BACHELOR

EM-Waves in Plasma

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EM-Waves in Plasma

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Abstract

The aim of this experiment is to find the electron density n_e of a plasma using EM-waves. A cylindrical vessel is used to contain a plasma. A magnetic field and an electric field are used to prevent electrons from the plasma from reaching the walls of the vessel. The magnetic field is created by a coil around the vessel and the electric field is made using a high power supply. EM-waves are sent into the vessel and received by antennas. An interferometer is used, so that an oscilloscope can measure the phase difference between the emitted EM-wave and the received EM-wave.

The phase difference is related to the electron density. In this experiment four different EM-waves are considered. Two waves which travel parallel to the external magnetic field and two waves that travel perpendicular to the external magnetic field. For the parallel propagating waves the distinction is made between right handed polarized(RH-mode) and left handed polarized(LH-mode) waves. For the perpendicular propagating waves the distinction is made between the X-mode and the O-mode. The electric field of the waves in X-mode and O-mode are respectively perpendicular and parallel to the external magnetic field. For each wave type the phase difference is related differently to the electron density.

Three different methods, which can determine the electron density, give a relation between the phase difference and the electron density. The first method uses the fact that a minimum refractive index is needed for a EM-wave of 1297 MHz to propagate through the plasma. The second method uses the assumption that there is a linear relation between the electron density and the plasma current. The third method uses the magnetic field as a variable. This method assumes that the electron density does not change when the magnetic field increases.

Method 1,2 and 3 give the following relations respectively,

$$\phi_{\| RH} = (\sqrt{1 - \frac{4.779 \cdot 10^{-17} \cdot n_e}{1 - 1.07}} - 1) \cdot 2.72 + 0.95$$

$$\phi_{\| RH} = (\sqrt{1 - \frac{4.779 \cdot 10^{-17} \cdot n_e}{1 - 1.07}} - 1) \cdot 2.72 - 5 \cdot 10^3$$

$$\phi_{\| RH} = (\sqrt{1 - \frac{4.779 \cdot 10^{-17} \cdot n_e}{1 - 1.07}} - 1) \cdot 2.72 - 1.8 \cdot 10^3$$

Only the parallel propagating wave in RH-mode could be used for this experimental upset, because the other EM-wave won't propagate through the vessel.

Contents

1 Introduction

There are many different types of plasma. The sun, for example, is a nearly perfect sphere of plasma, where nuclear fusion takes place. A lot of energy is released when small nuclei fuse. This is constantly happening in the sun. It is useful to use fusion as an energy source, because it is very clean, in that it does not produce long-lived nuclear waste. Researches in ITER use this physics to find out whether this method can be used as a power source.[1]

For nuclear fusion to happen on earth, the plasma must have a temperature of 150.000.000 K. One way to heat plasma is to use high energetic EM-waves. The energy carried by high-frequency waves propagating into the plasma transfers energy to the plasma in the same way that microwaves transfer heat to food in a microwave oven. The wave increases the velocity of the particles' motion, and at the same time their temperature.[2]

Because EM-waves are in interest for situations like above, it is useful to know how waves behave in plasmas and for what purpose they can also be used. In this experiment EM-waves are used to determine the electron density n_e of a plasma. An electric and magnetic field are used to contain a plasma in a cylindrical vessel. The magnetic field is made by a coil around the vessel. An emitting antenna is used to send EM-waves through the plasma. Two receiving antennas each at a different place are used te receive a signal. An interferometer and an oscilloscope are used to measure the phase difference between the emitted signal and the received signal.

Because the electron density influences the refractive index of the plasma and therefore the phase difference between these signals, a theory can be made to determine the electron density. This theory is described in chapter 2. Here assumptions will be made on the Appleton-Hartree equation and a theory for the cutoff frequency for circular waveguide will be given. The experimental set-up and methodology are shown in chapter 3. In chapter 4 the results of the experiment are shown. In this Chapter the data will be discussed. A conclusion is given in chapter 5.

2 Theoretical background

In this chapter four simplified equations for a relation between the phase difference and the electron density will be derived. Two equations for EM-waves where the wave vector k is parallel to the external magnetic field, where one is left handed polarized and one is right handed polarized. The other relations are for two EM-waves where the wave vector is perpendicular to the external magnetic field, one for the extraordinary mode(X-mode) and one for the ordinary mode(O-mode). The electric field of the X-mode and O-mode are respectively perpendicular and parallel to the external magnetic field. These simplifications are made because one receiving antenna is placed so that it receives mostly parallel propagating EM-waves and one antenna is placed so measures mostly perpendicular propagating waves.

Then a minimum value for the refractive index, wherefore a particular EM-wave just fit in the vessel, will be determined. This is done so that one can know which EM-mode(s) can be received by one of the antennas

2.1 EM-waves in a Plasma

A description for the propagation of electromagnetic waves in a plasma can be derived from the Maxwell's equations. Assuming that the averaged electron velocities are much smaller than the speed of light, the refractive index of a plasma can be described by the Appleton-Hartree equation [3] :

$$N^2 = 1 - \frac{X(1-X)}{1 - X - \frac{1}{2}Y^2\cos^2(\theta) \pm C} \tag{1}$$

where,

$$C = \sqrt{(\frac{1}{2}Y^2\sin^2(\theta))^2 + (1-X)^2Y^2\cos^2(\theta)}$$

$$X = \frac{\omega_{pe}^2}{\omega^2} \quad \text{and} \quad Y = \frac{\omega_{ce}}{\omega} \tag{2}$$

with θ the angle between the wave vector k and the external magnetic field B. ω is the angular frequency, which is defined as $\omega = 2\pi \cdot f$, where f is the frequency. X, Y and C are just used to simplify equation 1. ω_{pe} and ω_{ce} are usually referred to as respectively the plasma frequency and the electron cyclotron frequency. They are defined by the following definitions 3.

$$\omega_{pe} = \sqrt{\frac{n_e e^2}{\epsilon_0 m_e}} \quad ; \quad \omega_{ce} = \frac{eB}{m_e} \tag{3}$$

where n_e is the electron density, e is the electron charge, ϵ_0 the permittivity of free space and m_e the electron mass.

Two different cases are considered. In the first case the k vector of the electromagnetic wave is parallel($\theta = 0$) to the external magnetic field. In the second case the k vector is perpendicular($\theta = \frac{\pi}{2}$) to the external magnetic field.

The $\pm$ sign in front of C in equation 1 relates to the polarization of the electromagnetic wave. For parallel propagating waves, the plus sign refers to the right handed mode(RH-mode) and the minus sign to the left handed mode(LH-mode). For perpendicular propagation, the plus sign refers to the X-mode and the minus sign to the O-mode.

Four different wave propagations can be distinguished. The Appleton-Hartree equation can be simplified for each case:

Table 1: Refractive Index Perpendicular Propagation

	X-mode	**O-mode**
perpendicular ($\theta = \frac{\pi}{2}$)	$N^2 = 1 - X$	$N^2 = 1 - \frac{X(1-X)}{1-X-Y^2}$

Table 2: Refractive Index Parallel Propagation

	LH-mode	**RH-mode**
parallel ($\theta = 0$)	$N^2 = 1 - \frac{X}{1+Y}$	$N^2 = 1 - \frac{X}{1-Y}$

Figure 1 and 2 are useful to check which modes you are dealing with for different refractive indexes.

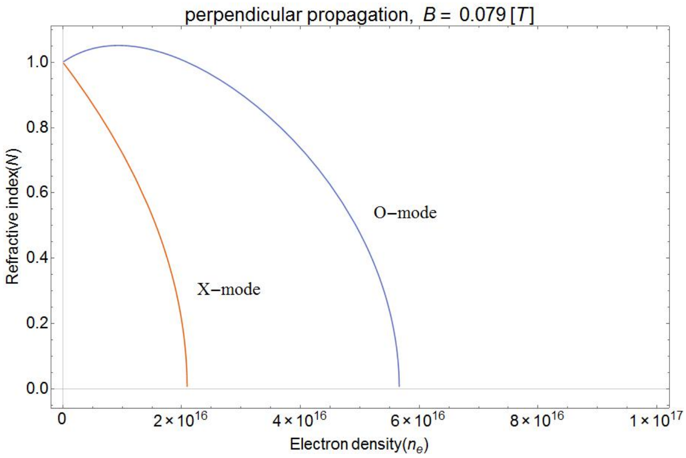

Figure 1: The refractive index N for a wave in the X-mode or O-mode, traveling perpendicular to the external magnetic field, is plotted over the electron density n_e. B=0.079 T.

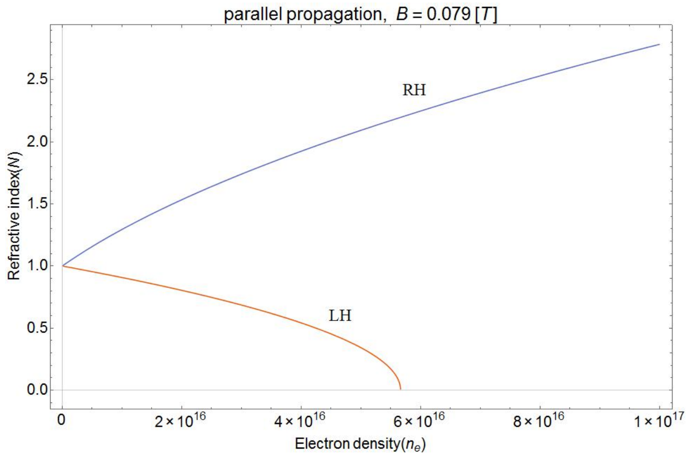

Figure 2: The refractive index N for a wave in the RH-mode or LH-mode, traveling parallel to the external magnetic field, is plotted over the electron density n_e. B=0.079 T.

To see how the graphics in of figure 1 and 2 depends on the magnetic field, graphics of the same modes are made for five different values of the magnetic field. See figure 3 and 4.

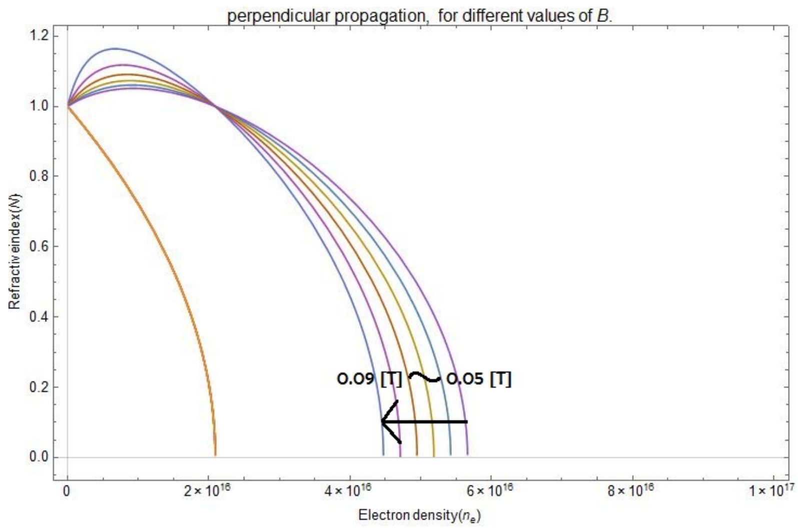

Figure 3: The refractive index N for a wave in the X-mode or O-mode, traveling perpendicular to the external magnetic field, is plotted over the electron density n_e. B=0.05;0.06;0.07;0.08;0.09 T.

The direction of the arrows shows the direction in which the magnetic field increases.

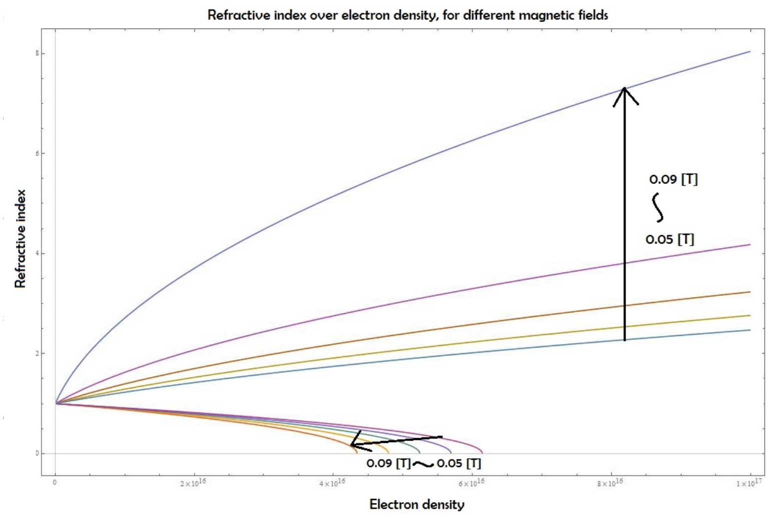

Figure 4: The refractive index N for a wave in the RH-mode or LH-mode, traveling parallel to the external magnetic field, is plotted over the electron density n_e. B=0.05;0.06;0.07;0.08;0.09 T.

2.2 Phase Difference related to Refractive Index

The refractive index is defined as,

$$N = \frac{c}{v_{ph}} \tag{4}$$

and gives a relation between the speed of light c and the phase velocity v_{ph}. The phase velocity changes in the plasma but the frequency stays the same, therefore the wavelength has to change. The difference between an electromagnetic wave traveling in vacuum or in a plasma can be found by measuring the phase difference $\Delta\Phi$:

$$\Delta\Phi = \int (k_{pl} - k_v)\, dl \tag{5}$$

here k_{pl} and k_v are the wave vectors in a plasma and in vacuum, respectively. Using the fact that the angular frequency ω is defined as $\omega = 2\pi \cdot f$, where f is the frequency and the wave vector k is defined as $k = \frac{2\pi}{\lambda}$, equation 5 can be rewritten as,

$$\Delta\Phi = \int (N - 1)\frac{\omega}{c}\, dl \tag{6}$$

Combining table 1, 2 and equation 6 leads to 4 equations for $\Delta\Phi$ as a function of X and Y. Assuming that the refractive index has the same value in the whole plasma($N \neq f(l)$), the equations for the phase differences becomes,

Table 3: Phase difference Perpendicular Propagation

	X-mode	O-mode
perpendicular	$\Delta\Phi = (\sqrt{1-X} - 1)\frac{\omega}{c} \cdot l$	$\Delta\Phi = (\sqrt{1 - \frac{X(1-X)}{1-X-Y^2}} - 1)\frac{\omega}{c} \cdot l$

Table 4: Phase difference Parallel Propagation

	LH-mode	**RH-mode**
parallel	$\Delta\Phi = (\sqrt{1 - \frac{X}{1+Y}} - 1)\frac{\omega}{c} \cdot l$	$\Delta\Phi = (\sqrt{1 - \frac{X}{1-Y}} - 1)\frac{\omega}{c} \cdot l$

where l is the propagation length for the wave in plasma. In order to use one of these relations for determining the electron density, which will be explained in the next chapter, it is important to know which EM-modes your receivers receive. In the next section a theory will be described which can determine a minimum refractive index using the cutoff frequency of a circular waveguide.

2.3 Cutoff frequency

When the wavelength of the transmitted EM-wave is bigger than the cutoff wavelength for a circular waveguide, the EM-wave won't propagate through it. The lower cutoff frequency (or wavelength) for a particular TE(Transverse Electric) or TM(Transverse Magnetic) mode in circular waveguide is determined by the following equation [4]:

$$\lambda_{c,mn} = \frac{2\pi r}{p_{mn}} \tag{7}$$

where r is the radius and p_{mn} is;

Table 5: TE_{mn}-modes

m	p_m1	p_m2	p_m3
0	3.832	7.016	10.174
1	1.841	5.331	8.536
2	3.054	6.706	9.9704

Table 6: TM_{mn}-modes

m	p_m1	p_m2	p_m3
0	2.405	5.520	8.654
1	3.832	7.016	10.174
2	5.135	8.417	11.620

The largest wave that fits in the vessel must have, according to equation 7, the smallest value for p_{mn}. This is the TE_{11} mode, see table 5 and 6.

When the vessel is filled with a plasma of refractive index N the wavelength changes by,

$$\lambda = \frac{\lambda_v}{N} \tag{8}$$

where λ_v is the wavelenght in vacuum. Using $\lambda_v = \frac{c}{f}$ and equation 8 an equation for the minimum value for the refractive index can be made:

$$N = \frac{c \cdot p_{mn}}{f \cdot 2\pi\, r} \tag{9}$$

In the next chapter a value for N will be determined using the properties of the setup used in this experiment. With this value for N one can use figure 1 and 2 to see which EM-mode(s) can be received.

3 Experimental set up

3.1 Equipments

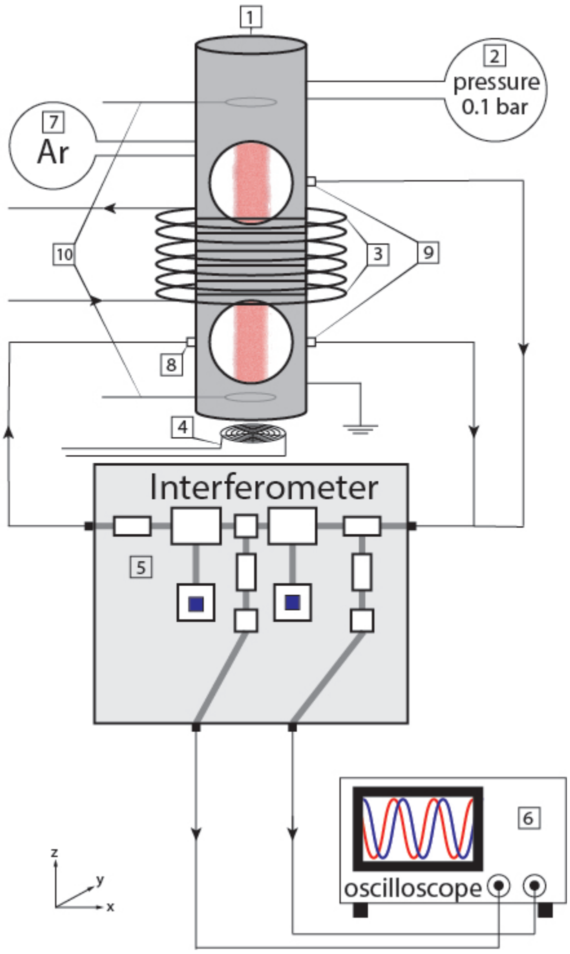

Figure 5: Schematic of the experiment: 1-Vacuum Vessel, 2-Vacuum Pump, 3-Coil, 4-Heating Element, 5-Interferometer, 6-Oscilloscope, 7-Argon Pump, 8-Lauching Antenna, 9-Receiving Antenna's, 10-High Voltage Cathode.

Figure 5 is a schematic of the experiment. In this experiment a vacuum vessel(*1*) is used to contain a cylindrical plasma. In order to bring electrons in the vessel , two Lanthanum Hexaboride LaB_6 cathodes(*10*) were used. The lower cathode, a 30 mm diameter and 3 mm thick LaB_6 disk, is being heated by a heating element(*4*). This lowers the work function for the electrons so more of them will travel into the tube. This vessel is grounded and forms the anode. In order to prevent electrons reaching the walls of the vessel a magnetic and electric field are used. A homogeneous magnetic field pointing in the z-direction is created using current carrying loops(*3*). This field lets electrons make a cyclic motion so they can't reach the side walls. To prevent electrons hitting the upper en lower walls, a high power supply, see figure 7 , is used to create a high negative electric potential of $-2\,keV$ over the upper and lower cathode(*10*). This electric field lets the electrons move up and down. Combining these motions, the electrons make helix motions up and down.

First the vessel is filled with Argon atoms by an Argon pump(*7*) and then the pressure is brought to $0.1\,bar$ by a vacuum pump(*2*). Argon is used because it has very low chemical reactivity. The vessel isn't complete vacuum, so there are still some Argon atoms. These atoms can get ionized as they collide with other particles, because the ionisation energy for Argon atoms is approximately $15.8\,eV$ [5] and the kinetic energy of the electrons are in the order of keV.

In order for the oscilloscope(*6*) to measure the phase difference between the signal emitted by the emitting antenna(*8*) and the signal from the received antennas(*9*), an interferometer(*5*) is used.

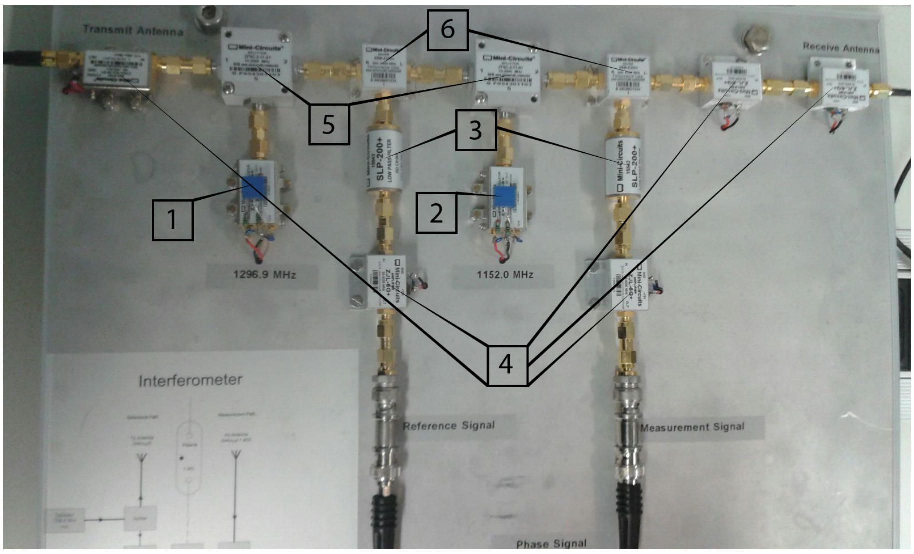

Figure 6: Interferometer: 1-1297 MHz Source, 2-1152 MHz Source, 3-Low-Pass Filters, 4-Amplifier, 5-Splitters, 6-Mixers

Figure 6 gives a closer look at the interferometer. The interferometer uses two EM-sources. One generates a frequency of 1297 MHz and the second one generates a frequency of 1152 MHz. The EM-waves of 1297 MHz are sent into the vessel, go through the plasma and will be received by the receiving antennas and go back to the interferometer.

The oscilloscope can't detect signals in the GHz-range. Therefore the two sources are mixed. This mixed wave has two components; one with the sum and one with the difference of the frequencies. A low pass filter is used so that only the low frequency component travels to the oscilloscope. Because the received signal that went through the plasma is weak, amplifiers are used to amplify the signal. All these adjustments to the waves don't influence the phase difference.

Besides the equipments in figures 5,6 and 7, water tubes are used to cool down the coil. When the coil becomes too hot the coil current I_c generator will turn off automatically. Also a current measurement device is used to keep track of the heating current I_h through the heating element. If the current is too high the cathode will not last. The coil and cathodes are connected to power supplies, which are not shown in figure 5.

Figure 7 gives a closer look the main power supplies.

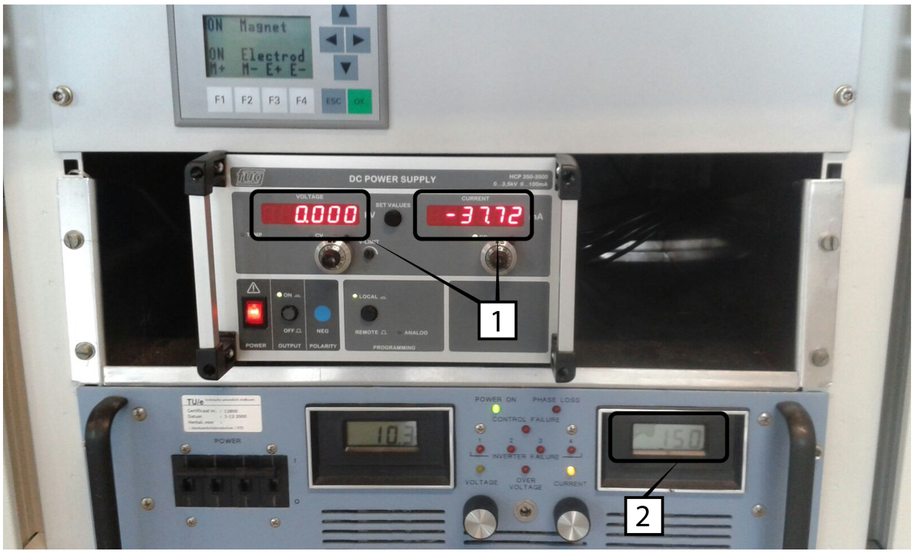

Figure 7: Power Supplies: 1-DC Power Supply, 2-Magnet Current Supply

On the upper left corner of the DC Power Supply the voltage on the cathodes is shown. During the experiment this had a value of -2 kV. On the upper right corner the plasma current I_{pl} is shown. The plasma current is the flow of charged particles from the plasma to the wall. On the upper right corner of the Magnet Current Supply the current through the coil is shown.

3.2 Methodology

In this section four different methods, which can determine the electron density, will be given. The first method uses the fact that a minimum refractive is needed for a EM-wave of 1297 MHz to propagate through the plasma. The second method uses the assumption that there is a linear relation between the electron density and the plasma current. The third method uses the magnetic field as a variable. This method assumes that the electron density does not change when the magnetic field increases. The fourth method combines parallel propagating waves and perpendicular propagating waves so a two system of equations can be solved.

One has to realize that the oscilloscope doesn't measure the phase difference $\Delta\Phi$ as described in Chapter 2. The measured phase has some offset ϕ_0. So the oscilloscope measures the phase difference, due to the difference in refractive index of the plasma, plus some offset,

$$\phi = \Delta\Phi + \phi_0 \tag{10}$$

One has to know the offset and mode of the EM-wave to have a complete relation between the measured phase and the electron density.

3.2.1 *Method 1*; Determine the Minimal Refractive Index

First an empty vessel is considered. Using the fact that the vessel has a radius of 2 cm, equation 7 tells us that the cutoff wavelength $\lambda_c \approx 7\,cm$. The wavelength of the emitted wave is,

$$\lambda = \frac{c}{f} \approx \frac{3 \cdot 10^8}{1297 \cdot 10^6} \approx 23\,cm \tag{11}$$

This wave is too big, so with no plasma no signal will be retrieved. The minimum refractive index is given by equation 9. This gives,

$$N = \frac{c \cdot p_{mn}}{f \cdot 2\pi\, r} \approx \frac{3 \cdot 10^8 \cdot 1.841}{1297 \cdot 10^6 \cdot 2\pi \cdot 2 \cdot 10^{-2}} \approx 3.3 \tag{12}$$

The minimum refractive index N can be measured by increasing the n_e until a received signal is measured. This can be done by slowly increasing the heating current, which will allow more electrons to fly in the plasma. According to theory this signal must come from the TE_{11} mode, because this is the first wave that fits. For this critical point the refractive index is, according to 12, approximately 3.3 .

According to figure 1 and 2 the only mode that travels through the plasma is the RH-mode which is in parallel direction. Using the refractive index for this mode, see table 2, the electron density at this point is approximately $1.46 \cdot 10^{17}$.

The definition of $\Delta\Phi$ can be plugged in equation 10, which gives,

$$\phi = (N-1)\frac{\omega}{c} \cdot l + \phi_0 \tag{13}$$

The measured phase at $N \approx 3.3$ left equation 13 with only one unknown parameter, ϕ_0. This can be calculated and the final relation becomes,

$$\phi_{\parallel RH} = (\sqrt{1 - \frac{X}{1-Y}} - 1)\frac{\omega}{c} \cdot l + \phi_0 \tag{14}$$

Plugging in the definitions for X and Y in equation 14 gives,

$$\phi_{\|RH} = (\sqrt{1 - \frac{\frac{e^2}{\epsilon_0\, m_e\, \omega^2} \cdot n_e}{1 - \frac{eB}{m_e\, \omega}}} - 1)\frac{\omega}{c} \cdot l + \phi_0 \tag{15}$$

l is in this case $0.1\,m$ and the magnetic field B inside the coil can be calculated with the following equation:

$$B = \frac{\mu_0 N I_c}{d} \tag{16}$$

where μ_0 is the permeability of vacuum, N the amount of windings in the coil, d the length of the coil and I_c the current through te coil. μ_0 is approximately $1.256 \cdot 10^{-6}$, $N = 189$ and $d \approx 0.4\,m$ and I_c was in this case set to $150\,A$. Therefore $B = 0.079\,T$

The only unknown parameter is the electron density, so that every measured phase is directly related to one particular electron density.

3.2.2 *Method 2*; Assuming a linear relation between n_e and I_{pl}

The assumption is made that there is a linear relation between the electron density n_e and the plasma current I_{pl}. So that,

$$n_e = K \cdot I_{pl} \tag{17}$$

where K is a constant. The aim of this part of the experiment is finding a value for K, because when K is known it is easy to calculate the electron density using equation 17. To do this, the equation for parallel propagation in the RH-mode is used, because we assume that the refractive index has to be higher than 3.3 in order to measure a signal. In this case the only signal that can be measured is the parallel propagating RH-mode. Equation 15 can be used in this case, because the same type of electro magnetic waves are measured. Combining equations 17 and 15 gives,

$$\phi_{\|RH} = (\sqrt{1 - \frac{\frac{e^2}{\epsilon_0\, m_e\, \omega^2} \cdot K \cdot I_{pl}}{1 - \frac{eB}{m_e\, \omega}}} - 1)\frac{\omega}{c} \cdot l + \phi_0 \tag{18}$$

Phase differences ϕ can be measured corresponding to different plasma currents I_{pl}. Equation 18 can be used to fit this data to obtain a value for K and ϕ_0.

3.2.3 *Method 3*; Magnetic field B as variable

For this method the assumption is made that the electron density doesn't vary when the magnetic field increases. In this case the value of n_e is fixed and B is used as a variable. Again assuming that $N > 3.3$, only the parallel propagating RH-mode is considered. Equation 15 can be used, while the same type of EM-wave is considered.

$$\phi_{\|RH} = (\sqrt{1 - \frac{\frac{e^2}{\epsilon_0\, m_e\, \omega^2} \cdot n_e}{1 - \frac{eB}{m_e\, \omega}}} - 1)\frac{\omega}{c} \cdot l + \phi_0 \tag{19}$$

but in this case phase differences ϕ will be measured corresponding to different magnetic fields B. This is done by slowly increasing the coil current I_c from $100\,A$ to $150\,A$. Equation 19 can be used to fit the data and obtain a value for n_e and ϕ_0.

In figure 8 it is shown how the refractive index of a parallel propagating wave in RH-mode depends on the magnetic field for a constant electron density of $6.6 \cdot 10^{21}$.

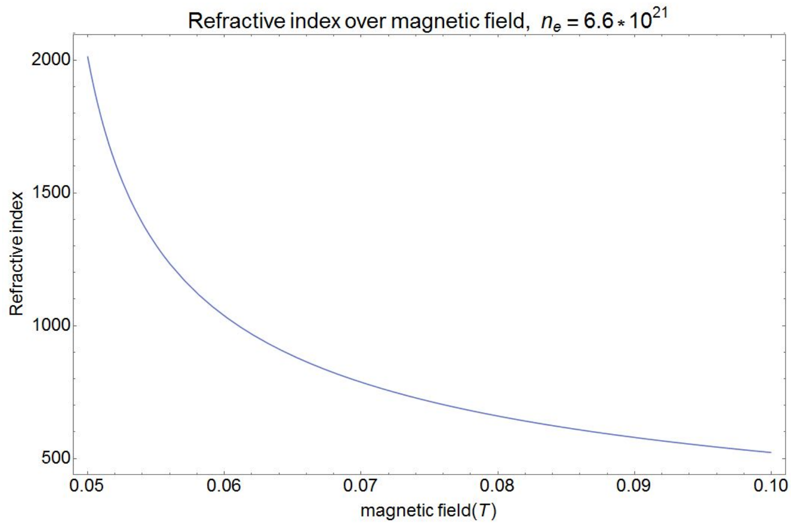

Figure 8: The refractive index is plotted over the magnetic field. $n_e = 6.6 \cdot 10^{21}$

3.2.4 *Method 4*; Parallel and perpendicular propagating wave

If one assumes that the refractive index has to be more that 3.3, this method will not work. But lets assume that $N < 3.3$, so that a perpendicular propagating wave in O-mode can be measured and simultaneously a parallel propagating wave in RH-mode. Combining equation 10 and $\Delta\Phi$ for both the O-mode and RH-mode, a set of two equations can be made,

$$\phi_{\parallel RH} = \left(\sqrt{1 - \frac{\frac{e^2}{\epsilon_0\, m_e\, \omega^2} \cdot n_e}{1 - \frac{eB}{m_e\, \omega}}} - 1\right)\frac{\omega}{c} \cdot l + \phi_0 \tag{20}$$

$$\phi_{\perp O} = \left(\sqrt{1 - \frac{\frac{e^2}{\epsilon_0\, m_e\, \omega^2} \cdot n_e \,(1 - \frac{e^2}{\epsilon_0\, m_e\, \omega^2} \cdot n_e)}{1 - \frac{e^2}{\epsilon_0\, m_e\, \omega^2} \cdot n_e - (\frac{eB}{m_e\, \omega})^2}} - 1\right)\frac{\omega}{c} \cdot l + \phi_0 \tag{21}$$

This set can be solved using Mathematica, for instance, which will give a value for n_e and ϕ_0. The disadvantage of this method is that one has to know which mode(s) an antenna receives.

4 Results and Discussion

In this chapter the result of the measurement described in the previous chapter will be given. In this experiment two receiving antennas were used, one(the upper receiver) which measure's mostly vertical(z-direction) propagating waves and one(the lower receiver) which measures mostly perpendicular(x-direction) propagating waves.

4.1 Minimum value for N

According to the theory in section 3.2.1 there shouldn't be a received signal when there is no plasma inside the vessel. This is indeed measured. To find to critical point where the value for $N = 3.3$, the plasma current was increased by slowly increasing the heating current until a signal is measured. At that specific point the measured phase difference is $101\,deg$. From equation 13 one can calculate that $\phi_0 = 95.0$. Equation 15 now can be written as,

$$\phi_{\|RH} = (\sqrt{1 - \frac{4.779 \cdot 10^{17} \cdot n_e}{1 - 1.07}} - 1) \cdot 2.72 + 0.95 \tag{22}$$

4.2 Determine a value for K

A set of measured phases corresponding to different plasma currents is made. This is done by slowly increasing the heating current, which increases the plasma current. The data from the upper receiver is shown in figure 9.

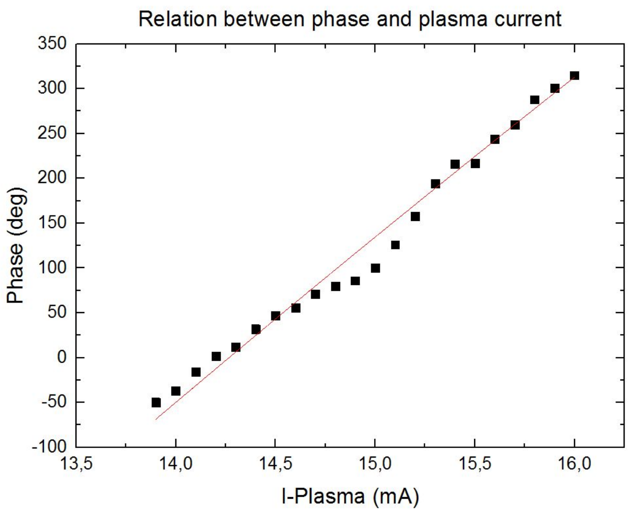

Figure 9: The function $y = y_0 + (\sqrt{1 - 6.765 \cdot 10^{-17} \cdot a \cdot x} - 1) \cdot 2.722$ is used to fit the data in Origin. Origin gives a value for the slope $y0$ and a. $a = -3.9 \cdot 10^{21} \pm 0.2 \cdot 10^{21}$ and $y_0 = -5 \cdot 10^3 \pm 0.1 \cdot 10^3$. $p = 0.1\,bar$ and $B = 0.079\,T$.

The data is measured in the I_{pl} range from 13.5 mA to 16.0 mA, because the signal in this range was strong. $a = K \approx -3.9 \cdot 10^{21}$. So for a plasma current of $14\,mA$ this means, according to equation 17, that $n_e \approx 4 \cdot 10^{19}$.

In section 3.2.1 the minimum electron density needed for this mode to propagate through the vessel is approximately $1.46 \cdot 10^{17}$. There is no contradiction between method 1 and 2 because $4 \cdot 10^{19} > 1.46 \cdot 10^{17}$.

There is a big difference in the offset ϕ_0 measured using method 1 and 2, which assumes that maybe a linear relation between the electron density and the plasma current can't be made or perhaps the assumption that the electron density is constant over the whole vessel isn't good. The final result of this method is,

$$\phi_{\|_{RH}} = (\sqrt{1 - \frac{4.779 \cdot 10^{-17} \cdot n_e}{1 - 1.07}} - 1) \cdot 2.72 - 5 \cdot 10^3 \tag{23}$$

Although, according to method 1, no perpendicular propagating wave in X-mode or O-mode can't be measured using this experimental set-up, it is useful to check this.

The data from the lower receiver is shown in figure 10

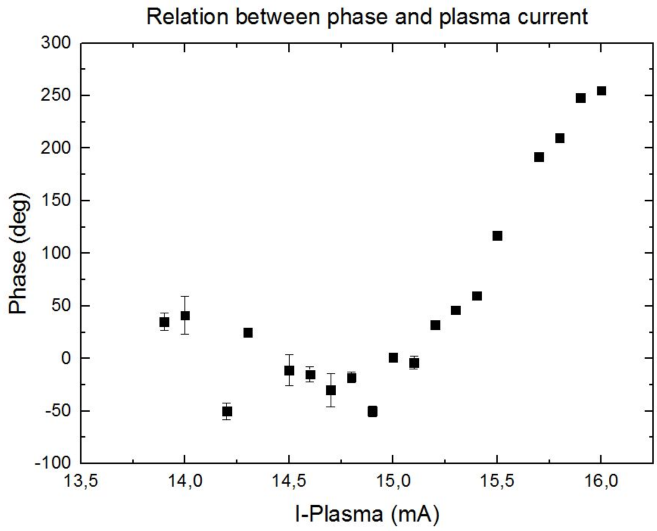

Figure 10: Data from the lower receiver. $p = 0.1\,bar$ and $I_c = 150\,A$

From table 3 one can see that this data doesn't matches one of the two equations for perpendicular propagation. none of these equation could be used to fit these data, which is in agreement with the theory from section 3.2.1. Probably the difference in the measured phase differences is due to other EM-modes that travel through the vessel.

4.3 The magnetic field B as variable

In this part of the experiment the magnetic field B is used as a variable. This is done by increasing the coil current. A set of measured phase differences corresponding to different coil currents is made. A measurement of the phase difference is made every four ampere. The data from the upper receiver is shown in figure 11

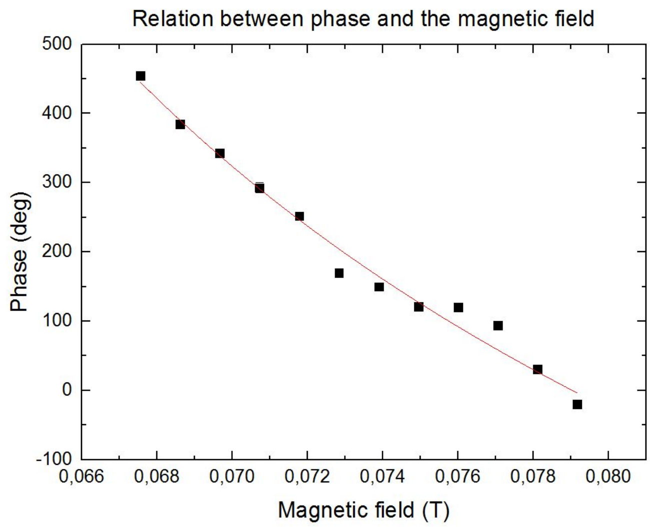

Figure 11: Data from the upper receiver. The function $y = y_0 + (\sqrt{1 - \frac{a}{1-21.5 \cdot I_c}} - 1) \cdot 2.722$ is used to fit the data. It follows that $a = 3 \cdot 10^5 \pm 0.2 \cdot 10^5$ and $y_0 = -1.8 \cdot 10^3 \pm 0.2 \cdot 10^3$. During the measurements $I_p = 14.1 \pm 0.2\, mA$ and $p = 0.1\, bar$.

$a = X = 4.78 \cdot 10^{-17} \cdot n_e \approx 3 \cdot 10^5$. It follows that $n_e \approx 6.6 \cdot 10^{21}$. The value of n_e using this method is a factor 100 bigger than the value of n_e using method 2. If it is assumed that the electron density n_e doesn't vary as the magnetic field increases, then both methods should give the same electron density. The plasma current is approximately $14\, mA$ during the measurements shown in figure 11 and in the previous section it is shown that according to method 2, $n_e \approx 4 \cdot 10^{19}$ for $I_p = 14\, mA$.

Perhaps the electron density changes when the magnetic field increases. To check this, method 2 can be used to find a value for the constant K for different measurement sets each with a different value for the magnetic field B. If K is the same for all measurement sets, it is most likely the electron density doesn't vary. But according to the difference in the measured electron density using method 2 and 3, it is more likely otherwise.

According to method 1 the electron density n_e for a parallel propagating EM-wave in RH-mode should be greater than the electron density corresponding to a refractive index of 3.3, which is $n_e \approx 1.46 \cdot 10^{17}$. $6.6 \cdot 10^{21} > 1.46 \cdot 10^{17}$, which means that there is no contradiction with what method 1 predicts. The final result of this method is,

$$\phi_{\|_{RH}} = (\sqrt{1 - \frac{4.779 \cdot 10^{-17} \cdot n_e}{1 - 1.07}} - 1) \cdot 2.72 - 1.8 \cdot 10^3 \tag{24}$$

Despite of the fact that method 1 predicts that no perpendicular propagation wave in X-mode or O-mode can be measured, it is still useful to check this. The same kind of measurement set is made as done above, but this time the lower antenna is used to measure a signal. Figure 12 shows the results of this measurement set:

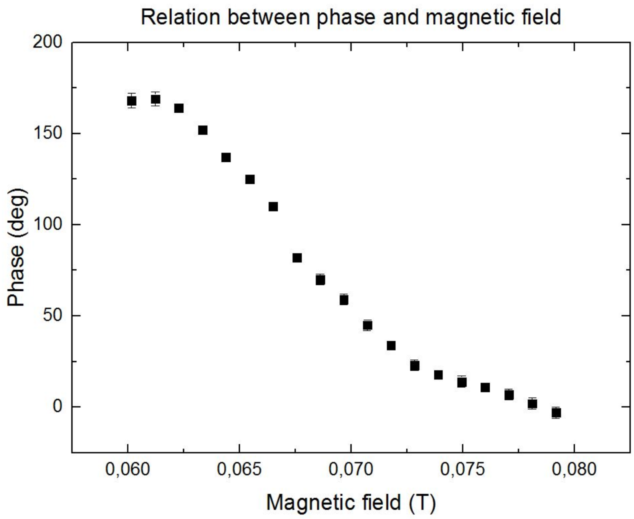

Figure 12: Data from the lower receiver. During the measurement $I_{pl} = 13.5 \pm 0.2\, mA$ and $p = 0.1\, bar$

Table 3 shows us that a wave traveling in a perpendicular direction and which is in X-mode doesn't depend op Y and therefore not on the magnetic field B. For this reason the differences in the measured phase differences couldn't be coming from the X-mode.

The data couldn't be fixed using the function for the measured phase difference according to the O-mode. Therefore it is likely that this difference in the measured phases isn't coming from the O-mode either. This is in agreement which what method 1 predicted.

Perhaps there are some other modes traveling through the vessel which are dependent on the magnetic field, which could explain the measured data.

4.4 Simultaneous Measurements

Method 4, described in the previous chapter, uses two equations corresponding to the parallel propagating EM-wave in RH-mode and the perpendicular propagating wave in O-mode to find the electron density n_e and the offset ϕ_0. Method 1 predicts that no O-mode can be measured and the results of the previous section are supporting this. Simultaneous measurements are done and gave, $\phi_{\|_{RH}} = 315$ and $\phi_{\perp_O} = 255$. These data can be plugged in equation 20 and 21. Mathematica is used to solve this two system of equations. Mathematica showed that no value for n_e and ϕ_0 is possible. This means that the measured phases don't correspond to the RH-mode and/or the O-mode. Which could be in alignment with method 1 if only the O-mode isn't measured, but that isn't sure.

5 Conclusion

Three of the four methods give a relationship between the measured phase difference and the electron density. These three relations are quite different, so this experiment doesn't give a reliable equation to measure the electron density. To check which method gives the best relationship, a Langmuir probe could be use to measure the electron density of the plasma. The method that predicts the value closest to the value which the Langmuir probe gives, is most likely the best method.

In order to do better measurements, EM-waves with different frequencies could be used. A frequency could be used so that perpendicular waves in O-mode could propagate through the plasma. According to equation 9, the frequency should be approximately $4.4 GHz$. In this case the minimum refractive index is approximately 1.01, so that according to figure 1 the O-mode will propagate. In this case method 2, 3 and 4 can be used to find the electron density. Method 4 can be used, because simultaneous measurements, in this case, *does* refer to the parallel propagating RH-mode and perpendicular propagating O-mode.

If this gives results as in this experiment, maybe a better theory for the cutoff frequency must be made. A theory that includes a radius dependency of the electron density and which includes the real shape of the vessel, which ins't a perfect cylinder. Also the Maxwell equations could be applied in this case which will give a more accurate equation for the propagation of EM-waves.

In this experiment unpolarized EM-waves are used. To make sure a particular mode is measured, polarized EM-waves could be used. In this case method 4 becomes more vulnerable, because there are more options for the system of two equations. For example, first a EM-wave in O-mode and then a EM-wave in X-mode could be sent in the vessel so that the equations for the measured phase corresponding to these modes could be used. One has to make sure that in this case a frequency is used, so that the minimum refractive index is less that 1 so both modes could propagate through the vessel.

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