Addendum to "Embedding theorems for infinite groups"

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MATHEMATICS

ADDENDUM TO "EMBEDDING THEOREMS FOR INFINITE GROUPS"

BY

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Mr. R. Jeurissen (Nijmegen) discovered an error in one of the applications in my paper "Embedding theorems for infinite groups" (Nederl. Akad. Wetensch. Proc. Ser. A. **60** = Indag. Math. **19** (1957), 560–569).

On page 563 two examples were given for the notion of symmetrically generated groups. The second one reads: (ii) Ω is the (restricted) direct product of $\mathbf{n}$ copies of H. Mr. Jeurissen noticed that this example requires that H is abelian, for otherwise the symmetry condition is violated as soon as $\mathbf{n}>1$.

A consequence is that in the second part of theorem 1.1 (the part about direct products) the restriction should be made that H is abelian.

The proof of theorem 4.1 is not valid, since it depends on the case that H is non-abelian. The author does not know whether the theorem remains correct, so the question whether the (restricted) direct product of $2^{\mathbf{m}}$ groups $\sum_{\mathbf{m}}$ can be embedded into $\sum_{\mathbf{m}}$ itself ($\mathbf{m}$ infinite), remains open.

Theorem 4.1 was used in the proof of theorem 4.3, but it is not difficult to replace the proof of theorem 4.3 by a correct one:

Theorem 4.3. If $\mathbf{m}$ is an infinite cardinal number, then every abelian group of order $2^{\mathbf{m}}$ can be embedded into $\sum_{\mathbf{m}}$.

Proof. Every abelian group can be embedded into a complete abelian group, and (assuming the axiom of choice) every complete abelian group is a direct product of groups of the type R, 2^∞, 3^∞, 5^∞, ... Here R is the rational group, and 2^∞, 3^∞, ... are the so-called quasicyclic groups (see A. G. Kurosh, The Theory of Groups, New York 1956, pp. 167 and 165). Let H be the direct product of R, 2^∞, 3^∞, 5^∞, ..., then every abelian group G can be embedded into a direct product of copies of H. If G has order $2^{\mathbf{m}}$ then we do not need more then $2^{\mathbf{m}}$ copies of H.

Since H is abelian, the direct product of $2^{\mathbf{m}}$ copies of H is symmetrically generated by these groups. Moreover, for any finite number k, the direct product of k copies of H can be embedded into $\sum_{\mathbf{m}}$ (since H^k is countable, and $\mathbf{m}$ is infinite, such an embedding is produced by the Cayley representation). Now applying theorem 3.1 we obtain that the direct product of $2^{\mathbf{m}}$ copies of H can be embedded into $(\sum_{\mathbf{m}})^{\mathbf{m}}$. The latter group can be considered as an intransitive permutation group on $\mathbf{m}^2$ objects, having $\mathbf{m}$ transitivity sets of $\mathbf{m}$ elements each. Thus $(\sum_{\mathbf{m}})^{\mathbf{m}}$ can be embedded into $\sum_{\mathbf{m}^2}$. As $\mathbf{m}^2=\mathbf{m}$, we finally obtain that G can be embedded into $\sum_{\mathbf{m}}$.

In connection with these corrections, the following changes should be made in the paper:

Page 561. Omit the first two lines, and replace them by: This result is an application of the following general theorem.

Page 561. Line 7 from top. Replace "If, for every finite k" by "If H is abelian, and if, for every finite k".

Page 563. Line 11 from top. After "$\mathbf{n}$ copies of H" add "if H is abelian".

Page 565. Delete Theorem 4.1 and its proof.

Page 566. Line 17 from top (in the Corollary): Omit the words "their direct product and".

Page 566. Replace the proof of theorem 4.3 by the one given above.

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