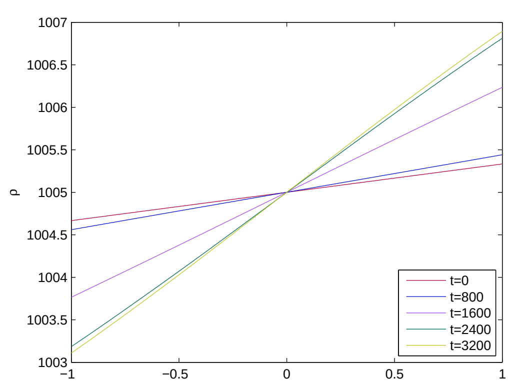

Figure 4.3: Plot of the density as a function of x at different times at $z = 0$.

The plot shows a linear relation between x and the density. This means that the initial linearity is conserved over time at this location. We can see that the slope of the lines increases with time, which means that the horizontal density gradient also increases over time, at $z = 0$.

The next step would be to look at the horizontal density gradient as a function of the vertical. In figure 4.4, the density gradient is plotted as a function of z at different times.

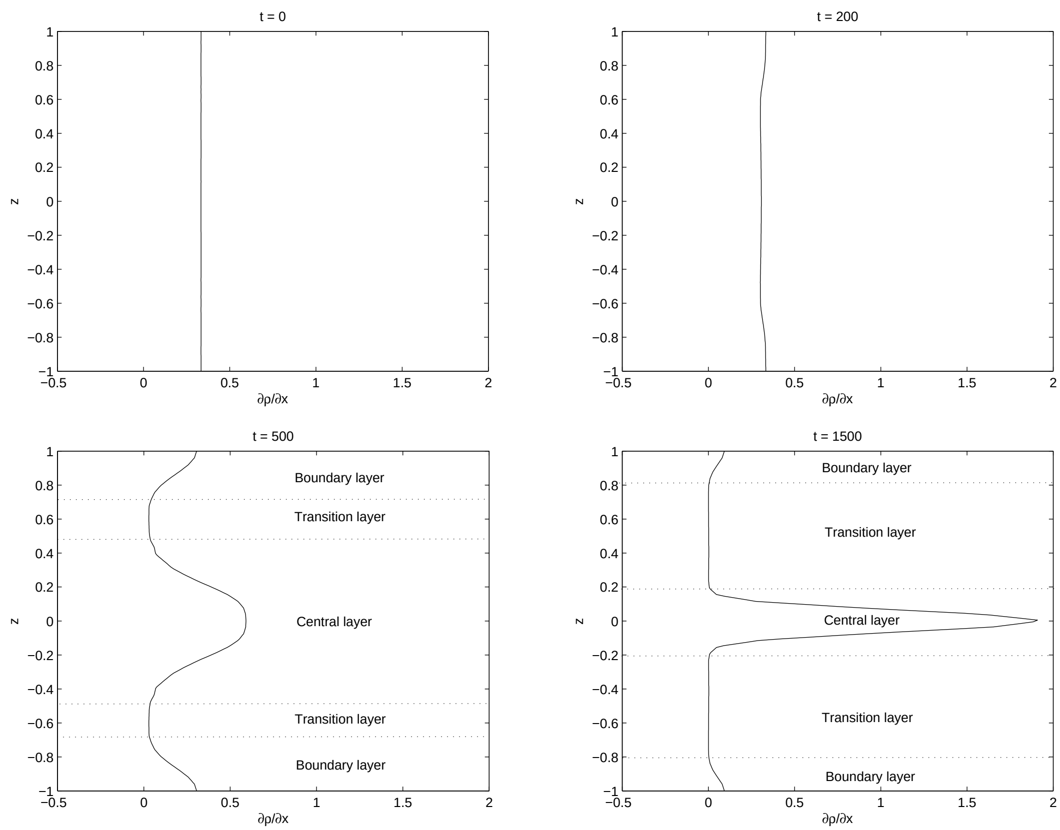

Figure 4.4: Multiple plots of the horizontal density gradient as a function of z at $x = 0$. After $t = 200$, we identify three layers.

Initially, the horizontal density gradient is independent of z, which was the initial condition of the system. After some time, the density profile is no longer independent of z but can be divided in three regions with each their own behaviour for the horizontal density gradient.

The first region is the central layer, where the horizontal density gradient increases over time until it reaches a fixed value, in agreement with what we have seen in figure 4.3.

The second region is the transition layer, where the density gradient decreases to zero rapidly. This is different from figure 4.3, where the density gradient increases in time. This means the horizontal density gradient does depend on the vertical.

The last region is the boundary layer, where the horizontal density gradient also decreases to zero but with a much slower speed. It decreases much slower, because the no-slip boundary conditon opposes the decrease in the gradient.

4.2 Stratification

We have seen from figure 4.1 and figure 4.2 that the water column stratifies and from figure 4.3 that the horizontal density gradient evolves as well. We now take a look at the evolution of the stratification in order to see if we can quantify it.

The stratification is defined as:

$$\rho_s = \frac{1}{2\Delta x} \int_{-\Delta x}^{\Delta x} \rho(x) dx, \tag{4.1}$$

where $2\Delta x$ is the length of the area under consideration. The value of ρ_s has been computed in the center of the channel and is presented in figure 4.5:

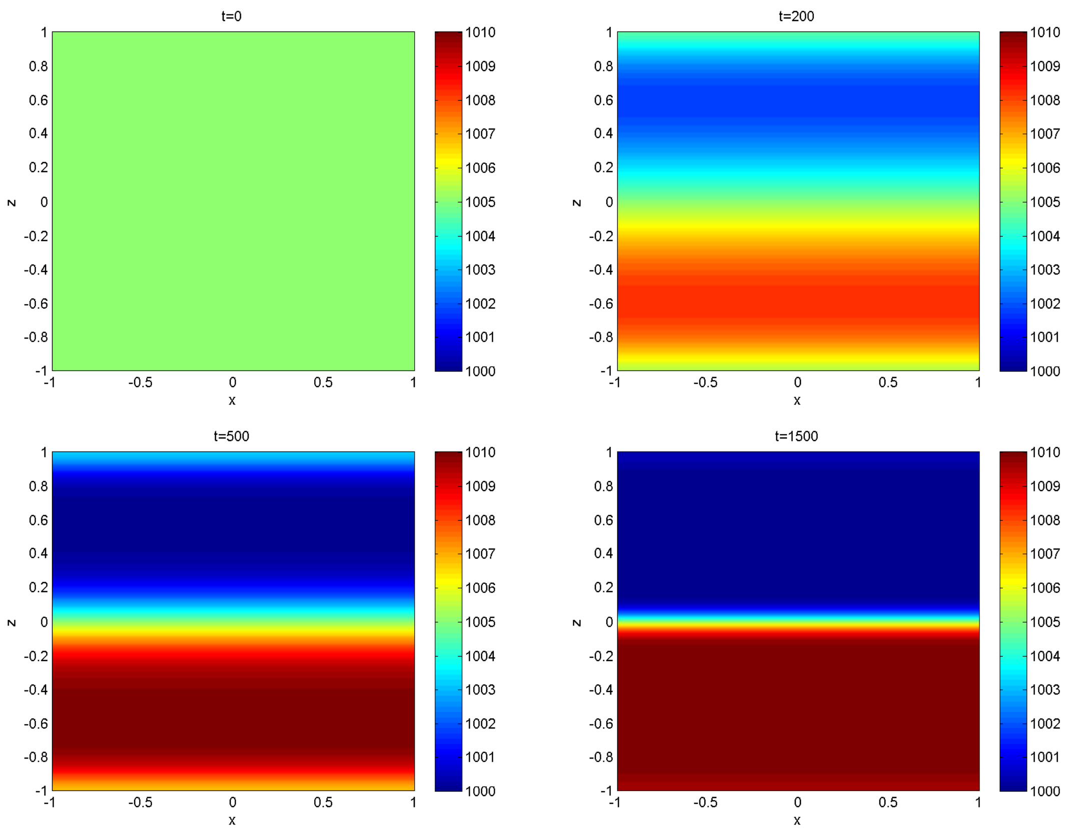

Figure 4.5: Stratification at the center of the channel. At $t = 0$, there is no stratification yet.

Initially, there is no stratification because there is no z dependence at $t = 0$. After $t = 1500$, the stratification changes little. Neglecting the plot at $t = 0$, the graphs in figure 4.5 look very similar to the corresponding plots in figure 4.2, except for the dependence on x. The stratification is independent of x by its definition, while the density still has a small dependence on it. The independence with respect to x means that the stratification can also be plotted as a function of z only; without any loss of information. This is done in figure 4.6 for $t = 1500$.

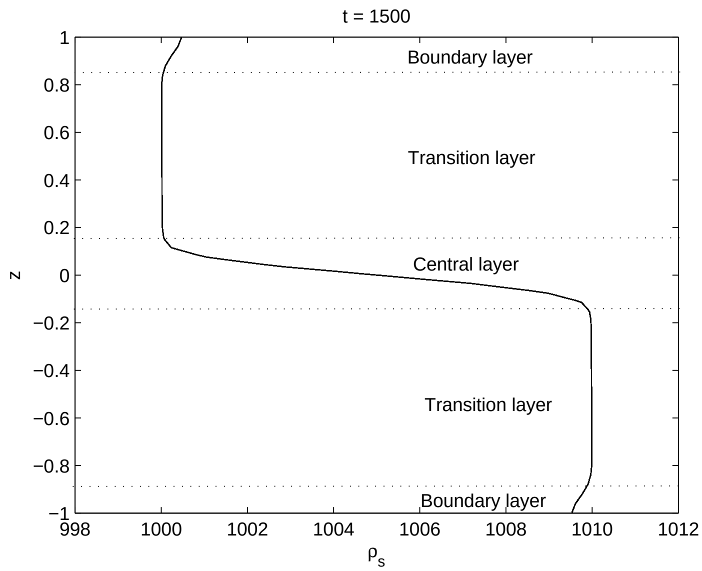

Figure 4.6: Stratification plotted as a function of z at $x = 0$.

If we compare figure 4.6 with the corresponding graph from figure 4.2, we see a similarity between ρ and ρ_s in the transition layer and in the boundary layer . However, in the center layer ρ_s slowly decreases over the vertical independently of x, which is different from the density.

Now that we know more about the stratification, we would like to test the validity of equation 2.11 and particularly its linear behaviour in x:

$$\rho = \rho_0 + f(\rho_s)\, x + \rho_s. \tag{4.2}$$

If we take ρ_0 and ρ_s to the left side:

$$\rho - \rho_0 - \rho_s = f(\rho_s)\, x \tag{4.3}$$

Now we use the fact that ρ_s is independent of x, so dividing by x we get:

$$(\rho - \rho_0 - \rho_s)/x = f(\rho_s). \tag{4.4}$$

Now we define a function $g(x, z)$ such that

$$g(x, z) = f(\rho_s). \tag{4.5}$$

The stratification depends only on z and t. This means that if we plot g as a function of x and z at multiple times, it should only show a dependence on z. This plot is presented in figure 4.7.

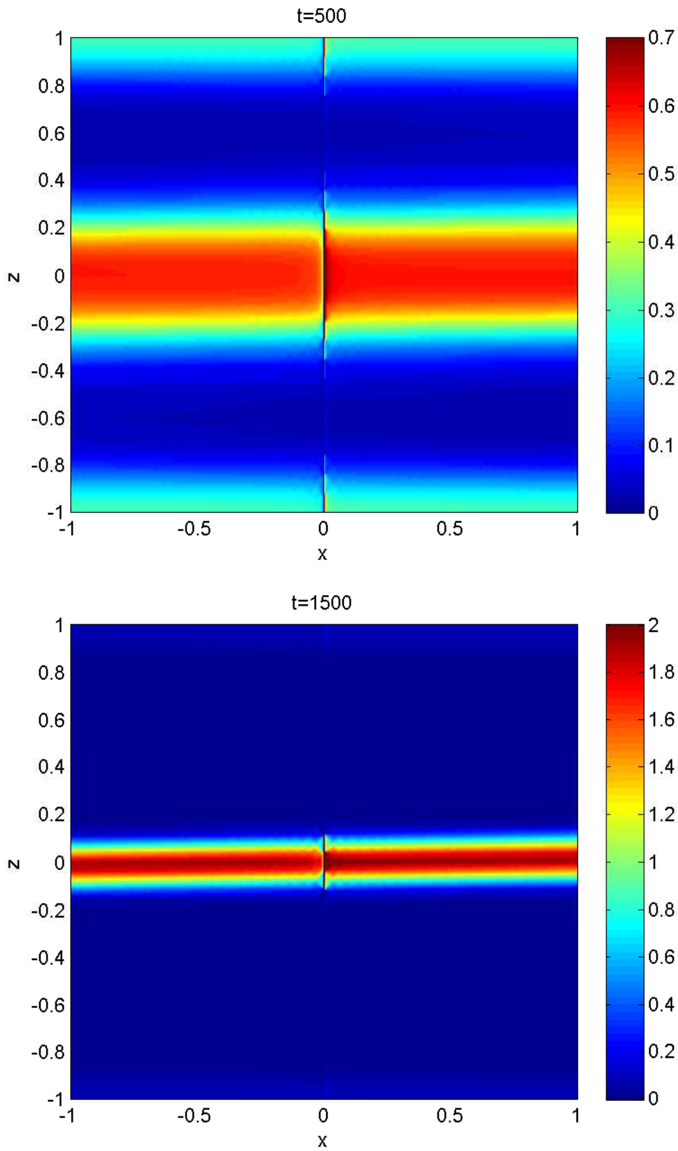

Figure 4.7: Plots of the defined function g as a function of x and z. At $x = 0$ there is a singularity because the function gets divided by 0, so the line at $x = 0$ should not be taken into account.

The plots for g at $t = 0$ and $t = 200$ do not give any information since the flow is not stratified yet, this is why they are not presented in figure 4.7.

The function g shows the expected behaviour in our plots: different horizontal layers start to appear. However, the layers are not completely horizontal. When looking closely in the last graph, the center region has a small slope from $x = -1$ to $x = 1$. This means there is still a small dependence on x, which is the error in our approximation.

We are now going to calculate this error for $t = 1500$ by calculationg the quantity

$$\frac{g(1,z) - g(-1,z)}{z}, \tag{4.6}$$

which gives us the error over z. Figure 4.8 shows the relation between this quantity and z at $t = 1500$.

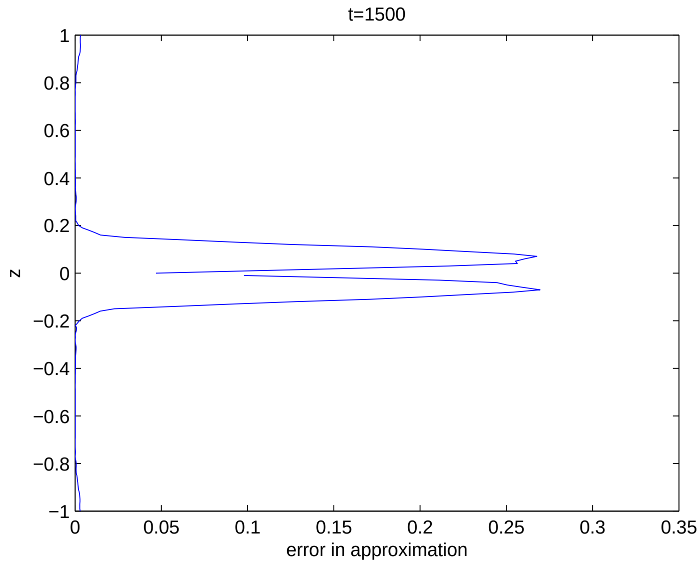

Figure 4.8: The error in the approximation as a function of z at $t = 1500$.

The error is small for $z > 0.2$ and $z < -0.2$, far from the central layer. Close to $z = 0$, the error is approximately 0.15 which is small compared to the corresponding value of $g = 2$. This means that in these regions, the linearity is conserved and our approximation holds. However, the two peaks at $z = 0.1$ and $z = -0.1$ both have an error of 0.27 with a corresponding value of $g = 1$. This means our approximation is still reasonable in this region, but not as valid as the other values for z.

4.3 Link between the horizontal density gradient and the stratification

We have seen that the initial linearity of the density is conserved and that the stratification was defined independent of x. Now we want to investigate the relation between the horizontal density gradient and the stratification by plotting them in the small region around $z = 0$ (figure 4.9).

In this region, the relation between the stratification and the density gradient seems to become linear after a few time steps, which continues until the horizontal density gradient reaches a value of approximately 1.95.

We can understand this behaviour with figure 4.4 at $t = 500$, when the layers are formed. When they are formed, we see all the lines increase linearly until they reache this value of 1.95, which would mean all our lines should be part of the peak in the central layer. The strange behaviour from figure 4.9 at the beginning could be the result of the layers being formed.

To check whether we are in the peak of the central layer, we need to increase or decrease z to see whether the final value of the horizontal density gradient decreases. In figure 4.10, only results for $z > 0$ are presented. However, we could also have plotted the results for $z < 0$ since the figure remains symmetrical around $\rho_s = 1005$ like in figure 4.9.

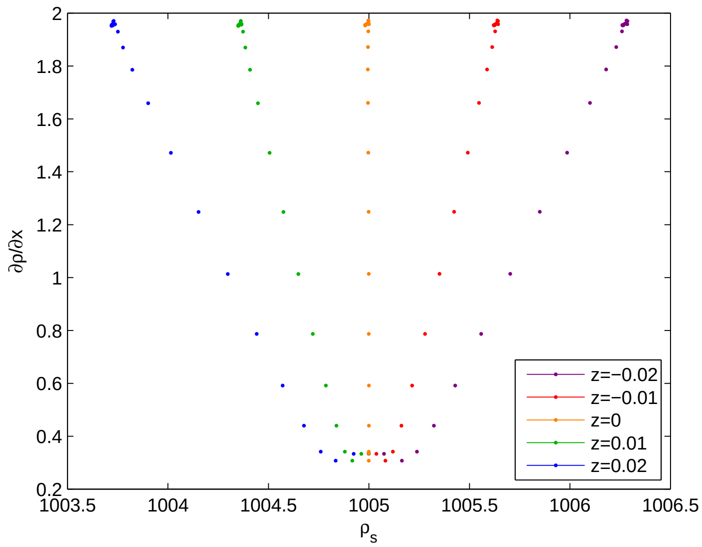

Figure 4.9: Horizontal density gradient plotted as a function of the stratification at different values for z, at $x = 0$. All the lines start at $\rho = 1005$ and $\partial\rho/\partial x = 0.333$, the initial conditions.

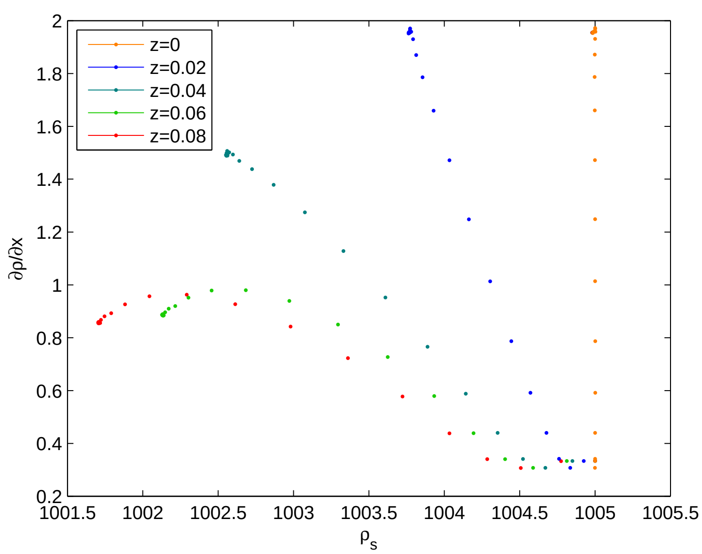

Figure 4.10: Horizontal density gradient plotted as a function of the stratification at different values for z, at $x = 0$. All the lines start at $\rho = 1005$ and $\partial\rho/\partial x = 0.333$, the initial conditions.

The linear relation is still visible but for a shorter range of ρ_s. We do see the expected change in the final value of the gradient, but the linear relation between the two seems to vanish at some point in time, which cannot be explained right now and requires more research.

Some other possible relationships have also been investigated, such as a dependency of $\partial\rho/\partial x$ on $\partial\rho/\partial z$, but did not lead to any hard conclusions. Such properties of the flow are beyond the scope of this project, but they could become part of future research.

Chapter 5

Conclusion

The estuaries form with their horizontal and vertical density gradients a complex system, generating a so called estuarine circulation in which the fresh water from the river flows at the top towards the sea and the salt water from the sea flows at the bottom into the river. These flows generate stratification that opposes mixing.

The aim of this project is to learn more about this stratification process in estuaries, by finding out whether the density in the estuary can be decomposed into a fraction due to a horizontal gradient and a fraction due to a vertical gradient and to check the linear properties of the density in the horizontal direction. The reason for doing this, is to provide tools for further numerical investigations on estuarine circulation.

In this project, the complex estuarine circulation was simplified to a two-dimensional model. This model was solved using numerical simulations without turbulence modelling.

It was found that the linear relationship between the density and the horizontal, initially imposed, is conserved over time even when the water column stratifies. Due to this conservation of linearity, the horizontal density gradient is independent of x to a good approximation and only depends on the vertical coordinate and the time.

Furthermore, three layers appear in the system where the density gradient has a different behaviour: the center layer, where the density increases in time until it reaches its final value. The boundary layer where the horizontal density gradient reaches zero in a long time due to the no-slip boundary conditions of the walls. The transition layer, that lies between the center layer and the boundary layer, where the horizontal gradient reaches zero in a short time.

The stratification was found to be dependent on the vertical coordinate and the time. We tried to verify the relation between the stratification and the horizontal density gradient to present a lead to new research oppurtunities. At some point during the stratification process, a linear relation between the horizontal density gradient and the stratification is observed. The reason for this is not found yet and requires more research. Overall, we can say that the formulation $\rho = \rho_0 + f(\rho_s)\ x + \rho_s$ is a good approximation for densities close to the fresh and salt water interface and far away from it.

Using other values for the dimensionless numbers or changing the top of the channel from a no-slip to a free surface might give more physical results. All these questions can be part of further research on the subject.

Chapter 6

Bibliography

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