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P. Kagan and R.M.M. Mattheij

CASA, Technical University Eindhoven, The Netherlands

Summary. Finite element approximation in space and Crank-Nicolson approximation in time are used to model incompressible creeping flow of molten glass with temperature dependent viscosity. Iso-P triangle elements and second degree approximation of temperature and velocity fields are applied. Localized thermal behavior is captured with adaptively refined unstructured mesh.

Key words: Glass pressing, thermal modeling, finite elements

1 Introduction

In bottle manufacturing, a gob of molten glass is formed into a parison in a plunger and mold machine. Most of the used analysis techniques are based on experimentally acquired knowledge [1]. On the other hand, mathematical modeling can prove a decisive factor in production optimization, e.g., [2], [3]. Therefore, this article reports on the study aiming at acquiring further insight into thermomechanical aspects of glass forming by means of numerically simulating the involved processes.

2 Physical model

Thermodynamics The glass density ρ and specific heat c_p are assumed constant based on the experimental data [4] and [6], respectively. Furthermore, the concept of "effective" heat conduction [6] is used to model heat transfer within the molten glass. That is, $\boldsymbol{q} = -\kappa \mathrm{grad}\,(T)$, where T is the absolute temperature of the glass. Moreover, heat convection within the glass and the viscous dissipation are neglected due to the creeping nature of the glass flow. Finally, boundary heat transfer is modeled according to $\boldsymbol{q} \cdot \boldsymbol{n} = \alpha \left[T - \bar{T}\right]$, where $\boldsymbol{n}$ is the unit vector normal to the boundary, $\bar{T}$ is the temperature of the boundary and α is the convection coefficient.

Fluid dynamics Based on extensive experimental data analyzed in [1] glass is modeled as an isotropic incompressible Newtonian viscous fluid. The Cauchy stress tensor $\boldsymbol{T}$ is $\boldsymbol{T} = -p\boldsymbol{I} + 2\mu\boldsymbol{D'}$, where p is the pressure, $\boldsymbol{I}$ is the unit second order tensor and $\boldsymbol{D'}$ is the deviatoric part of the velocity gradient. The dynamic viscosity μ depends on the temperature according to the experimentally established Fulcher relation $\log \frac{\mu}{\mu_0} = -a + b\frac{10^3}{T - T_0}$, where the numerical values of the constants depend on the glass composition. Furthermore, dimensions of a typical parison (length $\sim 0.1m$) and duration of a typical forming cycle (time $\sim 1s$) suggest that the glass flow is creeping, so inertial and convective accelerations are neglected. Nonpenetration and full slip boundary conditions are assumed along the mold and the plunger, and material surface kinematics is assumed along the free surface. Plunger kinematics is prescribed, that is, linearly diminishing plunger velocity is imposed. Finally, attention is focused on the axisymmetric problem described with reference to the cylindrical coordinates (r, z) along unit the vectors $(\boldsymbol{e}_r, \boldsymbol{e}_z)$, respectively.

Equations of motion In what follows P is the interior of the computational domain, ∂P is the boundary of P, ∂P^p, ∂P^f, ∂P^m and ∂P^s are parts of ∂P corresponding to plunger, free surface, mold and symmetry axis, respectively. The glass behavior is described by the solution of

$$\rho\dot{e} + \operatorname{div}(\boldsymbol{q}) = 0 \qquad @\ P, t > 0, \tag{1a}$$

$$\operatorname{div}(\boldsymbol{v}) = \boldsymbol{D} \cdot \boldsymbol{I} = 0 \qquad @\ P, \tag{1b}$$

$$\operatorname{div}(\boldsymbol{T}) = 0 \qquad @\ P, \tag{1c}$$

satisfying the thermal initial and boundary conditions

$$T = T^0 \qquad @\ t = 0, \tag{2a}$$

$$-\kappa \operatorname{grad}(T) \cdot \boldsymbol{n} = \alpha\left[T - \bar{T}\right] \qquad @\ \partial P^p, \partial P^f, \partial P^m, \tag{2b}$$

$$-\kappa \operatorname{grad}(T) \cdot \boldsymbol{n} = 0 \qquad @\ \partial P^s, \tag{2c}$$

and the flow boundary conditions

$$[\boldsymbol{v} - \bar{\boldsymbol{v}}] \cdot \boldsymbol{n} = 0, \qquad \boldsymbol{T} \cdot [\boldsymbol{t} \otimes \boldsymbol{n}] = 0 \quad @\ \partial P^p, \partial P^m, \partial P^s, \tag{3a}$$

$$\boldsymbol{T} \cdot [\boldsymbol{n} \otimes \boldsymbol{n}] = p_{ext}, \quad \boldsymbol{T} \cdot [\boldsymbol{t} \otimes \boldsymbol{n}] = 0 \quad @\ \partial P^f. \tag{3b}$$

In these expressions $\bar{\boldsymbol{v}}$ is the velocity of the boundary and $\boldsymbol{n}$ and $\boldsymbol{t}$ are the unit vectors normal and tangent to the corresponding components of ∂P, respectively.

Free surface kinematics The position of the free surface ∂P^f at time t is implicitly represented by $f = \bar{f}(r, z, t) = z - \bar{\eta}(r, t) = 0$. The material surface assumption together with a neglected convective term enforce the motion of the free surface in accordance with the solution of

$$\dot{f} = -\frac{\partial \eta}{\partial t} + z = 0 \qquad @\ t > 0, \tag{4}$$

$$\eta = 0 \qquad @\ t = 0. \tag{5}$$

3 Finite element model

Thermal problem The space finite element approximation of Equation (1a) is developed by means of Galerkin weighted residual formulation with six node triangle shape functions $N_j\,(r,z)$:

$$\boldsymbol{M}\dot{\boldsymbol{T}}+\boldsymbol{S}\boldsymbol{T}=\boldsymbol{F}\quad @\ t>0\ \Big| \tag{6a}$$

$$M_{ij}=\sum_e\int_{P^e}\rho c_p N_i N_j\ dv^e, \tag{6b}$$

$$S_{ij}=-\sum_e\int_{P^e}\mathrm{grad}\,(N_i)\cdot[-\kappa\mathrm{grad}\,(N_j)]\ dv^e+\sum_{e\in\partial P}\int_{\partial P^e}N_i\alpha N_j\ da^e, \tag{6c}$$

$$F_i=-\sum_{e\in\partial P}\int_{\partial P^e}N_i\alpha\bar{T}\ da^e, \tag{6d}$$

$$\boldsymbol{T}=\boldsymbol{T^0}\quad @\ t=0. \tag{6e}$$

In these expressions, P^e is a generic finite element domain and ∂P^e is its boundary. Equations (6a) describe an ordinary initial value problem that is conveniently solved by Crank-Nicolson finite difference scheme [5] with time step Δt:

$$\left[\boldsymbol{M}+\frac{1}{2}\Delta t\boldsymbol{S}\right]\boldsymbol{T^{n+1}}=-\Delta t\boldsymbol{F^n}+\left[\boldsymbol{M}-\frac{1}{2}\Delta t\boldsymbol{S}\right]\boldsymbol{T^n}. \tag{7}$$

Finally, boundary and initial conditions are imposed and the resulting algebraic linear equations are solved leading to the nodal values $\boldsymbol{T^{n+1}}$ of the glass temperature at time t^{n+1}.

Flow problem Following the guidelines of Babuška-Brezzi condition [7] the pressure p is approximated in terms of three node triangular shape functions $N_j^p\,(r,z)$, while the velocity components v_r and v_z are approximated in terms of six node triangle shape functions $N_j^v\,(r,z)$. The mixed finite element approximation of Equations (1b), (1c) is

$$\boldsymbol{A}\boldsymbol{X}=\boldsymbol{0}\ \Big|\ \boldsymbol{A_{ij}}=\begin{bmatrix}\boldsymbol{S_{ij}^{vv}} & \boldsymbol{S_{ij}^{vp}}\\ \boldsymbol{S_{ij}^{pv}} & \boldsymbol{0}\end{bmatrix},\boldsymbol{X_j}=\begin{bmatrix}\boldsymbol{V_j}\\ P_j\end{bmatrix},\boldsymbol{V_j}=\begin{bmatrix}V_j^r\\ V_j^z\end{bmatrix}, \tag{8a}$$

$$\boldsymbol{S_{ij}^{vv}}=\sum_e\int_{P^e}-\begin{bmatrix}N_{i,r}^v & 0 & \frac{1}{r}N_i^v & N_{i,z}^v\\ 0 & N_{i,z}^v & 0 & N_{i,r}^v\end{bmatrix}\mu\begin{bmatrix}2N_{j,r}^v & 0\\ 0 & 2N_{j,z}^v\\ \frac{2}{r}N_j^v & 0\\ N_{j,z}^v & N_{j,r}^v\end{bmatrix}\ dv^e, \tag{8b}$$

$$\boldsymbol{S_{ij}^{vp}}=\sum_e\int_{P^e}-N_i^v\begin{bmatrix}N_{j,r}^p\\ N_{j,z}^p\end{bmatrix}\ dv^e, \tag{8c}$$

$$\boldsymbol{S_{ij}^{pv}}=\sum_e\int_{P^e}N_i^p\left[N_{j,r}^v+\tfrac{1}{r}N_j^v\ N_{j,z}^v\right]\ dv^e. \tag{8d}$$

In these expressions the subscript comma designates partial derivative with respect to the corresponding coordinate. Finally, boundary conditions are imposed and the resulting algebraic linear equations are solved leading to the nodal values of the glass velocity and pressure. Rank of the coefficient matrix must be monitored carefully while solving Equations (8a) since the described flow finite element fails the patch test when too many velocity degrees of freedom are constrained [7].

Mesh deformation problem Thermomechanical behavior of the glass during the pressing involves large displacements of the material. Consequently, the Lagrangian finite element mesh used for solving the thermomechanical problem undergoes significant distortion and most certainly becomes invalid unless special care is taken. At present, complete reconstruction of the mesh is used followed by projecting the temperature from deformed mesh onto new mesh by means of Taylor series truncated after the linear term.

4 Results

The initial geometry of mold and plunger used at this stage of the study are depicted in metric units on the left of Figure 4. The finite element mesh was adaptively refined toward the external boundaries of the computational domain in order to resolve sharp changes of the temperature and the viscosity. Consequent subplots of this picture present the calculated temperature and the radial and axial velocities, respectively. The mesh and the calculated temperature at the half pressing time $t = 0.6s$ and at the end of the pressing $t = 1.2s$ are presented on the left and the right of Figure 4, respectively.

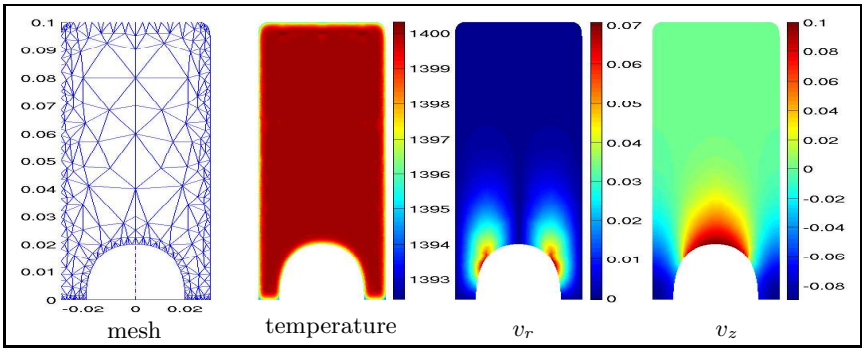

Fig. 1. Mesh and velocity components at $t = 0$, temperature at $t = 0.03s$

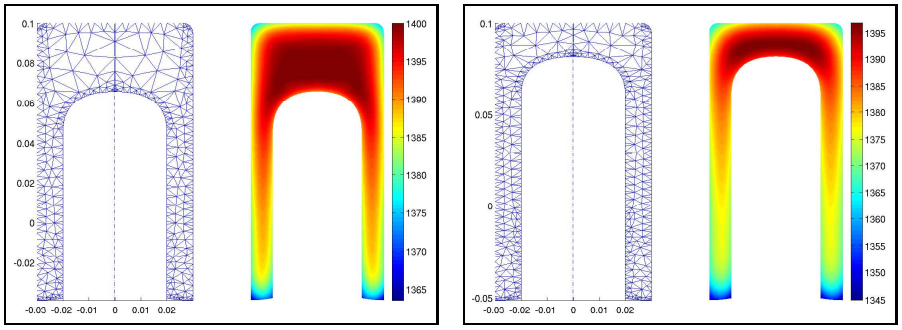

Fig. 2. Mesh and temperature: left $t = 0.6s$, right $t = 1.2s$

5 Conclusions

The results presented in this article extend our insight into the behavior of the molten glass during the pressing stage of bottle manufacturing. Two major observations are obvious already at this early stage of the study. First, the glass temperature is significantly affected by the heat convection across the rigid boundaries and the free surface. Second, the changes of the glass viscosity due to the thermal inhomogeneity considerably affect the glass flow pattern. From the conceptual point of view, our analysis shows real potential for the purposes of process optimization.

References

1. C.L. Babcock. *Silicate glass technology methods*. John Wiley, 1977.
2. J.M.A. Cesar de Sa. Numerical modeling of glass forming processes. *Engineering Computations*, 3(4):266–275, 1986.
3. K. Laevsky. *Pressing of glass in bottle and jar manufacturing: numerical analysis and computation*. PhD thesis, Technical University of Eindhoven, The Netherlands, 2003.
4. I. Sawai and S. Inoue. Spezifisches gewicht der ternären gläser $CaO-Na_2O-SiO$ bei höher temperatur. *Journal of the Society of Chemical Industry of Japan, Transactions*, 43(2):47B–49B, 1940.
5. G. Strang and G.J. Fix. *An analysis of the finite element methods*. Prentice-Hall, Englewood Cliffs, N.J., 1973.
6. A.F. Van Zee and C.L. Babcock. A method for the measurement of thermal diffusivity of molten glass. *Journal of American Ceramic Society*, 34(8):244–250, 1951.
7. O.C. Zienkiewicz and R.L. Taylor. *The finite element method*. Butterworth Heinmann, 5 edition, 2000.