Production to order

EINDHOVEN UNIVERSITY OF TECHNOLOGY

Faculty of Mathematics and Computing Science

Memorandum COSOR 87-07

Production to order

by

Nico P. Dellaert

Eindhoven, the Netherlands

April 1987

Production to order

Nico P. Dellaert

University of Technology
Eindhoven

Introduction

In process industry we often meet a situation in which different types of products can be manufactured on one machine and that there are many possible varieties of each type of product, so that no safety stocks can be kept. Usually the machine has to be rebuilded, or some other work has to be done before the production of another type can be started or at the beginning of a new period. This rebuilding-time is often quite large compared with the manufacturing-time of an order, whereas the rebuilding-time between the different varieties of the same type of product is rather small, or can be neglected. Because of the large rebuilding-time one would like to manufacture a large lot of the same type without rebuilding. On the other hand there is the interest in short and accurate delivery-times for the orders as well as the capacity-restrictions, storage costs and possible production failures that ask for rather small lotsizes.

This type of problem in all kinds of varieties has been the subject of many articles. Problems with deterministic demand have been studied by Maes and van Wassenhove (1986), Dixon and Silver (1981), Lambrecht and Vanderveken (1979) and many others. Much less research has been performed for the case of stochastic demand. Here the main contributors are Virgin and Lee (1978) and Graves (1980).

If the delivery-times have to be short and accurate, the exact demand will be known only a short period in advance. Suppose the clients are assigned to several groups and that the delivery-times for clients belonging to the same group are equal to the same constant. Then there may be a group of clients that always obtains a delivery-time of one period, another group that obtains a delivery-time of two periods and so on. This implies that at the end of a period the demand for the next period is known exactly, but the demand for later periods is not completely known, since there are one or more groups of clients that can still order demand for those periods.

Now an interesting problem is how to assign clients to the different groups, given a certain structure of costs and rewards, in order to maximize the profits. This assignment is of course related with the production-strategy that is used: if some type of product is manufactured nearly every period, it will be easier to assign a lot of clients to groups that obtain a short delivery-time than if that type of product is manufactured only very rarely, but in large series.

In this paper we will consider which kind of production-strategy can be used best as a tool for this kind of problems. The required strategy must give a good indication of the minimal average costs for a given demand distribution and should be easy to compute. Therefore we consider a simple scheduling model with one type of product on one machine where the main aspects are production to order in combination with a demand by different groups of clients, set-up times and unconstrained capacity. For different situations we will compare methods for Markov-Decision-Problems and heuristics inspired by the Wagner-Whitin and Silver-Meal algorithm.

After a description of the problem, modelled as a Markov-Decision-Problem, we will describe the strategies and their complexity and consider their performance in some examples. We will see that all heuristics perform reasonably well and that the main differences between the heuristics are in the complexity of the decision rules and the possibility to compute the average costs. This leads to the choice of a simple strategy, where all decisions are based only on the demand for the first period. Moreover this type of strategy enables us to calculate the average costs in complex situations.

1. Description of the problem

For the demand we assume that we have N different groups of clients, where clients of group i, $1 \leqslant i \leqslant N$, always require delivery-time i for their products, so the due-dates are independent of the production-schedule, but depend only on the priority-rules for the different clients and on the arrival-dates. We assume that the demand distribution is known and stationary. Once a client is assigned to a certain group, his delivery-time is always the same and the order-flow, and thereby the costs, can be controlled by the assigning of new clients to certain groups. In practice, it will not always be optimal that delivery-times are independent of the production-schedule and that, although there are different groups of clients, penalty costs and holding costs are independent of the clients.

At the beginning of each period we know the exact demand for that period and a part of the demand for later periods. Now we can decide to produce or decide to wait with production. If we produce, then we have set-up costs s and holding costs h per unit of product per period for orders that are manufactured too soon. If we wait with production, we have penalty costs p per unit of product per period for orders that are delivered too late. We assume that once a decision is taken, it is not changed. Furthermore we assume that we express the demand in units of products and that the products are made according to customer specifications, so that only the known demand can be manufactured.

Our object is now to minimize the long-term average costs per period. We will first model the problem as a Markov-Decision-Problem, so we can use a known method to determine the optimal strategy.

2. Description of the model

In order to give a good description of the problem, we model it as a Markov-Decision-Problem (MDP). We call the state we observe, $r=(r_0,r_1,...,r_{N-1})$, the residual demand-vector. This residual demand-vector contains the original demand for the current period and the next N-1 periods, but the products that already have been manufactured for these periods are left out. Because backlogging is permitted, r_0 also contains the demand of earlier periods that is not yet produced. The costs do not depend on the arrival-times of the orders, so therefore this residual demand provides all the necessary information. The set of all possible states is denoted by R.

Each state $r \in R$ is associated with a finite non-empty set of actions $A(r)$. Since we have no capacity-constraints, we will always produce the residual demand of an integral number of periods. Therefore, the meaning of action a is that we produce the known demand for a periods, that is $\sum_{i=0}^{a-1} r_i$ products. Action 0 means that we do not produce. To determine the action space, we choose $A(r)$, $r \in R$, in a reasonable way:

$A(r)=\{0\}$ if r_0 equals 0, since it is clearly optimal to wait with production;
$A(r)=\{1,..,N\}$ if $r_0 p > s$, since it is clearly optimal to produce because the production costs are less than the penalty costs;
$A(r)=\{0,...,N\}$ for all other $r \in R$.

Every period clients of group i can order a demand of $0,1,...,M_i$-1 or M_i units of product. We assume that the probability that they order j units during one period, denoted by d_{ij}, is known. Then $J := \{0,1,..,M_1\} \times .... \times \{0,1,..,M_N\}$ is the set of all possible one-period demands $(j_1,j_2,...,j_N)$. Let $Q_a(r)$ be the next-period state if we have taken action a in state r and we have no new demand during the current period. Then, for instance, $Q_0(r) = (r_0+r_1,r_2,..,r_{N-1},0)$ and $Q_3(r) = (0,0,r_3,...,r_{N-1},0)$.

On observing state r and choosing action $a \in A(r)$ we enter state

$$t = Q_a(r) + (j_1, j_2, \ldots, j_N) \tag{2.1}$$

with probability P^a_{rt}, which equals $\prod_{i=1}^{N} d_{ij_i}$ for all $(j_1, j_2, \ldots, j_N) \in J$, and 0 elsewhere.

The one-stage costs of taking action a on observing state r have the following form:

$$q^a_r = s + \sum_{i=1}^{a-1} i\, r_i\, h \qquad \text{if } a > 0 \tag{2.2}$$

$$q^a_r = r_0\, p \qquad \text{if } a = 0. \tag{2.3}$$

3. Optimal strategy

By the choice of the action space, we have that all stationary policies have transition-probability matrices representing recurrent aperiodic Markov-chains. Therefore we can use the method of successive iteration, as described by Odoni(1969), to determine the minimal expected costs per transition, denoted by g. Defining the quantity $v_n(r)$ as the total expected costs from the next n transitions if the current state is r and if an optimal policy is followed, the iteration scheme takes the following form:

$$v_{n+1}(r) = \min_{a \in A(r)} [q^a_r + \sum_{s \in R} P^a_{rs}\, v_n(s)] \quad ; r \in R,\ n = 0,1,2.. \tag{3.1}$$

Define $x_n(r)$ by:

$$x_n(r) = v_{n+1}(r) - v_n(r). \tag{3.2}$$

Then according to Odoni for any choice of starting conditions $v_0(r)$:

(i) $v_n(r) = ng + v(r) + O_n(r), r \in R$, where $O_n(r) \to 0$ and where the $v(r)$ satisfy:

$$v(r) = \min_{a \in A(r)} [q^a_r + \sum_{s \in R} P^a_{rs}\, v(s) - g]. \tag{3.3}$$

(ii) $x_n(r) \to g,\ r \in R$.

(iii) $L_2(n) = \max_{r \in R} x_n(r)$ is monotone decreasing in n to g.

(iv) $L_1(n) = \min_{r \in R} x_n(r)$ is monotone increasing in n to g.

(v) Any strategy achieving the minima in (3.1) for all $r \in R$ for all $n \geqslant$ some n_0 achieves the minima in (3.3) for all $r \in R$ and has the minimal costs per transition.

Beginning with $v_0(r)=0$ for all $r \in R$, we repeat (3.1) until a satisfactory degree of convergence is achieved. It follows that g may be estimated from $L_1(n) \leqslant g \leqslant L_2(n)$ as $g \sim [L_1(n) + L_2(n)]/2$. The range decreases with n and the estimate becomes nearly exact for large n.

3.1. Computational characteristics

For the computational efforts two aspects are important:
- the number of elementary operations required in each iteration step;
- the number of iteration steps.

If $r_0 p > s$, then the actions and costs do not depend on r_0, so therefore the number of states we have to consider is:

$$M := (\left\lfloor \frac{s}{p} \right\rfloor + 2) \prod_{i=2}^{N} (\sum_{l=i}^{N} M_l + 1), \tag{3.1.1}$$

where $\lfloor x \rfloor$ denotes the lower integer value of x. In every state we have to consider either 1, N or $N+1$ actions. Therefore for choosing the right action in every iteration step we require a number of elementary operations proportional to NM. Instead of determining the second term of (3.1) for every state, it suffices to determine this value for every possible $Q_a(r)$, the demand that is left after action a. We have to consider

$$N_1 := (\left\lfloor \frac{s}{p} \right\rfloor + 2) \prod_{i=3}^{N} (\sum_{l=i}^{N} M_l + 1) \tag{3.1.2}$$

possible states for $Q_{a(r)}$ and from every state we can enter

$$N_2 := \prod_{l=1}^{N} (M_l + 1) \tag{3.1.3}$$

other states. Therefore the total number of elementary operations in each iteration step is proportional to $NM + N_1 N_2$, which is generally smaller than M^2 and also smaller than the number of NM^2 that Odoni mentions.

It is well known that for this kind of problems the behaviour of $L_2(n) - L_1(n)$ can be described as:

$$L_2(n) - L_1(n) \leqslant 2\alpha^n g \tag{3.1.4}$$

where α is the absolute value of the one but largest eigenvalue of the transition matrix P, that represents the optimal strategy. In our examples the value of α varied from 0.3 to 0.9, so for an accuray of $0.0001g$, we need 8 to 88 iteration steps. Concluding we can say that the total number of elementary operations is less than $100M^2$. Although this seems not so bad, it may soon be impossible to compute the optimal strategy, since M is always greater than $N!$. Therefore we will also consider some heuristics.

4. Heuristic procedures.

Because it might not always be possible to determine the optimal strategy, especially in more complex situations, we also consider three heuristic approaches: a Silver-Meal like approach, a Wagner-Whitin like approach and a strategy we call the (x, T)-strategy, that is related with the (s,S)-strategy in inventory control. Before we describe the three different strategies, we will define some variables that we will use in more than one strategy.

4.1. Notation

We define the following variables:

- u_i is the expected demand of clients of group i during one period:

$$u_i = \sum_{j=0}^{M_i} j d_{ij} .$$

- b_{il} is the probability that clients belonging to the groups 1,2,...,i order a total amount of l units of product for one specific delivery period:

$$b_{il} = \sum_{j \in J} \left(\prod_{k=1}^{i} d_{k j_k} \quad with \ \sum_{k=1}^{i} j_k = l \right).$$

- e_i is the expected value of the i+1-th component of the residual demand, if no part of this demand has been produced:

$$e_i = \sum_{l=i+1}^{N} u_l .$$

4.2. The Silver-Meal approach

Applying the idea of the Silver-Meal algorithm, we divide the expected costs of an action by the number of periods involved in this action and we choose that action for which the quotient is minimal. The direct costs of action a are given by q_r^a. However, if we produce the demand for two or more periods ($a \geqslant 2$), it is reasonable to suppose that we will not produce during the next a-1 periods. This implies that the demand for these periods, that arrives during the current period and the following a-2 periods, will be delivered too late, so we will have to pay penalty-costs for this demand. Under the assumption that the next production takes place a periods after the current period, the expected penalty-costs during the next a-1 periods depend on a only and can be described by the penalty-function $p(a)$:

$$p(a) = p \sum_{i=2}^{a} (a+1-i) \sum_{l=1}^{i-1} u_l \quad a = 2,3,\ldots,N \tag{4.2.1}$$

$$p(a) = 0 \quad \text{for all other } a. \tag{4.2.2}$$

Using this penalty function, q_{r_a} for the direct costs and $\delta(a)$ as a function that equals 1 for $a = 0$ and 0 elsewhere, the strategy takes the following form:
if we observe a state $r \in R$ we take that action a for which

$$\frac{q_r^a + p(a)}{a + \delta(a)} \tag{4.2.3}$$

is minimal over $a \in A(r)$.

4.3. The Wagner-Whitin approach

When using the Wagner-Whitin approach, we consider all possible action-sequences $a_1, a_2, \ldots, a_H$ during the first H periods (H$\leqslant$N) and we choose the first action of the action-sequence with minimal expected costs to be taken this period. Compared with a deterministic scheduling problem with known demands and a fixed horizon, we have two extra problems: we have some unknown demand for the next periods and we have a moving horizon, which implies that it might be interesting to produce for periods after the horizon H.

For the first problem we replace the unknown future demands by their expected value and for the second problem we assume some simple strategy π, with average costs g_π, to be used after the H periods to measure the effect of an action-sequence on later periods. One of the simplest possibilities to measure this salvage value is by defining the following salvage function L(.): for every period after the horizon H for which a part of the demand has been produced, we obtain a bonus g_π and we pay some extra penalty costs, using the penalty-function $p(a)$.

Let $\mathbf{A}_H$ be the set of all possible action-sequences with H elements. Then, given an action-sequence $\mathbf{A} \in \mathbf{A}_H$, we denote by l_0 the last period in which we produced and by j_0 the last period for which we produced a part of the demand and we define:

$$L(\mathbf{A}) = 0 \qquad \text{if } j_0 \leqslant H \tag{4.3.1}$$

$$L(\mathbf{A}) = p(j_0 - l_0 + 1) - p(H - l_0 + 1) - (j_0 - H) g_\pi \quad \text{if } j_0 > H \tag{4.3.2}$$

A possible value of g_π follows from a strategy π in which we produce every T periods:

$$g_\pi = \frac{s + \sum_{i=1}^{T-1} i e_i + p(T)}{T} \tag{4.3.3}$$

The number of action-sequences we have to consider can be limited in two ways:
- we don't produce during periods for which already a part of the demand has been produced;
- we remove action-sequences in which we produced a smaller amount at higher costs than in one of the others. Now the strategy takes the following form: given a state $r \in R$ determine the action-sequence $\mathbf{A} \in \mathbf{A}_H$ for which:

$$\sum_{i=1}^{H} q_{s_i}^{a_i} + L(\mathbf{A}) \tag{4.3.4}$$

is minimal over the elements of $\mathbf{A}_H$. Here:
- $s_1 = r = (r_0, r_1, \ldots, r_{N-1})$ is the demand in the first period.
- $s_i = Q_{a_{i-1}}(s_{i-1}) + (u_1, \ldots, u_N)$ $i = 2, \ldots H$, is the expected demand during the i-th period.
- $a_1 \in A(r)$ is the action during the first period.
- $a_i = 0$ if $\max_{j<i}[j + a_j] > i$, which implies that we do not produce during the i-th period if a part of the demand for this period has already been produced.
- $a_i \in \{0, 1, \ldots, N\}$ otherwise.

Now the first action of the action sequence **A** is taken for the current period.

4.4. The (x,T)-strategy

Following the idea of (s,S)-strategies we consider the following approach. Produce the residual demand of the next T periods as soon as the demand for the current period is at least x. This strategy is called (x, T)-strategy. In the unconstrained capacity situation it is rather easy to determine the average costs for certain values of x and T. Of course we are interested in finding the optimal choices for x and T. We do this by computing the average costs per period for several values of x and T. Hereby we make use of the following property, proven in Dellaert (1987):

Property 4.1 : for a given value of T the optimal value of x satisfies:

$$x \leqslant \left\lfloor \frac{g(x, T)}{p} \right\rfloor + 1, \tag{4.4.1}$$

where $g(x, T)$ denotes the average costs per period using the pair (x, T).

To compute the $g(x, T)$ for some pair (x, T), we consider a finite recurrent Markov-chain, with states $(i,j) \in \{1, 2, \ldots, T-1\} \times \{0, 1, \ldots, x\}$, where
i= min {number of periods passed since the last production period, $T-1$} and
j= min {demand for the current period, x}. We assume $T \geqslant 2$.

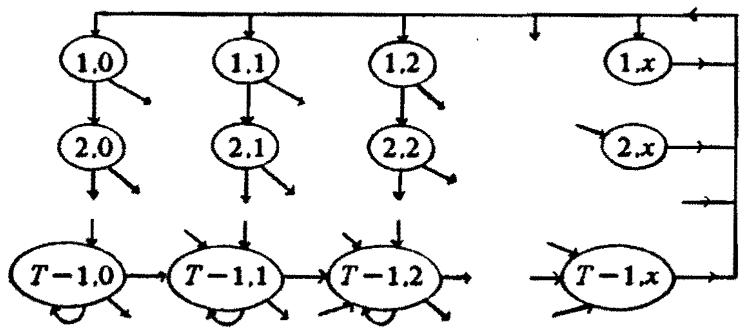

Figure 4.4.1 *The states of the Markov chain* for *the* (x,T)*−strategy.*

We use the following notations:

- q_{ij} is the probability to be in state (i,j);
- c_{ij} is the expected cost in state (i,j);
- $X = \sum_{i=1}^{T-1} q_{ix}$ is the probability that we produce in an arbitrary period; X^{-1} is the expected number of periods between two production periods.

Because of the special structure of the Markov-chain we can express every q_{ij} in X very easily:

$$q_{1j} = b_{1j} X \qquad 0 \leqslant j < x \tag{4.4.2}$$

$$q_{1x} = \Big(1 - \sum_{k=0}^{x-1} b_{1k} \Big) X \tag{4.4.3}$$

$$q_{ij} = \sum_{k=0}^{j} q_{i-1k}\, b_{ij-k} \qquad 0 \leqslant j < x\ ,\ 2 \leqslant i \leqslant T-1 \tag{4.4.4}$$

$$q_{ix} = \sum_{k=0}^{x-1} q_{i-1k} - q_{ik} \qquad 2 \leqslant i \leqslant T-2 \tag{4.4.5}$$

and then modifying q_{T-1j} for later periods:

$$q_{T-1j} = \Big(q_{T-1j} + \sum_{k=0}^{j-1} q_{T-1k}\, b_{Nj-k} \Big) (1 - b_{N0})^{-1} \quad 0 \leqslant j < x \tag{4.4.6}$$

$$q_{T-1x} = X - \sum_{i=1}^{T-2} q_{ix} \tag{4.4.7}$$

Since the sum of all probabilities equals 1, we know the value of X and thereby every q_{ij}.
The expected costs in state (i,j) are:

$$c_{ij} = pj \qquad 1 \leqslant i \leqslant T-1, \qquad 0 \leqslant j < x \tag{4.4.8}$$

$$c_{ix} = s + h \sum_{j=1}^{T-1} je_j - h \sum_{j=i+1}^{T-1} (j-i)\, e_j \tag{4.4.9}$$

$$= s + h \sum_{j=1}^{T-1} e_j \min(i,j) \qquad 1 \leqslant i \leqslant T-1 \tag{4.4.10}$$

The average costs of the strategy for this pair (x,T) are now given by:

$$g(x,T) = \sum_{i=1}^{T-1} \sum_{j=0}^{x} q_{ij}\, c_{ij} . \tag{4.4.11}$$

Initially we set T=N and $x = \left\lfloor \frac{s}{p} \right\rfloor + 1$ and determine $g(x,T)$. Then we decrease x according to (4.4.1) until x does not change anymore. The only values we have to recalculate are X, the q_{ix}'s and $g(x,T)$. Using the same x, we decrease T by 1, recalculating X, c_{ix}, q_{T-1j} and $g(x,T)$. For this T we determine the optimal x-value and we repeat this procedure until T equals 1 or if by decreasing T the average costs increase.

4.5. Computational characteristics

For the Silver-Meal like strategy as well as for the Wagner-Whitin like strategy the calculation of the average costs consists of two parts. First we have to determine the action that is chosen for every state. Then we have to calculate the average costs for the set of actions we have found.

The maximum number of actions we have to consider to determine the best action for a certain demand is:

- N_S+1 for the Silver-Meal like strategy, where N_S is the smallest number for which $p(N_S+1) - p(N_S) \geqslant s$, or N if no such value exists. If N_S is smaller than N, then we do not have to consider the demand for the periods $N_S+1,...,N$, since it will never be interesting to produce this demand.
- $(N+1)H(H+1)/2$ for the Wagner-Whitin like strategy with H periods, because during each of the H periods we have to consider $N+1$ actions and their costs during the first H periods.

Computing the average costs of the Silver-Meal like strategy and the Wagner-Whitin like strategy is almost as complex as computing the average costs of the optimal strategy. This is because we use the same iteration scheme, of course with only one possible action for each state. The number of states we have to consider for the Wagner-Whitin like strategy is the same as for the optimal strategy, whereas the number of considered states in the Silver-Meal like strategy, for N_S <N, is much smaller:

$$M_S := (\left\lfloor \frac{s}{p} \right\rfloor + 2) \prod_{i=2}^{N_S} (\sum_{l=i}^{N} M_l + 1). \tag{4.5.1}$$

Therefore the total number of actions we have to consider is proportional to:
- $N_S\ M_S$ for the Silver-Meal like strategy;
- NMH^2 for the Wagner-Whitin like strategy.

The actions have to be determined only once, but in every iteration step the value of the second term in the iterations scheme must be evaluated. For the Wagner-Whitin like strategy the number of elementary operations per iteration step is proportional to $N_1 N_2$. In the Silver-Meal like strategy we have to consider

$$N_{S1} := (\left\lfloor \frac{s}{p} \right\rfloor + 2) \prod_{i=3}^{N_S} (\sum_{l=i}^{N} M_l + 1) \tag{4.5.2}$$

possible states for $Q_a(r)$ and from every state we can enter

$$N_{S2} := \prod_{i=1}^{N_S - 1} (M_i + 1) (\sum_{l=N_S}^{N} M_l + 1) \tag{4.5.3}$$

other states. As we will see in the next section, both strategies are usually nearly-optimal, hence the transition matrix P will show much resemblance with the transition matrix for the optimal strategy and therefore the number of iteration steps we need for a certain accuracy will be roughly the same as for the optimal strategy. If we denote the number of iteration steps by I, the total number of elementary operations is proportional to:
- $N_S M_S + IN_{S1} N_{S2}$ for the Silver-Meal like strategy;
- $NMH^2 + IN_1 N_2$ for the Wagner-Whitin like strategy.

Computing the average costs of the optimal (x,T)-strategy is far more easier. For every pair (x,T) for which we have to determine $g(x,T)$ the number of operations is proportional to xT. If we start with the initial values T=N and $x = \left\lfloor \frac{s}{p} \right\rfloor + 1$, then it takes at most (N-1) + $\left\lfloor \frac{s}{p} \right\rfloor$ decreasing steps for x or T to reach the optimal pair (x,T). Therefore the total amount of elementary steps is proportional to:

$$N \left\lfloor \frac{s}{p} \right\rfloor (N + \left\lfloor \frac{s}{p} \right\rfloor). \tag{4.5.4}$$

If we compare all computational efforts for determining the average costs then we can conclude the following:
- determining the average costs of the Wagner-Whitin strategy takes a little less effort than for the optimal strategy, because we only have to determine the actions for each state once.
- determing the average costs of the Silver-Meal strategy can give a considerable reduction of the computational effort if $N_S < N$. However for larger values of N or the M_i's the

computation will still become impossible.
- determining the average costs of the optimal (x,T)-strategy takes almost no effort; for extremely large values of N we can take a much smaller initial value for T and for large values of $\frac{s}{p}$ we can take a much smaller initial value for x.

5. Numerical results

In order to study the performance of each of the strategies, we will determine the average costs for some examples. The size of the examples is small, so we can compute the costs of each strategy. We will consider the following strategies:
- OPT, the optimal strategy;
- SM, the Silver-Meal like strategy;
- WW(H), the Wagner-Whitin like strategy with H periods;
- XT, the optimal (x,T)- strategy.

We have considered two sets of examples. In the first set we have N=4, $p=3$, $h=1$, $d_{i1}=1-d_{i0}=c$ for i=1,..,4. For some different values of c and s we have the following results:

c-value s-value	0.25 3.25	0.25 8.00	0.50 6.50	0.50 16.00	0.75 9.75	0.75 24.00
strategy	average costs per period					
OPT	1.8895	3.7147	4.5357	8.1705	7.0425	12.6002
SM	1.9002	3.7173	4.5392	8.1793	7.0445	12.6054
WW(1)	1.8954	3.7830	4.5384	8.2135	7.0432	12.6054
WW(2)	1.8940	3.7770	4.5384	8.1910	7.0432	12.6020
WW(3)	1.9002	3.7815	4.5438	8.1861	7.0425	12.6020
WW(4)	1.9002	3.7815	4.5438	8.2002	7.0425	12.6020
XT	1.9074	3.7326	4.5392	8.1965	7.0451	12.6125

Table 5.1 The average costs of the strategies in the first set of examples.

In the second set of examples we have N=2, $p=3, h=1$ and Poisson-distributed demand with parameter 1 for i=1,2. For three different values of s we obtained the following results:

s-value	7	8	9
strategy	average costs per period		
OPT	4.8498	5.3335	5.8113
SM	4.8537	5.3365	5.8261
WW(1)	4.8574	5.3335	5.8258
WW(2)	4.8574	5.3335	5.8258
XT	4.8767	5.3807	5.8847

Table 5.2 The average costs of the strategies in the second set of examples.

From these examples one might get the impression that the SM-strategy is always better than the optimal (x,T)-strategy. This however, is not the case: if in the second set of examples we take a Poisson-distributed demand with parameter 1.01 and set-up costs $s = 9$ then the average costs of the Silver-Meal strategy are 5.9308, whereas the average costs of the optimal (x,T)-strategy are 5.9082.

6. Conclusions

The examples showed us that the heuristic strategies perform very well. The difference in average costs between these strategies and the optimal strategy is always less than a few percents. It might be interesting to see how the differences between the optimal strategy and the heuristics arise:

- in the SM-strategy we produce less often: we wait too long with production and if we produce, we produce too much;
- the XT-strategy contains the same elements as the SM-strategy, but usually even stronger;
- in the WW-strategy we wait too long with production and if we produce, we produce less than the optimal amount.

Now which of the heuristic approaches must be preferred ? Although the decision-rules for SM and for instance WW(2) are not very complex and their performance is usually slightly better than the optimal (x,T)-strategy, the disadvantage to be unable to compute the average costs makes them less useful for situations in which we have to take decisions about assigning clients or other problems. In those situations we would need simulation to get a reasonable insight in the average costs. The (x,T)-strategy however, offers a very simple decision rule with known average costs, and must therefore be preferred. In Dellaert (1987) some examples of the use of the (x,T)-strategy for assigning clients are given. In that paper we also give some simple rules to improve the (x,T)-strategy until it is nearly optimal.

Literature

Dellaert N.P. and J.Wessels (1986), Production Scheduling with uncertain demand, Operations Research Proceedings 1985,247-252.

Dellaert N.P. (1987), The use of the (x,T)-strategy for production on order, Memorandum COSOR, University of Technology Eindhoven, to appear.

Dixon P.S. and E.A.Silver (1981), A heuristic solution procedure for the multi item, single level, limited capacity, lotsizing problem, Journal of Operations Management 2, 23-39.

Graves S.C. (1980), The multi-product production cycling problem, AIIE Transactions 12, 233-240.

Lambrecht M.R. and H.Vanderveken (1979), Heuristic procedures for the single operation multi item loading problem, AIIE Transactions 11, 319-326.

Maes J. and L.N.van Wassenhove (1986), A simple heuristic for the multi item single level capacitated lotsizing problem, Operations Research Letters 4, 265-273.

Odoni A. (1969), On finding the maximal gain for Markov decision processes, Oper. Res. 17, 857-860.

Virgin R.C. and T.N.Lee (1978), Scheduling rules for the Multiple Product Single Machine System with Stochastic Demand, Infor 16, 64-73.