

Duplication of constants in process algebra

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Duplication of Constants in Process Algebra

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Abstract

The constant 0 (or δ , *nil*) has different roles in process algebra: on the one hand, it serves as the identity element of alternative composition, on the other hand, it stands for a blocked atomic action or for livelock. When extensions with timing are considered, these roles diverge. We argue that it is better to use two separate constants $\hat{0}$ and 0 for the different usages.

With respect to the termination constant 1 (or ϵ , *skip*), the situation is comparable: on the one hand, it serves as the identity element of sequential composition, on the other hand, it serves as the identity element of parallel composition, and stands for a skipped atomic action. We have separate constants $\hat{1}$ and 1 for the different usages.

1 Introduction

In the design of a process algebra, both operational intuition and the resulting set of laws play an important role. On the one hand, the operational intuition gives us what is observable about a behavior: the execution of a (visible) action, termination, or (in theories with quantitative time) the passage of time. On the other hand, the resulting set of laws turn the theory into an algebra, and we look for instance for identity elements for the basic operators.

In a process algebra without quantitative timing, consider a process that starts with the execution of an atomic action a . This means we observe the execution of a at some moment, and then, the process continues with the remainder. Interpreting this in a theory with timing, we say a occurs at some unspecified moment of time, i.e. we may observe some passage of time first, and then the execution of a . Stated differently, we interpret a as a *delayable* action. Besides this delayable a , a theory with timing will also contain undelayable actions.

Next, consider choice, e.g. consider a process $a.x + b.y$ that either starts with the execution of a or with the execution of b . In the process algebra ACP or CCS, the intuition is that the choice is made by the execution of an action, and not at any time before. Interpreted in a timed theory, a and b occur at unspecified moments of time, maybe a occurs after 2 time units and b occurs after 3 time units. Then, after 1 unit of time, the choice is not made, and both options are still open. This is called *time-determinism* or *time factorization* in timed process algebra: passage of time as such does not make a choice.

Now, some theories with timing use so-called *strong time-determinism*: if b happens to occur later than a , then b cannot be chosen, and a has to occur. We feel this is not in accordance with untimed theories: a choice not to do b cannot be taken before any action execution, as this is opposed to time-determinism. But then, we arrive at so-called *weak time-determinism*: it is possible to delay past the execution time of a , but then a choice is made not to do a , and b will be executed. To repeat, adherence to weak time-determinism means that passage of time is possible in a choice context as long as at least one component allows this delay. This means adding an option with more delay adds more options in a choice context, and the identity element for choice should not be delayable at all.

The identity element of choice in untimed process algebra is the inaction process 0 (also called δ or *nil*) that is characterized by no action execution and no termination. Interpreted in a timed

setting, the question is whether or not 0 allows passage of time. Since 0 stands for a blocked atomic action (0 is the process $a.x$ when execution of a is blocked), and a is delayable, we take also 0 to be delayable. But on the other hand, we adopt weak time-determinism, and thus, the identity element is a process that does not allow passage of time. We take a different constant $\hat{0}$ for this identity element. This was also done in [BB91, BM02], where the notation $\hat{\delta}$ was used for this purpose.

The empty process 1 (also called ϵ or *skip*) denoting successful termination or *skip* has not been studied nearly as well as the unsuccessful termination constant. The untimed theory was investigated in [KV85, BG87, Vra97]. In the context of ACP-like process algebras the empty process in a timed setting is mentioned in [Gro91, Ver97, BV97]. In [Ver97, BV97] a relative-time, discrete-time process algebra has been extended with both a non-delayable and a delayable successful termination constant. The hybrid process algebra HyPA from [CR05] contains a non-delayable successful termination constant.

As is the case for 0 in the untimed theory, also the process 1 has more roles. On the one hand, it serves as the identity element for sequential composition, on the other hand, it stands for the process that executes no actions but terminates at some unspecified time, and as such acts as the identity element of parallel composition. Assuming that we want the embedding of untimed process algebra into timed process algebra where atomic actions and 0 are delayable timed constants, it is impossible to use only one timed successful termination constant for both roles. This is explained as follows. Suppose that we want to treat the untimed successful termination constant as being non-delayable in the timed setting. Then, the timed interpretation of the untimed identity $1 + 0 = 1$ is not valid anymore as the left-hand side of the identity is delayable and the right-hand side is not! Thus, the interpretation of 1 must be a delayable constant. Such a delayable constant cannot act as an identity element for sequential composition: 1 followed by a non-delayable a adds an arbitrary delay before the execution of a , so is not the same as the non-delayable a . Hence, with timing, if 1 represents the successful termination constant that allows passage of time, we introduce a new constant $\hat{1}$, called the *terminated process*, that is the identity element for sequential composition. The delayable 1 can still act as the identity element of parallel composition, as a delay can only occur in a parallel composition if all components allow this delay.

The process $\hat{1}$ denotes a terminated process: termination has taken place, so no parallel activity can precede the termination. With this constant, we finally have a complete interpretation of the constant process a of ACP in a timed setting: upon executing the action, what remains is $\hat{1}$.

In [Bae03], it was established that action constants make embedding of untimed into timed theories difficult, and it was suggested to use action prefixing instead. This was subsequently worked out in [BMR05, BB05, BBR06]. We follow this approach here, so we start out from the theory TCP.

Thus, we have separated out two different roles of the basic constants 0 and 1 . By having 0 stand for a blocked atomic action, and having $\hat{0}$ for the identity element of alternative composition, and at the same time having 1 as the identity element of parallel composition, and $\hat{1}$ as the identity element of sequential composition, it becomes easier to define timed extensions in different ways: discrete time or dense time, relative time or absolute time. In all of these cases, the four basic constants 0 , $\hat{0}$, 1 , and $\hat{1}$ keep their respective roles. As an example, we work out the theory in the case of relative discrete time. We also worked out the variants for dense time and absolute time, but do not present these in the current paper.

In Section 2, we present the untimed process algebra TCP. In Section 3, a discrete relative timing extension of TCP, called TCP_{drt} , is presented. In Section 4, we sketch how absolute time and dense time extensions of TCP can be obtained. Then, in Section 5, we introduce the extension of TCP with the new constants $\hat{0}$ and $\hat{1}$. The resulting process algebra is called TCP^\bullet . In Section 6, a discrete relative timing extension of TCP^\bullet is given. It is called $\text{TCP}_{\text{drt}}^\bullet$.

2 Untimed Process Algebra

We start out from the Theory of Communicating Processes, TCP, of [BBR06], see also [BB05, BMR05].

This process algebra is parameterized by a set A of (atomic) actions, and a *communication function* γ on A . The function γ is a partial binary, commutative and associative function on A , and when $\gamma(a, b) = c$, then a and b are matching actions, that when they synchronize yield the resulting action c . The signature of TCP contains the following elements:

- *inaction* 0 . This is the process that cannot perform any action and cannot terminate. Operationally, it is characterized by having no operational rules at all. Inaction is the identity element of alternative composition. It is often called deadlock and denoted δ in ACP-style process algebra, and *nil* in CCS.
- *termination* 1 . This is the process that cannot perform any action, but can only terminate successfully. Termination is the identity element of sequential composition and at the same time of parallel composition. This process is called the empty process or skip, and denoted ϵ in ACP-style process algebra, and SKIP in CSP.
- for each action $a \in A$, the *action prefix* operator $a.$. The process $a.x$ executes action a and next continues with the execution of x .
- alternative composition $+$. The process $x + y$ executes either x or y , but not both. The choice is resolved upon execution of the first action.
- sequential composition \cdot . The process $x \cdot y$ first executes x , and upon termination of x starts the execution of y .
- parallel composition \parallel . The process $x \parallel y$ interleaves the actions of processes x and y (denoted by auxiliary operator \parallel) and synchronizes communicating actions and termination (denoted by auxiliary operator \mid).
- encapsulation ∂_H blocks the execution of actions from $H \subseteq A$, and is used to enforce communication.

The axioms of TCP are presented in Table 1. Axioms A1-A10 are the axioms of the theory TSP, Theory of Sequential Processes (see [BBR06]). Alternative composition is commutative, associative, idempotent and has identity element 0 . Sequential composition is associative and has identity element 1 . Sequential composition distributes over alternative composition from the right, but not from the left. 0 is a left-zero for sequential composition, but not a right-zero. Action prefixing always binds strongest, alternative composition always binds weakest. The subtheory BSP of TSP is obtained by omitting sequential composition and the axioms involving it.

Axioms M, LM1-LM4 and CM1-CM6 axiomatize parallel composition. Parallel composition is split up, using auxiliary operators left-merge (\parallel) and communication merge (\mid). The axioms follow the structure of BSP-terms. Axioms SC1-SC8 are the axioms of Standard Concurrency: they list useful properties of parallel operators, such as the commutativity and associativity of parallel composition. 1 is the identity element of parallel composition.

Finally, D1-D5 axiomatize the encapsulation operator, again following the structure of BSP-terms.

In [Bae03], by means of the operational rules of Table 2, an operational semantics is given for closed TCP-terms defining binary relations $- \xrightarrow{a} -$ (for $a \in A$), and a unary relation (predicate) $- \downarrow$. Intuitively, these have the following meaning:

- $x \xrightarrow{a} x'$ means that x evolves into x' by executing atomic action a ;
- $x \downarrow$ means that x has an option to terminate successfully.

The axioms introduced before are meant to identify processes that are strongly bisimilar.

Table 1: Axioms of TCP ($a, b, c \in A$, $H \subseteq A$)

$x + y = y + x$	A1	$x + 0 = x$	A6
$(x + y) + z = x + (y + z)$	A2	$0 \cdot x = 0$	A7
$x + x = x$	A3	$1 \cdot x = x$	A8
$(x + y) \cdot z = x \cdot z + y \cdot z$	A4	$x \cdot 1 = x$	A9
$(x \cdot y) \cdot z = x \cdot (y \cdot z)$	A5	$a \cdot x \cdot y = a \cdot (x \cdot y)$	A10
$x \parallel y = x \ll y + y \ll x + x \mid y$	M	$x \mid y = y \mid x$	SC1
		$x \parallel 1 = x$	SC2
$0 \ll x = 0$	LM1	$1 \mid x + 1 = 1$	SC3
$1 \ll x = 0$	LM2	$(x \parallel y) \parallel z = x \parallel (y \parallel z)$	SC4
$a \cdot x \ll y = a \cdot (x \parallel y)$	LM3	$(x \mid y) \mid z = x \mid (y \mid z)$	SC5
$(x + y) \ll z = x \ll z + y \ll z$	LM4	$(x \ll y) \ll z = x \ll (y \parallel z)$	SC6
		$(x \mid y) \ll z = x \mid (y \ll z)$	SC7
$0 \mid x = 0$	CM1	$x \ll 0 = x \cdot 0$	SC8
$(x + y) \mid z = x \mid z + y \mid z$	CM2		
$1 \mid 1 = 1$	CM3	$\partial_H(0) = 0$	D1
$a \cdot x \mid 1 = 0$	CM4	$\partial_H(1) = 1$	D2
$a \cdot x \mid b \cdot y = c \cdot (x \parallel y)$ if $\gamma(a, b) = c$	CM5	$\partial_H(a \cdot x) = 0$ if $a \in H$	D3
$a \cdot x \mid b \cdot y = 0$ if $\gamma(a, b)$ not defined	CM6	$\partial_H(a \cdot x) = a \cdot \partial_H(x)$ otherwise	D4
		$\partial_H(x + y) = \partial_H(x) + \partial_H(y)$	D5

Table 2: Deduction rules for TCP ($a, b, c \in A$, $H \subseteq A$)

	$a \cdot x \xrightarrow{a} x$	$1 \downarrow$		
$\frac{x \xrightarrow{a} x'}{x + y \xrightarrow{a} x'}$	$\frac{y \xrightarrow{a} y'}{x + y \xrightarrow{a} y'}$	$\frac{x \downarrow}{x + y \downarrow}$	$\frac{y \downarrow}{x + y \downarrow}$	
$\frac{x \xrightarrow{a} x'}{x \cdot y \xrightarrow{a} x' \cdot y}$	$\frac{x \downarrow \quad y \xrightarrow{a} y'}{x \cdot y \xrightarrow{a} y'}$	$\frac{x \downarrow \quad y \downarrow}{x \cdot y \downarrow}$		
$\frac{x \xrightarrow{a} x'}{x \parallel y \xrightarrow{a} x' \parallel y}$	$\frac{y \xrightarrow{a} y'}{x \parallel y \xrightarrow{a} x \parallel y'}$	$\frac{x \xrightarrow{a} x' \quad y \xrightarrow{b} y' \quad \gamma(a, b) = c}{x \parallel y \xrightarrow{c} x' \parallel y'}$	$\frac{x \downarrow \quad y \downarrow}{x \parallel y \downarrow}$	
$\frac{x \xrightarrow{a} x'}{x \ll y \xrightarrow{a} x' \ll y}$	$\frac{x \xrightarrow{a} x' \quad y \xrightarrow{b} y' \quad \gamma(a, b) = c}{x \mid y \xrightarrow{c} x' \mid y'}$	$\frac{x \downarrow \quad y \downarrow}{x \mid y \downarrow}$		
$\frac{x \xrightarrow{a} x' \quad a \notin H}{\partial_H(x) \xrightarrow{a} \partial_H(x')}$	$\frac{x \downarrow}{\partial_H(x) \downarrow}$			

Definition 2.1 (Strong bisimilarity) *A symmetric, binary relation R on processes is called a strong bisimulation relation if for all process terms p and q such that $(p, q) \in R$ we have*

- *if $p \downarrow$ then $q \downarrow$;*
- *for all $a \in A$ and process terms p' : if $p \xrightarrow{a} p'$, then there exists a process term q' such that $q \xrightarrow{a} q'$ and $(p', q') \in R$.*

Two processes p and q are strongly bisimilar, notation $p \rightleftharpoons q$, if there exists a strong bisimulation relation R such that $(p, q) \in R$.

The notion of strong bisimilarity on closed TCP-terms is both an equivalence and a congruence for all the operators of the process algebra TCP. As we have used the standard definition of strong bisimilarity, congruence is for free (follows from the format of the deduction rules).

Theorem 2.2 (Equivalence) *Strong bisimilarity is an equivalence relation.*

Theorem 2.3 (Congruence) *Strong bisimilarity is a congruence for the operators of the process algebra TCP.*

Proof. The deduction system is in path format and hence strong bisimilarity is a congruence [Ver95, Fok94]. □

We establish that the structure of transition systems modulo strong bisimilarity is a model for our axioms, or, put differently, that our axioms are sound with respect to the set of closed terms modulo strong bisimilarity. We also prove that the axiomatization is complete.

Theorem 2.4 (Soundness) *The process algebra TCP is a sound axiomatization of strong bisimilarity on closed TCP-terms.*

Theorem 2.5 (Completeness) *The process algebra TCP is a complete axiomatization of strong bisimilarity on closed TCP-terms.*

Proof. For a proof of this theorem we refer to [BBR06]. □

The process algebra TCP is generic, in the sense that most features of commonly used process algebras can be embedded in it. For details of this, see [BB05].

3 Discrete relative timing

Let us now first consider the timing extension with relative, discrete timing. We take this version of a timed theory, as it is the easiest one to explain. The following syntax elements are added:

- *Current time slice inaction $\underline{0}$.* This process cannot execute any action, cannot terminate, and cannot let time pass to the next time slice. The process $\underline{0}$ can now be called *any time inaction*; this process allows any passing of time.
- *Current time slice termination $\underline{1}$.* This process cannot execute any action, cannot let time pass, but can terminate in the current time slice. The process $\underline{1}$ is *any time termination*, and allows passing of time.
- *Current time slice action prefix $\underline{a}..$.* The process $\underline{a}..x$ executes a in the current time slice and continues with x . It does not allow passing of time. The process $a.x$ allows any passing of time before the execution of a .

- *Unit delay prefix* $\underline{\underline{\sigma}}.$. The process $\underline{\underline{\sigma}}.x$ can pass to the next time slice and there start the execution of x .
- *Unit time-out operator* v^1 . This is an auxiliary operator used in the axiomatization. Process $v^1(x)$ does not allow passage to the next time slice, and only allows an initial action or termination of x in the current time slice.

Note that the unit delay prefix binds equally strong as action prefix.

The axioms of TCP_{drt} are given in Table 3. The interpretation of alternative composition with respect to timing is called *weak time-determinism*: in $x + y$, if both components can let time pass, then the process can let time pass and no choice is made, this is expressed in axiom DRTF (Discrete Relative Time Factorization); if one component can let time pass, but the other component cannot, then the process can let time pass; doing this, the other component is discarded. To give an example, the process $\underline{\underline{1}} + 0$ can either terminate in the current time slice, or let time pass and turn into 0. This implies that 0 is no longer the identity element of alternative composition (it is only so for processes that allow an arbitrary initial delay). This role is taken over by $\underline{\underline{0}}$, see axiom A6DR.

The process $\underline{\underline{0}}$ is a left-zero for sequential composition (A7DR). Although axiom A7 from TCP that states that 0 is a left-zero for sequential composition is not present in TCP_{drt} , it is derivable from the other axioms: $\text{TCP}_{\text{drt}} \vdash 0 \cdot x \stackrel{\text{DT2}}{=} (1 \cdot \underline{\underline{0}}) \cdot x \stackrel{\text{A5}}{=} 1 \cdot (\underline{\underline{0}} \cdot x) \stackrel{\text{A7DR}}{=} 1 \cdot \underline{\underline{0}} \stackrel{\text{DT2}}{=} 0$.

The interpretation of sequential composition is consistent with relative timing: $x \cdot y$ will start y in the time slice in which x terminates. 1 is no longer the identity element of sequential composition, $1 \cdot x$ can start x in an arbitrary time slice. This role is taken over by $\underline{\underline{1}}$, see axioms A8DR and A9DR.

A parallel composition can let time pass if both components allow the delay. Termination of the parallel composition only occurs in case both components can terminate successfully. Therefore, 1 still is the identity element.

The first 10 axioms A1-A5 and A6DR-A10DR now correspond to A1-A10 of TCP, substituting double underlined elements for their untimed counterparts. Process 1 is characterized by the recursive equation DT1: termination takes place in the current time slice, or a delay is executed and we are back where we started. The process 1 in turn can be used to define the untimed counterparts of $\underline{\underline{0}}$ and $\underline{\underline{a}}$, see DT2 and DT3: $1 \cdot x$ will add an arbitrary delay to the start of x . All the axioms of TCP involving 0 , 1 , or a can now be derived, using just some axioms for 1 . The axioms for parallel composition M, LM1DR-LM3DR, LM4, CM1DR, CM2, CM3DR-CM6DR, SC1, SC2, SC3DR, and SC4-SC7 are like their untimed counterparts. Axioms LM5DR-LM7DR describe a delay of left-merge: this can happen when both sides allow the delay (LM6DR, discarding the part on the right-hand side that has to start in the current time slice). If the right-hand side does not allow a delay, nothing can happen (LM5DR). Finally, LM7DR is added. This axiom allows to derive $a.x \underline{\underline{\parallel}} y = a.(x \parallel y)$ for untimed y (i.e. processes with an arbitrary initial delay, that can be written in the form $1 \cdot y'$ or even $1 \cdot v^1(y')$). For communication merge, a delay is only possible when both components allow this (CM7DR, CM8DR). Axiom CM9DR is needed to derive CM1 and CM3-CM6 of TCP for untimed processes.

Notice this approach to parallel composition with timing is slightly different from [BB05, BMR05], but it is the same as in [BBR06]. Here, we allow a delay of a parallel composition precisely when both components allow this delay. As a consequence, the process 1 is the identity element of parallel composition. This makes the intuition and the operational semantics of parallel composition simpler, and separates out different roles of 1 .

The axioms of encapsulation are straightforward, and the axioms of the unit time-out operator block any initial delay (RTO5, RTO6).

Definition 3.1 (Basic terms) *Basic terms are defined inductively as follows:*

- $\underline{\underline{0}}$ and $\underline{\underline{1}}$ are basic terms;
- 0 and 1 are basic terms;

Table 3: Axioms of TCP_{drt} ($a, b, c \in A$, $H \subseteq A$)

$x + y = y + x$	A1	$x + \underline{0} = x$	A6DR
$(x + y) + z = x + (y + z)$	A2	$\underline{0} \cdot x = \underline{0}$	A7DR
$x + x = x$	A3	$\underline{1} \cdot x = x$	A8DR
$(x + y) \cdot z = x \cdot z + y \cdot z$	A4	$x \cdot \underline{1} = x$	A9DR
$(x \cdot y) \cdot z = x \cdot (y \cdot z)$	A5	$\underline{a}.x \cdot y = \underline{a}.(x \cdot y)$	A10DR
$\underline{\sigma}.(x + y) = \underline{\sigma}.x + \underline{\sigma}.y$	DRTF	$\underline{\sigma}.x \cdot y = \underline{\sigma}.(x \cdot y)$	DRA10
$\underline{1} = \underline{1} + \underline{\sigma}.1$	DT1	$\underline{1} \cdot \underline{1} = \underline{1}$	DT4
$0 = \underline{1} \cdot \underline{0}$	DT2	$\underline{1} \cdot \underline{\sigma}.x = \underline{\sigma}.(1 \cdot x)$	DT5
$a.x = \underline{1} \cdot \underline{a}.x$	DT3	$\underline{1} \cdot (x + y) = \underline{1} \cdot x + \underline{1} \cdot y$	DT6
$x \parallel y = x \parallel y + y \parallel x + x \mid y$	M		
$\underline{0} \parallel x = \underline{0}$	LM1DR	$\underline{0} \mid x = \underline{0}$	CM1DR
$\underline{1} \parallel x = \underline{0}$	LM2DR	$(x + y) \mid z = x \mid z + y \mid z$	CM2
$\underline{a}.x \parallel y = \underline{a}.(x \parallel y)$	LM3DR	$\underline{1} \mid \underline{1} = \underline{1}$	CM3DR
$(x + y) \parallel z = x \parallel z + y \parallel z$	LM4	$\underline{a}.x \mid \underline{1} = \underline{0}$	CM4DR
$\underline{\sigma}.x \parallel v^1(y) = \underline{0}$	LM5DR	$\underline{a}.x \mid \underline{b}.y = \underline{c}.(x \parallel y)$ if $\gamma(a, b) = c$	CM5DR
$\underline{\sigma}.x \parallel (v^1(y) + \underline{\sigma}.z) = \underline{\sigma}.(x \parallel z)$	LM6DR	$\underline{a}.x \mid \underline{b}.y = \underline{0}$ if $\gamma(a, b)$ not defined	CM6DR
$(1 \cdot x) \parallel (1 \cdot v^1(y)) =$ $= 1 \cdot (x \parallel (1 \cdot v^1(y)))$	LM7DR	$\underline{\sigma}.x \mid v^1(y) = \underline{0}$	CM7DR
$x \mid y = y \mid x$	SC1	$\underline{\sigma}.x \mid \underline{\sigma}.y = \underline{\sigma}.(x \mid y)$	CM8DR
$x \parallel 1 = x$	SC2	$(1 \cdot x) \mid (1 \cdot y) =$ $= 1 \cdot (x \mid (1 \cdot y) + (1 \cdot x) \mid y)$	CM9DR
$\underline{1} \parallel x + \underline{1} = \underline{1}$	SC3DR	$(x \parallel y) \parallel z = x \parallel (y \parallel z)$	SC6
$(x \parallel y) \parallel z = x \parallel (y \parallel z)$	SC4	$(x \mid y) \parallel z = x \mid (y \parallel z)$	SC7
$(x \mid y) \mid z = x \mid (y \mid z)$	SC5		
$\partial_H(\underline{0}) = \underline{0}$	D1DR	$v^1(\underline{0}) = \underline{0}$	RTO1
$\partial_H(\underline{1}) = \underline{1}$	D2DR	$v^1(\underline{1}) = \underline{1}$	RTO2
$\partial_H(\underline{a}.x) = \underline{0}$ if $a \in H$	D3DR	$v^1(\underline{a}.x) = \underline{a}.x$	RTO3
$\partial_H(\underline{a}.x) = \underline{a}.\partial_H(x)$ otherwise	D4DR		
$\partial_H(x + y) = \partial_H(x) + \partial_H(y)$	D5	$v^1(x + y) = v^1(x) + v^1(y)$	RTO4
$\partial_H(\underline{\sigma}.x) = \underline{\sigma}.\partial_H(x)$	D6DR	$v^1(\underline{\sigma}.x) = \underline{0}$	RTO5
$\partial_H(1 \cdot x) = \underline{1} \cdot \partial_H(x)$	D7DR	$v^1(1 \cdot x) = v^1(x)$	RTO6

- for $a \in A$ and basic term p , $\underline{a}.p$ and $a.p$ are basic terms;
- for basic term p , $\underline{\sigma}.p$ is a basic term;
- for basic terms p and q , $p + q$ is a basic term.

Theorem 3.2 (Elimination) For closed TCP_{drt} -term p , there exists a basic term q such that $\text{TCP}_{\text{drt}} \vdash p = q$.

Proof. For the proof of this statement we refer to Appendix A.1. \square

The operational semantics of TCP_{drt} can be given by just adding to the rules of TCP. We add one relation $_ \xrightarrow{1} _$, executing a ‘tick’. Intuitively, there is the following meaning:

- $x \xrightarrow{1} x'$ means that x delays to the next time slice and evolves into x' .

The rules are given in Table 4.

Table 4: Deduction rules for TCP_{drt} ($a \in A$ and $H \subseteq A$)

$\underline{a}.x \xrightarrow{a} x$	$\underline{1} \downarrow$	$\underline{\sigma}.x \xrightarrow{1} x$	$a.x \xrightarrow{1} a.x$	$1 \xrightarrow{1} 1$	$0 \xrightarrow{1} 0$
$\frac{x \xrightarrow{1} x' \quad y \not\xrightarrow{1}}{x + y \xrightarrow{1} x'}$	$\frac{y \xrightarrow{1} y' \quad x \not\xrightarrow{1}}{x + y \xrightarrow{1} y'}$	$\frac{x \xrightarrow{1} x' \quad y \xrightarrow{1} y'}{x + y \xrightarrow{1} x' + y'}$			
$\frac{x \xrightarrow{1} x' \quad x \not\downarrow}{x \cdot y \xrightarrow{1} x' \cdot y}$	$\frac{x \xrightarrow{1} x' \quad y \not\xrightarrow{1}}{x \cdot y \xrightarrow{1} x' \cdot y}$	$\frac{x \xrightarrow{1} x' \quad x \downarrow \quad y \xrightarrow{1} y'}{x \cdot y \xrightarrow{1} x' \cdot y + y'}$	$\frac{x \not\xrightarrow{1} \quad x \downarrow \quad y \xrightarrow{1} y'}{x \cdot y \xrightarrow{1} y'}$		
$\frac{x \xrightarrow{1} x' \quad y \xrightarrow{1} y'}{x \parallel y \xrightarrow{1} x' \parallel y'}$	$\frac{x \xrightarrow{1} x' \quad y \xrightarrow{1} y'}{x \ll y \xrightarrow{1} x' \ll y'}$	$\frac{x \xrightarrow{1} x' \quad y \xrightarrow{1} y'}{x \mid y \xrightarrow{1} x' \mid y'}$			
$\frac{x \xrightarrow{1} x'}{\partial_H(x) \xrightarrow{1} \partial_H(x')}$	$\frac{x \xrightarrow{a} x'}{v^1(x) \xrightarrow{a} x'}$	$\frac{x \downarrow}{v^1(x) \downarrow}$			

Theorem 3.3 (Congruence) Strong bisimilarity is a congruence for the operators of the process algebra TCP_{drt} .

Proof. The deduction system is stratifiable and in panth format and hence strong bisimilarity is a congruence [Ver95]. \square

Theorem 3.4 (Soundness) The process algebra TCP_{drt} is a sound axiomatization of strong bisimilarity on closed TCP_{drt} -terms.

We establish that the structure of transition systems modulo strong bisimilarity is a model for our axioms, or, put differently, that our axioms are sound with respect to the set of closed terms modulo strong bisimilarity. We also prove that the axiomatization is complete.

Theorem 3.5 (Completeness) The process algebra TCP_{drt} is a complete axiomatization of strong bisimilarity on closed TCP_{drt} -terms.

Proof. For a proof of this theorem we refer to Appendix A.2. □

Next, we compare the theories TCP and TCP_{drt}. TCP_{drt} is not an equationally conservative extension¹ of TCP since the axioms of TCP are not contained in the axioms of TCP_{drt}. However, it is an *equationally conservative ground-extension* (see [MR05a] for a definition).

Theorem 3.6 (Equational Conservativity) TCP_{drt} is an equationally conservative ground-extension of TCP, i.e., for all closed TCP-terms p and q , $\text{TCP} \vdash p = q$ if and only if $\text{TCP}_{\text{drt}} \vdash p = q$.

Proof. The proof is given using meta-theory from [MR05a] in Appendix A.3 □

4 Absolute timing and dense timing

We can set up a variant of the theory of the previous section for absolute timing instead of relative timing. Instead of the relative timing syntax elements $\underline{0}$, $\underline{1}$, $\underline{a.}$, and $\underline{\sigma.}$, we have absolute timing elements:

- *First time slice inaction* $\underline{0}$. This process cannot execute any action, cannot terminate, and cannot let time pass to the second time slice.
- *First time slice termination* $\underline{1}$. This process cannot execute any action, cannot let time pass, but can terminate in the first time slice.
- *First time slice action prefix* $\underline{a.}$. The process $\underline{a.x}$ executes a in the first time slice and continues with x in the first time slice.
- *Unit time shift prefix* $\underline{\sigma.}$. The process $\underline{\sigma.x}$ will shift the time slices in x by 1. Thus, $\underline{\sigma.a.\sigma.1}$ will execute a in the second time slice and terminate in the third time slice.

Many things will go as before, but there are some notable differences. Consider the term $\underline{\sigma.a.1} \cdot \underline{b.1}$: the first component will execute a in the second time slice, followed by termination in the second time slice. The second component wants to execute b in the first time slice, but this is impossible, as we cannot go back in time. We see that upon termination in the second time slice, there is a *time inconsistency*. We will assume that a process will deadlock immediately upon encountering a time inconsistency. Different from the undelayable deadlock constant $\underline{0}$ from the relative time theory from Section 3 or the undelayable deadlock constant $\underline{0}$ from the absolute timing theory, this immediately deadlock will not even allow undelayable actions in a parallel component. In [BB91, BM02], the notation δ is introduced for this deadlocked process. Adding this process to the current theory will necessitate that it becomes the identity element of alternative composition, not $\underline{0}$ or $\underline{1}$.

We can integrate relative and absolute timing by going to parametric timing. We refer to [BB96, BB97, BM02] for more information on parametric timing. We omit giving axioms and operational rules of the absolute and parametric time theories, as these are not essential for the discussion of this paper.

Next, we can also set up a variant of the theory TCP_{drt} of the previous section by replacing discrete timing by dense timing. Instead of the discrete timing syntax elements $\underline{0}$, $\underline{1}$, $\underline{a.}$, and $\underline{\sigma.}$, we have dense timing elements:

- *Current time point inaction* $\tilde{0}$. This process cannot execute any action, cannot terminate, and cannot let time progress beyond the current point of time. This process can be used as the identity element of alternative composition in theories with dense time.

¹For a definition of this notion we refer to [Ver94].

- *Current time point termination* $\tilde{\mathbb{1}}$. This process cannot execute any action, cannot let time progress beyond the current point of time, but can terminate at the current point of time. This process can be used as the identity element of sequential composition in theories with dense time.
- *Current time point action prefix* $\tilde{a} \dots$. This process must execute a at the current point of time, and continues with the remainder at the current point of time.
- *Relative delay prefix* $\tilde{\sigma}^t \dots$. The process $\tilde{\sigma}^t . x$ will delay for t time units beyond the current point of time ($t \in \mathbb{R}^{\geq 0}$), and then continue with x .

Again, we omit giving axioms and operational rules of the relative time dense time theory.

Finally, we can set up a variant of the theory with dense timing for absolute timing instead of relative timing. Instead of the relative timing syntax elements $\tilde{0}$, $\tilde{\mathbb{1}}$, $\tilde{a} \dots$, and $\tilde{\sigma} \dots$, we have absolute timing elements:

- *Inaction at time 0* $\underline{0}$. This process cannot execute any action, cannot terminate, and does not allow delay to a point of time after 0.
- *Termination at time 0* $\underline{\mathbb{1}}$. This process cannot execute any action, cannot let time pass, but can terminate at time 0.
- *Action prefix at time 0* $\underline{a} \dots$. The process $\underline{a} . x$ executes a at time 0 and continues with x at time 0.
- *Time shift prefix* $\underline{\sigma}^t \dots$. The process $\underline{\sigma}^t . x$ will shift the time points in x by t . Thus, $\underline{\sigma}^t . \underline{a} . \underline{\sigma}^s . \underline{\mathbb{1}}$ will execute a at time t and terminate at time $t + s$.

Again, timing inconsistencies can occur, as in the discrete time case. Embedding of the relative time theory into the absolute time theory can again be achieved by time parametrization. Moreover, note that we can embed the discrete absolute time theory into the dense absolute time theory:

$$\begin{aligned} \underline{0} &= \tilde{\sigma}^{(0,1)} . \tilde{0} \\ \underline{\mathbb{1}} &= \tilde{\sigma}^{(0,1)} . \tilde{\mathbb{1}} \\ \underline{a} . x &= \tilde{\sigma}^{(0,1)} . \tilde{a} . x. \end{aligned}$$

Here, the prefix operator $\tilde{\sigma}^{(0,1)} . x$ allows any delay t with $0 \leq t < 1$ before continuing with x .

Note that a similar embedding of the discrete relative time theory into the dense relative time theory cannot be achieved because we do not know where the current point of time is within the time slice, we do not know how far the end of the time slice is away. This embedding can only be achieved by going via the parametric time theory. For further details, see [BM02].

5 Inconsistent states

We see that in the untimed, discrete time and dense time theories, each time there is a different constant for the identity element of alternative composition (resp. 0 , $\underline{0}$, $\tilde{0}$, $\underline{0}$, or $\tilde{0}$). Besides this, there is a need for an additional constant $\hat{0}$ denoting a timing inconsistency, the deadlocked process. It turns out we can take $\hat{0}$ to be the identity element of alternative composition in all of the cases.

A similar situation occurs with the identity element of sequential composition. We get constants 1 , $\underline{\mathbb{1}}$, $\tilde{\mathbb{1}}$, $\underline{\mathbb{1}}$ or $\tilde{\mathbb{1}}$ acting as this identity element in the untimed, discrete time and dense time theories. It turns out we can take a new constant $\hat{\mathbb{1}}$ to be this identity element in all of the cases.

In this section we show this can be achieved in the untimed process theory with the additional constants $\hat{0}$ and $\hat{\mathbb{1}}$. In the next section, we look at the discrete relative time process theory with

these so-called inconsistent processes. As mentioned in the introduction, it is also possible to develop absolute time and dense time variants, but we do not present these here.

Starting out from the syntax of TCP, we add two additional constants:

- *The deadlocked process $\dot{0}$.* Identity element of alternative composition. In contrast, the constant 0 stands for a blocked or encapsulated action ($\partial_{\{a\}}(a.x)$), and for livelock (not discussed here).
- *The terminated process $\dot{1}$.* Identity element of sequential composition. In contrast, the constant 1 stands for a skipped or abstracted action, and the identity element of parallel composition.

The axioms of TCP[•] are presented in Table 5.

Table 5: Axioms of TCP[•]($a, b, c \in A, H \subseteq A$)

$x + y = y + x$	A1	$x + \dot{0} = x$	A6•
$(x + y) + z = x + (y + z)$	A2	$\dot{0} \cdot x = \dot{0}$	A7•
$x + x = x$	A3	$\dot{1} \cdot x = x$	A8•
$(x + y) \cdot z = x \cdot z + y \cdot z$	A4	$x \cdot \dot{1} = x$	A9•
$(x \cdot y) \cdot z = x \cdot (y \cdot z)$	A5	$a.x \cdot y = a.(x \cdot y)$	A10
$1 + \dot{1} = 1$	DOT1	$1 \cdot 1 = 1$	DOT3
$0 = 1 \cdot \dot{0}$	DOT2	$1 \cdot a.x = a.x$	DOT4
		$1 \cdot (x + y) = 1 \cdot x + 1 \cdot y$	DOT5
$x \parallel y = x \parallel y \parallel x + x \mid y$	M		
$\dot{0} \parallel x = \dot{0}$	LM1•	$\dot{0} \mid x = \dot{0}$	CM1•
$\dot{1} \parallel x = \dot{0}$	LM2•	$(x + y) \mid z = x \mid z + y \mid z$	CM2
$a.x \parallel (1 \cdot y) = a.(x \parallel (1 \cdot y))$	LM3•	$\dot{1} \mid \dot{1} = \dot{1}$	CM3•
$(x + y) \parallel z = x \parallel z + y \parallel z$	LM4	$a.x \mid \dot{1} = \dot{0}$	CM4•
$x \parallel \dot{0} = \dot{0}$	LM5•	$a.x \mid b.y = c.(x \parallel y)$ if $\gamma(a, b) = c$	CM5
$x \parallel (y + \dot{1}) = x \parallel y$	LM6•	$a.x \mid b.y = 0$ if $\gamma(a, b)$ not defined	CM6
$0 \parallel (y + 0) = 0$	LM7•	$1 \mid \dot{1} = \dot{1}$	CM7•
$1 \parallel (y + 0) = 0$	LM8•	$0 \mid (y + 0) = 0$	CM1a
$x \mid y = y \mid x$	SC1	$0 \mid \dot{1} = \dot{0}$	CM1b
$x \parallel 1 = x$	SC2	$1 \mid 1 = 1$	CM3
$\dot{1} \mid x + \dot{1} = \dot{1}$	SC3•	$a.x \mid 1 = 0$	CM4
$(x \parallel y) \parallel z = x \parallel (y \parallel z)$	SC4	$(x \parallel y) \parallel z = x \parallel (y \parallel z)$	SC6
$(x \mid y) \mid z = x \mid (y \mid z)$	SC5	$(x \mid y) \parallel z = x \mid (y \parallel z)$	SC7
$\partial_H(\dot{0}) = \dot{0}$	D1•	$\partial_H(a.x) = 0$ if $a \in H$	D3
$\partial_H(\dot{1}) = \dot{1}$	D2•	$\partial_H(a.x) = a.\partial_H(x)$ otherwise	D4
$\partial_H(0) = 0$	D1	$\partial_H(x + y) = \partial_H(x) + \partial_H(y)$	D5
$\partial_H(1) = 1$	D2		

Note that any term except $\dot{0}, \dot{1}$ can be written in the form $x + 0$ (see also Lemma B.3). Moreover, any term that can be written without occurrence of $\dot{0}$ and $\dot{1}$ can be written in the form $1 \cdot x$ (see also Lemma B.4). The difference is exhibited by the term of the form $0 + \dot{1}$ that contains a $\dot{1}$ summand that cannot be eliminated. Notice that in axiom LM6•, any possibly occurring $\dot{1}$ summand on the right is removed: the left-merge will execute an action from the left component, if possible; by doing so, the possibility of having terminated already is removed.

We state and prove that any closed TCP[•]-term is derivably equal to a so-called basic term. A basic term is a term with a more restricted syntax than allowed by the signature of TCP[•]. Typically, sequential and parallel composition (and the auxiliary operators for parallel composition) and encapsulation do not occur.

Definition 5.1 (Basic terms) *Basic terms are defined inductively as follows:*

- $\dot{0}$ and $\dot{1}$ are basic terms;
- 0 and 1 are basic terms;
- for $a \in A$ and basic term p , $a.p$ is a basic term;
- for basic terms p and q , $p + q$ is a basic term.

Theorem 5.2 (Elimination) *For closed TCP[•]-term p , there exists a basic term q such that $\text{TCP}^\bullet \vdash p = q$.*

Proof. For the proof of this statement we refer to Appendix B.1. □

In the operational semantics, the term $0 + \dot{1}$ needs to be distinguished from the term 1 (it is possible to equate these terms: the algebra becomes simpler, but the semantics of parallel composition is changed and the axiom cannot be maintained in extensions with timing). Both are consistent, and have a termination option. But for the first term, this termination option has already materialized at the current point of time, and for the second term, this termination option can take place at some arbitrary time in the future. As will be made explicit in the next section, when we add timing, by delaying, the first term evolves to 0 and the second term evolves to 1 . In this section, we do not look at timing, and phrase things differently: the consistent part of $0 + \dot{1}$ is 0 , and the consistent part of 1 is 1 . When this process is placed in a parallel composition with a process that starts with the execution of an atomic action a , then upon execution of a only the consistent part of the other component is kept, since any option of having terminated before, is past.

Operationally, we make the difference by means of an additional predicate $\overset{0}{\vdash}$: $x \overset{0}{\vdash} x'$ will mean that x is consistent, and that x' is the TCP-part of x , i.e., $x = 1 \cdot x' + x$ and $x' = 1 \cdot x'$. Thus, the operational semantics is given by adding one extra relation to the operational semantics of TCP: $_ \overset{0}{\vdash} _$.

The term deduction system for TCP[•] consists of the deduction rules for TCP (from Table 2) except for the first two rules of parallel composition and the first rule for left-merge and additionally the deduction rules from Table 6. The deduction rules for merge and left-merge that have been omitted are replaced by similar rules where the action execution only takes place if the other component is consistent. Upon action execution the inconsistent part of the component that does not execute the action is removed.

Theorem 5.3 (Congruence) *Strong bisimilarity is a congruence for the operators of the process algebra TCP[•].*

Proof. The deduction system is stratifiable and in panth format and hence strong bisimilarity is a congruence [Ver95]. □

Theorem 5.4 (Soundness) *The process algebra TCP[•] is a sound axiomatization of strong bisimilarity on closed TCP[•]-terms.*

We establish that the structure of transition systems modulo strong bisimilarity is a model for our axioms, or, put differently, that our axioms are sound with respect to the set of closed terms modulo strong bisimilarity. We also prove that the axiomatization is complete.

Theorem 5.5 (Completeness) *The process algebra TCP[•] is a complete axiomatization of strong bisimilarity on closed TCP[•]-terms.*

Table 6: Additional deduction rules for TCP^\bullet ($a, b, c \in A$, $H \subseteq A$)

$a.x \xrightarrow{0} a.x$		$0 \xrightarrow{0} 0$	$1 \xrightarrow{0} 1$	$\dot{1} \downarrow$
$\frac{x \xrightarrow{0} x' \quad y \xrightarrow{0} y'}{x + y \xrightarrow{0} x' + y'}$	$\frac{x \xrightarrow{0} x' \quad y \not\xrightarrow{0}}{x + y \xrightarrow{0} x'}$	$\frac{y \xrightarrow{0} y' \quad x \not\xrightarrow{0}}{x + y \xrightarrow{0} y'}$		
$\frac{x \xrightarrow{0} x' \quad x \not\downarrow}{x \cdot y \xrightarrow{0} x' \cdot y}$	$\frac{x \xrightarrow{0} x' \quad y \xrightarrow{0}}{x \cdot y \xrightarrow{0} x' \cdot y}$	$\frac{x \xrightarrow{0} x' \quad x \downarrow \quad y \xrightarrow{0} y'}{x \cdot y \xrightarrow{0} x' \cdot y + y'}$	$\frac{x \not\xrightarrow{0} \quad x \downarrow \quad y \xrightarrow{0} y'}{x \cdot y \xrightarrow{0} y'}$	
$\frac{x \xrightarrow{a} x' \quad y \xrightarrow{0} y'}{x \parallel y \xrightarrow{a} x' \parallel y'}$	$\frac{x \xrightarrow{0} x' \quad y \xrightarrow{a} y'}{x \parallel y \xrightarrow{a} x' \parallel y'}$	$\frac{x \xrightarrow{a} x' \quad y \xrightarrow{0} y'}{x \parallel y \xrightarrow{a} x' \parallel y'}$		
$\frac{x \xrightarrow{0} x' \quad y \xrightarrow{0} y'}{x \parallel y \xrightarrow{0} x' \parallel y'}$	$\frac{x \xrightarrow{0} x' \quad y \xrightarrow{0} y'}{x \parallel y \xrightarrow{0} x' \parallel y'}$	$\frac{x \xrightarrow{0} x' \quad y \xrightarrow{0} y'}{x \mid y \xrightarrow{0} x' \mid y'}$		
		$\frac{x \xrightarrow{0} x'}{\partial_H(x) \xrightarrow{0} \partial_H(x')}$		

Proof. For a proof of this theorem we refer to Appendix B.2. \(\square\)

Next, we compare the theories TCP and TCP^\bullet . TCP^\bullet is not an equationally conservative extension of TCP since the axioms of TCP are not contained in the axioms of TCP^\bullet . However, it is an equationally conservative ground-extension.

Theorem 5.6 (Equational Conservativity) *TCP^\bullet is an equationally conservative ground-extension of TCP , i.e., for all closed TCP -terms p and q , $\text{TCP} \vdash p = q$ if and only if $\text{TCP}^\bullet \vdash p = q$.*

Proof. The proof is given in Appendix B.3. \(\square\)

With the addition of the new constants, it also becomes possible to embed the process algebra ACP of [BK84, BW90] into TCP in such a way that timed extensions can be done conservatively. The crux is to interpret the constant atomic actions a of ACP by $a.\dot{1}$ in TCP^\bullet . Thus we have achieved a theory TCP^\bullet into which ACP with termination can be embedded, as suggested above, and which can be extended with timing in a conservative way as shown in the next section. Operationally, ACP has a deduction rule $a \xrightarrow{a} \surd$. Here, we have an extension of ACP , where \surd can be treated as a process (viz., $\dot{1}$).

6 Discrete relative timing with inconsistent state

$\text{TCP}_{\text{drt}}^\bullet$ now is a conservative extension of TCP^\bullet (not just a conservative ground-extension), we add the axioms from Table 7 to the axioms of TCP^\bullet . Most of these axioms are similar to or simple reformulations of the axioms of TCP_{drt} . The use of axiom DR10 makes a lot of axioms derivable. As an example, we have $\underline{\sigma}.x \parallel (v^1(y) + \underline{0}) = \underline{\sigma}.x \parallel (v^1(y) + \underline{\sigma}.\dot{0}) = \underline{\sigma}.(x \parallel \dot{0}) = \underline{\sigma}.\dot{0} = \underline{0}$. Another example: from DR1 we can derive $0 = \underline{\sigma}.\dot{0}$, so $v^1(0) = v^1(\underline{\sigma}.\dot{0}) = \underline{\sigma}.\dot{0} = \underline{0}$.

We state and prove that any closed TCP^\bullet -term is derivably equal to a so-called basic term. A basic term is a term with a more restricted syntax than allowed by the signature of TCP^\bullet . Typically, sequential and parallel composition (and the auxiliary operators for parallel composition) and encapsulation and unit time-out do not occur.

Table 7: Additional axioms of $\text{TCP}_{\text{drt}}^\bullet$ ($a, b, c \in A$, $H \subseteq A$)

$1 = \underline{1} + \underline{\sigma}.1$	DT1	$1 \cdot \underline{\sigma}.x = \underline{\sigma}.(1 \cdot x)$	DR7•
$\underline{0} = \underline{1} \cdot \dot{0}$	DR2•	$1 \cdot \underline{1} = 1$	DR8
$\underline{a}.x = \underline{1} \cdot \underline{a}.x$	DR3•	$1 \cdot \underline{a}.x = a.x$	DR9
$\underline{1} \cdot 1 = 1$	DR4a•	$\underline{0} = \underline{\sigma}.\dot{0}$	DR10
$\underline{1} \cdot \underline{1} = \underline{1}$	DR4b•	$\underline{1} + \dot{1} = \underline{1}$	DR11
$\underline{1} \cdot \underline{\sigma}.x = \underline{\sigma}.x$	DR5•	$\underline{a}.x \cdot y = \underline{a}.(x \cdot y)$	A10DR
$\underline{1} \cdot (x + y) = \underline{1} \cdot x + \underline{1} \cdot y$	DR6•	$\underline{\sigma}.x \cdot y = \underline{\sigma}.(x \cdot y)$	DRA10
$\underline{\sigma}.(x + y) = \underline{\sigma}.x + \underline{\sigma}.y$	DRTF		
$\underline{1} \parallel (x + \underline{0}) = \underline{0}$	LM2DR•	$\underline{1} \mid \underline{1} = \underline{1}$	CM3DR
$\underline{a}.x \parallel (\underline{1} \cdot y) = \underline{a}.(x \parallel (\underline{1} \cdot y))$	LM3DR•	$\underline{a}.x \mid \underline{1} = \underline{0}$	CM4DR
$\underline{\sigma}.x \parallel (v^1(y) + \underline{\sigma}.z) = \underline{\sigma}.(x \parallel z)$	LM6DR	$\underline{a}.x \mid \underline{b}.y = \underline{c}.(x \parallel y)$ if $\gamma(a, b) = c$	CM5DR
$\underline{1} \mid x + \underline{1} = \underline{1}$	SC3DR	$\underline{a}.x \mid \underline{b}.y = \underline{0}$ if $\gamma(a, b)$ not defined	CM6DR
$v^1(\underline{1}) = \underline{1}$	RTO2	$\underline{\sigma}.x \mid (v^1(y) + \underline{\sigma}.z) = \underline{\sigma}.(x \mid z)$	CM7DR•
$v^1(\underline{a}.x) = \underline{a}.x$	RTO3	$\underline{0} \mid \dot{1} = \dot{0}$	CM8DR•
$v^1(x + y) = v^1(x) + v^1(y)$	RTO4	$\underline{a}.x \mid \dot{1} = \dot{0}$	CM9DR•
$v^1(\underline{\sigma}.x) = \underline{0}$	RTO5	$\underline{\sigma}.x \mid \dot{1} = \dot{0}$	CM10DR•
$v^1(\dot{0}) = \dot{0}$	RTO1•	$\partial_H(\underline{1}) = \underline{1}$	D2DR
$v^1(\dot{1}) = \dot{1}$	RTO2•	$\partial_H(\underline{a}.x) = \underline{0}$ if $a \in H$	D3DR
		$\partial_H(\underline{a}.x) = \underline{a}.\partial_H(x)$ otherwise	D4DR
		$\partial_H(\underline{\sigma}.x) = \underline{\sigma}.\partial_H(x)$	D6DR

Definition 6.1 (Basic terms) *Basic terms are defined inductively as follows:*

- $\dot{0}$ and $\dot{1}$ are basic terms;
- $\underline{0}$ and $\underline{1}$ are basic terms;
- 0 and 1 are basic terms;
- for $a \in A$ and basic term p , $\underline{a}.p$ and $a.p$ are basic terms;
- for basic term p , $\underline{\sigma}.p$ is a basic term;
- for basic terms p and q , $p + q$ is a basic term.

Theorem 6.2 (Elimination) *For closed $\text{TCP}_{\text{drt}}^\bullet$ -term p , there exists a basic term q such that $\text{TCP}_{\text{drt}}^\bullet \vdash p = q$.*

Proof. For the proof of this statement we refer to Appendix C.2. □

 Table 8: Additional reduction rules for $\text{TCP}_{\text{drt}}^\bullet$ ($a \in A$)

$\underline{0} \xrightarrow{0} \underline{0}$	$\underline{1} \xrightarrow{0} \underline{1}$	$\underline{a}.x \xrightarrow{0} \underline{a}.x$	$\underline{\sigma}.x \xrightarrow{0} \underline{\sigma}.x$	$\frac{x \xrightarrow{0} x'}{v^1(x) \xrightarrow{0} v^1(x')}$
	$\underline{0} \xrightarrow{1} \dot{0}$	$\underline{1} \xrightarrow{1} \dot{0}$	$\underline{a}.x \xrightarrow{1} \dot{0}$	$\frac{x \xrightarrow{1} x'}{v^1(x) \xrightarrow{1} \dot{0}}$

The term deduction system for $\text{TCP}_{\text{drt}}^\bullet$ consists of the deduction rules for TCP^\bullet , the deduction rules from Table 4, and additionally the deduction rules from Table 8. Note that, as in the case of TCP^\bullet , the first two deduction rules for parallel composition and the first deduction rule for left-merge from Table 2 are not part of the deduction rules of $\text{TCP}_{\text{drt}}^\bullet$.

Theorem 6.3 (Congruence) *Strong bisimilarity is a congruence for the operators of the process algebra $\text{TCP}_{\text{drt}}^\bullet$.*

Proof. The deduction system is stratifiable and in panth format and hence strong bisimilarity is a congruence [Ver95]. \square

Theorem 6.4 (Soundness) *The process algebra $\text{TCP}_{\text{drt}}^\bullet$ is a sound axiomatization of strong bisimilarity on closed $\text{TCP}_{\text{drt}}^\bullet$ -terms.*

We establish that the structure of transition systems modulo strong bisimilarity is a model for our axioms, or, put differently, that our axioms are sound with respect to the set of closed terms modulo strong bisimilarity. We also prove that the axiomatization is complete.

Theorem 6.5 (Completeness) *The process algebra $\text{TCP}_{\text{drt}}^\bullet$ is a complete axiomatization of strong bisimilarity on closed $\text{TCP}_{\text{drt}}^\bullet$ -terms.*

Proof. For a proof of this theorem we refer to Appendix C.3. \square

Theorem 6.6 (Equational Conservativity) *$\text{TCP}_{\text{drt}}^\bullet$ is an equationally conservative extension [Ver94] of TCP^\bullet , i.e., the axioms of TCP^\bullet are contained in the axioms of $\text{TCP}_{\text{drt}}^\bullet$ and for all closed TCP^\bullet -terms p and q , $\text{TCP}^\bullet \vdash p = q$ if and only if $\text{TCP}_{\text{drt}}^\bullet \vdash p = q$.*

Proof. The proof is given in Appendix C.4. \square

Theorem 6.7 (Equational Conservativity) *$\text{TCP}_{\text{drt}}^\bullet$ is an equationally conservative ground-extension of TCP_{drt} , i.e., for all closed TCP_{drt} -terms p and q , $\text{TCP}_{\text{drt}} \vdash p = q$ if and only if $\text{TCP}_{\text{drt}}^\bullet \vdash p = q$.*

Proof. The proof is given in Appendix C.5. \square

As was the case for the equation $1 = 0 + \dot{1}$ in TCP^\bullet , an interesting additional equation for $\text{TCP}_{\text{drt}}^\bullet$ is $\underline{1} = \underline{0} + \dot{1}$. If we add this equation, then we have to remove axiom LM6 \bullet , so we change the semantics of parallel composition. Also, further extensions of the theory, for instance with dense timing, become more difficult. On the other hand, both the algebra and the operational semantics become simpler. The axioms DR2 \bullet , DR4a \bullet -DR6 \bullet , DR8, DR11, CM3DR, CM4DR, RTO2 and D2DR become derivable from the other axioms. Operationally, we do not need the additional relation $\overset{0}{\mapsto}$.

7 Concluding remarks

We have introduced process algebras TCP^\bullet and $\text{TCP}_{\text{drt}}^\bullet$ with both successful and unsuccessful termination constants in which, on the one hand, the roles of the identity element for alternative composition and livelock, and on the other hand the identity elements for sequential and parallel

composition are separated. The different timed process algebras are now equationally conservative ground-extensions of TCP.

The main difference between the process algebras presented in this paper and the ACP-like process algebras from literature is that here explicit termination (i.e., action prefix) is used instead of action constants.

The approach to parallel composition with timing is slightly different from [BB05, BMR05], but it is the same as in [BBR06]. Here, we allow a delay of a parallel composition precisely when both components allow this delay. As a consequence, the process 1 is the identity element of parallel composition. This makes the intuition and the operational semantics of parallel composition simpler, and separates out different roles of 1. This is also a major difference with the timed ACP-like process algebras of [Ver97, BV97].

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A Theorems for TCP_{drt}

A.1 Proof of elimination theorem for TCP_{drt}

In this appendix we prove that any closed TCP_{drt} -term is derivably equal to a basic term.

Theorem A.1 (Elimination of sequential composition) *For basic terms p_1 and p_2 , there exists a basic term q such that $\text{TCP}_{\text{drt}} \vdash p_1 \cdot p_2 = q$.*

Proof. By induction on the structure of basic term p_1 .

1. $p_1 \equiv \underline{0}$. Then $\text{TCP}_{\text{drt}} \vdash p_1 \cdot p_2 \equiv \underline{0} \cdot p_2 \stackrel{\text{A7DR}}{=} \underline{0}$.
2. $p_1 \equiv \underline{1}$. Then $\text{TCP}_{\text{drt}} \vdash p_1 \cdot p_2 \equiv \underline{1} \cdot p_2 \stackrel{\text{A8DR}}{=} p_2$.
3. $p_1 \equiv 0$. Then $\text{TCP}_{\text{drt}} \vdash p_1 \cdot p_2 = 0 \cdot p_2 \stackrel{\text{DT2}}{=} (1 \cdot \underline{0}) \cdot p_2 \stackrel{\text{A5}}{=} 1 \cdot (\underline{0} \cdot p_2) \stackrel{\text{A7DR}}{=} 1 \cdot \underline{0} \stackrel{\text{DT2}}{=} 0$.
4. $p_1 \equiv 1$. By induction on the structure of basic term p we prove that there exists a basic term r such that $\text{TCP}_{\text{drt}} \vdash 1 \cdot p = r$.
 - (a) $p \equiv \underline{0}$. Then $\text{TCP}_{\text{drt}} \vdash 1 \cdot p \equiv 1 \cdot \underline{0} \stackrel{\text{DT2}}{=} 0$.
 - (b) $p \equiv \underline{1}$. Then $\text{TCP}_{\text{drt}} \vdash 1 \cdot p \equiv 1 \cdot \underline{1} \stackrel{\text{A9DR}}{=} 1$.
 - (c) $p \equiv 0$. Then $\text{TCP}_{\text{drt}} \vdash 1 \cdot p \equiv 1 \cdot 0 \stackrel{\text{DT2}}{=} 1 \cdot (1 \cdot \underline{0}) \stackrel{\text{A5}}{=} (1 \cdot 1) \cdot \underline{0} \stackrel{\text{DT4}}{=} 1 \cdot \underline{0} \stackrel{\text{DT2}}{=} 0$.
 - (d) $p \equiv 1$. Then $\text{TCP}_{\text{drt}} \vdash 1 \cdot p \equiv 1 \cdot 1 \stackrel{\text{DT4}}{=} 1$.
 - (e) $p \equiv \underline{a.p'}$ for some $a \in A$ and basic term p' . Then $\text{TCP}_{\text{drt}} \vdash 1 \cdot p \equiv 1 \cdot \underline{a.p'} \stackrel{\text{DT3}}{=} a.p'$.
 - (f) $p \equiv a.p'$ for some $a \in A$ and basic term p' . Then $\text{TCP}_{\text{drt}} \vdash 1 \cdot p \equiv 1 \cdot a.p' \stackrel{\text{DT3}}{=} 1 \cdot (1 \cdot \underline{a.p'}) \stackrel{\text{A5}}{=} (1 \cdot 1) \cdot \underline{a.p'} \stackrel{\text{DT4}}{=} 1 \cdot \underline{a.p'} \stackrel{\text{DT3}}{=} a.p'$.
 - (g) $p \equiv \underline{\sigma.p'}$ for some basic term p' . By induction we have the existence of a basic term r' such that $\text{TCP}_{\text{drt}} \vdash 1 \cdot p' = r'$. Then, $\text{TCP}_{\text{drt}} \vdash 1 \cdot p \equiv 1 \cdot \underline{\sigma.p'} \stackrel{\text{DT5}}{=} \underline{\sigma} \cdot (1 \cdot p') = \underline{\sigma} \cdot r'$.
 - (h) $p \equiv p' + p''$ for some basic terms p' and p'' . By induction we have the existence of basic terms r' and r'' such that $\text{TCP}_{\text{drt}} \vdash 1 \cdot p' = r'$ and $\text{TCP}_{\text{drt}} \vdash 1 \cdot p'' = r''$. Then $\text{TCP}_{\text{drt}} \vdash 1 \cdot p \equiv 1 \cdot (p' + p'') \stackrel{\text{DT6}}{=} 1 \cdot p' + 1 \cdot p'' \stackrel{\text{ih}}{=} r' + r''$.

Using this lemma we have the existence of a basic term r such that $\text{TCP}_{\text{drt}} \vdash 1 \cdot p_2 = r$. Then $\text{TCP}_{\text{drt}} \vdash p_1 \cdot p_2 \equiv 1 \cdot p_2 \stackrel{\text{lemma}}{=} r$.

5. $p_1 \equiv \underline{a.p'_1}$ for some $a \in A$ and basic term p'_1 . By induction we have the existence of a basic term q' such that $\text{TCP}_{\text{drt}} \vdash p'_1 \cdot p_2 = q'$. Then $\text{TCP}_{\text{drt}} \vdash p_1 \cdot p_2 \equiv (\underline{a.p'_1}) \cdot p_2 \stackrel{\text{A10DR}}{=} \underline{a} \cdot (p'_1 \cdot p_2) \stackrel{\text{ih}}{=} \underline{a} \cdot q'$.
6. $p_1 \equiv a.p'_1$ for some $a \in A$ and basic term p'_1 . By induction we have the existence of a basic term q' such that $\text{TCP}_{\text{drt}} \vdash p'_1 \cdot p_2 = q'$. Then $\text{TCP}_{\text{drt}} \vdash p_1 \cdot p_2 \equiv a.p'_1 \cdot p_2 \stackrel{\text{DT3}}{=} (1 \cdot \underline{a.p'_1}) \cdot p_2 \stackrel{\text{A5}}{=} 1 \cdot (\underline{a.p'_1} \cdot p_2) \stackrel{\text{A10DR}}{=} 1 \cdot \underline{a} \cdot (p'_1 \cdot p_2) \stackrel{\text{ih}}{=} 1 \cdot \underline{a} \cdot q' \stackrel{\text{DT3}}{=} a \cdot q'$.
7. $p_1 \equiv \underline{\sigma.p'_1}$ for some basic term p'_1 . By induction we have the existence of a basic term q' such that $\text{TCP}_{\text{drt}} \vdash p'_1 \cdot p_2 = q'$. Then $\text{TCP}_{\text{drt}} \vdash p_1 \cdot p_2 \equiv (\underline{\sigma.p'_1}) \cdot p_2 \stackrel{\text{DRA10}}{=} \underline{\sigma} \cdot (p'_1 \cdot p_2) \stackrel{\text{ih}}{=} \underline{\sigma} \cdot q'$.
8. $p_1 \equiv p'_1 + p''_1$ for some basic terms p'_1 and p''_1 . By induction we have the existence of basic terms r' and r'' such that $\text{TCP}_{\text{drt}} \vdash p'_1 \cdot p_2 = r'$ and $\text{TCP}_{\text{drt}} \vdash p''_1 \cdot p_2 = r''$. Then $\text{TCP}_{\text{drt}} \vdash p_1 \cdot p_2 \equiv (p'_1 + p''_1) \cdot p_2 \stackrel{\text{A4}}{=} p'_1 \cdot p_2 + p''_1 \cdot p_2 = r' + r''$.

Observe that in each of the above cases the last term in the derivation is indeed a basic term. \square

We define $|\cdot|$ for basic terms p and q as follows: $|\underline{0}| = |\underline{1}| = |0| = |1| = 1$, $|\underline{a.p}| = |a.p| = |\underline{\sigma.p}| = |p| + 1$, and $|p + q| = |p| + |q|$. We define $p \leq q$ as $|p| \leq |q|$ and $p < q$ as $|p| < |q|$.

Lemma A.2 (Representation) *For basic term p ,*

1. $\text{TCP}_{\text{drt}} \vdash p = v^1(p')$ for some basic TCP_{drt} -term p' , or

2. $\text{TCP}_{\text{drt}} \vdash p = v^1(p') + \underline{\sigma}.p''$ for some basic TCP_{drt} -terms p' and p'' such that $p'' \leq p$.

Proof. By induction on the structure of basic term p .

1. $p \equiv \underline{0}$. Then $\text{TCP}_{\text{drt}} \vdash p \equiv \underline{0} \stackrel{\text{RTO1}}{=} v^1(\underline{0})$.
2. $p \equiv \underline{1}$. Then $\text{TCP}_{\text{drt}} \vdash p \equiv \underline{1} \stackrel{\text{RTO2}}{=} v^1(\underline{1})$.
3. $p \equiv 0$. Then $\text{TCP}_{\text{drt}} \vdash p \equiv 0 \stackrel{\text{DT2}}{=} 1 \cdot \underline{0} \stackrel{\text{DT1}}{=} (\underline{1} + \underline{\sigma}.1) \cdot \underline{0} \stackrel{\text{A4}}{=} \underline{1} \cdot \underline{0} + \underline{\sigma}.1 \cdot \underline{0} \stackrel{\text{A8DR, DRA10}}{=} \underline{0} + \underline{\sigma}.(1 \cdot \underline{0}) \stackrel{\text{RTO1, DT2}}{=} v^1(\underline{0}) + \underline{\sigma}.0$.
4. $p \equiv 1$. Then $\text{TCP}_{\text{drt}} \vdash p \equiv 1 \stackrel{\text{DT1}}{=} \underline{1} + \underline{\sigma}.1 \stackrel{\text{RTO1}}{=} v^1(\underline{1}) + \underline{\sigma}.1$.
5. $p \equiv \underline{a}.p'$ for some $a \in A$ and basic term p' . Then $\text{TCP}_{\text{drt}} \vdash p \equiv \underline{a}.p' \stackrel{\text{RTO3}}{=} v^1(\underline{a}.p')$.
6. $p \equiv a.p'$ for some $a \in A$ and basic term p' . Then $\text{TCP}_{\text{drt}} \vdash p \equiv a.p' \stackrel{\text{DT3}}{=} 1 \cdot \underline{a}.p' \stackrel{\text{DT1}}{=} (\underline{1} + \underline{\sigma}.1) \cdot \underline{a}.p' \stackrel{\text{A4}}{=} \underline{1} \cdot \underline{a}.p' + \underline{\sigma}.1 \cdot \underline{a}.p' \stackrel{\text{A8DR, DRA10}}{=} \underline{a}.p' + \underline{\sigma}.(1 \cdot \underline{a}.p') \stackrel{\text{RTO3, DT3}}{=} v^1(\underline{a}.p') + \underline{\sigma}.a.p'$.
7. $p \equiv \underline{\sigma}.p'$ for some basic term p' . Then $\text{TCP}_{\text{drt}} \vdash p \equiv \underline{\sigma}.p' \stackrel{\text{A6DR, A1}}{=} \underline{0} + \underline{\sigma}.p' \stackrel{\text{RTO1}}{=} v^1(\underline{0}) + \underline{\sigma}.p'$.
8. $p \equiv p' + p''$ for some basic term $p' + p''$. By induction, for both p' and p'' we have two cases. This results in the following four cases:
 - (a) $\text{TCP}_{\text{drt}} \vdash p' = v^1(p'_1)$ for some closed TCP_{drt} -term p'_1 and $\text{TCP}_{\text{drt}} \vdash p'' = v^1(p''_1)$ for some closed TCP_{drt} -term p''_1 . Then $\text{TCP}_{\text{drt}} \vdash p \equiv p' + p'' = v^1(p'_1) + v^1(p''_1) \stackrel{\text{RTO4}}{=} v^1(p'_1 + p''_1)$.
 - (b) $\text{TCP}_{\text{drt}} \vdash p' = v^1(p'_1) + \underline{\sigma}.p'_2$ for some closed TCP_{drt} -term p'_1 and basic term p'_2 and $\text{TCP}_{\text{drt}} \vdash p'' = v^1(p''_1)$ for some closed TCP_{drt} -term p''_1 . Then $\text{TCP}_{\text{drt}} \vdash p \equiv p' + p'' = (v^1(p'_1) + \underline{\sigma}.p'_2) + v^1(p''_1) \stackrel{\text{A1, A2}}{=} (v^1(p'_1) + v^1(p''_1)) + \underline{\sigma}.p'_2 \stackrel{\text{RTO4}}{=} v^1(p'_1 + p''_1) + \underline{\sigma}.p'_2$.
 - (c) $\text{TCP}_{\text{drt}} \vdash p' = v^1(p'_1)$ for some closed TCP_{drt} -term p'_1 and $\text{TCP}_{\text{drt}} \vdash p'' = v^1(p''_1) + \underline{\sigma}.p''_2$ for some closed TCP_{drt} -term p''_1 and basic term p''_2 . Then $\text{TCP}_{\text{drt}} \vdash p \equiv p' + p'' = v^1(p'_1) + (v^1(p''_1) + \underline{\sigma}.p''_2) \stackrel{\text{A2}}{=} (v^1(p'_1) + v^1(p''_1)) + \underline{\sigma}.p''_2 \stackrel{\text{RTO4}}{=} v^1(p'_1 + p''_1) + \underline{\sigma}.p''_2$.
 - (d) $\text{TCP}_{\text{drt}} \vdash p' = v^1(p'_1) + \underline{\sigma}.p'_2$ for some closed TCP_{drt} -term p'_1 and basic term p'_2 and $\text{TCP}_{\text{drt}} \vdash p'' = v^1(p''_1) + \underline{\sigma}.p''_2$ for some closed TCP_{drt} -term p''_1 and basic term p''_2 . Then $\text{TCP}_{\text{drt}} \vdash p \equiv p' + p'' = (v^1(p'_1) + \underline{\sigma}.p'_2) + (v^1(p''_1) + \underline{\sigma}.p''_2) \stackrel{\text{A1, A2}}{=} (v^1(p'_1) + v^1(p''_1)) + (\underline{\sigma}.p'_2 + \underline{\sigma}.p''_2) \stackrel{\text{RTO4, DRTF}}{=} v^1(p'_1 + p''_1) + \underline{\sigma}.(p'_2 + p''_2)$ and note that $p'_2 + p''_2 \leq p$.

□

For the elimination of \ll , in the case where the left-hand side is delayable, we need a representation lemma. Note the subtle difference with the previous representation lemma where the term following the delay operator was allowed to be of equal size. Here it is not!

Lemma A.3 (Representation) *For basic term p ,*

1. $\text{TCP}_{\text{drt}} \vdash p = v^1(p')$ for some basic TCP_{drt} -term p' ,
2. $\text{TCP}_{\text{drt}} \vdash p = v^1(p') + \underline{\sigma}.p''$ for some basic TCP_{drt} -terms p' and p'' such that $p'' < p$, or
3. $\text{TCP}_{\text{drt}} \vdash p = 1 \cdot v^1(p')$ for some basic term p' such that $p' \leq p$.

Proof. By induction on the structure of basic term p .

1. $p \equiv \underline{0}$. Then $\text{TCP}_{\text{drt}} \vdash p \equiv \underline{0} \stackrel{\text{RTO1}}{=} v^1(\underline{0})$.
2. $p \equiv \underline{1}$. Then $\text{TCP}_{\text{drt}} \vdash p \equiv \underline{1} \stackrel{\text{RTO2}}{=} v^1(\underline{1})$.
3. $p \equiv 0$. Then $\text{TCP}_{\text{drt}} \vdash p \equiv 0 \stackrel{\text{DT2}}{=} 1 \cdot \underline{0} \stackrel{\text{RTO1}}{=} 1 \cdot v^1(\underline{0})$. Note that $\underline{0} \leq 0$.
4. $p \equiv 1$. Then $\text{TCP}_{\text{drt}} \vdash p \equiv 1 \stackrel{\text{A9DR}}{=} 1 \cdot \underline{1} \stackrel{\text{RTO2}}{=} 1 \cdot v^1(\underline{1})$. Note that $\underline{1} \leq 1$.
5. $p \equiv \underline{a.p'}$ for some $a \in A$ and basic term p' . Then $\text{TCP}_{\text{drt}} \vdash p \equiv \underline{a.p'} \stackrel{\text{RTO3}}{=} v^1(\underline{a.p'})$.
6. $p \equiv a.p'$ for some $a \in A$ and basic term p' . Then $\text{TCP}_{\text{drt}} \vdash p \equiv a.p' \stackrel{\text{DT3}}{=} 1 \cdot \underline{a.p'} \stackrel{\text{RTO3}}{=} 1 \cdot v^1(\underline{a.p'})$. Note that $\underline{a.p'} \leq a.p'$.
7. $p \equiv \underline{\sigma.p'}$ for some basic term p' . Then $\text{TCP}_{\text{drt}} \vdash p \equiv \underline{\sigma.p'} \stackrel{\text{A6DR,A1}}{=} \underline{0} + \underline{\sigma.p'} \stackrel{\text{RTO1}}{=} v^1(\underline{0}) + \underline{\sigma.p'}$. Observe that indeed $p' < p$.
8. $p \equiv p' + p''$ for some basic term $p' + p''$. By induction, for each of p' and p'' we have three cases. This results in the following cases:
 - (a) $\text{TCP}_{\text{drt}} \vdash p' = v^1(p'_1)$ for some closed TCP_{drt} -term p'_1 and $\text{TCP}_{\text{drt}} \vdash p'' = v^1(p''_1)$ for some closed TCP_{drt} -term p''_1 . Then $\text{TCP}_{\text{drt}} \vdash p \equiv p' + p'' = v^1(p'_1) + v^1(p''_1) \stackrel{\text{RTO4}}{=} v^1(p'_1 + p''_1)$.
 - (b) $\text{TCP}_{\text{drt}} \vdash p' = v^1(p'_1) + \underline{\sigma.p'_2}$ for some closed TCP_{drt} -term p'_1 and basic term p'_2 such that $p'_2 < p'$ and $\text{TCP}_{\text{drt}} \vdash p'' = v^1(p''_1)$ for some closed TCP_{drt} -term p''_1 . Then $\text{TCP}_{\text{drt}} \vdash p \equiv p' + p'' = (v^1(p'_1) + \underline{\sigma.p'_2}) + v^1(p''_1) \stackrel{\text{A1,A2}}{=} (v^1(p'_1) + v^1(p''_1)) + \underline{\sigma.p'_2} \stackrel{\text{RTO4}}{=} v^1(p'_1 + p''_1) + \underline{\sigma.p'_2}$. Note that $p'_2 < p$.
 - (c) $\text{TCP}_{\text{drt}} \vdash p' = 1 \cdot v^1(p'_1)$ for some basic term p'_1 such that $p'_1 \leq p'$ and $\text{TCP}_{\text{drt}} \vdash p'' = v^1(p''_1)$ for some closed TCP_{drt} -term p''_1 . Then $\text{TCP}_{\text{drt}} \vdash p \equiv p' + p'' = 1 \cdot v^1(p'_1) + v^1(p''_1) \stackrel{\text{DT1,A4,A8DR,DRA10}}{=} v^1(p'_1) + \underline{\sigma.(1 \cdot v^1(p'_1))} + v^1(p''_1) \stackrel{\text{A1,A2,RTO4}}{=} v^1(p'_1 + p''_1) + \underline{\sigma.(1 \cdot v^1(p'_1))} = v^1(p'_1 + p''_1) + \underline{\sigma.p'}$. Note that $p' < p$.
 - (d) $\text{TCP}_{\text{drt}} \vdash p' = v^1(p'_1)$ for some closed TCP_{drt} -term p'_1 and $\text{TCP}_{\text{drt}} \vdash p'' = v^1(p''_1) + \underline{\sigma.p''_2}$ for some closed TCP_{drt} -term p''_1 and basic term p''_2 such that $p''_2 < p''$. Similar to the second case.
 - (e) $\text{TCP}_{\text{drt}} \vdash p' = v^1(p'_1) + \underline{\sigma.p'_2}$ for some closed TCP_{drt} -term p'_1 and basic term p'_2 such that $p'_2 < p'$ and $\text{TCP}_{\text{drt}} \vdash p'' = v^1(p''_1) + \underline{\sigma.p''_2}$ for some closed TCP_{drt} -term p''_1 and basic term p''_2 such that $p''_2 < p''$. Then $\text{TCP}_{\text{drt}} \vdash p \equiv p' + p'' = (v^1(p'_1) + \underline{\sigma.p'_2}) + (v^1(p''_1) + \underline{\sigma.p''_2}) \stackrel{\text{A1,A2}}{=} (v^1(p'_1) + v^1(p''_1)) + (\underline{\sigma.p'_2} + \underline{\sigma.p''_2}) \stackrel{\text{RTO4,DRTF}}{=} v^1(p'_1 + p''_1) + \underline{\sigma.(p'_2 + p''_2)}$ and note that $p'_2 + p''_2 < p$.
 - (f) $\text{TCP}_{\text{drt}} \vdash p' = 1 \cdot v^1(p'_1)$ for some basic term p'_1 such that $p'_1 \leq p'$ and $\text{TCP}_{\text{drt}} \vdash p'' = v^1(p''_1) + \underline{\sigma.p''_2}$ for some closed TCP_{drt} -term p''_1 and basic term p''_2 such that $p''_2 < p''$. Then $\text{TCP}_{\text{drt}} \vdash p \equiv p' + p'' = 1 \cdot v^1(p'_1) + v^1(p''_1) + \underline{\sigma.p''_2} \stackrel{\text{DT1,A4,A8DR,DRA10}}{=} v^1(p'_1) + \underline{\sigma.(1 \cdot v^1(p'_1))} + v^1(p''_1) + \underline{\sigma.p''_2} \stackrel{\text{A1,A2,RTO4,DRTF}}{=} v^1(p'_1 + p''_1) + \underline{\sigma.(1 \cdot v^1(p'_1) + p''_2)} = v^1(p'_1 + p''_1) + \underline{\sigma.(p' + p''_2)}$. Note that $p' + p''_2 < p$.
 - (g) $\text{TCP}_{\text{drt}} \vdash p' = v^1(p'_1)$ for some closed TCP_{drt} -term p'_1 and $\text{TCP}_{\text{drt}} \vdash p'' = 1 \cdot v^1(p''_1)$ for some basic term p''_1 such that $p''_1 \leq p''$. Similar to the third case.
 - (h) $\text{TCP}_{\text{drt}} \vdash p' = v^1(p'_1) + \underline{\sigma.p'_2}$ for some closed TCP_{drt} -term p'_1 and basic term p'_2 such that $p'_2 < p'$ and $\text{TCP}_{\text{drt}} \vdash p'' = 1 \cdot v^1(p''_1)$ for some basic term p''_1 such that $p''_1 \leq p''$. Similar to the sixth case.

- (i) $\text{TCP}_{\text{drt}} \vdash p' = 1 \cdot v^1(p'_1)$ for some basic term p'_1 such that $p'_1 \leq p'$ and $\text{TCP}_{\text{drt}} \vdash p'' = 1 \cdot v^1(p''_1)$ for some basic term p''_1 such that $p''_1 \leq p''$. Then $\text{TCP}_{\text{drt}} \vdash p' + p'' = 1 \cdot v^1(p'_1) + 1 \cdot v^1(p''_1) \stackrel{\text{DT6}}{=} 1 \cdot (v^1(p'_1) + v^1(p''_1)) \stackrel{\text{RTO4}}{=} 1 \cdot v^1(p'_1 + p''_1)$. Note that $p'_1 + p''_1 \leq p$.

□

Theorem A.4 (Elimination of parallel composition operators) *For basic terms p_1 and p_2 ,*

1. *there exists a basic term q such that $\text{TCP}_{\text{drt}} \vdash p_1 \parallel p_2 = q$;*
2. *there exists a basic term q such that $\text{TCP}_{\text{drt}} \vdash p_1 \mid p_2 = q$;*
3. *there exists a basic term q such that $\text{TCP}_{\text{drt}} \vdash p_1 \parallel\!\! \parallel p_2 = q$.*

Proof. These three statements are proven simultaneously using induction on the number of symbols of basic terms p_1 and p_2 . When we apply an induction hypothesis we indicate to which statement it refers.

First we give the proof for statement (1), the elimination of \parallel . We use case distinction on the structure of basic term p_1 .

1. $p_1 \equiv \underline{0}$. $\text{TCP}_{\text{drt}} \vdash p_1 \parallel p_2 \equiv \underline{0} \parallel p_2 \stackrel{\text{LM1DR}}{=} \underline{0}$.
2. $p_1 \equiv \underline{1}$. $\text{TCP}_{\text{drt}} \vdash p_1 \parallel p_2 \equiv \underline{1} \parallel p_2 \stackrel{\text{LM2DR}}{=} \underline{0}$.
3. $p_1 \equiv 0$. According to Lemma A.3 we can distinguish three cases for p_2 . In the proofs below, we may use $\text{TCP}_{\text{drt}} \vdash 0 \stackrel{\text{DT2}}{=} 1 \cdot \underline{0} \stackrel{\text{DT1}}{=} (\underline{1} + \underline{\sigma}.1) \cdot \underline{0} \stackrel{\text{A4}}{=} \underline{1} \cdot \underline{0} + \underline{\sigma}.1 \cdot \underline{0} \stackrel{\text{A8DR}}{=} \underline{0} + \underline{\sigma}.1 \cdot \underline{0} \stackrel{\text{A2,A6DR}}{=} \underline{\sigma}.1 \cdot \underline{0} \stackrel{\text{DRA10}}{=} \underline{\sigma} \cdot (1 \cdot \underline{0}) \stackrel{\text{DT2}}{=} \underline{\sigma} \cdot 0$ (*).
 - (a) $\text{TCP}_{\text{drt}} \vdash p_2 = v^1(p'_2)$ for some basic term p'_2 . Then, $\text{TCP}_{\text{drt}} \vdash p_1 \parallel p_2 \equiv 0 \parallel p_2 = 0 \parallel v^1(p'_2) \stackrel{(*)}{=} \underline{\sigma}.0 \parallel v^1(p'_2) \stackrel{\text{LM5DR}}{=} \underline{0}$.
 - (b) $\text{TCP}_{\text{drt}} \vdash p_2 = v^1(p'_2) + \underline{\sigma}.p''_2$ for some basic term p'_2 and basic term p''_2 such that $p''_2 < p_2$. By induction hypothesis (1) we have $\text{TCP}_{\text{drt}} \vdash 0 \parallel p''_2 = q''$ for some basic term q'' . Then, $\text{TCP}_{\text{drt}} \vdash p_1 \parallel p_2 \equiv 0 \parallel p_2 = 0 \parallel (v^1(p'_2) + \underline{\sigma}.p''_2) \stackrel{(*)}{=} \underline{\sigma}.p \parallel (v^1(p'_2) + \underline{\sigma}.p''_2) \stackrel{\text{LM6DR}}{=} \underline{\sigma} \cdot (0 \parallel p''_2) \stackrel{\text{ih}(1)}{=} \underline{\sigma} \cdot q''$.
 - (c) $\text{TCP}_{\text{drt}} \vdash p_2 = 1 \cdot v^1(p'_2)$ for some basic term p'_2 . Then $\text{TCP}_{\text{drt}} \vdash p_1 \parallel p_2 \equiv 0 \parallel p_2 = 0 \parallel (1 \cdot v^1(p'_2)) \stackrel{\text{DT2}}{=} (1 \cdot \underline{0}) \parallel (1 \cdot v^1(p'_2)) \stackrel{\text{LM7DR}}{=} 1 \cdot (0 \parallel (1 \cdot v^1(p'_2))) \stackrel{\text{LM1DR}}{=} 1 \cdot \underline{0} \stackrel{\text{DT2}}{=} 0$.
4. $p_1 \equiv 1$. According to Lemma A.3 we can distinguish three cases for p_2 .
 - (a) $\text{TCP}_{\text{drt}} \vdash p_2 = v^1(p'_2)$ for some basic term p'_2 . Then, $\text{TCP}_{\text{drt}} \vdash p_1 \parallel p_2 \equiv 1 \parallel p_2 \stackrel{\text{DT1}}{=} (\underline{1} + \underline{\sigma}.1) \parallel p_2 \stackrel{\text{LM4}}{=} \underline{1} \parallel p_2 + \underline{\sigma}.1 \parallel p_2 \stackrel{\text{LM1DR,A6DR}}{=} \underline{\sigma}.1 \parallel p_2 = \underline{\sigma}.1 \parallel v^1(p'_2) \stackrel{\text{LM5DR}}{=} \underline{0}$.
 - (b) $\text{TCP}_{\text{drt}} \vdash p_2 = v^1(p'_2) + \underline{\sigma}.p''_2$ for some basic term p'_2 and basic term p''_2 such that $p''_2 < p_2$. By induction hypothesis (1) we have $\text{TCP}_{\text{drt}} \vdash 0 \parallel p''_2 = q''$ for some basic term q'' . Then, $\text{TCP}_{\text{drt}} \vdash p_1 \parallel p_2 \equiv 1 \parallel p_2 \stackrel{\text{DT1}}{=} (\underline{1} + \underline{\sigma}.1) \parallel p_2 \stackrel{\text{LM4}}{=} \underline{1} \parallel p_2 + \underline{\sigma}.1 \parallel p_2 \stackrel{\text{LM1DR,A6DR}}{=} \underline{\sigma}.1 \parallel p_2 = \underline{\sigma}.1 \parallel (v^1(p'_2) + \underline{\sigma}.p''_2) \stackrel{\text{LM6DR}}{=} \underline{\sigma} \cdot (1 \parallel p''_2) = \underline{\sigma} \cdot q''$.
 - (c) $\text{TCP}_{\text{drt}} \vdash p_2 = 1 \cdot v^1(p'_2)$ for some basic term p'_2 . Then $\text{TCP}_{\text{drt}} \vdash p_1 \parallel p_2 \equiv 1 \parallel p_2 \stackrel{\text{A9DR}}{=} (1 \cdot \underline{1}) \parallel p_2 = (1 \cdot \underline{1}) \parallel (1 \cdot v^1(p'_2)) \stackrel{\text{LM7DR}}{=} 1 \cdot (\underline{1} \parallel (1 \cdot v^1(p'_2))) \stackrel{\text{LM2DR}}{=} 1 \cdot \underline{0} \stackrel{\text{DT2}}{=} 0$.

5. $p_1 \equiv \underline{a}.p'_1$ for some $a \in A$ and basic term p'_1 . By induction hypothesis (3) there exists a basic term q' such that $\text{TCP}_{\text{drt}} \vdash p'_1 \parallel p_2 = q'$. Then, $\text{TCP}_{\text{drt}} \vdash p_1 \parallel p_2 \equiv \underline{a}.p'_1 \parallel p_2 \stackrel{\text{LM3DR}}{=} \underline{a}.(p'_1 \parallel p_2) \stackrel{\text{ih}(3)}{=} \underline{a}.q'$.
6. $p_1 \equiv a.p'_1$ for some $a \in A$ and basic term p'_1 . According to Lemma A.3 we can distinguish three cases for p_2 . In the proofs below, we may use $\text{TCP}_{\text{drt}} \vdash a.p'_1 \stackrel{\text{DT3}}{=} 1 \cdot \underline{a}.p'_1 \stackrel{\text{DT1}}{=} (\underline{1} + \underline{\sigma}.1) \cdot \underline{a}.p'_1 \stackrel{\text{A4}}{=} \underline{1} \cdot \underline{a}.p'_1 + \underline{\sigma}.1 \cdot \underline{a}.p'_1 \stackrel{\text{A8DR}}{=} \underline{a}.p'_1 + \underline{\sigma}.1 \cdot \underline{a}.p'_1 \stackrel{\text{DRA10}}{=} \underline{a}.p'_1 + \underline{\sigma}.(1 \cdot \underline{a}.p'_1) \stackrel{\text{DT3}}{=} \underline{a}.p'_1 + \underline{\sigma}.a.p'_1$ (*).
 - (a) $\text{TCP}_{\text{drt}} \vdash p_2 = v^1(p'_2)$ for some basic term p'_2 . By induction hypothesis (3) we have the existence of basic term q' such that $\text{TCP}_{\text{drt}} \vdash p'_1 \parallel p_2 = q'$. Then, $\text{TCP}_{\text{drt}} \vdash p_1 \parallel p_2 \equiv a.p'_1 \parallel p_2 \stackrel{(*)}{=} (\underline{a}.p'_1 + \underline{\sigma}.a.p'_1) \parallel p_2 \stackrel{\text{LM4}}{=} \underline{a}.p'_1 \parallel p_2 + \underline{\sigma}.a.p'_1 \parallel p_2 \stackrel{\text{LM3DR}}{=} \underline{a}.(p'_1 \parallel p_2) + \underline{\sigma}.a.p'_1 \parallel p_2 \stackrel{\text{ih}(3)}{=} \underline{a}.q' + \underline{\sigma}.a.p'_1 \parallel p_2 = \underline{a}.q' + \underline{\sigma}.a.p'_1 \parallel v^1(p'_2) \stackrel{\text{LM5DR}}{=} \underline{a}.q' + \underline{0} \stackrel{\text{A6DR}}{=} \underline{a}.q'$.
 - (b) $\text{TCP}_{\text{drt}} \vdash p_2 = v^1(p'_2) + \underline{\sigma}.p''_2$ for some basic term p'_2 and basic term p''_2 such that $p''_2 < p_2$. By induction hypothesis (1) we have the existence of basic term q'' such that $\text{TCP}_{\text{drt}} \vdash p_1 \parallel p''_2 = q''$. By induction hypothesis (3) we have the existence of basic term q' such that $\text{TCP}_{\text{drt}} \vdash p'_1 \parallel p_2 = q'$. Then, $\text{TCP}_{\text{drt}} \vdash p_1 \parallel p_2 \equiv a.p'_1 \parallel p_2 \stackrel{(*)}{=} (\underline{a}.p'_1 + \underline{\sigma}.a.p'_1) \parallel p_2 \stackrel{\text{LM4}}{=} \underline{a}.p'_1 \parallel p_2 + \underline{\sigma}.a.p'_1 \parallel p_2 \stackrel{\text{LM3DR}}{=} \underline{a}.(p'_1 \parallel p_2) + \underline{\sigma}.a.p'_1 \parallel p_2 \stackrel{\text{ih}(3)}{=} \underline{a}.q' + \underline{\sigma}.a.p'_1 \parallel p_2 = \underline{a}.q' + \underline{\sigma}.a.p'_1 \parallel (v^1(p'_2) + \underline{\sigma}.p''_2) \stackrel{\text{LM6DR}}{=} \underline{a}.q' + \underline{\sigma}.(a.p'_1 \parallel p''_2) = \underline{a}.q' + \underline{\sigma}.(p_1 \parallel p''_2) \stackrel{\text{ih}(1)}{=} \underline{a}.q' + q''$.
 - (c) $\text{TCP}_{\text{drt}} \vdash p_2 = 1 \cdot v^1(p'_2)$ for some basic term p'_2 such that $p'_2 \leq p_2$. By induction hypothesis (3) we have the existence of basic term q' such that $\text{TCP}_{\text{drt}} \vdash p'_1 \parallel p'_2 = q'$. Then $\text{TCP}_{\text{drt}} \vdash p_1 \parallel p_2 \equiv a.p'_1 \parallel p_2 \stackrel{\text{DT3}}{=} (1 \cdot \underline{a}.p'_1) \parallel p_2 = (1 \cdot \underline{a}.p'_1) \parallel (1 \cdot v^1(p'_2)) \stackrel{\text{LM7DR}}{=} 1 \cdot (\underline{a}.p'_1 \parallel (1 \cdot v^1(p'_2))) = 1 \cdot (\underline{a}.p'_1 \parallel p_2) \stackrel{\text{LM3DR}}{=} 1 \cdot \underline{a}.(p'_1 \parallel p_2) \stackrel{\text{ih}(3)}{=} 1 \cdot \underline{a}.q' \stackrel{\text{DT3}}{=} \underline{a}.q'$.
7. $p_1 \equiv \underline{\sigma}.p'_1$ for some basic term p'_1 . According to Lemma A.2 we can distinguish two cases for p_2 .
 - (a) $\text{TCP}_{\text{drt}} \vdash p_2 = v^1(p'_2)$ for some basic term p'_2 . Then $\text{TCP}_{\text{drt}} \vdash p_1 \parallel p_2 \equiv \underline{\sigma}.p'_1 \parallel p_2 = \underline{\sigma}.p'_1 \parallel v^1(p'_2) \stackrel{\text{LM5DR}}{=} \underline{0}$.
 - (b) $\text{TCP}_{\text{drt}} \vdash p_2 = v^1(p'_2) + \underline{\sigma}.p''_2$ for some basic terms p'_2 and p''_2 such that $p''_2 \leq p_2$. By induction hypothesis (1) we have the existence of basic term q' such that $\text{TCP}_{\text{drt}} \vdash p'_1 \parallel p''_2 = q'$. Then $\text{TCP}_{\text{drt}} \vdash p_1 \parallel p_2 \equiv \underline{\sigma}.p'_1 \parallel p_2 = \underline{\sigma}.p'_1 \parallel (v^1(p'_2) + \underline{\sigma}.p''_2) \stackrel{\text{LM6DR}}{=} \underline{\sigma}.(p'_1 \parallel p''_2) \stackrel{\text{ih}(1)}{=} \underline{\sigma}.q'$.
8. $p_1 \equiv p'_1 + p''_1$. By induction hypothesis (1) there exist basic terms q' and q'' such that $\text{TCP}_{\text{drt}} \vdash p'_1 \parallel p_2 = q'$ and $\text{TCP}_{\text{drt}} \vdash p''_1 \parallel p_2 = q''$. Then $\text{TCP}_{\text{drt}} \vdash p_1 \parallel p_2 \equiv (p'_1 + p''_1) \parallel p_2 \stackrel{\text{LM4}}{=} p'_1 \parallel p_2 + p''_1 \parallel p_2 \stackrel{\text{ih}(1)}{=} q' + q''$.

Then we prove statement (2), the elimination of $|$. We use case distinction on the structure of basic terms p_1 .

1. $p_1 \equiv \underline{0}$. $\text{TCP}_{\text{drt}} \vdash p_1 | p_2 \equiv \underline{0} | p_2 \stackrel{\text{CM1DR}}{=} \underline{0}$.
2. $p_1 \equiv \underline{1}$. We use case distinction on the structure of basic term p_2 .
 - (a) $p_2 \equiv \underline{0}$. $\text{TCP}_{\text{drt}} \vdash p_1 | p_2 \equiv p_1 | \underline{0} \stackrel{\text{SC1}}{=} \underline{0} | p_1 \stackrel{\text{CM1DR}}{=} \underline{0}$.
 - (b) $p_2 \equiv \underline{1}$. $\text{TCP}_{\text{drt}} \vdash p_1 | p_2 \equiv \underline{1} | \underline{1} \stackrel{\text{CM3DR}}{=} \underline{1}$.

- (c) $p_2 \equiv 0$. $\text{TCP}_{\text{drt}} \vdash p_1 | p_2 \equiv \underline{1} | 0 \stackrel{\text{DC1}}{=} 0 | \underline{1} \stackrel{\text{DT2}}{=} (1 \cdot \underline{0}) | \underline{1} \stackrel{\text{DT1}}{=} ((\underline{1} + \underline{\sigma}.1) \cdot \underline{0}) | \underline{1} \stackrel{\text{A4}}{=} (\underline{1} \cdot \underline{0} + \underline{\sigma}.1 \cdot \underline{0}) | \underline{1} \stackrel{\text{CM2}}{=} (\underline{1} \cdot \underline{0}) | \underline{1} + (\underline{\sigma}.1 \cdot \underline{0}) | \underline{1} \stackrel{\text{A8DR,DRA10}}{=} \underline{0} | \underline{1} + \underline{\sigma}.(1 \cdot \underline{0}) | \underline{1} \stackrel{\text{CM1DR,RTO2}}{=} \underline{0} + \underline{\sigma}.(1 \cdot \underline{0}) | v^1(\underline{1}) \stackrel{\text{CM7DR}}{=} \underline{0} + \underline{0}$.
- (d) $p_2 \equiv 1$. $\text{TCP}_{\text{drt}} \vdash p_1 | p_2 \equiv \underline{1} | 1 \stackrel{\text{SC1}}{=} 1 | \underline{1} \stackrel{\text{DT1}}{=} (\underline{1} + \underline{\sigma}.1) | \underline{1} \stackrel{\text{CM2}}{=} \underline{1} | \underline{1} + \underline{\sigma}.1 | \underline{1} \stackrel{\text{CM3DR,RTO2}}{=} \underline{1} + \underline{\sigma}.1 | v^1(\underline{1}) \stackrel{\text{CM7DR}}{=} \underline{1} + \underline{0} \stackrel{\text{A6DR}}{=} \underline{1}$.
- (e) $p_2 \equiv \underline{a}.p'_2$ for some $a \in A$ and basic term p'_2 . $\text{TCP}_{\text{drt}} \vdash p_1 | p_2 \equiv \underline{1} | \underline{a}.p'_2 \stackrel{\text{SC1}}{=} \underline{a}.p'_2 | \underline{1} \stackrel{\text{CM4DR}}{=} \underline{0}$.
- (f) $p_2 \equiv \underline{a}.p'_2$ for some $a \in A$ and basic term p'_2 . $\text{TCP}_{\text{drt}} \vdash p_1 | p_2 \equiv \underline{1} | \underline{a}.p'_2 \stackrel{\text{SC1}}{=} \underline{a}.p'_2 | \underline{1} \stackrel{\text{DT3,DT1,A4,CM2}}{=} (\underline{1} \cdot \underline{a}.p'_2) | \underline{1} + (\underline{\sigma}.1 \cdot \underline{a}.p'_2) | \underline{1} \stackrel{\text{A8DR,DRA10}}{=} \underline{a}.p'_2 | \underline{1} + \underline{\sigma}.(1 \cdot \underline{a}.p'_2) | \underline{1} \stackrel{\text{CM4DR,RTO2}}{=} \underline{0} + \underline{\sigma}.(1 \cdot \underline{a}.p'_2) | v^1(\underline{1}) \stackrel{\text{CM7DR}}{=} \underline{0} + \underline{0}$.
- (g) $p_2 \equiv p'_2 + p''_2$ for some basic terms p'_2 and p''_2 . By induction we have the existence of basic terms q' and q'' such that $\text{TCP}_{\text{drt}} \vdash p_1 | p'_2 = q'$ and $\text{TCP}_{\text{drt}} \vdash p_1 | p''_2 = q''$. Then $\text{TCP}_{\text{drt}} \vdash p_1 | p_2 \equiv p_1 | (p'_2 + p''_2) \stackrel{\text{SC1}}{=} (p'_2 + p''_2) | p_1 \stackrel{\text{CM2}}{=} p'_2 | p_1 + p''_2 | p_1 \stackrel{\text{SC1}}{=} p_1 | p'_2 + p_1 | p''_2 \stackrel{\text{ih(2)}}{=} q' + q''$.
3. $p_1 \equiv 0$. According to Lemma A.3 we can distinguish three cases for p_2 . In the proofs below, we may use $\text{TCP}_{\text{drt}} \vdash 0 \stackrel{\text{DT2}}{=} 1 \cdot \underline{0} \stackrel{\text{DT1}}{=} (\underline{1} + \underline{\sigma}.1) \cdot \underline{0} \stackrel{\text{A4}}{=} \underline{1} \cdot \underline{0} + \underline{\sigma}.1 \cdot \underline{0} \stackrel{\text{A8DR}}{=} \underline{0} + \underline{\sigma}.1 \cdot \underline{0} \stackrel{\text{A1,A6DR}}{=} \underline{\sigma}.1 \cdot \underline{0} \stackrel{\text{DRA10}}{=} \underline{\sigma}.(1 \cdot \underline{0}) \stackrel{\text{DT2}}{=} \underline{\sigma}.0$ (DD).
- (a) $\text{TCP}_{\text{drt}} \vdash p_2 = v^1(p'_2)$ for some basic term p'_2 . Then, $\text{TCP}_{\text{drt}} \vdash p_1 | p_2 \equiv 0 | p_2 \stackrel{(*)}{=} \underline{\sigma}.0 | p_2 = \underline{\sigma}.0 | v^1(p'_2) \stackrel{\text{CM7DR}}{=} \underline{0}$.
- (b) $\text{TCP}_{\text{drt}} \vdash p_2 = v^1(p'_2) + \underline{\sigma}.p''_2$ for some basic term p'_2 and basic term p''_2 such that $p''_2 < p_2$. By induction hypothesis (2) we have the existence of basic term q'' such that $\text{TCP}_{\text{drt}} \vdash p_1 | p''_2 = q''$. Then, $\text{TCP}_{\text{drt}} \vdash p_1 | p_2 \equiv 0 | p_2 \stackrel{\text{DD}}{=} \underline{\sigma}.0 | p_2 = \underline{\sigma}.0 | (v^1(p'_2) + \underline{\sigma}.p''_2) \stackrel{\text{SC1,CM2,SC1}}{=} \underline{\sigma}.0 | v^1(p'_2) + \underline{\sigma}.0 | \underline{\sigma}.p''_2 \stackrel{\text{CM7DR,CM8DR}}{=} \underline{0} + \underline{\sigma}.(0 | p''_2) = \underline{0} + \underline{\sigma}.(p_1 | p''_2) \stackrel{\text{ih(2)}}{=} \underline{0} + \underline{\sigma}.q''$.
- (c) $\text{TCP}_{\text{drt}} \vdash p_2 = 1 \cdot v^1(p'_2)$ for some basic term p'_2 such that $p'_2 \leq p_2$. Then $\text{TCP}_{\text{drt}} \vdash p_1 | p_2 \equiv 0 | p_2 \stackrel{\text{DT2}}{=} (1 \cdot \underline{0}) | p_2 = (1 \cdot \underline{0}) | (1 \cdot v^1(p'_2)) \stackrel{\text{CM9DR}}{=} 1 \cdot (\underline{0} | (1 \cdot v^1(p'_2))) + (1 \cdot \underline{0}) | v^1(p'_2) \stackrel{\text{CM1DR,DT2}}{=} 1 \cdot (\underline{0} + 0 | v^1(p'_2)) \stackrel{\text{DD}}{=} 1 \cdot (\underline{0} + \underline{\sigma}.0 | v^1(p'_2)) \stackrel{\text{CM7DR}}{=} 1 \cdot (\underline{0} + \underline{0}) \stackrel{\text{A3}}{=} 1 \cdot \underline{0} \stackrel{\text{DT2}}{=} 0$.
4. $p_1 \equiv 1$. We use case distinction on the structure of basic term p_2 . We omit the cases where p_2 is of the form $\underline{0}$, $\underline{1}$, 0 , or $p'_2 + p''_2$ since these are symmetrical to previous cases.
- (a) $p_2 \equiv 1$. We already have shown before that $\text{TCP}_{\text{drt}} \vdash \underline{1} | 1 = \underline{1}$ (*). $\text{TCP}_{\text{drt}} \vdash p_1 | p_2 \equiv 1 | 1 \stackrel{\text{A9DR}}{=} (1 \cdot \underline{1}) | (1 \cdot \underline{1}) \stackrel{\text{CM9DR}}{=} 1 \cdot (\underline{1} | (1 \cdot \underline{1}) + (1 \cdot \underline{1}) | \underline{1}) \stackrel{\text{SC1,A3}}{=} 1 \cdot (\underline{1} | (1 \cdot \underline{1})) \stackrel{\text{A9DR}}{=} 1 \cdot (\underline{1} | 1) \stackrel{(*)}{=} 1 \cdot \underline{1} \stackrel{\text{A9DR}}{=} 1$.
- (b) $p_2 \equiv \underline{a}.p'_2$ for some $a \in A$ and basic term p'_2 . $\text{TCP}_{\text{drt}} \vdash p_1 | p_2 \equiv 1 | \underline{a}.p'_2 \stackrel{\text{DT1,CM4}}{=} \underline{1} | \underline{a}.p'_2 + \underline{\sigma}.1 | \underline{a}.p'_2 \stackrel{\text{SC1,CM4DR,A6DR}}{=} \underline{\sigma}.1 | \underline{a}.p'_2 \stackrel{\text{RTO3,CM7DR}}{=} \underline{0}$.
- (c) $p_2 \equiv \underline{a}.p'_2$ for some $a \in A$ and basic term p'_2 . We already have shown before that $\text{TCP}_{\text{drt}} \vdash \underline{1} | \underline{a}.p'_2 = \underline{0}$ (*) and $\text{TCP}_{\text{drt}} \vdash 1 | \underline{a}.p'_2 = \underline{0}$ (**). Then, $\text{TCP}_{\text{drt}} \vdash p_1 | p_2 \equiv 1 | \underline{a}.p'_2 \stackrel{\text{A9DR,DT3}}{=} (1 \cdot \underline{1}) | (1 \cdot \underline{a}.p'_2) \stackrel{\text{CM9DR}}{=} 1 \cdot (\underline{1} | (1 \cdot \underline{a}.p'_2) + (1 \cdot \underline{1}) | \underline{a}.p'_2) \stackrel{\text{DT3,A9DR}}{=} 1 \cdot (\underline{1} | \underline{a}.p'_2 + 1 | \underline{a}.p'_2) \stackrel{(*),(**)}{=} 1 \cdot (\underline{0} + \underline{0}) \stackrel{\text{A3,DT2}}{=} 0$.
- (d) $p_2 \equiv \underline{\sigma}.p'_2$ for some basic term p'_2 . We already have shown before that $\text{TCP}_{\text{drt}} \vdash \underline{1} | \underline{\sigma}.p'_2 = \underline{0}$ (*). By induction hypothesis (2) we have the existence of basic term q' such

that $\text{TCP}_{\text{drt}} \vdash 1 \mid p'_2 = q'$. Then, $\text{TCP}_{\text{drt}} \vdash p_1 \mid p_2 \equiv 1 \mid \underline{\sigma}.p'_2 \stackrel{\text{DT1,CM2}}{=} \underline{1} \mid \underline{\sigma}.p'_2 + \underline{\sigma}.1 \mid \underline{\sigma}.p'_2 \stackrel{(*)}{=} \underline{0} + \underline{\sigma}.(1 \mid p'_2) \stackrel{\text{A6DR,ih(2)}}{=} \underline{\sigma}.q'$.

5. $p_1 \equiv \underline{a}.p'_1$ for some $a \in A$ and basic term p'_1 . We use case distinction on the structure of basic term p_2 . We omit the cases where p_2 is of the form $\underline{0}$, $\underline{1}$, 0 , 1 or $p'_2 + p''_2$ since these are symmetrical to previous cases.

(a) $p_2 \equiv \underline{b}.p'_2$ for some $b \in A$ and basic term p'_2 . By the induction hypothesis for statement (3) we have the existence of basic term q' such that $\text{TCP}_{\text{drt}} \vdash p'_1 \parallel p'_2 = q'$. In case $\gamma(a, b)$ is not defined, $\text{TCP}_{\text{drt}} \vdash p_1 \mid p_2 \equiv \underline{a}.p'_1 \mid \underline{b}.p'_2 \stackrel{\text{CM6DR}}{=} \underline{0}$. In case $\gamma(a, b) = c$, $\text{TCP}_{\text{drt}} \vdash p_1 \mid p_2 \equiv \underline{a}.p'_1 \mid \underline{b}.p'_2 \stackrel{\text{CM5DR}}{=} \underline{c}.(p'_1 \parallel p'_2) \stackrel{\text{ih(3)}}{=} \underline{c}.q'$.

(b) $p_2 \equiv b.p'_2$ for some $b \in A$ and basic term p'_2 . We use $\text{TCP}_{\text{drt}} \vdash a.x \stackrel{\text{DT3,DT1,A4,A8DR,DRA10}}{\underline{a}.x + \underline{\sigma}.a.x}$ (DAP). By the induction hypothesis for statement (3) we have the existence of basic term q' such that $\text{TCP}_{\text{drt}} \vdash p'_1 \parallel p'_2 = q'$. In case $\gamma(a, b)$ is not defined, $\text{TCP}_{\text{drt}} \vdash p_1 \mid p_2 \equiv \underline{a}.p'_1 \mid b.p'_2 \stackrel{\text{DAP,SC1,CM2}}{\underline{a}.p'_1 \mid \underline{b}.p'_2 + \underline{a}.p'_1 \mid \underline{\sigma}.b.p'_2} \stackrel{\text{CM6DR,A6DR}}{\underline{a}.p'_1 \mid \underline{\sigma}.b.p'_2} \stackrel{\text{SC1,RTO3}}{\underline{\sigma}.b.p'_2 \mid v^1(\underline{a}.p'_1)} \stackrel{\text{CM7DR}}{=} \underline{0}$. In case $\gamma(a, b) = c$, $\text{TCP}_{\text{drt}} \vdash p_1 \mid p_2 \equiv \underline{a}.p'_1 \mid b.p'_2 \stackrel{\text{DAP,SC1,CM2}}{\underline{a}.p'_1 \mid \underline{b}.p'_2 + \underline{a}.p'_1 \mid \underline{\sigma}.b.p'_2} \stackrel{\text{CM5DR}}{\underline{c}.(p'_1 \parallel p'_2) + \underline{a}.p'_1 \mid \underline{\sigma}.b.p'_2} \stackrel{\text{ih(3)}}{=} \underline{c}.q' + \underline{a}.p'_1 \mid \underline{\sigma}.b.p'_2 \stackrel{\text{SC1,RTO3}}{\underline{c}.q' + \underline{a}.p'_1 \mid \underline{\sigma}.b.p'_2} \stackrel{\text{CM7DR}}{\underline{c}.q' + \underline{0}} \stackrel{\text{A6DR}}{=} \underline{c}.q'$.

(c) $p_2 \equiv \underline{\sigma}.p'_2$ for some basic term p'_2 . $\text{TCP}_{\text{drt}} \vdash p_1 \mid p_2 \equiv \underline{a}.p'_1 \mid \underline{\sigma}.p'_2 \stackrel{\text{SC1}}{\underline{\sigma}.p'_2 \mid \underline{a}.p'_1} \stackrel{\text{RTO3}}{\underline{\sigma}.p'_2 \mid v^1(\underline{a}.p'_1)} \stackrel{\text{CM7DR}}{=} \underline{0}$.

6. $p_1 \equiv a.p'_1$ for some $a \in A$ and basic term p'_1 . We use case distinction on the structure of basic term p_2 . We omit the cases where p_2 is of the form $\underline{0}$, $\underline{1}$, 0 , 1 , $\underline{b}.p'_2$ or $p'_2 + p''_2$ since these are symmetrical to previous cases.

(a) $p_2 \equiv b.p'_2$ for some $b \in A$ and basic term p'_2 . In case $\gamma(a, b)$ is not defined, we have already proved before that $\text{TCP}_{\text{drt}} \vdash \underline{a}.p'_1 \mid b.p'_2 = \underline{0}$ (*) and $\text{TCP}_{\text{drt}} \vdash a.p'_1 \mid \underline{b}.p'_2 = \underline{0}$ (**). Then, $\text{TCP}_{\text{drt}} \vdash p_1 \mid p_2 \equiv a.p'_1 \mid b.p'_2 \stackrel{\text{DT3}}{=} (1 \cdot \underline{a}.p'_1) \mid (1 \cdot \underline{b}.p'_2) \stackrel{\text{CM9DR}}{=} 1 \cdot (\underline{a}.p'_1 \mid (1 \cdot \underline{b}.p'_2) + (1 \cdot \underline{a}.p'_1) \mid \underline{b}.p'_2) \stackrel{\text{DT3}}{=} 1 \cdot (\underline{a}.p'_1 \mid b.p'_2 + a.p'_1 \mid \underline{b}.p'_2) \stackrel{(*),(**)}{=} 1 \cdot (\underline{0} + \underline{0}) \stackrel{\text{A3,DT2}}{=} \underline{0}$.

In case $\gamma(a, b) = c$, we have already proved before that $\text{TCP}_{\text{drt}} \vdash \underline{a}.p'_1 \mid b.p'_2 = \underline{c}.(p'_1 \parallel p'_2)$ (*) and $\text{TCP}_{\text{drt}} \vdash a.p'_1 \mid \underline{b}.p'_2 = \underline{c}.(p'_1 \parallel p'_2)$ (**). By the induction hypothesis for statement (3) we have the existence of basic term q' such that $\text{TCP}_{\text{drt}} \vdash p'_1 \parallel p'_2 = q'$. Then, $\text{TCP}_{\text{drt}} \vdash p_1 \mid p_2 \equiv a.p'_1 \mid b.p'_2 \stackrel{\text{DT3}}{=} (1 \cdot \underline{a}.p'_1) \mid (1 \cdot \underline{b}.p'_2) \stackrel{\text{CM9DR}}{=} 1 \cdot (\underline{a}.p'_1 \mid (1 \cdot \underline{b}.p'_2) + (1 \cdot \underline{a}.p'_1) \mid \underline{b}.p'_2) \stackrel{\text{DT3}}{=} 1 \cdot (\underline{a}.p'_1 \mid b.p'_2 + a.p'_1 \mid \underline{b}.p'_2) \stackrel{(*),(**),A3}{=} 1 \cdot \underline{c}.(p'_1 \parallel p'_2) \stackrel{\text{ih(3)}}{=} 1 \cdot \underline{c}.q' \stackrel{\text{DT3}}{=} \underline{c}.q'$.

(b) $p_2 \equiv \underline{\sigma}.p'_2$ for some basic term p'_2 . By induction hypothesis (2) there exists a basic term q' such that $\text{TCP}_{\text{drt}} \vdash p_1 \mid p'_2 = q'$. We have already proved before that $\text{TCP}_{\text{drt}} \vdash \underline{a}.p'_1 \mid \underline{\sigma}.p'_2 = \underline{0}$. $\text{TCP}_{\text{drt}} \vdash p_1 \mid p_2 \equiv \underline{a}.p'_1 \mid \underline{\sigma}.p'_2 \stackrel{\text{DT3,DT1,A4,CM2,A8DR,DRA10}}{\underline{a}.p'_1 \mid \underline{\sigma}.p'_2 + \underline{\sigma}.a.p'_1 \mid \underline{\sigma}.p'_2} \stackrel{(*),A6DR}{=} \underline{\sigma}.a.p'_1 \mid \underline{\sigma}.p'_2 \stackrel{\text{CM8DR}}{=} \underline{\sigma}.(a.p'_1 \mid p'_2) \stackrel{\text{ih(2)}}{=} \underline{\sigma}.q'$.

7. $p_1 \equiv \underline{\sigma}.p'_1$ for some basic term p'_1 . We use case distinction on the structure of basic term p_2 . We omit the cases where p_2 is of the form $\underline{0}$, $\underline{1}$, 0 , 1 , $\underline{a}.p'_2$, $a.p'_2$ or $p'_2 + p''_2$ since these are symmetrical to previous cases.

(a) $p_2 \equiv \underline{\sigma}.p'_2$ for some basic term p'_2 . By induction hypothesis (2) we have the existence of basic term q' such that $\text{TCP}_{\text{drt}} \vdash p'_1 \mid p'_2 = q'$. Then $\text{TCP}_{\text{drt}} \vdash p_1 \mid p_2 \equiv \underline{\sigma}.p'_1 \mid \underline{\sigma}.p'_2 \stackrel{\text{CM8DR}}{=} \underline{\sigma}.(p'_1 \mid p'_2) \stackrel{\text{ih(2)}}{=} \underline{\sigma}.q'$.

8. $p_1 \equiv p'_1 + p''_1$ for some basic terms p'_1 and p''_1 . By the induction hypothesis for statement (2), we have the existence of basic terms q' and q'' such that $\text{TCP}_{\text{drt}} \vdash p'_1 \mid p_2 = r'$ and $\text{TCP}_{\text{drt}} \vdash p''_1 \mid p_2 = r''$. Then $\text{TCP}_{\text{drt}} \vdash p_1 \mid p_2 \equiv (p'_1 + p''_1) \mid p_2 \stackrel{\text{CM2}}{=} p'_1 \mid p_2 + p''_1 \mid p_2 \stackrel{\text{ih}(2)}{=} r' + r''$.

Finally, statement (3) follows straightforwardly from the previous statements: By induction on statements (1) and (2) we have the existence of basic terms q_1 , q_2 , and q_3 such that $\text{TCP}_{\text{drt}} \vdash p_1 \parallel p_2 = q_1$, $\text{TCP}_{\text{drt}} \vdash p_2 \parallel p_1 = q_2$, and $\text{TCP}_{\text{drt}} \vdash p_1 \mid p_2 = q_3$. Then $\text{TCP}_{\text{drt}} \vdash p_1 \parallel p_2 \stackrel{\text{M}}{=} p_1 \parallel p_2 + p_2 \parallel p_1 \mid p_2 \stackrel{\text{ih}(1), \text{ih}(2)}{=} q_1 + q_2 + q_3$. \square

Theorem A.5 (Elimination of encapsulation) *For basic terms p and $H \subseteq A$, there exists a basic term q such that $\text{TCP}_{\text{drt}} \vdash \partial_H(p) = q$.*

Proof. Trivial, by induction on the structure of basic term p and the derivable equalities $0 = 1 \cdot \underline{0}$, $1 = 1 \cdot \underline{1}$, and $a.x = 1 \cdot \underline{a.x}$. \square

Theorem A.6 (Elimination of time-out operator) *For basic terms p , there exists a basic term q such that $\text{TCP}_{\text{drt}} \vdash v^1(p) = q$.*

Proof. Trivial, by induction on the structure of basic term p and the derivable equalities $0 = 1 \cdot \underline{0}$, $1 = 1 \cdot \underline{1}$, and $a.x = 1 \cdot \underline{a.x}$. \square

A.2 Completeness of TCP_{drt}

Note that the term deduction system for TCP_{drt} is such that all action and time transitions that can be derived starting from a basic term always result in a basic term. We do not prove this statement formally, and will use it silently in the remainder.

Lemma A.7 (Towards completeness) *For arbitrary basic TCP_{drt} -terms p and p' and arbitrary action $a \in A$*

1. if $p \downarrow$, then $\text{TCP}_{\text{drt}} \vdash p = \underline{1} + p$;
2. if $p \xrightarrow{a} p'$, then $\text{TCP}_{\text{drt}} \vdash p = \underline{a.p'} + p$;
3. if $p \xrightarrow{1} p'$, then $p' \equiv p$ or $p' < p$;
4. if $p \xrightarrow{1} p'$, then $\text{TCP}_{\text{drt}} \vdash p = \underline{\sigma.p'} + p$;
5. if $p \xrightarrow{1} p$, then $\text{TCP}_{\text{drt}} \vdash p = 1 \cdot p$.

Proof. Easy; by induction on the structure of basic TCP_{drt} -term p . \square

Theorem A.8 *The process algebra TCP_{drt} is a complete axiomatization of strong bisimilarity on closed TCP_{drt} -terms.*

Proof. By the elimination theorem for TCP_{drt} it suffices to prove this theorem for basic terms only. We use induction on the structure of basic terms p and q and use case analysis on the structure of basic term p to prove that $p + q \Leftrightarrow q$ implies $\text{TCP}_{\text{drt}} \vdash p + q = q$.

1. $p \equiv \underline{0}$. Then $\text{TCP}_{\text{drt}} \vdash p + q \equiv \underline{0} + q \stackrel{\text{A1,A6DR}}{=} q$.
2. $p \equiv \underline{1}$. Then $p + q \downarrow$, and since $p + q \Leftrightarrow q$ also $q \downarrow$. By Lemma A.7.1, we have $\text{TCP}_{\text{drt}} \vdash q = \underline{1} + q$. Then, $\text{TCP}_{\text{drt}} \vdash p + q \equiv \underline{1} + q = q$.
3. $p \equiv 0$. Then $p \xrightarrow{1} p$, and therefore also $p + q \xrightarrow{1}$ and since $p + q \Leftrightarrow q$ also $q \xrightarrow{1}$. By Lemma A.7.3, we can distinguish two cases:
 - (a) $q \xrightarrow{1} q$. By Lemma A.7.5 we have $\text{TCP}_{\text{drt}} \vdash q = 1 \cdot q$. Then $\text{TCP}_{\text{drt}} \vdash p + q = 0 + q \stackrel{\text{DT2}}{=} 1 \cdot \underline{0} + q = 1 \cdot \underline{0} + 1 \cdot q \stackrel{\text{DT6}}{=} 1 \cdot (\underline{0} + q) \stackrel{\text{A1,A6DR}}{=} 1 \cdot q = q$.
 - (b) $q \xrightarrow{1} q'$ for some $q' < q$. Then $p + q \xrightarrow{1} p + q'$ and therefore, since $p + q \Leftrightarrow q$, we need to have $p + q' \Leftrightarrow q'$. By induction we then have $\text{TCP}_{\text{drt}} \vdash p + q' = q'$. By Lemma A.7.4 we have $\text{TCP}_{\text{drt}} \vdash q = \underline{\sigma}.q' + q$. Then $\text{TCP}_{\text{drt}} \vdash p + q = 0 + q \stackrel{\text{DD}}{=} \underline{\sigma}.0 + q = \underline{\sigma}.0 + \underline{\sigma}.q' + q \stackrel{\text{A2,DRTF}}{=} \underline{\sigma}.(0 + q') + q = \underline{\sigma}.(p + q') + q = \underline{\sigma}.q' + q = q$.
4. $p \equiv 1$. From $p \downarrow$ we have $q \downarrow$. Therefore, by Lemma A.7.1, we have $\text{TCP}_{\text{drt}} \vdash q = \underline{1} + q$. Then $p \xrightarrow{1} p$, and therefore also $p + q \xrightarrow{1}$ and since $p + q \Leftrightarrow q$ also $q \xrightarrow{1}$. By Lemma A.7.3, we can distinguish two cases:
 - (a) $q \xrightarrow{1} q$. By Lemma A.7.5 we have $\text{TCP}_{\text{drt}} \vdash q = 1 \cdot q$. Then $\text{TCP}_{\text{drt}} \vdash p + q = 1 + q \stackrel{\text{A9DR}}{=} 1 \cdot \underline{1} + q = 1 \cdot \underline{1} + 1 \cdot q \stackrel{\text{DT6}}{=} 1 \cdot (\underline{1} + q) = 1 \cdot q = q$.
 - (b) $q \xrightarrow{1} q'$ for some $q' < q$. Then $p + q \xrightarrow{1} p + q'$ and therefore, since $p + q \Leftrightarrow q$, we need to have $p + q' \Leftrightarrow q'$. By induction we then have $\text{TCP}_{\text{drt}} \vdash p + q' = q'$. By Lemma A.7.4 we have $\text{TCP}_{\text{drt}} \vdash q = \underline{\sigma}.q' + q$. Then $\text{TCP}_{\text{drt}} \vdash p + q = 1 + q \stackrel{\text{A9DR}}{=} \underline{1} + \underline{\sigma}.1 + q \stackrel{\text{A1,A2}}{=} \underline{\sigma}.1 + q + \underline{1} = \underline{\sigma}.1 + q = \underline{\sigma}.1 + \underline{\sigma}.q' + q \stackrel{\text{A2,DRTF}}{=} \underline{\sigma}.(1 + q') + q = \underline{\sigma}.(p + q') + q = \underline{\sigma}.q' + q = q$.
5. $p \equiv \underline{a}.p'$ for some $a \in A$ and basic term p' . Then $p + q \xrightarrow{a} p'$, and since $p + q \Leftrightarrow q$ we have $q \xrightarrow{a} q'$ for some q' such that $p' \Leftrightarrow q'$. Then due to soundness of axiom A3 and congruence of bisimilarity w.r.t. alternative composition we also have $p' + q' \Leftrightarrow q'$ and $q' + p' \Leftrightarrow p'$. By induction we then have $\text{TCP}_{\text{drt}} \vdash p' + q = q'$ and $\text{TCP}_{\text{drt}} \vdash q' + p' = p'$. Therefore, we also have $\text{TCP}_{\text{drt}} \vdash p' = q' + p' = p' + q' = q'$. By Lemma A.7.2, we have $\text{TCP}_{\text{drt}} \vdash q = \underline{a}.q' + q$. Then, $\text{TCP}_{\text{drt}} \vdash p + q \equiv \underline{a}.p' + q \stackrel{\text{ih}}{=} \underline{a}.q' + q = q$.
6. $p \equiv a.p'$ for some $a \in A$ and basic term p' . Then $p + q \xrightarrow{a} p'$, and since $p + q \Leftrightarrow q$ we have $q \xrightarrow{a} q'$ for some q' such that $p' \Leftrightarrow q'$. Then due to soundness of axiom A3 and congruence of bisimilarity w.r.t. alternative composition we also have $p' + q' \Leftrightarrow q'$ and $q' + p' \Leftrightarrow p'$. By induction we then have $\text{TCP}_{\text{drt}} \vdash p' + q = q'$ and $\text{TCP}_{\text{drt}} \vdash q' + p' = p'$. Therefore, we also have $\text{TCP}_{\text{drt}} \vdash p' = q' + p' = p' + q' = q'$. By Lemma A.7.2, we have $\text{TCP}_{\text{drt}} \vdash q = \underline{a}.q' + q$. Also $p \xrightarrow{1} p$, and therefore also $p + q \xrightarrow{1}$ and since $p + q \Leftrightarrow q$ also $q \xrightarrow{1}$. By Lemma A.7.3, we can distinguish two cases:
 - (a) $q \xrightarrow{1} q$. By Lemma A.7.5 we have $\text{TCP}_{\text{drt}} \vdash q = 1 \cdot q$. Then $\text{TCP}_{\text{drt}} \vdash p + q = a.p' + q \stackrel{\text{DT3}}{=} 1 \cdot \underline{a}.p' + q = 1 \cdot \underline{a}.p' + 1 \cdot q \stackrel{\text{DT6}}{=} 1 \cdot (\underline{a}.p' + q) = 1 \cdot (\underline{a}.q' + q) = 1 \cdot q = q$.
 - (b) $q \xrightarrow{1} q''$ for some $q'' < q$. Then $p + q \xrightarrow{1} p + q''$ and therefore, since $p + q \Leftrightarrow q$, we need to have $p + q'' \Leftrightarrow q''$. By induction we then have $\text{TCP}_{\text{drt}} \vdash p + q'' = q''$. By Lemma A.7.4 we have $\text{TCP}_{\text{drt}} \vdash q = \underline{\sigma}.q' + q$. Then $\text{TCP}_{\text{drt}} \vdash p + q = a.p' + q \stackrel{\text{DT3}}{=} 1 \cdot \underline{a}.p' + q \stackrel{\text{DT1,A4,A8DR}}{=} \underline{a}.p' + \underline{\sigma}.1 \cdot \underline{a}.p' + q \stackrel{\text{DRA10,DT3}}{=} \underline{a}.p' + \underline{\sigma}.a.p' + q = \underline{a}.p' + \underline{\sigma}.p + q = \underline{a}.p' + \underline{\sigma}.p + \underline{\sigma}.q'' + q \stackrel{\text{DRTF}}{=} \underline{a}.p' + \underline{\sigma}.(p + q'') + q = \underline{a}.p' + \underline{\sigma}.q'' + q = \underline{a}.p' + q = \underline{a}.q' + q = q$.

7. $p \equiv \underline{\sigma}.p'$ for some basic term p' . From $p \xrightarrow{1} p'$, and the fact that $p + q \Leftrightarrow q$ it follows that there is some q' such that $q \xrightarrow{1} q'$ and $p' + q' \Leftrightarrow q'$. By induction we then have $\text{TCP}_{\text{drt}} \vdash p' + q' = q'$. Also, by Lemma A.7.4 we have $\text{TCP}_{\text{drt}} \vdash q = \underline{\sigma}.q' + q$. Then $\text{TCP}_{\text{drt}} \vdash p + q = \underline{\sigma}.p' + q = \underline{\sigma}.p' + \underline{\sigma}.q' + q \stackrel{\text{DRTF}}{=} \underline{\sigma}.(p' + q') + q = \underline{\sigma}.q' + q = q$.
8. $p \equiv p' + p''$ for some basic terms p' and p'' . From $p + q \Leftrightarrow q$ it follows that both $p' + q \Leftrightarrow q$ and $p'' + q \Leftrightarrow q$. Then, by induction it follows that $\text{TCP}_{\text{drt}} \vdash p' + q = q$ and $\text{TCP}_{\text{drt}} \vdash p'' + q = q$. Then, $\text{TCP}_{\text{drt}} \vdash p + q \equiv (p' + p'') + q \stackrel{\text{A2}}{=} p' + (p'' + q) \stackrel{\text{ih}}{=} p' + q \stackrel{\text{ih}}{=} q$.

□

A.3 Proof of conservativity of TCP_{drt} w.r.t. TCP

We cannot apply the meta-theorems for equational conservativity from the literature that rely on the operational conservativity of the term deduction systems (see [Ver94, FV98, AFV01, Mid01]) since there is a new transition relation that can be derived for some old terms.

Using Theorem 6 of [MR05b] (or [Mou05, Theorem 6.51]), to conclude that TCP_{drt} is an equationally conservative ground-extension of TCP in case we already know that both TCP and TCP_{drt} are sound and complete, it suffices to prove that the term deduction system for TCP_{drt} is an *orthogonal extension* of the term deduction system for TCP.

For the term deduction system for TCP_{drt} to be an orthogonal extension of the term deduction system of TCP, we need to prove that (1) the derivability of all old transition relations and predicates for old terms in the two term deduction systems coincides, and (2) that bisimilarity on old terms in the two term deduction systems coincides.

For the first proof obligation we have the following reasoning. All derivations in the term deduction system for TCP are also derivations in the term deduction system for TCP_{drt} since the deduction rules of the first are contained in the latter. For the other implication, note that all new deduction rules are either about the new transition relation $\xrightarrow{1}$ or about new syntax. Hence these can also not contribute to new facts about old terms and transition relations or predicates.

For the second proof obligation we have the following reasoning. First, note that with respect to the old transition relations and predicates, i.e. the action transitions and termination relation, the two term deduction systems coincide as reasoned before. Thus it remains to prove that also the new time transitions cannot discriminate between old terms.

We can prove (but won't do so explicitly) the following facts: (1) every closed TCP-term has a time transition, (2) for any time transition $p \xrightarrow{1} p'$ of an old term p , it holds that $p \Leftrightarrow p'$ w.r.t. the term deduction system for TCP_{drt} . For this latter statement we need to prove the statement that $p \downarrow$ implies $p \Leftrightarrow p + \underline{1}$ for closed TCP-terms.

B Theorems for TCP^\bullet

B.1 Proof of elimination theorem for TCP^\bullet

In this appendix we prove that any closed TCP^\bullet -term is derivably equal to a so-called basic term. A basic term is a term with a more restricted syntax than allowed by the signature of TCP^\bullet . Typically, sequential composition, parallel composition (and the auxiliary operators for parallel composition) and encapsulation do not occur in such basic terms.

Definition B.1 (Basic terms) *Basic terms are defined inductively as follows:*

1. $\dot{0}$ and $\dot{1}$ are basic terms;
2. 0 and 1 are basic terms;

3. for $a \in A$ and basic term p , $a.p$ is a basic term;
4. for basic terms p and q , $p + q$ is a basic term.

Theorem B.2 (Elimination of sequential composition) For basic terms p_1 and p_2 , there exists a basic term q such that $\text{TCP}^\bullet \vdash p_1 \cdot p_2 = q$.

Proof. By induction on the structure of basic term p_1 .

1. $p_1 \equiv \dot{0}$. Then $\text{TCP}^\bullet \vdash p_1 \cdot p_2 \equiv \dot{0} \cdot p_2 \stackrel{\text{A7}^\bullet}{\equiv} \dot{0}$.
2. $p_1 \equiv \dot{1}$. Then $\text{TCP}^\bullet \vdash p_1 \cdot p_2 \equiv \dot{1} \cdot p_2 \stackrel{\text{A8}^\bullet}{\equiv} p_2$.
3. $p_1 \equiv 0$. Then $\text{TCP}^\bullet \vdash p_1 \cdot p_2 = 0 \cdot p_2 \stackrel{\text{DOT2}}{\equiv} (1 \cdot \dot{0}) \cdot p_2 \stackrel{\text{A5}}{\equiv} 1 \cdot (\dot{0} \cdot p_2) \stackrel{\text{A7}^\bullet}{\equiv} 1 \cdot \dot{0} \stackrel{\text{DOT2}}{\equiv} 0$.
4. $p_1 \equiv 1$. By induction on the structure of basic term p we prove that there exists a basic term r such that $\text{TCP}^\bullet \vdash 1 \cdot p = r$.
 - (a) $p \equiv \dot{0}$. Then $\text{TCP}^\bullet \vdash 1 \cdot p \equiv 1 \cdot \dot{0} \stackrel{\text{DOT2}}{\equiv} 0$.
 - (b) $p \equiv \dot{1}$. Then $\text{TCP}^\bullet \vdash 1 \cdot p \equiv 1 \cdot \dot{1} \stackrel{\text{A9}^\bullet}{\equiv} 1$.
 - (c) $p \equiv 0$. Then $\text{TCP}^\bullet \vdash 1 \cdot p \equiv 1 \cdot 0 \stackrel{\text{DOT2}}{\equiv} 1 \cdot (1 \cdot \dot{0}) \stackrel{\text{A5}}{\equiv} (1 \cdot 1) \cdot \dot{0} \stackrel{\text{DOT3}}{\equiv} 1 \cdot \dot{0} \stackrel{\text{DOT2}}{\equiv} 0$.
 - (d) $p \equiv 1$. Then $\text{TCP}^\bullet \vdash 1 \cdot p \equiv 1 \cdot 1 \stackrel{\text{DOT3}}{\equiv} 1$.
 - (e) $p \equiv a.p'$ for some $a \in A$ and basic term p' . Then $\text{TCP}^\bullet \vdash 1 \cdot p \equiv 1 \cdot a.p' \stackrel{\text{DOT4}}{\equiv} a.p'$.
 - (f) $p \equiv p' + p''$ for some basic terms p' and p'' . By induction we have the existence of basic terms r' and r'' such that $\text{TCP}^\bullet \vdash 1 \cdot p' = r'$ and $\text{TCP}^\bullet \vdash 1 \cdot p'' = r''$. Then $\text{TCP}^\bullet \vdash 1 \cdot p \equiv 1 \cdot (p' + p'') \stackrel{\text{DOT5}}{\equiv} 1 \cdot p' + 1 \cdot p'' \stackrel{\text{ih}}{\equiv} r' + r''$.

Using this lemma we have the existence of a basic term r such that $\text{TCP}^\bullet \vdash 1 \cdot p_2 = r$. Then $\text{TCP}^\bullet \vdash p_1 \cdot p_2 \equiv 1 \cdot p_2 \stackrel{\text{lemma}}{\equiv} r$.

5. $p_1 \equiv a.p'_1$ for some $a \in A$ and basic term p'_1 . By induction we have the existence of a basic term q' such that $\text{TCP}^\bullet \vdash p'_1 \cdot p_2 = q'$. Then $\text{TCP}^\bullet \vdash p_1 \cdot p_2 \equiv a.p'_1 \cdot p_2 \stackrel{\text{A5}}{\equiv} a.(p'_1 \cdot p_2) \stackrel{\text{ih}}{\equiv} a.q'$.
6. $p_1 \equiv p'_1 + p''_1$ for some basic terms p'_1 and p''_1 . By induction we have the existence of basic terms r' and r'' such that $\text{TCP}^\bullet \vdash p'_1 \cdot p_2 = r'$ and $\text{TCP}^\bullet \vdash p''_1 \cdot p_2 = r''$. Then $\text{TCP}^\bullet \vdash p_1 \cdot p_2 \equiv (p'_1 + p''_1) \cdot p_2 \stackrel{\text{A4}}{\equiv} p'_1 \cdot p_2 + p''_1 \cdot p_2 = r' + r''$.

Observe that in each of the above cases the last term in the derivation is indeed a basic term. \square

Lemma B.3 (Representation I) For basic term p , $\text{TCP}^\bullet \vdash p = \dot{0}$, $\text{TCP}^\bullet \vdash p = \dot{1}$, or $\text{TCP}^\bullet \vdash p = p + 0$.

Proof. By induction on the structure of basic term p .

1. $p \equiv \dot{0}$. Trivial.
2. $p \equiv \dot{1}$. Trivial.
3. $p \equiv 0$. Then $\text{TCP}^\bullet \vdash p \equiv 0 \stackrel{\text{A3}}{\equiv} 0 + 0 \equiv p + 0$.
4. $p \equiv 1$. Then $\text{TCP}^\bullet \vdash p \equiv 1 \stackrel{\text{A9}^\bullet}{\equiv} 1 \cdot \dot{1} \stackrel{\text{A6}^\bullet}{\equiv} 1 \cdot (\dot{1} + \dot{0}) \stackrel{\text{DOT5}}{\equiv} 1 \cdot \dot{1} + 1 \cdot \dot{0} \stackrel{\text{A9}^\bullet}{\equiv} 1 + 1 \cdot \dot{0} \stackrel{\text{DOT2}}{\equiv} 1 + 0 \equiv p + 0$.
5. $p \equiv a.p'$ for some $a \in A$ and basic term p' . Then $\text{TCP}^\bullet \vdash p \equiv a.p' \stackrel{\text{DOT4}}{\equiv} 1 \cdot a.p' \stackrel{\text{A6}^\bullet}{\equiv} 1 \cdot (a.p' + \dot{0}) \stackrel{\text{DOT5}}{\equiv} 1 \cdot a.p' + 1 \cdot \dot{0} \stackrel{\text{DOT4}}{\equiv} a.p' + 1 \cdot \dot{0} \stackrel{\text{DOT2}}{\equiv} a.p' + 0 \equiv p + 0$.

6. $p \equiv p' + p''$ for some basic terms p' and p'' . By induction we have $\text{TCP}^\bullet \vdash p' = \dot{0}$, $\text{TCP}^\bullet \vdash p' = \dot{1}$, or $\text{TCP}^\bullet \vdash p' = p' + 0$. By induction we have $\text{TCP}^\bullet \vdash p'' = \dot{0}$, $\text{TCP}^\bullet \vdash p'' = \dot{1}$, or $\text{TCP}^\bullet \vdash p'' = p'' + 0$. Obviously, the following cases are to be considered:

- (a) $\text{TCP}^\bullet \vdash p' = \dot{0}$ and $\text{TCP}^\bullet \vdash p'' = \dot{0}$. Then $\text{TCP}^\bullet \vdash p \equiv p' + p'' = \dot{0} + \dot{0} \stackrel{\text{A3}}{\equiv} \dot{0}$.
- (b) $\text{TCP}^\bullet \vdash p' = \dot{0}$ and $\text{TCP}^\bullet \vdash p'' = \dot{1}$. Then $\text{TCP}^\bullet \vdash p \equiv p' + p'' = \dot{0} + \dot{1} \stackrel{\text{A1, A6}^\bullet}{\equiv} \dot{1}$.
- (c) $\text{TCP}^\bullet \vdash p' = \dot{0}$ and $\text{TCP}^\bullet \vdash p'' = p'' + 0$. Then $\text{TCP}^\bullet \vdash p \equiv p' + p'' = p' + (p'' + 0) \stackrel{\text{A2}}{\equiv} (p' + p'') + 0 \equiv p + 0$.
- (d) $\text{TCP}^\bullet \vdash p' = \dot{1}$ and $\text{TCP}^\bullet \vdash p'' = \dot{0}$. Similar to case 2.
- (e) $\text{TCP}^\bullet \vdash p' = \dot{1}$ and $\text{TCP}^\bullet \vdash p'' = \dot{1}$. Then $\text{TCP}^\bullet \vdash p \equiv p' + p'' = \dot{1} + \dot{1} \stackrel{\text{A3}}{\equiv} \dot{1}$.
- (f) $\text{TCP}^\bullet \vdash p' = \dot{1}$ and $\text{TCP}^\bullet \vdash p'' = p'' + 0$. Similar to case 3.
- (g) $\text{TCP}^\bullet \vdash p' = p' + 0$. Then $\text{TCP}^\bullet \vdash p \equiv p' + p'' = (p' + 0) + p'' \stackrel{\text{A2, A1, A2}}{\equiv} (p' + p'') + 0 \equiv p + 0$.

□

We define $|p|$ for basic terms p as follows: $|\dot{0}| = |\dot{1}| = |0| = |1| = 1$ and $|a.p| = |p| + 1$. We define $p < q$ as $|p| < |q|$.

Lemma B.4 (Representation II) *For basic term p , $\text{TCP}^\bullet \vdash p = \dot{0}$, $\text{TCP}^\bullet \vdash p = \dot{1}$, $\text{TCP}^\bullet \vdash p = 1 \cdot p$, or $\text{TCP}^\bullet \vdash p = q + \dot{1}$ for some basic term q such that $q < p$.*

Proof. By induction on the structure of basic term p .

- 1. $p \equiv \dot{0}$. Trivial.
- 2. $p \equiv \dot{1}$. Trivial.
- 3. $p \equiv 0$. Then $\text{TCP}^\bullet \vdash p \equiv 0 \stackrel{\text{DOT2}}{\equiv} 1 \cdot \dot{0} \stackrel{\text{DOT3}}{\equiv} (1 \cdot 1) \cdot \dot{0} \stackrel{\text{A5}}{\equiv} 1 \cdot (1 \cdot \dot{0}) \stackrel{\text{DOT2}}{\equiv} 1 \cdot 0 \equiv 1 \cdot p$.
- 4. $p \equiv 1$. Then $\text{TCP}^\bullet \vdash p \equiv 1 \stackrel{\text{DOT3}}{\equiv} 1 \cdot 1 \equiv 1 \cdot p$.
- 5. $p \equiv a.p'$ for some $a \in A$ and basic term p' . Then $\text{TCP}^\bullet \vdash p \equiv a.p' \stackrel{\text{DOT4}}{\equiv} 1 \cdot a.p' \equiv 1 \cdot p$.
- 6. $p \equiv p' + p''$ for some basic terms p' and p'' . Then, by induction we have $\text{TCP}^\bullet \vdash p' = \dot{0}$, $\text{TCP}^\bullet \vdash p' = \dot{1}$, $\text{TCP}^\bullet \vdash p' = 1 \cdot p'$ or $\text{TCP}^\bullet \vdash p' = q' + \dot{1}$ for some q' such that $q' < p'$. Also by induction we have $\text{TCP}^\bullet \vdash p'' = \dot{0}$, $\text{TCP}^\bullet \vdash p'' = \dot{1}$, $\text{TCP}^\bullet \vdash p'' = 1 \cdot p''$, or $\text{TCP}^\bullet \vdash p'' = q'' + \dot{1}$ for some q'' such that $q'' < p''$. The following cases can be distinguished
 - 1 $\text{TCP}^\bullet \vdash p' = \dot{0}$ and $\text{TCP}^\bullet \vdash p'' = \dot{0}$. Then $\text{TCP}^\bullet \vdash p \equiv p' + p'' \stackrel{\text{ih}}{\equiv} \dot{0} + \dot{0} \stackrel{\text{A3}}{\equiv} \dot{0}$.
 - 2 $\text{TCP}^\bullet \vdash p' = \dot{0}$ and $\text{TCP}^\bullet \vdash p'' = \dot{1}$. Then $\text{TCP}^\bullet \vdash p \equiv p' + p'' \stackrel{\text{ih}}{\equiv} \dot{0} + \dot{1} \stackrel{\text{A2}}{\equiv} \dot{1} + \dot{0} \stackrel{\text{A6}^\bullet}{\equiv} \dot{1}$.
 - 3 $\text{TCP}^\bullet \vdash p' = \dot{0}$ and $\text{TCP}^\bullet \vdash p'' = 1 \cdot p''$. Then $\text{TCP}^\bullet \vdash p \equiv p' + p'' \stackrel{\text{ih}}{\equiv} \dot{0} + 1 \cdot p'' \stackrel{\text{A1}}{\equiv} 1 \cdot p'' + \dot{0} \stackrel{\text{A6}^\bullet}{\equiv} 1 \cdot p'' \stackrel{\text{A6}^\bullet}{\equiv} 1 \cdot (p'' + \dot{0}) \stackrel{\text{A2}}{\equiv} 1 \cdot (\dot{0} + p'') \stackrel{\text{ih}}{\equiv} 1 \cdot (p' + p'') \equiv 1 \cdot p$.
 - 4 $\text{TCP}^\bullet \vdash p' = \dot{0}$ and $\text{TCP}^\bullet \vdash p'' = q'' + \dot{1}$ for some q'' such that $q'' < p''$. Then $\text{TCP}^\bullet \vdash p \equiv p' + p'' \stackrel{\text{ih}}{\equiv} \dot{0} + (q'' + \dot{1}) \stackrel{\text{A1, A2}}{\equiv} (\dot{0} + q'') + \dot{1} \stackrel{\text{A1, A6}^\bullet}{\equiv} q'' + \dot{1}$. Observe that $q'' < p$ follows from $q'' < p''$.
 - 5 $\text{TCP}^\bullet \vdash p' = \dot{1}$ and $\text{TCP}^\bullet \vdash p'' = \dot{0}$. Similar to case (2).
 - 6 $\text{TCP}^\bullet \vdash p' = \dot{1}$ and $\text{TCP}^\bullet \vdash p'' = \dot{1}$. Then $\text{TCP}^\bullet \vdash p \equiv p' + p'' \stackrel{\text{ih}}{\equiv} \dot{1} + \dot{1} \stackrel{\text{A3}}{\equiv} \dot{1}$.
 - 7 $\text{TCP}^\bullet \vdash p' = \dot{1}$ and $\text{TCP}^\bullet \vdash p'' = 1 \cdot p''$. Then $\text{TCP}^\bullet \vdash p \equiv p' + p'' \stackrel{\text{ih}}{\equiv} \dot{1} + 1 \cdot p'' \stackrel{\text{A1}}{\equiv} 1 \cdot p'' + \dot{1} = p'' + \dot{1}$. Observe that $p'' < p$.

- 8 $\text{TCP}^\bullet \vdash p' = \dot{1}$ and $\text{TCP}^\bullet \vdash p'' = q'' + \dot{1}$ for some q'' such that $q'' < p''$. Then $\text{TCP}^\bullet \vdash p \equiv p' + p'' \stackrel{\text{ih}}{=} \dot{1} + (q'' + \dot{1}) \stackrel{\text{A1-A3}}{=} q'' + \dot{1}$. Observe that $q'' < p$ follows from $q'' < p''$.
- 9 $\text{TCP}^\bullet \vdash p' = 1 \cdot p'$ and $\text{TCP}^\bullet \vdash p'' = \dot{0}$. Similar to case (3).
- 10 $\text{TCP}^\bullet \vdash p' = 1 \cdot p'$ and $\text{TCP}^\bullet \vdash p'' = \dot{1}$. Similar to case (7).
- 11 $\text{TCP}^\bullet \vdash p' = 1 \cdot p'$ and $\text{TCP}^\bullet \vdash p'' = 1 \cdot p''$. Then $\text{TCP}^\bullet \vdash p \equiv p' + p'' = 1 \cdot p' + 1 \cdot p'' \stackrel{\text{DOT5}}{=} 1 \cdot (p' + p'') \equiv 1 \cdot p$.
- 12 $\text{TCP}^\bullet \vdash p' = 1 \cdot p'$ and $\text{TCP}^\bullet \vdash p'' = q'' + \dot{1}$ for some basic term q'' such that $q'' < r''$. Then $\text{TCP}^\bullet \vdash p \equiv p' + p'' = p' + (q'' + \dot{1}) \stackrel{\text{A2}}{=} (p' + q'') + \dot{1}$. Note that $p' + q'' < p' + p''$ follows from $q'' < p''$.
- 13-16 $\text{TCP}^\bullet \vdash p' = q' + \dot{1}$ for some basic term q' such that $q' < p'$. Then $\text{TCP}^\bullet \vdash p \equiv p' + p'' = (q' + \dot{1}) + p'' \stackrel{\text{A1,A2}}{=} (q' + p'') + \dot{1}$. Note that $q' + p'' < p' + p''$ follows from $q' < p'$.

□

Theorem B.5 (Elimination of parallel composition operators) *For basic terms p_1 and p_2 ,*

1. *there exists a basic term q such that $\text{TCP}^\bullet \vdash p_1 \parallel p_2 = q$;*
2. *there exists a basic term q such that $\text{TCP}^\bullet \vdash p_1 | p_2 = q$;*
3. *there exists a basic term q such that $\text{TCP}^\bullet \vdash p_1 \parallel p_2 = q$.*

Proof. These three statements are proven simultaneously using induction on the number of symbols of basic terms p_1 and p_2 . When we apply an induction hypothesis we indicate to which statement it refers.

First we give the proof for statement (1), the elimination of \parallel . We use case distinction on the structure of basic term p_1 .

1. $p_1 \equiv \dot{0}$. $\text{TCP}^\bullet \vdash p_1 \parallel p_2 \equiv \dot{0} \parallel p_2 \stackrel{\text{LM1}^\bullet}{=} \dot{0}$.
2. $p_1 \equiv \dot{1}$. $\text{TCP}^\bullet \vdash p_1 \parallel p_2 \equiv \dot{1} \parallel p_2 \stackrel{\text{LM2}^\bullet}{=} \dot{0}$.
3. $p_1 \equiv 0$. According to Lemma B.3 we can distinguish three cases for p_2 :
 - (a) $\text{TCP}^\bullet \vdash p_2 = \dot{0}$. Then $\text{TCP}^\bullet \vdash p_1 \parallel p_2 = p_1 \parallel \dot{0} \stackrel{\text{LM5}^\bullet}{=} \dot{0}$.
 - (b) $\text{TCP}^\bullet \vdash p_2 = \dot{1}$. Then $\text{TCP}^\bullet \vdash p_1 \parallel p_2 = p_1 \parallel \dot{1} \stackrel{\text{A6}^\bullet, \text{A1}}{=} p_1 \parallel (\dot{0} + \dot{1}) \stackrel{\text{LM6}^\bullet}{=} p_1 \parallel \dot{0} \stackrel{\text{LM5}^\bullet}{=} \dot{0}$.
 - (c) $\text{TCP}^\bullet \vdash p_2 = p_2 + 0$. Then $\text{TCP}^\bullet \vdash p_1 \parallel p_2 \equiv 0 \parallel p_2 = 0 \parallel (p_2 + 0) \stackrel{\text{LM8}^\bullet}{=} 0$.
4. $p_1 \equiv 1$. According to Lemma B.3 we can distinguish three cases for p_2 :
 - (a) $\text{TCP}^\bullet \vdash p_2 = \dot{0}$. Then $\text{TCP}^\bullet \vdash p_1 \parallel p_2 = p_1 \parallel \dot{0} \stackrel{\text{LM5}^\bullet}{=} \dot{0}$.
 - (b) $\text{TCP}^\bullet \vdash p_2 = \dot{1}$. Then $\text{TCP}^\bullet \vdash p_1 \parallel p_2 = p_1 \parallel \dot{1} \stackrel{\text{A6}^\bullet, \text{A2}}{=} p_1 \parallel (\dot{0} + \dot{1}) \stackrel{\text{LM6}^\bullet}{=} p_1 \parallel \dot{0} \stackrel{\text{LM5}^\bullet}{=} \dot{0}$.
 - (c) $\text{TCP}^\bullet \vdash p_2 = p_2 + 0$. Then $\text{TCP}^\bullet \vdash p_1 \parallel p_2 \equiv 1 \parallel p_2 = 1 \parallel (p_2 + 0) \stackrel{\text{LM9}^\bullet}{=} 0$.
5. $p_1 \equiv a.p'_1$ for some $a \in A$ and basic term p'_1 . According to Lemma B.4 we can distinguish four cases for p_2 :
 - (a) $\text{TCP}^\bullet \vdash p_2 = \dot{0}$. Then $\text{TCP}^\bullet \vdash p_1 \parallel p_2 = p_1 \parallel \dot{0} \stackrel{\text{LM5}^\bullet}{=} \dot{0}$.
 - (b) $\text{TCP}^\bullet \vdash p_2 = \dot{1}$. Then $\text{TCP}^\bullet \vdash p_1 \parallel p_2 = p_1 \parallel \dot{1} \stackrel{\text{A6}^\bullet, \text{A1}}{=} p_1 \parallel (\dot{0} + \dot{1}) \stackrel{\text{LM6}^\bullet}{=} p_1 \parallel \dot{0} \stackrel{\text{LM5}^\bullet}{=} \dot{0}$.

- (c) $\text{TCP}^\bullet \vdash p_2 = 1 \cdot p_2$. By induction there exists a basic term q' such that $\text{TCP}^\bullet \vdash p_1 \parallel p_2 = q'$. Then $\text{TCP}^\bullet \vdash p_1 \parallel p_2 \equiv a.p'_1 \parallel p_2 = a.p'_1 \parallel (1 \cdot p_2) \stackrel{\text{LM3}^\bullet}{=} a.(p'_1 \parallel (1 \cdot p_2)) = a.(p'_1 \parallel p_2) \stackrel{\text{ih(3)}}{=} a.q'$.
- (d) $\text{TCP}^\bullet \vdash p_2 = p'_2 + \dot{1}$ for some basic term p'_2 such that $p'_2 < p_2$. By induction there exists a basic term q' such that $\text{TCP}^\bullet \vdash p_1 \parallel p'_2 = q'$. Then $\text{TCP}^\bullet \vdash p_1 \parallel p_2 = p_1 \parallel (p'_2 + \dot{1}) \stackrel{\text{LM6}^\bullet}{=} p_1 \parallel p'_2 \stackrel{\text{ih(1)}}{=} q'$.
- 6. $p_1 \equiv p'_1 + p''_1$. By induction there exist basic terms q' and q'' such that $\text{TCP}^\bullet \vdash p_1 \parallel p_2 = q'$ and $\text{TCP}^\bullet \vdash p'_1 \parallel p_2 = q''$. Then $\text{TCP}^\bullet \vdash p_1 \parallel p_2 \equiv (p'_1 + p''_1) \parallel p_2 \stackrel{\text{LM4}}{=} p'_1 \parallel p_2 + p''_1 \parallel p_2 \stackrel{\text{ih(1)}}{=} q' + q''$.

Then we prove statement (2), the elimination of $|$. We use case distinction on the structure of basic terms p_1 .

1. $p_1 \equiv \dot{0}$. $\text{TCP}^\bullet \vdash p_1 | p_2 \equiv \dot{0} | p_2 \stackrel{\text{CM1}^\bullet}{=} \dot{0}$.
2. $p_1 \equiv \dot{1}$. We use case distinction on the structure of basic term p_2 .
 - (a) $p_2 \equiv \dot{0}$. $\text{TCP}^\bullet \vdash p_1 | p_2 \equiv p_1 | \dot{0} \stackrel{\text{SC1}}{=} \dot{0} | p_1 \stackrel{\text{CM1}^\bullet}{=} \dot{0}$.
 - (b) $p_2 \equiv \dot{1}$. $\text{TCP}^\bullet \vdash p_1 | p_2 \equiv \dot{1} | \dot{1} \stackrel{\text{CM3}^\bullet}{=} \dot{1}$.
 - (c) $p_2 \equiv 0$. $\text{TCP}^\bullet \vdash p_1 | p_2 \equiv \dot{1} | 0 \stackrel{\text{SC1}}{=} 0 | \dot{1} \stackrel{\text{CM1b}}{=} \dot{0}$.
 - (d) $p_2 \equiv 1$. $\text{TCP}^\bullet \vdash p_1 | p_2 \equiv \dot{1} | 1 \stackrel{\text{SC1}}{=} 1 | \dot{1} \stackrel{\text{CM7}^\bullet}{=} \dot{1}$.
 - (e) $p_2 \equiv a.p'_2$ for some $a \in A$ and basic term p'_2 . $\text{TCP}^\bullet \vdash p_1 | p_2 \equiv \dot{1} | a.p'_2 \stackrel{\text{SC1}}{=} a.p'_2 | \dot{1} \stackrel{\text{CM4}^\bullet}{=} \dot{0}$.
 - (f) $p_2 \equiv p'_2 + p''_2$ for some basic terms p'_2 and p''_2 . By induction we have the existence of basic terms q' and q'' such that $\text{TCP}^\bullet \vdash p_1 | p'_2 = q'$ and $\text{TCP}^\bullet \vdash p_1 | p''_2 = q''$. Then $\text{TCP}^\bullet \vdash p_1 | p_2 \equiv p_1 | (p'_2 + p''_2) \stackrel{\text{SC1}}{=} (p'_2 + p''_2) | p_1 \stackrel{\text{CM2}}{=} p'_2 | p_1 + p''_2 | p_1 \stackrel{\text{SC1}}{=} p_1 | p'_2 + p_1 | p''_2 \stackrel{\text{ih(2)}}{=} q' + q''$.
3. $p_1 \equiv 0$. By Lemma B.3 we can distinguish three cases for p_2 .
 - (a) $\text{TCP}^\bullet \vdash p_2 = \dot{0}$. $\text{TCP}^\bullet \vdash p_1 | p_2 \equiv 0 | p_2 = 0 | \dot{0} \stackrel{\text{SC1}}{=} \dot{0} | 0 \stackrel{\text{CM1}^\bullet}{=} \dot{0}$.
 - (b) $\text{TCP}^\bullet \vdash p_2 = \dot{1}$. $\text{TCP}^\bullet \vdash p_1 | p_2 \equiv 0 | p_2 = 0 | \dot{1} \stackrel{\text{CM1b}}{=} \dot{0}$.
 - (c) $\text{TCP}^\bullet \vdash p_2 = p_2 + 0$. $\text{TCP}^\bullet \vdash p_1 | p_2 \equiv 0 | p_2 = 0 | (p_2 + 0) \stackrel{\text{CM1a}}{=} 0$.
4. $p_1 \equiv 1$. We use case distinction on the structure of basic term p_2 .
 - (a) $p_2 \equiv \dot{0}$. $\text{TCP}^\bullet \vdash p_1 | p_2 \equiv p_1 | \dot{0} \stackrel{\text{SC1}}{=} \dot{0} | p_1 \stackrel{\text{CM1}^\bullet}{=} \dot{0}$.
 - (b) $p_2 \equiv \dot{1}$. $\text{TCP}^\bullet \vdash p_1 | p_2 \equiv 1 | \dot{1} \stackrel{\text{CM7}^\bullet}{=} \dot{1}$.
 - (c) $p_2 \equiv 0$. We may use $\text{TCP}^\bullet \vdash 1 + 0 \stackrel{\text{A9}^\bullet}{=} 1 \cdot \dot{1} + 0 \stackrel{\text{DOT2}}{=} 1 \cdot \dot{1} + 1 \cdot \dot{0} \stackrel{\text{DOT5}}{=} 1 \cdot (\dot{1} + \dot{0}) \stackrel{\text{A6}^\bullet}{=} 1 \cdot \dot{1} \stackrel{\text{A9}^\bullet}{=} 1$
(*) $\text{TCP}^\bullet \vdash p_1 | p_2 \equiv 1 | 0 \stackrel{\text{SC1}}{=} 0 | 1 \stackrel{(*)}{=} 0 | (1 + 0) \stackrel{\text{CM1a}}{=} 0$.
 - (d) $p_2 \equiv 1$. $\text{TCP}^\bullet \vdash p_1 | p_2 \equiv 1 | 1 \stackrel{\text{CM3}}{=} 1$.
 - (e) $p_2 \equiv a.p'_2$ for some $a \in A$ and basic term p'_2 . $\text{TCP}^\bullet \vdash p_1 | p_2 \equiv 1 | a.p'_2 \stackrel{\text{SC1}}{=} a.p'_2 | 1 \stackrel{\text{CM4}}{=} 0$.
 - (f) $p_2 \equiv p'_2 + p''_2$ for some basic terms p'_2 and p''_2 . By induction we have the existence of basic terms q' and q'' such that $\text{TCP}^\bullet \vdash p_1 | p'_2 = q'$ and $\text{TCP}^\bullet \vdash p_1 | p''_2 = q''$. Then $\text{TCP}^\bullet \vdash p_1 | p_2 \equiv p_1 | (p'_2 + p''_2) \stackrel{\text{SC1}}{=} (p'_2 + p''_2) | p_1 \stackrel{\text{CM2}}{=} p'_2 | p_1 + p''_2 | p_1 \stackrel{\text{SC1}}{=} p_1 | p'_2 + p_1 | p''_2 \stackrel{\text{ih(2)}}{=} q' + q''$.
5. $p_1 \equiv a.p'_1$ for some $a \in A$ and basic term p'_1 . We use case distinction on the structure of basic term p_2 .

- (a) $p_2 \equiv \dot{0}$. $\text{TCP}^\bullet \vdash p_1 \mid p_2 \equiv p_1 \mid \dot{0} \stackrel{\text{SC1}}{\equiv} \dot{0} \mid p_1 \stackrel{\text{CM1}\bullet}{\equiv} \dot{0}$.
- (b) $p_2 \equiv \dot{1}$. $\text{TCP}^\bullet \vdash p_1 \mid p_2 \equiv a.p'_1 \mid \dot{1} \stackrel{\text{CM4}\bullet}{\equiv} \dot{0}$.
- (c) $p_2 \equiv 0$. We can derive $\text{TCP}^\bullet \vdash a.x \stackrel{\text{DOT4}}{\equiv} 1.a.x \stackrel{\text{A6}\bullet}{\equiv} 1.(a.x + \dot{0}) \stackrel{\text{DOT5}}{\equiv} 1.a.x + 1.\dot{0} \stackrel{\text{DOT4, DOT2}}{\equiv} a.x + 0$ (*). Then, $\text{TCP}^\bullet \vdash p_1 \mid p_2 \equiv a.p'_1 \mid 0 \stackrel{\text{SC1}}{\equiv} 0 \mid a.p'_1 \stackrel{(*)}{\equiv} 0 \mid (a.p'_1 + 0) \stackrel{\text{CM1a}}{\equiv} 0$.
- (d) $p_2 \equiv 1$. $\text{TCP}^\bullet \vdash p_1 \mid p_2 \equiv a.p'_1 \mid 1 \stackrel{\text{CM4}}{\equiv} 0$.
- (e) $p_2 \equiv b.p'_2$ for some $b \in A$ and basic term p'_2 . By the induction hypothesis for statement (3) we have the existence of basic term q' such that $\text{TCP}^\bullet \vdash p'_1 \parallel p'_2 = q'$. In case $\gamma(a, b)$ is not defined, $\text{TCP}^\bullet \vdash p_1 \mid p_2 \equiv a.p'_1 \mid b.p'_2 \stackrel{\text{CM6}\bullet}{\equiv} 0$. In case $\gamma(a, b) = c$, $\text{TCP}^\bullet \vdash p_1 \mid p_2 \equiv a.p'_1 \mid b.p'_2 \stackrel{\text{CM5}}{\equiv} c.(p'_1 \parallel p'_2) \stackrel{\text{ih}(3)}{\equiv} c.q'$.
- (f) $p_2 \equiv p'_2 + p''_2$ for some basic terms p'_2 and p''_2 . By induction we have the existence of basic terms q' and q'' such that $\text{TCP}^\bullet \vdash p_1 \mid p'_2 = q'$ and $\text{TCP}^\bullet \vdash p_1 \mid p''_2 = q''$. Then $\text{TCP}^\bullet \vdash p_1 \mid p_2 \equiv p_1 \mid (p'_2 + p''_2) \stackrel{\text{SC1}}{\equiv} (p'_2 + p''_2) \mid p_1 \stackrel{\text{CM2}}{\equiv} p'_2 \mid p_1 + p''_2 \mid p_1 \stackrel{\text{SC1}}{\equiv} p_1 \mid p'_2 + p_1 \mid p''_2 \stackrel{\text{ih}(2)}{\equiv} q' + q''$.
6. $p_1 \equiv p'_1 + p''_1$ for some basic terms p'_1 and p''_1 . By the induction hypothesis for statement (2), we have the existence of basic terms q' and q'' such that $\text{TCP}^\bullet \vdash p'_1 \mid p_2 = r'$ and $\text{TCP}^\bullet \vdash p''_1 \mid p_2 = r''$. Then $\text{TCP}^\bullet \vdash p_1 \mid p_2 \equiv (p'_1 + p''_1) \mid p_2 \stackrel{\text{CM2}}{\equiv} p'_1 \mid p_2 + p''_1 \mid p_2 \stackrel{\text{ih}(2)}{\equiv} r' + r''$.

Finally, statement (3) follows straightforwardly from the previous statements: By induction on statements (1) and (2) we have the existence of basic terms q_1 , q_2 , and q_3 such that $\text{TCP}^\bullet \vdash p_1 \parallel p_2 = q_1$, $\text{TCP}^\bullet \vdash p_2 \parallel p_1 = q_2$, and $\text{TCP}^\bullet \vdash p_1 \mid p_2 = q_3$. Then $\text{TCP}^\bullet \vdash p_1 \parallel p_2 \stackrel{\text{M}}{\equiv} p_1 \parallel p_2 + p_2 \parallel p_1 + p_1 \mid p_2 \stackrel{\text{ih}(1), \text{ih}(2)}}{\equiv} q_1 + q_2 + q_3$. \square

Theorem B.6 (Elimination of encapsulation) *For basic terms p and $H \subseteq A$, there exists a basic term q such that $\text{TCP}^\bullet \vdash \partial_H(p) = q$.*

Proof. Trivial, by induction on the structure of basic term p . \square

B.2 Completeness of TCP^\bullet

Note that the term deduction system for TCP^\bullet is such that all action and consistency transitions that can be derived starting from a basic term always result in a basic term. We do not prove this statement formally, and will use it silently in the remainder.

Lemma B.7 (Towards completeness) *For arbitrary closed TCP^\bullet -terms p and p' and arbitrary action $a \in A$*

1. if $p \downarrow$, then $\text{TCP}^\bullet \vdash p = \dot{1} + p$;
2. if $p \xrightarrow{a} p'$, then $\text{TCP}^\bullet \vdash p = a.p' + p$;
3. if $p \xrightarrow{0} p'$, then $p' \equiv p$ or $p' < p$;
4. if $p \xrightarrow{0} p'$, then $\text{TCP}^\bullet \vdash p = 1 \cdot p' + p$.

Proof. Easy; by induction on the structure of basic TCP^\bullet -term p . \square

Theorem B.8 *The process algebra TCP^\bullet is a complete axiomatization of strong bisimilarity on closed TCP^\bullet -terms.*

Proof. By the elimination theorem for TCP^\bullet it suffices to prove this theorem for basic terms only. We use induction on the structure of basic term p to prove that $p+q \Leftrightarrow q$ implies $\text{TCP}^\bullet \vdash p+q = q$.

1. $p \equiv \dot{0}$. Then $\text{TCP}^\bullet \vdash p+q \equiv \dot{0}+q \stackrel{\text{A1}}{=} q + \dot{0} \stackrel{\text{A6}^\bullet}{=} q$.
2. $p \equiv \dot{1}$. Then $p+q \downarrow$, and since $p+q \Leftrightarrow q$ also $q \downarrow$. By Lemma B.7.1, we have $\text{TCP}^\bullet \vdash q = \dot{1}+q$. Then, $\text{TCP}^\bullet \vdash p+q \equiv \dot{1}+q = q$.
3. $p \equiv 0$. Then $p+q \xrightarrow{0}$, and since $p+q \Leftrightarrow q$ also $q \xrightarrow{0} q'$ for some basic term q' . By Lemma B.7.4 we have $\text{TCP}^\bullet \vdash q = 1 \cdot q' + q$. Then, $\text{TCP}^\bullet \vdash p+q \equiv 0+q \stackrel{\text{DOT2}}{=} 1 \cdot \dot{0} + q = 1 \cdot \dot{0} + 1 \cdot q' + q \stackrel{\text{DOT5}}{=} 1 \cdot (\dot{0} + q') + q \stackrel{\text{A6}^\bullet}{=} 1 \cdot q' + q = q$.
4. $p \equiv 1$. Then $p+q \downarrow$, and since $p+q \Leftrightarrow q$ also $q \downarrow$. By Lemma B.7.1, we have $\text{TCP}^\bullet \vdash q = \dot{1}+q$. Also, $p+q \xrightarrow{0}$, and since $p+q \Leftrightarrow q$ also $q \xrightarrow{0}$. Based on Lemma B.7.3 we distinguish the following two cases.
 - (a) $q \xrightarrow{0} q$. By Lemma B.7.4 we have $\text{TCP}^\bullet \vdash q = 1 \cdot q + q$. Then $\text{TCP}^\bullet \vdash p+q \equiv 1+q = 1+1 \cdot q + q \stackrel{\text{A9}^\bullet}{=} 1 \cdot \dot{1} + 1 \cdot q + q \stackrel{\text{DOT5}}{=} 1 \cdot (\dot{1} + q) + q = 1 \cdot q + q = q$.
 - (b) $q \xrightarrow{0} q'$ for some basic term $q' < q$. We have $p+q' \Leftrightarrow q'$ and by induction we therefore obtain $\text{TCP}^\bullet \vdash p+q' = q'$. Also, by Lemma B.7.4, we have $\text{TCP}^\bullet \vdash q = 1 \cdot q' + q$. Then, $\text{TCP}^\bullet \vdash p+q \equiv 1+q \stackrel{\text{DOT3}}{=} 1 \cdot 1 + q = 1 \cdot 1 + 1 \cdot q' + q \stackrel{\text{DOT5}}{=} 1 \cdot (1 + q') + q = 1 \cdot (p+q') + q = 1 \cdot q' + q = q$.

Then, $\text{TCP}^\bullet \vdash p+q \equiv 1+q \stackrel{\text{A1}}{=} q+1 = q$.

5. $p \equiv a.p'$ for some $a \in A$ and basic term p' . Then $p+q \xrightarrow{a} p'$, and since $p+q \Leftrightarrow q$ we have $q \xrightarrow{a} q'$ for some q' such that $p' \Leftrightarrow q'$. Then due to soundness of axiom A3 and congruence of bisimilarity w.r.t. alternative composition we also have $p'+q' \Leftrightarrow q'$ and $q'+p' \Leftrightarrow p'$. By induction we then have $\text{TCP}^\bullet \vdash p'+q = q'$ and $\text{TCP}^\bullet \vdash q'+p' = p'$. Therefore, we also have $\text{TCP}^\bullet \vdash p' = q'+p' = p'+q' = q'$. By Lemma B.7.2, we have $\text{TCP}^\bullet \vdash q = a.q' + q$. Then, $\text{TCP}^\bullet \vdash p+q \equiv a.p' + q = a.q' + q = q$.
6. $p \equiv p'+p''$ for some basic terms p' and p'' . From $p+q \Leftrightarrow q$ it follows that both $p'+q \Leftrightarrow q$ and $p''+q \Leftrightarrow q$. Then, by induction it follows that $\text{TCP}^\bullet \vdash p'+q = q$ and $\text{TCP}^\bullet \vdash p''+q = q$. Then, $\text{TCP}^\bullet \vdash p+q \equiv (p'+p'')+q \stackrel{\text{A2}}{=} p' + (p''+q) \stackrel{\text{ih}}{=} p'+q \stackrel{\text{ih}}{=} q$.

□

B.3 Proof of conservativity of TCP^\bullet w.r.t. TCP

Again, using Theorem 6 of [MR05b] (or [Mou05, Theorem 6.51]), to conclude that TCP^\bullet is an equationally conservative ground-extension of TCP in case we already know that both TCP and TCP^\bullet are sound and complete, it suffices to prove that the term deduction system for TCP^\bullet is an *orthogonal extension* of the term deduction system for TCP.

For the term deduction system for TCP^\bullet to be an orthogonal extension of the term deduction system of TCP, we need to prove that (1) the derivability of all old transition relations and predicates for old terms in the two term deduction systems coincides, and (2) that bisimilarity on old terms in the two term deduction systems coincides.

For the first proof obligation we have the following reasoning. First, all derivations in the term deduction system for TCP are also derivations in the term deduction system for TCP^\bullet . This can

be seen as follows: We can easily prove that for any closed TCP-term p there is some term p' such that $p \xrightarrow{0} p'$ is derivable in the term deduction system for TCP^\bullet . As a consequence, the deduction rules defining action transitions in Table 6 reduce to the deduction rules that were omitted from the term deduction system of TCP. Thus all derivations from the term deduction system of TCP can be mimicked in the term deduction system for TCP^\bullet .

For the other implication, note that all new deduction rules are either about the new transition relation $\xrightarrow{0}$ or about new syntax or for closed TCP-terms reduce to rules from the term deduction system for TCP. Hence these can also not contribute to new facts about old terms and transition relations or predicates.

For the second proof obligation we have the following reasoning. First, note that with respect to the old transition relations and predicates, i.e. the action transitions and termination relation, the two term deduction systems coincide as reasoned before. Thus it remains to prove that also the new consistency transitions cannot discriminate between old terms.

We can prove (but won't do so explicitly) the following facts: (1) every closed TCP-term has a consistency transition, (2) for any consistency transition $p \xrightarrow{0} p'$ of an old term p , it holds that $p \rightleftharpoons p'$ w.r.t. the term deduction system for TCP^\bullet . For this latter statement we need to prove the statement that $p \downarrow$ implies $p \rightleftharpoons p + \dot{1}$ for closed TCP-terms.

C Theorems for $\text{TCP}_{\text{drt}}^\bullet$

C.1 Useful identities for $\text{TCP}_{\text{drt}}^\bullet$

Lemma C.1 *The following identities are derivable from $\text{TCP}_{\text{drt}}^\bullet$.*

1. $\text{TCP}_{\text{drt}}^\bullet \vdash \underline{0} \cdot x = \underline{0}$ (LU1);
2. $\text{TCP}_{\text{drt}}^\bullet \vdash 0 \cdot x = 0$ (LU2);
3. $\text{TCP}_{\text{drt}}^\bullet \vdash \underline{1} = \underline{1} + \underline{0}$ (U1);
4. $\text{TCP}_{\text{drt}}^\bullet \vdash \underline{a}.x = \underline{a}.x + \underline{0}$ (U2);
5. $\text{TCP}_{\text{drt}}^\bullet \vdash 0 = \underline{\sigma}.0$ (DD);
6. $\text{TCP}_{\text{drt}}^\bullet \vdash a.x = \underline{a}.x + \underline{\sigma}.a.x$ (DAP);
7. $\text{TCP}_{\text{drt}}^\bullet \vdash \underline{\sigma}.x \mid \underline{\sigma}.y = \underline{\sigma}.(x \mid y)$ (CM7DR \bullet);

Proof.

1. $\text{TCP}_{\text{drt}}^\bullet \vdash \underline{0} \cdot x \stackrel{\text{DR2}\bullet}{=} (\underline{1} \cdot \dot{0}) \cdot x \stackrel{\text{A5}}{=} \underline{1} \cdot (\dot{0} \cdot x) \stackrel{\text{A7}\bullet}{=} \underline{1} \cdot \dot{0} \stackrel{\text{DR2}\bullet}{=} \underline{0};$
2. $\text{TCP}_{\text{drt}}^\bullet \vdash 0 \cdot x \stackrel{\text{DOT2}}{=} (1 \cdot \dot{0}) \cdot x \stackrel{\text{A5}}{=} 1 \cdot (\dot{0} \cdot x) \stackrel{\text{A7}\bullet}{=} 1 \cdot \dot{0} \stackrel{\text{DOT2}}{=} 0;$
3. $\text{TCP}_{\text{drt}}^\bullet \vdash \underline{1} \stackrel{\text{DR4b}\bullet}{=} \underline{1} \cdot \underline{1} \stackrel{\text{A6}\bullet}{=} \underline{1} \cdot (\underline{1} + \dot{0}) \stackrel{\text{DR6}\bullet}{=} \underline{1} \cdot \underline{1} + \underline{1} \cdot \dot{0} \stackrel{\text{DR4b}\bullet}{=} \underline{1} + \underline{1} \cdot \dot{0} \stackrel{\text{DR2}\bullet}{=} \underline{1} + \underline{0};$
4. $\text{TCP}_{\text{drt}}^\bullet \vdash \underline{a}.x \stackrel{\text{DR3}\bullet}{=} \underline{1} \cdot \underline{a}.x \stackrel{\text{A6}\bullet}{=} \underline{1} \cdot (\underline{a}.x + \dot{0}) \stackrel{\text{DR6}\bullet}{=} \underline{1} \cdot \underline{a}.x + \underline{1} \cdot \dot{0} \stackrel{\text{DR3}\bullet}{=} \underline{a}.x + \underline{1} \cdot \dot{0} \stackrel{\text{DR2}\bullet}{=} \underline{a}.x + \underline{0};$
5. $\text{TCP}_{\text{drt}}^\bullet \vdash 0 \stackrel{\text{DOT2}}{=} 1 \cdot \dot{0} \stackrel{\text{DT1}}{=} (\underline{1} + \underline{\sigma}.1) \cdot \dot{0} \stackrel{\text{A4}}{=} \underline{1} \cdot \dot{0} + \underline{\sigma}.1 \cdot \dot{0} \stackrel{\text{DR2}\bullet}{=} \underline{0} + \underline{\sigma}.1 \cdot \dot{0} \stackrel{\text{DR10}}{=} \underline{\sigma} \cdot \dot{0} + \underline{\sigma}.1 \cdot \dot{0} \stackrel{\text{DRA10}}{=} \underline{\sigma} \cdot \dot{0} + \underline{\sigma} \cdot (1 \cdot \dot{0}) \stackrel{\text{DRTF}}{=} \underline{\sigma} \cdot (\dot{0} + 1 \cdot \dot{0}) \stackrel{\text{A1,A6}\bullet}{=} \underline{\sigma} \cdot (1 \cdot \dot{0}) \stackrel{\text{DOT2}}{=} \underline{\sigma} \cdot 0;$
6. $\text{TCP}_{\text{drt}}^\bullet \vdash a.x \stackrel{\text{DR9}}{=} 1 \cdot \underline{a}.x \stackrel{\text{DT1}}{=} (\underline{1} + \underline{\sigma}.1) \cdot \underline{a}.x \stackrel{\text{A4}}{=} \underline{1} \cdot \underline{a}.x + \underline{\sigma}.1 \cdot \underline{a}.x \stackrel{\text{DR3}\bullet}{=} \underline{a}.x + \underline{\sigma}.1 \cdot \underline{a}.x \stackrel{\text{DRA10}}{=} \underline{a}.x + \underline{\sigma} \cdot (1 \cdot \underline{a}.x) \stackrel{\text{DR9}}{=} \underline{a}.x + \underline{\sigma} \cdot \underline{a}.x;$
7. $\text{TCP}_{\text{drt}}^\bullet \vdash \underline{\sigma}.x \mid \underline{\sigma}.y \stackrel{\text{A6}\bullet, \text{A1}}{=} \underline{\sigma}.x \mid (\dot{0} + \underline{\sigma}.y) \stackrel{\text{RTO1}\bullet}{=} \underline{\sigma}.x \mid (v^1(\dot{0}) + \underline{\sigma}.y) \stackrel{\text{CM7DR}\bullet}{=} \underline{\sigma} \cdot (x \mid y);$

□

Lemma C.2 *The following identity is derivable from $\text{TCP}_{\text{drt}}^\bullet$ for basic terms p*

$$1. \text{TCP}_{\text{drt}}^\bullet \vdash \underline{\underline{1}} \cdot v^1(p) = v^1(\underline{\underline{1}} \cdot p) \quad (\text{TN}).$$

Proof. The statement is proven by induction on the structure of basic term p . □

C.2 Proof of elimination theorem for $\text{TCP}_{\text{drt}}^\bullet$

In this appendix we prove that any closed $\text{TCP}_{\text{srt}}^\bullet$ -term is derivably equal to a basic term.

Theorem C.3 (Elimination of sequential composition) *For basic terms p_1 and p_2 , there exists a basic term q such that $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \cdot p_2 = q$.*

Proof. By induction on the structure of basic term p_1 .

1. $p_1 \equiv \dot{0}$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \cdot p_2 \equiv \dot{0} \cdot p_2 \stackrel{\text{A7}^\bullet}{=} \dot{0}$.
2. $p_1 \equiv \dot{1}$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \cdot p_2 \equiv \dot{1} \cdot p_2 \stackrel{\text{A8}^\bullet}{=} \dot{1}$.
3. $p_1 \equiv \underline{\underline{0}}$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \cdot p_2 \equiv \underline{\underline{0}} \cdot p_2 \stackrel{\text{LU1}}{=} \underline{\underline{0}}$.
4. $p_1 \equiv \underline{\underline{1}}$. By induction on the structure of basic term p we prove that there exists a basic term r such that $\text{TCP}_{\text{drt}}^\bullet \vdash \underline{\underline{1}} \cdot p = r$.
 - (a) $p \equiv \dot{0}$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash \underline{\underline{1}} \cdot p \equiv \underline{\underline{1}} \cdot \dot{0} \stackrel{\text{DR2}^\bullet}{=} \underline{\underline{0}}$.
 - (b) $p \equiv \dot{1}$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash \underline{\underline{1}} \cdot p \equiv \underline{\underline{1}} \cdot \dot{1} \stackrel{\text{A9}^\bullet}{=} \underline{\underline{1}}$.
 - (c) $p \equiv \underline{\underline{0}}$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash \underline{\underline{1}} \cdot p \equiv \underline{\underline{1}} \cdot \underline{\underline{0}} \stackrel{\text{DR2}^\bullet}{=} \underline{\underline{1}} \cdot (\underline{\underline{1}} \cdot \dot{0}) \stackrel{\text{A5}}{=} (\underline{\underline{1}} \cdot \underline{\underline{1}}) \cdot \dot{0} \stackrel{\text{DR4b}^\bullet}{=} \underline{\underline{1}} \cdot \dot{0} \stackrel{\text{DR2}^\bullet}{=} \underline{\underline{0}}$.
 - (d) $p \equiv \underline{\underline{1}}$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash \underline{\underline{1}} \cdot p \equiv \underline{\underline{1}} \cdot \underline{\underline{1}} \stackrel{\text{DR4b}^\bullet}{=} \underline{\underline{1}}$.
 - (e) $p \equiv 0$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash \underline{\underline{1}} \cdot p \equiv \underline{\underline{1}} \cdot 0 \stackrel{\text{DOT2}}{=} \underline{\underline{1}} \cdot (1 \cdot \dot{0}) \stackrel{\text{A5}}{=} (\underline{\underline{1}} \cdot 1) \cdot \dot{0} \stackrel{\text{DR4a}^\bullet}{=} 1 \cdot \dot{0} \stackrel{\text{DOT2}}{=} 0$.
 - (f) $p \equiv 1$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash \underline{\underline{1}} \cdot p \equiv \underline{\underline{1}} \cdot 1 \stackrel{\text{DR4a}^\bullet}{=} 1$.
 - (g) $p \equiv \underline{a}.p'$ for some $a \in A$ and basic term p' . Then $\text{TCP}_{\text{drt}}^\bullet \vdash \underline{\underline{1}} \cdot p \equiv \underline{\underline{1}} \cdot (\underline{a}.p') \stackrel{\text{DR3}^\bullet}{=} \underline{\underline{1}} \cdot \underline{a}.p'$.
 - (h) $p \equiv a.p'$ for some $a \in A$ and basic term p' . Then $\text{TCP}_{\text{drt}}^\bullet \vdash \underline{\underline{1}} \cdot p \equiv \underline{\underline{1}} \cdot a.p' \stackrel{\text{DR9}}{=} \underline{\underline{1}} \cdot (1 \cdot \underline{a}.p') \stackrel{\text{A5}}{=} (\underline{\underline{1}} \cdot 1) \cdot \underline{a}.p' \stackrel{\text{DR4a}^\bullet}{=} 1 \cdot \underline{a}.p' \stackrel{\text{DR9}}{=} a.p'$.
 - (i) $p \equiv \underline{\sigma}.p'$ for some basic term p' . Then, $\text{TCP}_{\text{drt}}^\bullet \vdash \underline{\underline{1}} \cdot p \equiv \underline{\underline{1}} \cdot \underline{\sigma}.p' \stackrel{\text{DR5}^\bullet}{=} \underline{\underline{\sigma}}.p'$.
 - (j) $p \equiv p' + p''$ for some basic terms p' and p'' . By induction we have the existence of basic terms r' and r'' such that $\text{TCP}_{\text{drt}}^\bullet \vdash \underline{\underline{1}} \cdot p' = r'$ and $\text{TCP}_{\text{drt}}^\bullet \vdash \underline{\underline{1}} \cdot p'' = r''$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash \underline{\underline{1}} \cdot p \equiv \underline{\underline{1}} \cdot (p' + p'') \stackrel{\text{DR6}^\bullet}{=} \underline{\underline{1}} \cdot p' + \underline{\underline{1}} \cdot p'' \stackrel{\text{ih}}{=} r' + r''$.

Using this lemma we have the existence of a basic term r such that $\text{TCP}_{\text{drt}}^\bullet \vdash \underline{\underline{1}} \cdot p_2 = r$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \cdot p_2 \equiv \underline{\underline{1}} \cdot p_2 \stackrel{\text{lemma}}{=} r$.

$$5. p_1 \equiv 0. \text{Then } \text{TCP}_{\text{drt}}^\bullet \vdash p_1 \cdot p_2 = 0 \cdot p_2 \stackrel{\text{LU2}}{=} 0.$$

6. $p_1 \equiv 1$. By induction on the structure of basic term p we prove that there exists a basic term r such that $\text{TCP}_{\text{drt}}^\bullet \vdash 1 \cdot p = r$.

- (a) $p \equiv \dot{0}$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash 1 \cdot p \equiv 1 \cdot \dot{0} \stackrel{\text{DOT2}}{\equiv} 0$.
- (b) $p \equiv \dot{1}$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash 1 \cdot p \equiv 1 \cdot \dot{1} \stackrel{\text{A9}^\bullet}{\equiv} 1$.
- (c) $p \equiv \underline{0}$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash 1 \cdot p \equiv 1 \cdot \underline{0} \stackrel{\text{DR2}^\bullet}{\equiv} 1 \cdot (\underline{1} \cdot \dot{0}) \stackrel{\text{A5}}{\equiv} (1 \cdot \underline{1}) \cdot \dot{0} \stackrel{\text{DR8}}{\equiv} 1 \cdot \dot{0} \stackrel{\text{DOT2}}{\equiv} 0$.
- (d) $p \equiv \underline{1}$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash 1 \cdot p \equiv 1 \cdot \underline{1} \stackrel{\text{DR8}}{\equiv} 1$.
- (e) $p \equiv 0$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash 1 \cdot p \equiv 1 \cdot 0 \stackrel{\text{DOT2}}{\equiv} 1 \cdot (1 \cdot \dot{0}) \stackrel{\text{A5}}{\equiv} (1 \cdot 1) \cdot \dot{0} \stackrel{\text{DOT3}}{\equiv} 1 \cdot \dot{0} \stackrel{\text{DOT2}}{\equiv} 0$.
- (f) $p \equiv 1$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash 1 \cdot p \equiv 1 \cdot 1 \stackrel{\text{DOT3}}{\equiv} 1$.
- (g) $p \equiv \underline{a}.p'$ for some $a \in A$ and basic term p' . Then $\text{TCP}_{\text{drt}}^\bullet \vdash 1 \cdot p \equiv 1 \cdot \underline{a}.p' \stackrel{\text{DR9}}{\equiv} a.p'$.
- (h) $p \equiv a.p'$ for some $a \in A$ and basic term p' . Then $\text{TCP}_{\text{drt}}^\bullet \vdash 1 \cdot p \equiv 1 \cdot a.p' \stackrel{\text{DOT4}}{\equiv} a.p'$.
- (i) $p \equiv \underline{\sigma}.p'$ for some basic term p' . By induction we have the existence of a basic term r' such that $\text{TCP}_{\text{drt}}^\bullet \vdash 1 \cdot p' = r'$. Then, $\text{TCP}_{\text{drt}}^\bullet \vdash 1 \cdot p \equiv 1 \cdot \underline{\sigma}.p' \stackrel{\text{DR7}^\bullet}{\equiv} \underline{\sigma} \cdot (1 \cdot p') = \underline{\sigma}.r'$.
- (j) $p \equiv p' + p''$ for some basic terms p' and p'' . By induction we have the existence of basic terms r' and r'' such that $\text{TCP}_{\text{drt}}^\bullet \vdash 1 \cdot p' = r'$ and $\text{TCP}_{\text{drt}}^\bullet \vdash 1 \cdot p'' = r''$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash 1 \cdot p \equiv 1 \cdot (p' + p'') \stackrel{\text{DOT5}}{\equiv} 1 \cdot p' + 1 \cdot p'' \stackrel{\text{ih}}{\equiv} r' + r''$.

Using this lemma we have the existence of a basic term r such that $\text{TCP}_{\text{drt}}^\bullet \vdash 1 \cdot p_2 = r$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \cdot p_2 \equiv 1 \cdot p_2 \stackrel{\text{lemma}}{\equiv} r$.

- 7. $p_1 \equiv \underline{a}.p'_1$ for some $a \in A$ and basic term p'_1 . By induction we have the existence of a basic term q' such that $\text{TCP}_{\text{drt}}^\bullet \vdash p'_1 \cdot p_2 = q'$. Then $\text{TCP}^\bullet \vdash p_1 \cdot p_2 \equiv \underline{a}.p'_1 \cdot p_2 \stackrel{\text{A10DR}}{\equiv} \underline{a} \cdot (p'_1 \cdot p_2) \stackrel{\text{ih}}{\equiv} \underline{a}.q'$.
- 8. $p_1 \equiv a.p'_1$ for some $a \in A$ and basic term p'_1 . By induction we have the existence of a basic term q' such that $\text{TCP}_{\text{drt}}^\bullet \vdash p'_1 \cdot p_2 = q'$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \cdot p_2 \equiv a.p'_1 \cdot p_2 \stackrel{\text{A10}}{\equiv} a \cdot (p'_1 \cdot p_2) \stackrel{\text{ih}}{\equiv} a.q'$.
- 9. $p_1 \equiv \underline{\sigma}.p'_1$ for some basic term p'_1 . By induction we have the existence of a basic term q' such that $\text{TCP}_{\text{drt}}^\bullet \vdash p'_1 \cdot p_2 = q'$. Then $\text{TCP}^\bullet \vdash p_1 \cdot p_2 \equiv \underline{\sigma}.p'_1 \cdot p_2 \stackrel{\text{DRA10}}{\equiv} \underline{\sigma} \cdot (p'_1 \cdot p_2) \stackrel{\text{ih}}{\equiv} \underline{\sigma}.q'$.
- 10. $p_1 \equiv p'_1 + p''_1$ for some basic terms p'_1 and p''_1 . By induction we have the existence of basic terms r' and r'' such that $\text{TCP}_{\text{drt}}^\bullet \vdash p'_1 \cdot p_2 = r'$ and $\text{TCP}_{\text{drt}}^\bullet \vdash p''_1 \cdot p_2 = r''$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \cdot p_2 \equiv (p'_1 + p''_1) \cdot p_2 \stackrel{\text{A4}}{\equiv} p'_1 \cdot p_2 + p''_1 \cdot p_2 \stackrel{\text{ih}}{\equiv} r' + r''$.

Observe that in each of the above cases the last term in the derivation is indeed a basic term. \square

Lemma C.4 (Representation) For basic term p ,

- 1. $\text{TCP}_{\text{drt}}^\bullet \vdash p = \dot{0}$,
- 2. $\text{TCP}_{\text{drt}}^\bullet \vdash p = \dot{1}$,
- 3. $\text{TCP}_{\text{drt}}^\bullet \vdash p = \underline{1} \cdot p$, or
- 4. $\text{TCP}_{\text{drt}}^\bullet \vdash p = \underline{1} \cdot p + \dot{1}$.

Proof. Trivial, by induction on the structure of basic term p . \square

Lemma C.5 (Representation) For basic term p ,

1. $\text{TCP}_{\text{drt}}^\bullet \vdash p = \dot{0}$,
2. $\text{TCP}_{\text{drt}}^\bullet \vdash p = \dot{1}$, or
3. $\text{TCP}_{\text{drt}}^\bullet \vdash p = p + \underline{0}$.

Proof. According to Lemma C.4 we can distinguish four cases for basic term p .

1. $\text{TCP}_{\text{drt}}^\bullet \vdash p = \dot{0}$. Trivial.
2. $\text{TCP}_{\text{drt}}^\bullet \vdash p = \dot{1}$. trivial.
3. $\text{TCP}_{\text{drt}}^\bullet \vdash p = \underline{1} \cdot p$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p = \underline{1} \cdot p \stackrel{\text{A6}\bullet}{=} \underline{1} \cdot (p + \dot{0}) \stackrel{\text{DR6}\bullet}{=} \underline{1} \cdot p + \underline{1} \cdot \dot{0} \stackrel{\text{DR2}\bullet}{=} \underline{1} \cdot p + \underline{0} = p + \underline{0}$.
4. $\text{TCP}_{\text{drt}}^\bullet \vdash p = \underline{1} \cdot p + \dot{1}$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p = \underline{1} \cdot p + \dot{1} \stackrel{\text{A6}\bullet}{=} \underline{1} \cdot (p + \dot{0}) + \dot{1} \stackrel{\text{DR6}\bullet}{=} \underline{1} \cdot p + \underline{1} \cdot \dot{0} + \dot{1} \stackrel{\text{DR2}\bullet}{=} \underline{1} \cdot p + \underline{0} + \dot{1} \stackrel{\text{A1}, \text{A2}}{=} \underline{1} \cdot p + \dot{1} + \underline{0} = p + \underline{0}$.

□

We define $|_|_$ for basic terms p and q as follows: $|\dot{0}| = |\dot{1}| = |\underline{0}| = |\underline{1}| = |0| = |1| = 1$, $|\underline{a} \cdot p| = |a \cdot p| = |\underline{\sigma} \cdot p| = |p| + 1$, and $|p + q| = |p| + |q|$. We define $p \leq q$ as $|p| \leq |q|$ and $p < q$ as $|p| < |q|$.

Lemma C.6 (Representation) For basic term p ,

1. $\text{TCP}_{\text{drt}}^\bullet \vdash p = \dot{0}$,
2. $\text{TCP}_{\text{drt}}^\bullet \vdash p = \dot{1}$, or
3. $\text{TCP}_{\text{drt}}^\bullet \vdash p = v^1(p') + \underline{\sigma} \cdot p''$ for some basic $\text{TCP}_{\text{drt}}^\bullet$ -terms p' and p'' such that $p'' \leq p$.

Proof. By induction on the structure of basic term p .

1. $p \equiv \dot{0}$. Trivial.
2. $p \equiv \dot{1}$. Trivial.
3. $p \equiv \underline{0}$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p \equiv \underline{0} \stackrel{\text{A6}\bullet, \text{A1}}{=} \dot{0} + \underline{0} \stackrel{\text{RTO1}\bullet, \text{DR10}}{=} v^1(\dot{0}) + \underline{\sigma} \cdot \dot{0}$. Note that $\dot{0} \leq \underline{0}$.
4. $p \equiv \underline{1}$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p \equiv \underline{1} \stackrel{\text{U1}}{=} \underline{1} + \underline{0} \stackrel{\text{RTO1}, \text{DR10}}{=} v^1(\underline{1}) + \underline{\sigma} \cdot \dot{0}$. Note that $\dot{0} \leq \underline{1}$.
5. $p \equiv 0$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p \equiv 0 \stackrel{\text{DOT2}}{=} 1 \cdot \dot{0} \stackrel{\text{DT1}}{=} (\underline{1} + \underline{\sigma} \cdot 1) \cdot \dot{0} \stackrel{\text{A4}}{=} \underline{1} \cdot \dot{0} + \underline{\sigma} \cdot 1 \cdot \dot{0} \stackrel{\text{DR2}\bullet, \text{DR7}\bullet}{=} \underline{0} + \underline{\sigma} \cdot (1 \cdot \dot{0}) \stackrel{\text{RTO5}, \text{DOT2}}{=} v^1(\underline{\sigma} \cdot \dot{0}) + \underline{\sigma} \cdot 0$. Observe that $0 \leq 0$.
6. $p \equiv 1$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p \equiv 1 \stackrel{\text{DT1}}{=} \underline{1} + \underline{\sigma} \cdot 1 \stackrel{\text{RTO2}}{=} v^1(\underline{1}) + \underline{\sigma} \cdot 1$. Observe that $1 \leq 1$.
7. $p \equiv \underline{a} \cdot p'$ for some $a \in A$ and basic term p' . Then $\text{TCP}_{\text{drt}}^\bullet \vdash p \equiv \underline{a} \cdot p' \stackrel{\text{U2}}{=} \underline{a} \cdot p' + \underline{0} \stackrel{\text{RTO3}, \text{DR10}}{=} v^1(\underline{a} \cdot p') + \underline{\sigma} \cdot \dot{0}$. Observe that $\dot{0} \leq \underline{a} \cdot p'$.
8. $p \equiv a \cdot p'$ for some $a \in A$ and basic term p' . Then $\text{TCP}_{\text{drt}}^\bullet \vdash p \equiv a \cdot p' \stackrel{\text{DAP}}{=} \underline{a} \cdot p' + \underline{\sigma} \cdot a \cdot p' \stackrel{\text{RTO3}}{=} v^1(\underline{a} \cdot p') + \underline{\sigma} \cdot a \cdot p'$. Note that $a \cdot p' \leq a \cdot p'$.
9. $p \equiv \underline{\sigma} \cdot p'$ for some basic term p' . Then $\text{TCP}_{\text{drt}}^\bullet \vdash p \equiv \underline{\sigma} \cdot p' \stackrel{\text{A6}\bullet, \text{A1}}{=} \dot{0} + \underline{\sigma} \cdot p' \stackrel{\text{RTO1}\bullet}{=} v^1(\dot{0}) + \underline{\sigma} \cdot p'$. Note that $p' \leq \underline{\sigma} \cdot p'$.

10. $p \equiv p' + p''$ for some basic term $p' + p''$. By induction, for both p' and p'' we have three cases. In case $\text{TCP}_{\text{drt}}^\bullet \vdash p' = \dot{0}$ or $\text{TCP}_{\text{drt}}^\bullet \vdash p'' = \dot{0}$, the proof is trivial and therefore omitted. The following are the remaining cases:

- (a) $\text{TCP}_{\text{drt}}^\bullet \vdash p' = \dot{1}$ and $\text{TCP}_{\text{drt}}^\bullet \vdash p'' = \dot{1}$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p \equiv p' + p'' = \dot{1} + \dot{1} \stackrel{\text{A3}}{\equiv} \dot{1}$.
- (b) $\text{TCP}_{\text{drt}}^\bullet \vdash p' = v^1(p'_1) + \underline{\sigma}.p'_2$ for some basic TCP_{drt} -terms p'_1 and p'_2 such that $p'_2 \leq p'$ and $\text{TCP}_{\text{drt}}^\bullet \vdash p'' = \dot{1}$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p \equiv p' + p'' = (v^1(p'_1) + \underline{\sigma}.p'_2) + \dot{1} \stackrel{\text{RTO2}\bullet}{\equiv} (v^1(p'_1) + \underline{\sigma}.p'_2) + v^1(\dot{1}) \stackrel{\text{A1,A2}}{\equiv} (v^1(p'_1) + v^1(\dot{1})) + \underline{\sigma}.p'_2 \stackrel{\text{RTO4}}{\equiv} v^1(p'_1 + \dot{1}) + \underline{\sigma}.p'_2 = v^1(p'_1 + p'') + \underline{\sigma}.p'_2$. Note that $p'_2 \leq p$.
- (c) $\text{TCP}_{\text{drt}}^\bullet \vdash p' = \dot{1}$ and $\text{TCP}_{\text{drt}}^\bullet \vdash p'' = v^1(p''_1) + \underline{\sigma}.p''_2$ for some basic TCP_{drt} -terms p''_1 and p''_2 such that $p''_2 \leq p''$. Similar to the previous case.
- (d) $\text{TCP}_{\text{drt}}^\bullet \vdash p' = v^1(p'_1) + \underline{\sigma}.p'_2$ for some basic TCP_{drt} -terms p'_1 and p'_2 such that $p'_2 \leq p'$ and $\text{TCP}_{\text{drt}}^\bullet \vdash p'' = v^1(p''_1) + \underline{\sigma}.p''_2$ for some basic TCP_{drt} -terms p''_1 and p''_2 such that $p''_2 \leq p''$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p \equiv p' + p'' = (v^1(p'_1) + \underline{\sigma}.p'_2) + (v^1(p''_1) + \underline{\sigma}.p''_2) \stackrel{\text{A1,A2}}{\equiv} (v^1(p'_1) + v^1(p''_1)) + (\underline{\sigma}.p'_2 + \underline{\sigma}.p''_2) \stackrel{\text{RTO4,DRTF}}{\equiv} v^1(p'_1 + p''_1) + \underline{\sigma}.(p'_2 + p''_2)$ and note that $p'_2 + p''_2 \leq p$.

□

Lemma C.7 (Representation) *For basic term p ,*

- 1. $\text{TCP}_{\text{drt}}^\bullet \vdash p = \dot{0}$,
- 2. $\text{TCP}_{\text{drt}}^\bullet \vdash p = \dot{1}$,
- 3. $\text{TCP}_{\text{drt}}^\bullet \vdash p = v^1(p') + \underline{\sigma}.p''$ for some basic $\text{TCP}_{\text{drt}}^\bullet$ -terms p' and p'' such that $p'' < p$ or $p'' \equiv \dot{0}$, or
- 4. $\text{TCP}_{\text{drt}}^\bullet \vdash p = 1 \cdot v^1(p')$ for some basic term p' such that $p' \leq p$.

Proof. By induction on the structure of basic term p .

- 1. $p \equiv \dot{0}$. Trivial.
- 2. $p \equiv \dot{1}$. Trivial.
- 3. $p \equiv \underline{0}$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p \equiv \underline{0} \stackrel{\text{A6}\bullet, \text{A1}}{\equiv} \dot{0} + \underline{0} \stackrel{\text{RTO1}\bullet, \text{DR10}}{\equiv} v^1(\dot{0}) + \underline{\sigma}.\dot{0}$.
- 4. $p \equiv \underline{1}$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p \equiv \underline{1} \stackrel{\text{U1}}{\equiv} \underline{1} + \underline{0} \stackrel{\text{RTO1}, \text{DR10}}{\equiv} v^1(\underline{1}) + \underline{\sigma}.\dot{0}$.
- 5. $p \equiv 0$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p \equiv 0 \stackrel{\text{DOT2}}{\equiv} 1 \cdot \dot{0} \stackrel{\text{RTO1}\bullet}{\equiv} 1 \cdot v^1(\dot{0})$. Note that $\dot{0} \leq 0$.
- 6. $p \equiv 1$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p \equiv 1 \stackrel{\text{DR8}}{\equiv} 1 \cdot \underline{1} \stackrel{\text{RTO2}}{\equiv} 1 \cdot v^1(\underline{1})$. Note that $\underline{1} \leq 1$.
- 7. $p \equiv \underline{a}.p'$ for some $a \in A$ and basic term p' . Then $\text{TCP}_{\text{drt}}^\bullet \vdash p \equiv \underline{a}.p' \stackrel{\text{U2}}{\equiv} \underline{a}.p' + \underline{0} \stackrel{\text{RTO3}, \text{DR10}}{\equiv} v^1(\underline{a}.p') + \underline{\sigma}.\dot{0}$.
- 8. $p \equiv a.p'$ for some $a \in A$ and basic term p' . Then $\text{TCP}_{\text{drt}}^\bullet \vdash p \equiv a.p' \stackrel{\text{DT3}}{\equiv} 1 \cdot \underline{a}.p' \stackrel{\text{RTO3}}{\equiv} 1 \cdot v^1(\underline{a}.p')$. Note that $\underline{a}.p' \leq a.p'$.
- 9. $p \equiv \underline{\sigma}.p'$ for some basic term p' . Then $\text{TCP}_{\text{drt}}^\bullet \vdash p \equiv \underline{\sigma}.p' \stackrel{\text{A6}\bullet, \text{A1}}{\equiv} \dot{0} + \underline{\sigma}.p' \stackrel{\text{RTO1}\bullet}{\equiv} v^1(\dot{0}) + \underline{\sigma}.p'$. Observe that indeed $p' < p$.

10. $p \equiv p' + p''$ for some basic term $p' + p''$. By induction, for both p' and p'' we have four cases. In case $\text{TCP}_{\text{drt}}^\bullet \vdash p' = \dot{0}$ or $\text{TCP}_{\text{drt}}^\bullet \vdash p'' = \dot{0}$, the proof is trivial and therefore omitted. The following are the remaining cases:

- (a) $\text{TCP}_{\text{drt}}^\bullet \vdash p' = \dot{1}$ and $\text{TCP}_{\text{drt}}^\bullet \vdash p'' = \dot{1}$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p \equiv p' + p'' = \dot{1} + \dot{1} \stackrel{\text{A3}}{\equiv} \dot{1}$.
- (b) $\text{TCP}_{\text{drt}}^\bullet \vdash p' = v^1(p'_1) + \underline{\sigma}.p'_2$ for some basic TCP_{drt} -terms p'_1 and p'_2 such that $p'_2 < p'$ or $p'_2 \equiv \dot{0}$ and $\text{TCP}_{\text{drt}}^\bullet \vdash p'' = \dot{1}$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p \equiv p' + p'' = (v^1(p'_1) + \underline{\sigma}.p'_2) + \dot{1} \stackrel{\text{RTO2}^\bullet}{\equiv} (v^1(p'_1) + \underline{\sigma}.p'_2) + v^1(\dot{1}) \stackrel{\text{A1,A2}}{\equiv} (v^1(p'_1) + v^1(\dot{1})) + \underline{\sigma}.p'_2 \stackrel{\text{RTO4}}{\equiv} v^1(p'_1 + \dot{1}) + \underline{\sigma}.p'_2 = v^1(p'_1 + p'') + \underline{\sigma}.p'_2$. Note that $p'_2 < p$ or $p'_2 \equiv \dot{0}$.
- (c) $\text{TCP}_{\text{drt}}^\bullet \vdash p' = 1 \cdot v^1(p'_1)$ for some basic term p'_1 such that $p'_1 \leq p'$ and $\text{TCP}_{\text{drt}}^\bullet \vdash p'' = \dot{1}$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p \equiv p' + p'' = 1 \cdot v^1(p'_1) + \dot{1} \stackrel{\text{DT1,A4,DRA10}}{\equiv} \underline{1}.v^1(p'_1) + \underline{\sigma}.(1 \cdot v^1(p'_1)) + \dot{1} \stackrel{\text{TN}}{\equiv} v^1(p'_1) + \underline{\sigma}.(1 \cdot v^1(p'_1)) + \dot{1} \stackrel{\text{RTO1}^\bullet, \text{RTO3}}{\equiv} v^1(p'_1 + \dot{1}) + \underline{\sigma}.(1 \cdot v^1(p'_1)) = v^1(p'_1 + p'') + \underline{\sigma}.p'$. Note that $p' < p$.
- (d) $\text{TCP}_{\text{drt}}^\bullet \vdash p' = \dot{1}$ and $\text{TCP}_{\text{drt}}^\bullet \vdash p'' = v^1(p''_1) + \underline{\sigma}.p''_2$ for some basic TCP_{drt} -terms p''_1 and p''_2 such that $p''_2 < p''$ or $p''_2 \equiv \dot{0}$. Similar to the second case.
- (e) $\text{TCP}_{\text{drt}}^\bullet \vdash p' = v^1(p'_1) + \underline{\sigma}.p'_2$ for some basic TCP_{drt} -terms p'_1 and p'_2 such that $p'_2 < p'$ or $p'_2 \equiv \dot{0}$ and $\text{TCP}_{\text{drt}}^\bullet \vdash p'' = v^1(p''_1) + \underline{\sigma}.p''_2$ for some basic TCP_{drt} -terms p''_1 and p''_2 such that $p''_2 < p''$ or $p''_2 \equiv \dot{0}$. Here we only consider the case where $p'_2 < p'$ and $p''_2 < p''$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p \equiv p' + p'' = (v^1(p'_1) + \underline{\sigma}.p'_2) + (v^1(p''_1) + \underline{\sigma}.p''_2) \stackrel{\text{A1,A2}}{\equiv} (v^1(p'_1) + v^1(p''_1)) + (\underline{\sigma}.p'_2 + \underline{\sigma}.p''_2) \stackrel{\text{RTO4,DRTF}}{\equiv} v^1(p'_1 + p''_1) + \underline{\sigma}.(p'_2 + p''_2)$ and note that $p'_2 + p''_2 < p$.
- (f) $\text{TCP}_{\text{drt}}^\bullet \vdash p' = 1 \cdot v^1(p'_1)$ for some basic term p'_1 such that $p'_1 \leq p'$ and $\text{TCP}_{\text{drt}}^\bullet \vdash p'' = v^1(p''_1) + \underline{\sigma}.p''_2$ for some basic TCP_{drt} -terms p''_1 and p''_2 such that $p''_2 < p''$ or $p''_2 \equiv \dot{0}$. Here we only consider the case that $p''_2 < p''$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p \equiv p' + p'' = 1 \cdot v^1(p'_1) + v^1(p''_1) + \underline{\sigma}.p''_2 \stackrel{\text{DT1,A4,DRA10}}{\equiv} \underline{1}.v^1(p'_1) + \underline{\sigma}.(1 \cdot v^1(p'_1)) + v^1(p''_1) + \underline{\sigma}.p''_2 \stackrel{\text{TN}}{\equiv} v^1(p'_1) + \underline{\sigma}.(1 \cdot v^1(p'_1)) + v^1(p''_1) + \underline{\sigma}.p''_2 \stackrel{\text{RTO3,DRTF}}{\equiv} v^1(p'_1 + p''_1) + \underline{\sigma}.(1 \cdot v^1(p'_1) + p''_2) = v^1(p'_1 + p''_1) + \underline{\sigma}.(p' + p''_2)$. Note that $p' + p''_2 < p$.
- (g) $\text{TCP}_{\text{drt}}^\bullet \vdash p' = \dot{1}$ and $\text{TCP}_{\text{drt}}^\bullet \vdash p'' = 1 \cdot v^1(p''_1)$ for some basic TCP_{drt} -term p''_1 such that $p''_1 \leq p''$. Similar to the third case.
- (h) $\text{TCP}_{\text{drt}}^\bullet \vdash p' = v^1(p'_1) + \underline{\sigma}.p'_2$ for some basic TCP_{drt} -terms p'_1 and p'_2 such that $p'_2 < p'$ or $p'_2 \equiv \dot{0}$ and $\text{TCP}_{\text{drt}}^\bullet \vdash p'' = 1 \cdot v^1(p''_1)$ for some basic TCP_{drt} -term p''_1 such that $p''_1 \leq p''$. Similar to the sixth case.
- (i) $\text{TCP}_{\text{drt}}^\bullet \vdash p' = 1 \cdot v^1(p'_1)$ for some basic term p'_1 such that $p'_1 \leq p'$ and $\text{TCP}_{\text{drt}}^\bullet \vdash p'' = 1 \cdot v^1(p''_1)$ for some basic TCP_{drt} -term p''_1 such that $p''_1 \leq p''$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p \equiv p' + p'' = 1 \cdot v^1(p'_1) + 1 \cdot v^1(p''_1) \stackrel{\text{DOT5}}{\equiv} 1 \cdot (v^1(p'_1) + v^1(p''_1)) \stackrel{\text{RTO4}}{\equiv} 1 \cdot v^1(p'_1 + p''_1)$. Note that $p'_1 + p''_1 \leq p$.

□

Lemma C.8 (Representation) *For basic term p ,*

1. $\text{TCP}_{\text{drt}}^\bullet \vdash p = \dot{0}$,
2. $\text{TCP}_{\text{drt}}^\bullet \vdash p = \dot{1}$,
3. $\text{TCP}_{\text{drt}}^\bullet \vdash p = v^1(p') + \underline{\sigma}.p''$ for some basic $\text{TCP}_{\text{drt}}^\bullet$ -terms p' and p'' such that $p'' < p$ or $p'' \equiv \dot{0}$, or

4. $\text{TCP}_{\text{drt}}^\bullet \vdash p = p + 0$.

Proof. By Lemma C.7, we can distinguish four cases:

1. $\text{TCP}_{\text{drt}}^\bullet \vdash p = \dot{0}$. Trivial.
2. $\text{TCP}_{\text{drt}}^\bullet \vdash p = \dot{1}$. Trivial.
3. $\text{TCP}_{\text{drt}}^\bullet \vdash p = v^1(p') + \underline{\sigma}.p''$ for some basic $\text{TCP}_{\text{drt}}^\bullet$ -terms p' and p'' such that $p'' < p$ or $p'' \equiv \dot{0}$. Trivial.
4. $\text{TCP}_{\text{drt}}^\bullet \vdash p = 1 \cdot v^1(p')$ for some basic term p' such that $p' \leq p$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p = 1 \cdot v^1(p') \stackrel{\text{A6}\bullet}{\equiv} 1 \cdot (v^1(p') + \dot{0}) \stackrel{\text{DOT5}}{\equiv} 1 \cdot v^1(p') + 1 \cdot \dot{0} \stackrel{\text{DOT2}}{\equiv} p + 0$.

□

Theorem C.9 (Elimination of parallel composition operators) *For basic terms p_1 and p_2 ,*

1. *there exists a basic term q such that $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \parallel p_2 = q$;*
2. *there exists a basic term q such that $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \mid p_2 = q$;*
3. *there exists a basic term q such that $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \parallel p_2 = q$.*

Proof. These three statements are proven simultaneously using induction on the number of symbols of basic terms p_1 and p_2 . When we apply an induction hypothesis we indicate to which statement it refers.

First we give the proof for statement (1), the elimination of \parallel . We use case distinction on the structure of basic term p_1 .

1. $p_1 \equiv \dot{0}$. $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \parallel p_2 \equiv \dot{0} \parallel p_2 \stackrel{\text{LM1}\bullet}{\equiv} \dot{0}$.
2. $p_1 \equiv \dot{1}$. $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \parallel p_2 \equiv \dot{1} \parallel p_2 \stackrel{\text{LM2}\bullet}{\equiv} \dot{0}$.
3. $p_1 \equiv \underline{0}$. $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \parallel p_2 \equiv \underline{0} \parallel p_2$. Based on Lemma C.6, we can distinguish three cases for basic term p_2 .
 - (a) $\text{TCP}_{\text{drt}}^\bullet \vdash p_2 = \dot{0}$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \parallel p_2 = p_1 \parallel \dot{0} \stackrel{\text{LM5}\bullet}{\equiv} \dot{0}$.
 - (b) $\text{TCP}_{\text{drt}}^\bullet \vdash p_2 = \dot{1}$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \parallel p_2 = p_1 \parallel \dot{1} \stackrel{\text{A6}\bullet, \text{A1}}{\equiv} p_1 \parallel (\dot{0} + \dot{1}) \stackrel{\text{LM6}\bullet}{\equiv} p_1 \parallel \dot{0} \stackrel{\text{LM5}\bullet}{\equiv} \dot{0}$.
 - (c) $\text{TCP}_{\text{drt}}^\bullet \vdash p_2 = v^1(p'_2) + \underline{\sigma}.p''_2$ for some basic terms p'_2 and p''_2 such that $p''_2 \leq p_2$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \parallel p_2 \equiv \underline{0} \parallel p_2 \stackrel{\text{DR10}}{\equiv} \underline{\sigma}.\dot{0} \parallel p_2 = \underline{\sigma}.\dot{0} \parallel (v^1(p'_2) + \underline{\sigma}.p''_2) \stackrel{\text{LM6DR}}{\equiv} \underline{\sigma}.\dot{0} \parallel p''_2 \stackrel{\text{LM1}\bullet}{\equiv} \underline{\sigma}.\dot{0}$.
4. $p_1 \equiv \underline{1}$. According to Lemma C.5 we can distinguish three cases for p_2 :
 - (a) $\text{TCP}_{\text{drt}}^\bullet \vdash p_2 = \dot{0}$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \parallel p_2 = p_1 \parallel \dot{0} \stackrel{\text{LM5}\bullet}{\equiv} \dot{0}$.
 - (b) $\text{TCP}_{\text{drt}}^\bullet \vdash p_2 = \dot{1}$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \parallel p_2 = p_1 \parallel \dot{1} \stackrel{\text{A6}\bullet, \text{A1}}{\equiv} p_1 \parallel (\dot{0} + \dot{1}) \stackrel{\text{LM6}\bullet}{\equiv} p_1 \parallel \dot{0} \stackrel{\text{LM5}\bullet}{\equiv} \dot{0}$.
 - (c) $\text{TCP}_{\text{drt}}^\bullet \vdash p_2 = p_2 + \underline{0}$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \parallel p_2 \equiv \underline{1} \parallel p_2 = \underline{1} \parallel (p_2 + \underline{0}) \stackrel{\text{LM2DR}\bullet}{\equiv} \underline{0}$.
5. $p_1 \equiv 0$. According to Lemma C.8 we can distinguish four cases for p_2 :
 - (a) $\text{TCP}_{\text{drt}}^\bullet \vdash p_2 = \dot{0}$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \parallel p_2 = p_1 \parallel \dot{0} \stackrel{\text{LM5}\bullet}{\equiv} \dot{0}$.
 - (b) $\text{TCP}_{\text{drt}}^\bullet \vdash p_2 = \dot{1}$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \parallel p_2 = p_1 \parallel \dot{1} \stackrel{\text{A6}\bullet, \text{A2}}{\equiv} p_1 \parallel (\dot{0} + \dot{1}) \stackrel{\text{LM6}\bullet}{\equiv} p_1 \parallel \dot{0} \stackrel{\text{LM5}\bullet}{\equiv} \dot{0}$.

- (c) $\text{TCP}_{\text{drt}}^\bullet \vdash p_2 = v^1(p'_2) + \underline{\sigma}.p''_2$ for some basic terms p'_2 and p''_2 such that $p''_2 < p_2$ or $p''_2 \equiv \dot{0}$. Then, $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \parallel p_2 \equiv 0 \parallel p_2 \stackrel{\text{DD}}{=} \underline{\sigma}.0 \parallel p_2 = \underline{\sigma}.0 \parallel (v^1(p'_2) + \underline{\sigma}.p''_2) \stackrel{\text{LM6DR}}{=} \underline{\sigma}.(0 \parallel p''_2)$. In case $p''_2 \equiv \dot{0}$ we have $\text{TCP}_{\text{drt}}^\bullet \vdash \underline{\sigma}.(0 \parallel p''_2) \equiv \underline{\sigma}.(0 \parallel \dot{0}) \stackrel{\text{LM5}\bullet}{=} \underline{\sigma}.\dot{0}$. In case $p''_2 < p_2$, by induction hypothesis (1) we have the existence of basic term q'' such that $\text{TCP}_{\text{drt}}^\bullet \vdash 0 \parallel p''_2 = q''$. So in this case we have $\text{TCP}_{\text{drt}}^\bullet \vdash \underline{\sigma}.(0 \parallel p''_2) = \underline{\sigma}.q''$.
- (d) $\text{TCP}_{\text{drt}}^\bullet \vdash p_2 = p_2 + 0$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \parallel p_2 \equiv 0 \parallel p_2 = 0 \parallel (p_2 + 0) \stackrel{\text{LM7}\bullet}{=} 0$.
6. $p_1 \equiv 1$. According to Lemma C.8 we can distinguish four cases for p_2 :
- (a) $\text{TCP}_{\text{drt}}^\bullet \vdash p_2 = \dot{0}$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \parallel p_2 = p_1 \parallel \dot{0} \stackrel{\text{LM5}\bullet}{=} \dot{0}$.
- (b) $\text{TCP}_{\text{drt}}^\bullet \vdash p_2 = \dot{1}$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \parallel p_2 = p_1 \parallel \dot{1} \stackrel{\text{A6}\bullet, \text{A2}}{=} p_1 \parallel (\dot{0} + \dot{1}) \stackrel{\text{LM6}\bullet}{=} p_1 \parallel \dot{0} \stackrel{\text{LM5}\bullet}{=} \dot{0}$.
- (c) $\text{TCP}_{\text{drt}}^\bullet \vdash p_2 = v^1(p'_2) + \underline{\sigma}.p''_2$ for some basic terms p'_2 and p''_2 such that $p''_2 < p_2$ or $p''_2 \equiv \dot{0}$. According to the fourth case we have the existence of basic term q' such that $\text{TCP}_{\text{drt}}^\bullet \vdash \underline{1} \parallel p_2 = q'$. Then, $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \parallel p_2 \equiv 1 \parallel p_2 \stackrel{\text{DT1, LM4}}{=} \underline{1} \parallel p_2 + \underline{\sigma}.1 \parallel p_2 = q' + \underline{\sigma}.1 \parallel (v^1(p'_2) + \underline{\sigma}.p''_2) \stackrel{\text{LM6DR}}{=} q' + \underline{\sigma}.(1 \parallel p''_2)$. In case $p''_2 \equiv \dot{0}$ we have $\text{TCP}_{\text{drt}}^\bullet \vdash q' + \underline{\sigma}.(1 \parallel p''_2) \equiv q' + \underline{\sigma}.(1 \parallel \dot{0}) \stackrel{\text{LM5}\bullet}{=} q' + \underline{\sigma}.\dot{0}$. In case $p''_2 < p_2$, by induction hypothesis (1) we have the existence of basic term q'' such that $\text{TCP}_{\text{drt}}^\bullet \vdash 1 \parallel p''_2 = q''$. so then $\text{TCP}_{\text{drt}}^\bullet \vdash q' + \underline{\sigma}.(1 \parallel p''_2) \stackrel{\text{ih}(1)}{=} q' + \underline{\sigma}.q''$.
- (d) $\text{TCP}_{\text{drt}}^\bullet \vdash p_2 = p_2 + 0$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \parallel p_2 \equiv 1 \parallel p_2 = 1 \parallel (p_2 + 0) \stackrel{\text{LM8}\bullet}{=} 0$.
7. $p_1 \equiv \underline{a}.p'_1$ for some $a \in A$ and basic term p'_1 . According to Lemma C.4 we can distinguish four cases for p_2 :
- (a) $\text{TCP}_{\text{drt}}^\bullet \vdash p_2 = \dot{0}$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \parallel p_2 = p_1 \parallel \dot{0} \stackrel{\text{LM5}\bullet}{=} \dot{0}$.
- (b) $\text{TCP}_{\text{drt}}^\bullet \vdash p_2 = \dot{1}$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \parallel p_2 = p_1 \parallel \dot{1} \stackrel{\text{A6}\bullet, \text{A1}}{=} p_1 \parallel (\dot{0} + \dot{1}) \stackrel{\text{LM6}\bullet}{=} p_1 \parallel \dot{0} \stackrel{\text{LM5}\bullet}{=} \dot{0}$.
- (c) $\text{TCP}_{\text{drt}}^\bullet \vdash p_2 = \underline{1} \cdot p_2$. By induction hypothesis (3) there exists a basic term q' such that $\text{TCP}_{\text{drt}}^\bullet \vdash p'_1 \parallel p_2 = q'$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \parallel p_2 \equiv \underline{a}.p'_1 \parallel p_2 = \underline{a}.p'_1 \parallel (\underline{1} \cdot p_2) \stackrel{\text{LM3DR}\bullet}{=} \underline{a}.\underline{(p'_1 \parallel (\underline{1} \cdot p_2))} \stackrel{\text{ih}(3)}{=} \underline{a}.q'$.
- (d) $\text{TCP}_{\text{drt}}^\bullet \vdash p_2 = \underline{1} \cdot p_2 + \dot{1}$. By induction hypothesis (3) there exists a basic term q' such that $\text{TCP}_{\text{drt}}^\bullet \vdash p'_1 \parallel p_2 = q'$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \parallel p_2 \equiv \underline{a}.p'_1 \parallel p_2 = \underline{a}.p'_1 \parallel (\underline{1} \cdot p_2 + \dot{1}) \stackrel{\text{LM6}\bullet}{=} \underline{a}.p'_1 \parallel (\underline{1} \cdot p_2) \stackrel{\text{LM3DR}\bullet}{=} \underline{a}.\underline{(p'_1 \parallel (\underline{1} \cdot p_2))} \stackrel{\text{ih}(3)}{=} \underline{a}.q'$.
8. $p_1 \equiv a.p'_1$ for some $a \in A$ and basic term p'_1 . According to Lemma C.7 we can distinguish four cases for p_2 :
- (a) $\text{TCP}_{\text{drt}}^\bullet \vdash p_2 = \dot{0}$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \parallel p_2 = p_1 \parallel \dot{0} \stackrel{\text{LM5}\bullet}{=} \dot{0}$.
- (b) $\text{TCP}_{\text{drt}}^\bullet \vdash p_2 = \dot{1}$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \parallel p_2 = p_1 \parallel \dot{1} \stackrel{\text{A6}\bullet, \text{A1}}{=} p_1 \parallel (\dot{0} + \dot{1}) \stackrel{\text{LM6}\bullet}{=} p_1 \parallel \dot{0} \stackrel{\text{LM5}\bullet}{=} \dot{0}$.
- (c) $\text{TCP}_{\text{drt}}^\bullet \vdash p_2 = v^1(p'_2) + \underline{\sigma}.p''_2$ for some basic terms p'_2 and p''_2 such that $p''_2 < p_2$ or $p''_2 \equiv \dot{0}$. By the previous case we have the existence of a basic term q' such that $\text{TCP}_{\text{drt}}^\bullet \vdash \underline{a}.p'_1 \parallel p_2 = q'$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \parallel p_2 \equiv a.p'_1 \parallel p_2 \stackrel{\text{DAP, LM4}}{=} \underline{a}.p'_1 \parallel p_2 + \underline{\sigma}.a.p'_1 \parallel p_2 = q' + \underline{\sigma}.a.p'_1 \parallel p_2 = q' + \underline{\sigma}.p_1 \parallel (v^1(p'_2) + \underline{\sigma}.p''_2) \stackrel{\text{LM6DR}}{=} q' + \underline{\sigma}.\underline{(p_1 \parallel p''_2)}$. In case $p''_2 \equiv \dot{0}$ we have $\text{TCP}_{\text{drt}}^\bullet \vdash q' + \underline{\sigma}.\underline{(p_1 \parallel p''_2)} \equiv q' + \underline{\sigma}.\underline{(p_1 \parallel \dot{0})} \stackrel{\text{LM5}\bullet}{=} q' + \dot{0}$. In case $p''_2 < p_2$, by induction hypothesis (3) there exists a basic term q'' such that $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \parallel p''_2 = q''$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash q' + \underline{\sigma}.\underline{(p_1 \parallel p''_2)} \stackrel{\text{ih}}{=} q' + \underline{\sigma}.q''$.

- (d) $\text{TCP}_{\text{drt}}^\bullet \vdash p_2 = 1 \cdot v^1(p'_2)$ for some basic term p'_2 such that $p'_2 \leq p_2$. By induction hypothesis (3) there exists a basic term q' such that $\text{TCP}_{\text{drt}}^\bullet \vdash p'_1 \parallel p'_2 = q'$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \parallel p_2 \equiv a.p'_1 \parallel p_2 \stackrel{a}{=} .p'_1 \parallel (1 \cdot v^1(p'_2)) \stackrel{\text{LM3}^\bullet}{=} a.(p'_1 \parallel (1 \cdot v^1(p'_2))) = a.(p'_1 \parallel p_2) \stackrel{\text{ih}(3)}{=} a.q'$.
9. $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \equiv \underline{\sigma}.p'_1$ for some basic term p'_1 . Based on Lemma C.6, we can distinguish three cases for basic term p_2 .
- (a) $\text{TCP}_{\text{drt}}^\bullet \vdash p_2 = \dot{0}$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \parallel p_2 = p_1 \parallel \dot{0} \stackrel{\text{LM5}^\bullet}{=} \dot{0}$.
- (b) $\text{TCP}_{\text{drt}}^\bullet \vdash p_2 = \dot{1}$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \parallel p_2 = p_1 \parallel \dot{1} \stackrel{\text{A6}^\bullet, \text{A1}}{=} p_1 \parallel (\dot{0} + \dot{1}) \stackrel{\text{LM6}^\bullet}{=} p_1 \parallel \dot{0} \stackrel{\text{LM5}^\bullet}{=} \dot{0}$.
- (c) $\text{TCP}_{\text{drt}}^\bullet \vdash p_2 = v^1(p'_2) + \underline{\sigma}.p''_2$ for some basic terms p'_2 and p''_2 such that $p''_2 \leq p_2$. By induction hypothesis (1) we have the existence of basic term q'' such that $\text{TCP}_{\text{drt}}^\bullet \vdash p'_1 \parallel p''_2 = q''$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \parallel p_2 \equiv \underline{\sigma}.p'_1 \parallel p_2 = \underline{\sigma}.p'_1 \parallel (v^1(p'_2) + \underline{\sigma}.p''_2) \stackrel{\text{LM6DR}}{=} \underline{\sigma}.(p'_1 \parallel p''_2) \stackrel{\text{ih}(1)}{=} \underline{\sigma}.q''$.
10. $p_1 \equiv p'_1 + p''_1$. By induction there exist basic terms q' and q'' such that $\text{TCP}_{\text{drt}}^\bullet \vdash p'_1 \parallel p_2 = q'$ and $\text{TCP}_{\text{drt}}^\bullet \vdash p''_1 \parallel p_2 = q''$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \parallel p_2 \equiv (p'_1 + p''_1) \parallel p_2 \stackrel{\text{LM4}}{=} p'_1 \parallel p_2 + p''_1 \parallel p_2 \stackrel{\text{ih}(1)}{=} q' + q''$.

Then we prove statement (2), the elimination of $|$. We use case distinction on the structure of basic term p_1 .

1. $p_1 \equiv \dot{0}$. $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 | p_2 \equiv \dot{0} | p_2 \stackrel{\text{CM1}^\bullet}{=} \dot{0}$.
2. $p_1 \equiv \underline{0}$. By Lemma C.6 we can distinguish three cases for p_2 .
 - (a) $\text{TCP}_{\text{drt}}^\bullet \vdash p_2 = \dot{0}$. Similar to case (1) using SC1.
 - (b) $\text{TCP}_{\text{drt}}^\bullet \vdash p_2 = \dot{1}$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 | p_2 \equiv \underline{0} | \dot{1} \stackrel{\text{CM8DR}^\bullet}{=} \dot{0}$.
 - (c) $\text{TCP}_{\text{drt}}^\bullet \vdash p_2 = v^1(p'_2) + \underline{\sigma}.p''_2$ for some basic $\text{TCP}_{\text{drt}}^\bullet$ -terms p'_2 and p''_2 such that $p''_2 \leq p_2$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 | p_2 \equiv \underline{0} | p_2 \stackrel{\text{DR10}}{=} \underline{\sigma}.\dot{0} | p_2 = \underline{\sigma}.\dot{0} | (v^1(p'_2) + \underline{\sigma}.p''_2) \stackrel{\text{CM7DR}^\bullet}{=} \underline{\sigma}.\dot{0} | p''_2 \stackrel{\text{CM1}^\bullet}{=} \underline{\sigma}.\dot{0}$.
3. $p_1 \equiv 0$. By Lemma C.8 we can distinguish four cases for p_2 .
 - (a) $\text{TCP}_{\text{drt}}^\bullet \vdash p_2 = \dot{0}$. Similar to case (1) using SC1.
 - (b) $\text{TCP}_{\text{drt}}^\bullet \vdash p_2 = \dot{1}$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 | p_2 \equiv 0 | p_2 = 0 | \dot{1} \stackrel{\text{CM1b}}{=} \dot{0}$.
 - (c) $\text{TCP}_{\text{drt}}^\bullet \vdash p_2 = v^1(p'_2) + \underline{\sigma}.p''_2$ for some basic $\text{TCP}_{\text{drt}}^\bullet$ -terms p'_2 and p''_2 such that $p''_2 < p_2$ or $p''_2 \equiv \dot{0}$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 | p_2 \equiv 0 | p_2 \stackrel{\text{DD}}{=} \underline{\sigma}.0 | p_2 = \underline{\sigma}.0 | (v^1(p'_2) + \underline{\sigma}.p''_2) = \underline{\sigma}.(0 | p''_2)$. In case $p''_2 \equiv \dot{0}$, we have $\underline{\sigma}.\dot{0} | p''_2 \equiv \underline{\sigma}.\dot{0} | \dot{0} \stackrel{\text{SC1}}{=} \underline{\sigma}.\dot{0} | p \stackrel{\text{CM1}^\bullet}{=} \underline{\sigma}.\dot{0}$. In case $p''_2 < p_2$, by induction hypothesis (2) we have the existence of basic term q'' such that $\text{TCP}_{\text{drt}}^\bullet \vdash 0 | p''_2 = q''$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash \underline{\sigma}.\dot{0} | p''_2 \stackrel{\text{ih}}{=} \underline{\sigma}.q''$.
 - (d) $\text{TCP}_{\text{drt}}^\bullet \vdash p_2 = p_2 + 0$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 | p_2 \equiv 0 | p_2 = 0 | (p_2 + 0) \stackrel{\text{CM1a}}{=} 0$.
4. $p_1 \equiv p'_1 + p''_1$ for some basic terms p'_1 and p''_1 . By the induction hypothesis for statement (2), we have the existence of basic terms q' and q'' such that $\text{TCP}_{\text{drt}}^\bullet \vdash p'_1 | p_2 = r'$ and $\text{TCP}_{\text{drt}}^\bullet \vdash p''_1 | p_2 = r''$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 | p_2 \equiv (p'_1 + p''_1) | p_2 \stackrel{\text{CM2}}{=} p'_1 | p_2 + p''_1 | p_2 \stackrel{\text{ih}(2)}{=} r' + r''$.
5. $p_1 \equiv \dot{1}$. We use case distinction on the structure of basic term p_2 .
 - (a) $p_2 \equiv \dot{0}$. Similar to case (1) using axiom SC1.

- (b) $p_2 \equiv \dot{\mathbf{i}}$. $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 | p_2 \equiv \dot{\mathbf{i}} | \dot{\mathbf{i}} \stackrel{\text{CM3}^\bullet}{\equiv} \dot{\mathbf{i}}$.
- (c) $p_2 \equiv \underline{\mathbf{0}}$. Similar to case (2b) using axiom SC1.
- (d) $p_2 \equiv \underline{\mathbf{1}}$. $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 | p_2 \equiv \dot{\mathbf{i}} | \underline{\mathbf{1}} \stackrel{\text{SC1}}{\equiv} \underline{\mathbf{1}} | \dot{\mathbf{i}} \stackrel{\text{DR11}}{\equiv} (\underline{\mathbf{1}} + \dot{\mathbf{i}}) | \dot{\mathbf{i}} \stackrel{\text{CM2}}{\equiv} \underline{\mathbf{1}} | \dot{\mathbf{i}} + \dot{\mathbf{i}} | \dot{\mathbf{i}} \stackrel{\text{CM3}^\bullet}{\equiv} \underline{\mathbf{1}} | \dot{\mathbf{i}} + \dot{\mathbf{i}} \stackrel{\text{SC1}}{\equiv} \dot{\mathbf{i}} | \underline{\mathbf{1}} + \dot{\mathbf{i}} \stackrel{\text{SC3}^\bullet}{\equiv} \dot{\mathbf{i}}$.
- (e) $p_2 \equiv \mathbf{0}$. Similar to case (3b) using axiom SC1.
- (f) $p_2 \equiv \mathbf{1}$. $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 | p_2 \equiv \dot{\mathbf{i}} | \mathbf{1} \stackrel{\text{SC1}}{\equiv} \mathbf{1} | \dot{\mathbf{i}} \stackrel{\text{CM7}^\bullet}{\equiv} \dot{\mathbf{i}}$.
- (g) $p_2 \equiv \underline{a}.p'_2$ for some $a \in A$ and basic term p'_2 . $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 | p_2 \equiv \dot{\mathbf{i}} | \underline{a}.p'_2 \stackrel{\text{SC1}}{\equiv} \underline{a}.p'_2 | \dot{\mathbf{i}} \stackrel{\text{CM9DR}^\bullet}{\equiv} \dot{\mathbf{0}}$.
- (h) $p_2 \equiv a.p'_2$ for some $a \in A$ and basic term p'_2 . $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 | p_2 \equiv \dot{\mathbf{i}} | a.p'_2 \stackrel{\text{SC1}}{\equiv} a.p'_2 | \dot{\mathbf{i}} \stackrel{\text{CM4}^\bullet}{\equiv} \dot{\mathbf{0}}$.
- (i) $p_2 \equiv \underline{\sigma}.p'_2$ for some basic term p'_2 . $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 | p_2 \equiv \dot{\mathbf{i}} | \underline{\sigma}.p'_2 \stackrel{\text{SC1}}{\equiv} \underline{\sigma}.p'_2 | \dot{\mathbf{i}} \stackrel{\text{CM10DR}^\bullet}{\equiv} \dot{\mathbf{0}}$.
- (j) $p_2 \equiv p'_2 + p''_2$ for some basic terms p'_2 and p''_2 . Similar to case (4) using axiom SC1.

6. $p_1 \equiv \underline{\mathbf{1}}$. We use case distinction on the structure of basic term p_2 . We omit the cases that are similar to a previous case using axiom SC1.

- (a) $p_2 \equiv \underline{\mathbf{1}}$. $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 | p_2 \equiv \underline{\mathbf{1}} | \underline{\mathbf{1}} \stackrel{\text{CM3DR}}{\equiv} \underline{\mathbf{1}}$.
- (b) $p_2 \equiv \mathbf{1}$. $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 | p_2 \equiv \underline{\mathbf{1}} | \mathbf{1} \stackrel{\text{SC1}}{\equiv} \mathbf{1} | \underline{\mathbf{1}} \stackrel{\text{DT1,CM2}}{\equiv} \underline{\mathbf{1}} | \underline{\mathbf{1}} + \underline{\sigma}.\mathbf{1} | \underline{\mathbf{1}} \stackrel{\text{CM3DR}}{\equiv} \underline{\mathbf{1}} + \underline{\sigma}.\mathbf{1} | \underline{\mathbf{1}} \stackrel{\text{A1,SC1}}{\equiv} \underline{\mathbf{1}} | \underline{\sigma}.\mathbf{1} + \underline{\mathbf{1}} \stackrel{\text{SC3DR}}{\equiv} \underline{\mathbf{1}}$.
- (c) $p_2 \equiv \underline{a}.p'_2$ for some $a \in A$ and basic term p'_2 . $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 | p_2 \equiv \underline{\mathbf{1}} | \underline{a}.p'_2 \stackrel{\text{SC1}}{\equiv} \underline{a}.p'_2 | \underline{\mathbf{1}} \stackrel{\text{CM4DR}}{\equiv} \dot{\mathbf{0}}$.
- (d) $p_2 \equiv a.p'_2$ for some $a \in A$ and basic term p'_2 . $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 | p_2 \equiv \underline{\mathbf{1}} | a.p'_2 \stackrel{\text{SC1}}{\equiv} a.p'_2 | \underline{\mathbf{1}} \stackrel{\text{DAP}}{\equiv} (\underline{a}.p'_2 + \underline{\sigma}.a.p'_2) | \underline{\mathbf{1}} \stackrel{\text{CM2}}{\equiv} \underline{a}.p'_2 | \underline{\mathbf{1}} + \underline{\sigma}.a.p'_2 | \underline{\mathbf{1}} \stackrel{\text{CM4DR}}{\equiv} \underline{\mathbf{0}} + \underline{\sigma}.a.p'_2 | \underline{\mathbf{1}} \stackrel{\text{RTO2}}{\equiv} \underline{\mathbf{0}} + \underline{\sigma}.a.p'_2 | v^1(\underline{\mathbf{1}}) = \underline{\mathbf{0}} + \underline{\sigma}.a.p'_2 | (v^1(\underline{\mathbf{1}}) + \underline{\sigma}.\dot{\mathbf{0}}) \stackrel{\text{CM7DR}^\bullet}{\equiv} \underline{\mathbf{0}} + \underline{\mathbf{0}}$.
- (e) $p_2 \equiv \underline{\sigma}.p'_2$ for some basic term p'_2 . $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 | p_2 \equiv \underline{\mathbf{1}} | \underline{\sigma}.p'_2 \stackrel{\text{SC1}}{\equiv} \underline{\sigma}.p'_2 | \underline{\mathbf{1}} \stackrel{\text{RTO2}}{\equiv} \underline{\sigma}.p'_2 | v^1(\underline{\mathbf{1}}) = \underline{\sigma}.p'_2 | (v^1(\underline{\mathbf{1}}) + \underline{\sigma}.\dot{\mathbf{0}}) \stackrel{\text{CM7DR}^\bullet}{\equiv} \underline{\mathbf{0}}$.

7. $p_1 \equiv \mathbf{1}$. We use case distinction on the structure of basic term p_2 . We omit the cases that are similar to a previous case using axiom SC1.

- (a) $p_2 \equiv \mathbf{1}$. $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 | p_2 \equiv \mathbf{1} | \mathbf{1} \stackrel{\text{CM3}}{\equiv} \mathbf{1}$.
- (b) $p_2 \equiv \underline{a}.p'_2$ for some $a \in A$ and basic term p'_2 . $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 | p_2 \equiv \mathbf{1} | p_2 \stackrel{\text{DT1,CM2}}{\equiv} \underline{\mathbf{1}} | p_2 + \underline{\sigma}.\mathbf{1} | p_2 \stackrel{\text{SC1}}{\equiv} p_2 | \underline{\mathbf{1}} + \underline{\sigma}.\mathbf{1} | p_2 = \underline{a}.p'_2 | \underline{\mathbf{1}} + \underline{\sigma}.\mathbf{1} | \underline{a}.p'_2 \stackrel{\text{CM4DR,RTO3}}{\equiv} \underline{\mathbf{0}} + \underline{\sigma}.\mathbf{1} | v^1(\underline{a}.p'_2) = \underline{\mathbf{0}} + \underline{\sigma}.\mathbf{1} | (v^1(\underline{a}.p'_2) + \underline{\sigma}.\dot{\mathbf{0}}) \stackrel{\text{CM7DR}^\bullet}{\equiv} \underline{\mathbf{0}} + \underline{\mathbf{0}}$.
- (c) $p_2 \equiv a.p'_2$ for some $a \in A$ and basic term p'_2 . $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 | p_2 \equiv \mathbf{1} | a.p'_2 \stackrel{\text{SC1}}{\equiv} a.p'_2 | \mathbf{1} \stackrel{\text{CM4}}{\equiv} \mathbf{0}$.
- (d) $p_2 \equiv \underline{\sigma}.p'_2$ for some basic term p'_2 . By induction hypothesis (2) we have the existence of basic term q' such that $\text{TCP}_{\text{drt}}^\bullet \vdash \mathbf{1} | p'_2 = q'$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 | p_2 \equiv \mathbf{1} | \underline{\sigma}.p'_2 \stackrel{\text{SC1}}{\equiv} \underline{\sigma}.p'_2 | \mathbf{1} \stackrel{\text{DT1}}{\equiv} \underline{\sigma}.p'_2 | (\underline{\mathbf{1}} + \underline{\sigma}.\mathbf{1}) \stackrel{\text{RTO2}}{\equiv} \underline{\sigma}.p'_2 | (v^1(\underline{\mathbf{1}}) + \underline{\sigma}.\mathbf{1}) \stackrel{\text{CM7DR}^\bullet}{\equiv} \underline{\sigma}.p'_2 | \mathbf{1} \stackrel{\text{SC1}}{\equiv} \underline{\sigma}.p'_2 | \mathbf{1} \stackrel{\text{ih}(2)}{\equiv} \underline{\sigma}.q'$.

8. $p_1 \equiv \underline{a}.p'_1$ for some $a \in A$ and basic term p'_1 . We use case distinction on the structure of basic term p_2 . We omit the cases that are similar to a previous case using axiom SC1.

- (a) $p_2 \equiv \underline{b}.p'_2$ for some $b \in A$ and basic term p'_2 . By induction hypothesis (3) we have the existence of basic term q' such that $\text{TCP}_{\text{drt}}^\bullet \vdash p'_1 | p'_2 = q'$. Then, $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 | p_2 \equiv \underline{a}.p'_1 | \underline{b}.p'_2$. In case $\gamma(a, b) = c$ we have $\text{TCP}_{\text{drt}}^\bullet \vdash \underline{a}.p'_1 | \underline{b}.p'_2 \stackrel{\text{CM5DR}}{\equiv} \underline{c}.(p'_1 | p'_2) \stackrel{\text{ih}}{\equiv} \underline{c}.q'$. In case $\gamma(a, b)$ is not defined, we have $\text{TCP}_{\text{drt}}^\bullet \vdash \underline{a}.p'_1 | \underline{b}.p'_2 \stackrel{\text{CM6DR}}{\equiv} \underline{\mathbf{0}}$.

- (b) $p_2 \equiv b.p'_2$ for some $b \in A$ and basic term p'_2 . According to the previous item we have the existence of a basic term q' such that $\text{TCP}_{\text{drt}}^\bullet \vdash \underline{a}.p'_1 \mid \underline{b}.p'_2 = q'$. Then, $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \mid p_2 \equiv \underline{a}.p'_1 \mid b.p'_2 \stackrel{\text{SC1}}{\equiv} \underline{a}.p'_1 \mid \underline{b}.p'_2 \stackrel{\text{DAP}}{\equiv} \underline{a}.p'_1 \mid (\underline{b}.p'_2 + \underline{\sigma}.b.p'_2) \mid \underline{a}.p'_1 \stackrel{\text{CM2,SC1}}{\equiv} \underline{a}.p'_1 \mid \underline{b}.p'_2 + \underline{\sigma}.b.p'_2 \mid \underline{a}.p'_1 = q' + \underline{\sigma}.b.p'_2 \mid \underline{a}.p'_1 \stackrel{\text{RTO3}}{\equiv} q' + \underline{\sigma}.b.p'_2 \mid v^1(\underline{a}.p'_1) = q' + \underline{\sigma}.b.p'_2 \mid (v^1(\underline{a}.p'_1) + \underline{\sigma}.1) \stackrel{\text{CM7DR}\bullet}{\equiv} q' + \underline{0}$.
- (c) $p_2 \equiv \underline{\sigma}.p'_2$ for some basic term p'_2 . Then $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \mid p_2 \stackrel{\text{SC1}}{\equiv} p_2 \mid p_1 \equiv \underline{\sigma}.p'_2 \mid \underline{a}.p'_1 \stackrel{\text{RTO3}}{\equiv} \underline{\sigma}.p'_2 \mid v^1(\underline{a}.p'_1) = \underline{\sigma}.p'_2 \mid (v^1(\underline{a}.p'_1) + \underline{\sigma}.0) \stackrel{\text{CM7DR}\bullet}{\equiv} \underline{0}$.
9. $p_1 \equiv a.p'_1$ for some $a \in A$ and basic term p'_1 . We use case distinction on the structure of basic term p_2 . We omit the cases that are similar to a previous case using axiom SC1.
- (a) $p_2 \equiv b.p'_2$ for some $b \in A$ and basic term p'_2 . By induction hypothesis (3) we have the existence of basic term q' such that $\text{TCP}_{\text{drt}}^\bullet \vdash p'_1 \parallel p'_2 = q'$. In case $\gamma(a, b)$ is not defined, $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \mid p_2 \equiv a.p'_1 \mid b.p'_2 \stackrel{\text{CM6}}{\equiv} 0$. In case $\gamma(a, b) = c$, $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \mid p_2 \equiv a.p'_1 \mid b.p'_2 \stackrel{\text{CM5}}{\equiv} c.(p'_1 \parallel p'_2) \stackrel{\text{ih}(3)}{\equiv} c.q'$.
- (b) $p_2 \equiv \underline{\sigma}.p'_2$ for some basic term p'_2 . By induction hypothesis (2) there exists a basic term q' such that $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \mid p'_2 = q'$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \mid p_2 \stackrel{\text{SC1}}{\equiv} p_2 \mid p_1 = \underline{\sigma}.p'_2 \mid a.p'_1 \stackrel{\text{DAP}}{\equiv} \underline{\sigma}.p'_2 \mid (\underline{a}.p'_1 + \underline{\sigma}.a.p'_1) \stackrel{\text{RTO3}}{\equiv} \underline{\sigma}.p'_2 \mid (v^1(\underline{a}.p'_1) + \underline{\sigma}.a.p'_1) \stackrel{\text{CM7DR}\bullet}{\equiv} \underline{\sigma}.(p'_2 \mid a.p'_1) = \underline{\sigma}.(p'_2 \mid p_1) \stackrel{\text{SC1}}{\equiv} \underline{\sigma}.(p_1 \mid p'_2) \stackrel{\text{ih}(2)}{\equiv} \underline{\sigma}.q'$.
10. $p_1 \equiv \underline{\sigma}.p'_1$ for some basic term p'_1 . We use case distinction on the structure of basic term p_2 . We omit the cases that are similar to a previous case using axiom SC1.
- (a) $p_2 \equiv \underline{\sigma}.p'_2$ for some basic term p'_2 . By induction hypothesis (2) there exists a basic term q' such that $\text{TCP}_{\text{drt}}^\bullet \vdash p'_1 \mid p'_2 = q'$. then, $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \mid p_2 = \underline{\sigma}.p'_1 \mid \underline{\sigma}.p'_2 \stackrel{\text{CM7DR}\bullet}{\equiv} \underline{\sigma}.(p'_1 \mid p'_2) \stackrel{\text{ih}(2)}{\equiv} \underline{\sigma}.q'$.

Finally, statement (3) follows straightforwardly from the previous statements: By induction on statements (1) and (2) we have the existence of basic terms q_1 , q_2 , and q_3 such that $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \parallel p_2 = q_1$, $\text{TCP}_{\text{drt}}^\bullet \vdash p_2 \parallel p_1 = q_2$, and $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \mid p_2 = q_3$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p_1 \parallel p_2 \stackrel{\text{M}}{\equiv} p_1 \parallel p_2 + p_2 \parallel p_1 + p_1 \mid p_2 \stackrel{\text{ih}(1), \text{ih}(2)}{\equiv} q_1 + q_2 + q_3$. \square

Theorem C.10 (Elimination of encapsulation) *For basic terms p and $H \subseteq A$, there exists a basic term q such that $\text{TCP}_{\text{drt}}^\bullet \vdash \partial_H(p) = q$.*

Proof. Trivial, by induction on the structure of basic term p . \square

Theorem C.11 (Elimination of time-out operator) *For basic terms p , there exists a basic term q such that $\text{TCP}_{\text{drt}}^\bullet \vdash v^1(p) = q$.*

Proof. Trivial, by induction on the structure of basic term p . \square

C.3 Completeness of $\text{TCP}_{\text{drt}}^\bullet$

Note that the term deduction system for $\text{TCP}_{\text{drt}}^\bullet$ is such that all action and time transitions that can be derived starting from a basic term always result in a basic term. We do not prove this statement formally, and will use it silently in the remainder.

Lemma C.12 (Towards completeness) *For arbitrary basic $\text{TCP}_{\text{drt}}^\bullet$ -terms p and p' and arbitrary action $a \in A$*

1. if $p \downarrow$, then $\text{TCP}_{\text{drt}}^\bullet \vdash p = \dot{1} + p$;
2. if $p \xrightarrow{a} p'$, then $\text{TCP}_{\text{drt}}^\bullet \vdash p = \underline{a}.p' + p$;
3. if $p \xrightarrow{1} p'$, then $p' \equiv p$ or $p' < p$;
4. if $p \xrightarrow{1} p'$, then $\text{TCP}_{\text{drt}}^\bullet \vdash p = \underline{\sigma}.p' + p$;
5. if $p \xrightarrow{1} p$, then $\text{TCP}_{\text{drt}}^\bullet \vdash p = 1 \cdot p$;
6. if $p \xrightarrow{0}$, then $\text{TCP}_{\text{drt}}^\bullet \vdash p = \underline{1} \cdot p + p$.

Proof. Easy; by induction on the structure of basic $\text{TCP}_{\text{drt}}^\bullet$ -term p . \(\square\)

Theorem C.13 *The process algebra $\text{TCP}_{\text{drt}}^\bullet$ is a complete axiomatization of strong bisimilarity on closed $\text{TCP}_{\text{drt}}^\bullet$ -terms.*

Proof. By the elimination theorem for $\text{TCP}_{\text{drt}}^\bullet$ it suffices to prove this theorem for basic terms only. We use induction on the structure of basic terms p and q and use case analysis on the structure of basic term p to prove that $p + q \Leftrightarrow q$ implies $\text{TCP}_{\text{drt}}^\bullet \vdash p + q = q$.

1. $p \equiv \dot{0}$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p + q \equiv \dot{0} + q \stackrel{A1, A6^\bullet}{=} q$.
2. $p \equiv \dot{1}$. Then $p + q \downarrow$, and since $p + q \Leftrightarrow q$ also $q \downarrow$. By Lemma C.12.1, we have $\text{TCP}_{\text{drt}}^\bullet \vdash q = \dot{1} + q$. Then, $\text{TCP}_{\text{drt}}^\bullet \vdash p + q \equiv \dot{1} + q = q$.
3. $p \equiv \underline{0}$. Then $p \xrightarrow{1} \dot{0}$, and therefore also $p + q \xrightarrow{1}$ and since $p + q \Leftrightarrow q$ also $q \xrightarrow{1}$. By Lemma C.12.4 we have $\text{TCP}_{\text{drt}}^\bullet \vdash q = \underline{\sigma}.q' + q$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p + q \equiv \underline{0} + q \stackrel{DR2^\bullet}{=} \underline{\sigma}.\dot{0} + q = \underline{\sigma}.\dot{0} + \underline{\sigma}.q' + q \stackrel{DRTF}{=} \underline{\sigma}.\dot{0} + \underline{\sigma}.q' + q \stackrel{A1, A6^\bullet}{=} \underline{\sigma}.(\dot{0} + q') + q \stackrel{A1, A6^\bullet}{=} \underline{\sigma}.q' + q = q$.
4. $p \equiv \underline{1}$. Then $p + q \downarrow$, and since $p + q \Leftrightarrow q$ also $q \downarrow$. By Lemma C.12.1, we have $\text{TCP}_{\text{drt}}^\bullet \vdash q = \dot{1} + q$. Also, $p \xrightarrow{0}$, and therefore also $p + q \xrightarrow{0}$ and since $p + q \Leftrightarrow q$ also $q \xrightarrow{0}$. By Lemma C.12.6, we have $\text{TCP}_{\text{drt}}^\bullet \vdash q = \underline{1} \cdot q + q$. Then, $\text{TCP}_{\text{drt}}^\bullet \vdash p + q \equiv \underline{1} + q \stackrel{A9^\bullet}{=} \underline{1} \cdot \dot{1} + q = \underline{1} \cdot \dot{1} + \underline{1} \cdot q + q \stackrel{DR6^\bullet}{=} \underline{1} \cdot (\dot{1} + q) + q = \underline{1} \cdot q + q = q$.
5. $p \equiv 0$. Then $p \xrightarrow{1} p$, and therefore also $p + q \xrightarrow{1}$ and since $p + q \Leftrightarrow q$ also $q \xrightarrow{1}$. By Lemma C.12.3, we can distinguish two cases:
 - (a) $q \xrightarrow{1} q$. By Lemma C.12.5 we have $\text{TCP}_{\text{drt}}^\bullet \vdash q = 1 \cdot q$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p + q = 0 + q \stackrel{DOT2}{=} 1 \cdot \dot{0} + q = 1 \cdot \dot{0} + 1 \cdot q \stackrel{DOT5}{=} 1 \cdot (\dot{0} + q) \stackrel{A1, A6DR}{=} 1 \cdot q = q$.
 - (b) $q \xrightarrow{1} q'$ for some $q' < q$. Then $p + q \xrightarrow{1} p + q'$ and therefore, since $p + q \Leftrightarrow q$, we need to have $p + q' \Leftrightarrow q'$. By induction we then have $\text{TCP}_{\text{drt}}^\bullet \vdash p + q' = q'$. By Lemma C.12.4 we have $\text{TCP}_{\text{drt}}^\bullet \vdash q = \underline{\sigma}.q' + q$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p + q = 0 + q \stackrel{DD}{=} \underline{\sigma}.0 + q = \underline{\sigma}.0 + \underline{\sigma}.q' + q \stackrel{DRTF}{=} \underline{\sigma}.(0 + q') + q = \underline{\sigma}.(p + q') + q = \underline{\sigma}.q' + q = q$.
6. $p \equiv 1$. From $p \downarrow$ we have $q \downarrow$. Therefore, by Lemma C.12.1, we have $\text{TCP}_{\text{drt}}^\bullet \vdash q = \dot{1} + q$. Then $p \xrightarrow{1} p$, and therefore also $p + q \xrightarrow{1}$ and since $p + q \Leftrightarrow q$ also $q \xrightarrow{1}$. By Lemma C.12.3, we can distinguish two cases:

- (a) $q \xrightarrow{1} q$. By Lemma C.12.5 we have $\text{TCP}_{\text{drt}}^\bullet \vdash q = 1 \cdot q$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p + q = 1 + q \stackrel{\text{A9}\bullet}{=} 1 \cdot \dot{1} + q = 1 \cdot \dot{1} + 1 \cdot q \stackrel{\text{DOT5}}{=} 1 \cdot (\dot{1} + q) = 1 \cdot q = q$.
- (b) $q \xrightarrow{1} q'$ for some $q' < q$. Then $p + q \xrightarrow{1} p + q'$ and therefore, since $p + q \Leftrightarrow q$, we need to have $p + q' \Leftrightarrow q'$. By induction we then have $\text{TCP}_{\text{drt}}^\bullet \vdash p + q' = q'$. By Lemma C.12.4 we have $\text{TCP}_{\text{drt}}^\bullet \vdash q = \underline{\sigma}.q' + q$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p + q = 1 + q \stackrel{\text{DT1}}{=} \underline{1} + \underline{\sigma}.1 + q \stackrel{\text{A1,A2}}{=} \underline{1} + q + \underline{\sigma}. = q + \underline{\sigma}.1 \stackrel{\text{A1}}{=} \underline{\sigma}.1 + q = \underline{\sigma}.1 + \underline{\sigma}.q' + q \stackrel{\text{DRTF}}{=} \underline{\sigma}.(1 + q') + q = \underline{\sigma}.(p + q') + q = \underline{\sigma}.q' + q = q$.
7. $p \equiv \underline{a}.p'$ for some $a \in A$ and basic term p' . Then $p + q \xrightarrow{a} p'$, and since $p + q \Leftrightarrow q$ we have $q \xrightarrow{a} q'$ for some q' such that $p' \Leftrightarrow q'$. Then due to soundness of axiom A3 and congruence of bisimilarity w.r.t. alternative composition we also have $p' + q' \Leftrightarrow q'$ and $q' + p' \Leftrightarrow p'$. By induction we then have $\text{TCP}_{\text{drt}}^\bullet \vdash p' + q = q'$ and $\text{TCP}_{\text{drt}}^\bullet \vdash q' + p' = p'$. Therefore, we also have $\text{TCP}_{\text{drt}}^\bullet \vdash p' = q' + p' = p' + q' = q'$. By Lemma C.12.2, we have $\text{TCP}_{\text{drt}}^\bullet \vdash q = \underline{a}.q' + q$. Then, $\text{TCP}_{\text{drt}}^\bullet \vdash p + q \equiv \underline{a}.p' + q \stackrel{\text{ih}}{=} \underline{a}.q' + q = q$.
8. $p \equiv a.p'$ for some $a \in A$ and basic term p' . Then $p + q \xrightarrow{a} p'$, and since $p + q \Leftrightarrow q$ we have $q \xrightarrow{a} q'$ for some q' such that $p' \Leftrightarrow q'$. Then due to soundness of axiom A3 and congruence of bisimilarity w.r.t. alternative composition we also have $p' + q' \Leftrightarrow q'$ and $q' + p' \Leftrightarrow p'$. By induction we then have $\text{TCP}_{\text{drt}}^\bullet \vdash p' + q = q'$ and $\text{TCP}_{\text{drt}}^\bullet \vdash q' + p' = p'$. Therefore, we also have $\text{TCP}_{\text{drt}}^\bullet \vdash p' = q' + p' = p' + q' = q'$. By Lemma C.12.2, we have $\text{TCP}_{\text{drt}}^\bullet \vdash q = \underline{a}.q' + q$. Also $p \xrightarrow{1} p$, and therefore also $p + q \xrightarrow{1}$ and since $p + q \Leftrightarrow q$ also $q \xrightarrow{1}$. By Lemma C.12.3, we can distinguish two cases:
- (a) $q \xrightarrow{1} q$. By Lemma C.12.5 we have $\text{TCP}_{\text{drt}}^\bullet \vdash q = 1 \cdot q$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p + q = a.p' + q \stackrel{\text{DR9}}{=} 1 \cdot \underline{a}.p' + q = 1 \cdot \underline{a}.p' + 1 \cdot q \stackrel{\text{DOT5}}{=} 1 \cdot (\underline{a}.p' + q) = 1 \cdot (\underline{a}.q' + q) = 1 \cdot q = q$.
- (b) $q \xrightarrow{1} q''$ for some $q'' < q$. Then $p + q \xrightarrow{1} p + q''$ and therefore, since $p + q \Leftrightarrow q$, we need to have $p + q'' \Leftrightarrow q''$. By induction we then have $\text{TCP}_{\text{drt}}^\bullet \vdash p + q'' = q''$. By Lemma C.12.4 we have $\text{TCP}_{\text{drt}}^\bullet \vdash q = \underline{\sigma}.q' + q$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p + q = a.p' + q \stackrel{\text{DAP}}{=} \underline{a}.p' + \underline{\sigma}.a.p' + q = \underline{a}.p' + \underline{\sigma}.p + q = \underline{a}.p' + \underline{\sigma}.p + \underline{\sigma}.q'' + q \stackrel{\text{DRTF}}{=} \underline{a}.p' + \underline{\sigma}.(p + q'') + q = \underline{a}.p' + \underline{\sigma}.q'' + q = \underline{a}.p' + q = \underline{a}.q' + q = q$.
9. $p \equiv \underline{\sigma}.p'$ for some basic term p' . From $p \xrightarrow{1} p'$, and the fact that $p + q \Leftrightarrow q$ it follows that there is some q' such that $q \xrightarrow{1} q'$ and $p' + q' \Leftrightarrow q'$. By induction we then have $\text{TCP}_{\text{drt}}^\bullet \vdash p' + q' = q'$. Also, by Lemma C.12.4 we have $\text{TCP}_{\text{drt}}^\bullet \vdash q = \underline{\sigma}.q' + q$. Then $\text{TCP}_{\text{drt}}^\bullet \vdash p + q = \underline{\sigma}.p' + q = \underline{\sigma}.p' + \underline{\sigma}.q' + q \stackrel{\text{DRTF}}{=} \underline{\sigma}.(p' + q') + q = \underline{\sigma}.q' + q = q$.
10. $p \equiv p' + p''$ for some basic terms p' and p'' . From $p + q \Leftrightarrow q$ it follows that both $p' + q \Leftrightarrow q$ and $p'' + q \Leftrightarrow q$. Then, by induction it follows that $\text{TCP}_{\text{drt}}^\bullet \vdash p' + q = q$ and $\text{TCP}_{\text{drt}}^\bullet \vdash p'' + q = q$. Then, $\text{TCP}_{\text{drt}}^\bullet \vdash p + q \equiv (p' + p'') + q \stackrel{\text{A2}}{=} p' + (p'' + q) \stackrel{\text{ih}}{=} p' + q \stackrel{\text{ih}}{=} q$.

□

C.4 Proof of conservativity of $\text{TCP}_{\text{drt}}^\bullet$ w.r.t. TCP^\bullet

We cannot apply the meta-theorems for equational conservativity from the literature that rely on the operational conservativity of the term deduction systems (see [Ver94, FV98, AFV01, Mid01]) since there is a new transition relation that can be derived for some old terms; e.g. $a.x \xrightarrow{1} a.x$.

Using Theorem 6 of [MR05b] (or [Mou05, Theorem 6.51]), to conclude that $\text{TCP}_{\text{drt}}^\bullet$ is an equationally conservative ground-extension of TCP^\bullet in case we already know that both TCP^\bullet and

$\text{TCP}_{\text{drt}}^\bullet$ are sound and complete, it suffices to prove that the term deduction system for $\text{TCP}_{\text{drt}}^\bullet$ is an *orthogonal extension* of the term deduction system for TCP^\bullet .

For the term deduction system for $\text{TCP}_{\text{drt}}^\bullet$ to be an orthogonal extension of the term deduction system of TCP^\bullet , we need to prove that (1) the derivability of all old transition relations and predicates for old terms in the two term deduction systems coincides, and (2) that bisimilarity on old terms in the two term deduction systems coincides.

For the first proof obligation we have the following reasoning. All derivations in the term deduction system for TCP^\bullet are also derivations in the term deduction system for $\text{TCP}_{\text{drt}}^\bullet$ since the deduction rules of the first are contained in the latter. For the other implication, note that all new deduction rules are either about the new transition relation $\overset{1}{\mapsto}$ or about new syntax. Hence these can also not contribute to new facts about old terms and transition relations or predicates.

For the second proof obligation we have the following reasoning. First, note that with respect to the old transition relations and predicates, i.e. the action transitions and termination relation, the two term deduction systems coincide as reasoned before. Thus it remains to prove that also the new time transitions cannot discriminate between old terms.

We can prove (but won't do so explicitly) the following facts: (1) for any closed TCP^\bullet -term p we have $p \overset{0}{\mapsto}$ iff $p \overset{1}{\mapsto}$, (2) for any time transition $p \overset{1}{\mapsto} p'$ of an old term p , it holds that $\text{TCP}_{\text{drt}}^\bullet \vdash p = p'$. For this latter statement we need to prove the statement that $p \downarrow$ implies $\text{TCP}_{\text{drt}} \vdash p = p + \dot{1}$ for closed TCP -terms.

Since we have shown that $\text{TCP}_{\text{drt}}^\bullet$ is an equationally conservative ground-extension of TCP^\bullet and the axioms of TCP^\bullet are contained in the axioms of $\text{TCP}_{\text{drt}}^\bullet$, it follows that $\text{TCP}_{\text{drt}}^\bullet$ is an equationally conservative extension of TCP^\bullet as well.

C.5 Proof of conservativity of $\text{TCP}_{\text{drt}}^\bullet$ w.r.t. TCP_{drt}

For the proof of this theorem we cannot even use the notion of orthogonal extension of [MR05b] since the term deduction system for $\text{TCP}_{\text{drt}}^\bullet$ allows for the derivation of action transitions between old terms that the term deduction system for TCP_{drt} does not allow to derive: $\underline{a}.1 \parallel (\underline{1} \cdot 1) \xrightarrow{a} 1 \parallel (\underline{1} \cdot 1 + 1)$.

However, a weaker statement about the relation between the term deduction systems at hand is possible: for all closed TCP_{drt} -terms p and q

1. if $p \xrightarrow{a} q$ can be derived from the term deduction system for TCP_{drt} , then $p \xrightarrow{a} q'$ can be derived from the term deduction system for $\text{TCP}_{\text{drt}}^\bullet$ for some closed TCP_{drt} -term q' such that $q \Leftrightarrow q'$;
2. if $p \xrightarrow{a} q$ can be derived from the term deduction system for $\text{TCP}_{\text{drt}}^\bullet$, then $p \xrightarrow{a} q'$ can be derived from the term deduction system for TCP_{drt} for some closed TCP_{drt} -term q' such that $q \Leftrightarrow q'$;

For time transitions between closed TCP_{drt} -terms and termination predicates on closed TCP_{drt} -terms the term deduction systems coincide.

Hence we can say that the derivability of all old transition relations and predicates for old terms in the two term deduction systems coincides upto bisimilarity in the right-hand side of action transitions.

Next we need to consider whether bisimilarity on old terms in the two term deduction systems coincides. As mentioned before, for the old transition relations and predicates on old terms this is not a problem. So, what about the new transition relation $\overset{0}{\mapsto}$ and about time transitions between an old and a new term (e.g., $\underline{0} \overset{1}{\mapsto} \dot{0}$)?

We can prove (but won't do so explicitly) the following facts: (1) every closed TCP_{drt} -term has a consistency transition, (2) for any consistency transition $p \overset{0}{\mapsto} p'$ of an old term p , it holds that $p \Leftrightarrow p'$. Therefore, the new transition relation $\overset{0}{\mapsto}$ does not change bisimilarity on closed TCP_{drt} -terms.

We can (but won't) prove that (1) for any closed TCP_{drt} -term p we have $p \stackrel{1}{\mapsto}$ from the term deduction system for $\text{TCP}_{\text{drt}}^\bullet$, and (2) for any closed TCP_{drt} -term p and any closed $\text{TCP}_{\text{drt}}^\bullet$ -term q such that we can derive $p \stackrel{1}{\mapsto} q$ from the term deduction system for $\text{TCP}_{\text{drt}}^\bullet$ that $q \rightleftharpoons \dot{0}$. Hence, also these transitions do not change bisimilarity.

From the above observations and the completeness of the theories TCP_{drt} and $\text{TCP}_{\text{drt}}^\bullet$ it follows that $\text{TCP}_{\text{drt}}^\bullet$ is an equationally conservative ground-extension of TCP_{drt} .