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EINDHOVEN UNIVERSITY OF TECHNOLOGY

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Completion of the squares in the finite horizon H^∞ control problem by measurement feedback

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The Netherlands

COMPLETION OF THE SQUARES IN THE FINITE HORIZON H^∞ CONTROL PROBLEM BY MEASUREMENT FEEDBACK

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ABSTRACT

In this paper we study the finite horizon version of the standard H^∞ control problem by measurement feedback. Given a finite-dimensional linear, time-invariant system, together with a positive real number γ, we obtain necessary and sufficient conditions for the existence of a possibly time-varying dynamic compensator such that the $L_2[0,T]$-induced norm of the closed loop operator is smaller than γ. These conditions are expressed in terms of a pair of quadratic differential inequalities, generalizing the well-known Riccati differential equations that were introduced recently in the context of finite horizon H^∞ control.

1. THE FINITE HORIZON H^∞ CONTROL PROBLEM

Consider the linear time-invariant system

$$\Sigma \qquad \begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Ew(t) \ , \\ y(t) &= C_1 x(t) + \qquad D_1 w(t) \ , \\ z(t) &= C_2 x(t) + D_2 u(t) \ , \end{aligned} \tag{1.1}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ the control input, $w \in \mathbb{R}^l$ an unknown disturbance input, $y \in \mathbb{R}^p$ the measured output and $z \in \mathbb{R}^q$ the output to be controlled. A, B, E, C_1, D_1, C_2 and D_2 are constant real matrices of appropriate dimensions. In addition, we assume that some fixed time interval $[0, T]$ is given. We shall be concerned with the existence and construction of dynamic compensators of the form

$$\Sigma_F \qquad \begin{aligned} \dot{p}(t) &= K(t)p(t) + L(t)y(t) \ , \\ u(t) &= M(t)p(t) + N(t)y(t) \ , \end{aligned} \tag{1.2}$$

where K, L, M and N are real, matrix valued, continuous functions on $[0, T]$. The feedback interconnection of Σ and Σ_F is the linear time-varying system Σ_{cl} described by

$$\begin{bmatrix} \dot{x}(t) \\ \dot{p}(t) \end{bmatrix} = \begin{bmatrix} A + BN(t)C_1 & BM(t) \\ L(t)C_1 & K(t) \end{bmatrix} \begin{bmatrix} x(t) \\ p(t) \end{bmatrix} + \begin{bmatrix} E + BN(t)D_1 \\ L(t)D_1 \end{bmatrix} w(t) \ ,$$

$$z(t) = (C_2 + D_2N(t)C_1 \quad D_2M(t)) \begin{bmatrix} x(t) \\ p(t) \end{bmatrix} + D_2N(t)\, D_1\, w(t) \ .$$

Let us denote the matrices appearing in these equations by $A_e(t)$, $E_e(t)$, $C_e(t)$ and $D_e(t)$, respectively. Obviously, if we put $x(0) = 0$, $p(0) = 0$, then the closed loop system Σ_{cl} defines a convolution operator $G_{cl}: L_2^l[0, T] \to L_2^q[0, T]$ given by

$$(G_{cl}\, w)(t) = z(t) = \int_0^t C_e(t)\, \Phi_e(t, \tau)\, E_e(\tau)\, w(\tau)\, d\tau + D_e(t)\, w(t) \ ,$$

where $\Phi_e(t, \tau)$ is the transition matrix of $A_e(t)$. Thus, the influence of disturbances $w \in L_2^l[0, T]$ on the output z can be measured by the operator norm of G_{cl}, given in the usual way by

$$\|G_{cl}\| := \sup \{ \frac{\|G_F w\|_2}{\|w\|_2} \mid 0 \neq w \in L_2^l[0, T] \} \ .$$

Here, $\|x\|_2$ denotes the $L_2[0, T]$ norm of the function x. The problem that we shall discuss in this paper is the following:

given $\gamma > 0$, find necessary and sufficient conditions for the existence of a dynamic compensator Σ_F such that $\|G_{cl}\| < \gamma$.

The problem as posed here will be referred to as the *finite horizon H^∞ control problem by measurement feedback*. This problem was studied before in [6] and [2]. In the latter references it is however assumed that the following conditions hold: D_1 is surjective, D_2 is injective. In the present paper we shall extend the results obtained in [6] and [2] to the case that D_1 and D_2 are arbitrary.

2. QUADRATIC DIFFERENTIAL INEQUALITIES

A central role in our study of the problem posed is played by what we shall call *the quadratic differential inequality*. let $\gamma > 0$ be given. For any differentiable matrix function $P : [0, T] \to \mathbb{R}^{n \times n}$, define $F_\gamma(P) : [0, T] \to \mathbb{R}^{(n+m)\times(n+m)}$ by

$$F_\gamma(P) := \begin{bmatrix} \dot{P} + A'P + PA + C_2'C_2 + \frac{1}{\gamma^2} PEE^T P & PB + C_2'D_2 \\ B'P + D_2'C_2 & D_2'D_2 \end{bmatrix} . \tag{2.1}$$

If $F_\gamma(P)(t) \geq 0$ for all $t \in [0, T]$, then we shall say that P satisfies the quadratic differential inequality (at γ). Also a dual version of (2.1) will be important to us: for any differentiable $Q : [0, T] \to \mathbb{R}^{n \times n}$ define $G_\gamma(Q) : [0, T] \to \mathbb{R}^{(n+p)\times(n+p)}$ by:

$$G_\gamma(Q) := \begin{bmatrix} -\dot{Q} + AQ + QA' + EE' + \frac{1}{\gamma^2} QC_2'C_2Q & QC_1' + ED_1' \\ C_1Q + D_1E' & D_1D_1' \end{bmatrix} . \tag{2.2}$$

If $G_\gamma(Q)(t) \geq 0$ for all $t \in [0, T]$ then we shall say that Q satisfies the dual quadratic differential inequality (at γ). In the sequel let

$$G(s) := C_2(Is - A)^{-1}B + D_2 \ ,$$

$$H(s) := C_1(Is - A)^{-1}E + D_1$$

denote the open loop transfer matrices from u to z and w to y, respectively. Furthermore, denote by normrank(G) and normrank(H) the ranks of these transfer matrices considered as matrices with entries in the field of real rational functions. We are now ready to state our main result:

THEOREM 2.1. Let $\gamma > 0$. The following two statements are equivalent:

(i) There exists a time-varying dynamic compensator Σ_F such that $\|G_{cl}\| < \gamma$,

(ii) There exist differentiable matrix functions P and $Q : [0, T] \to \mathbb{R}^{n \times n}$ such that

$$(a) \quad F_\gamma(P)(t) \geq 0 \ \ \forall t \in [0, T] \text{ and } P(T) = 0 , \tag{2.3}$$

(b) rank $F_\gamma(P)(t) = \text{normrank}(G)$ $\forall t \in [0, T]$, (2.4)

(c) $G_\gamma(Q)(t) \geq 0$ $\forall t \in [0, T]$ and $Q(0) = 0$, (2.5)

(d) rank $G_\gamma(Q)(t) = \text{normrank}(H)$ $\forall t \in [0, T]$, (2.6)

(e) $\gamma^2 I - Q(t)P(t)$ is invertible $\forall t \in [0, T]$. (2.7)

The aim of this paper is to outline the main steps and ideas involved in a proof of the latter theorem. For a more detailed discussion we would like to refer to [5].

It can be shown that, in general, if $F_\gamma(P) \geq 0$ on $[0, T]$ then

$$\text{rank } F_\gamma(P)(t) \geq \text{normrank}(G) \text{ on } [0, T]$$

and, likewise, if $G_\gamma(Q) \geq 0$ on $[0, T]$ then

$$\text{rank } G_\gamma(Q)(t) \geq \text{normrank}(H) \text{ on } [0, T] \ .$$

This means that the pair of conditions (2.3), (2.4) can be reformulated as: P is a rank-minimizing solution of the quadratic differential inequality at γ, satisfying the end-condition $P(T) = 0$. A similar restatement is valid for the conditions (2.5), (2.6). It can also be shown that if P satisfies (2.3) and (2.4) then it is *unique*. Also, this unique solution turns out to be *symmetric* for all $t \in [0, T]$. The same holds for Q satisfying (2.5) and (2.6).

We will now show that for the special case that D_1 and D_2 are assumed to be surjective and injective, respectively, our Theorem 2.1 specializes to the results obtained before in [6] and [2]. Indeed, if D_2 is injective then of course

$$\text{normrank}(G) = \text{rank } D_2 = m \ .$$

Denote

$$R_\gamma(P) := \dot{P} + A'P + PA + C_2'C_2 + \frac{1}{\gamma^2}PEE'P - (PB + C_2'D_2)(D_2'D_2)^{-1}(B'P + D_2'C_2) \ .$$

Clearly, $R_\gamma(P)$ is the Schur-complement of $D_2'D_2$ in $F_\gamma(P)$. Therefore we have

$$\text{rank } F_\gamma(P)(t) = m + \text{rank } R_\gamma(P)(t)$$

for all $t \in [0, T]$. This implies that the pair of conditions (2.3), (2.4) is equivalent to the condition: P is the solution of the *Riccati differential equation* $R_\gamma(P) = 0$ with terminal condition $P(T) = 0$. A similar statement holds for Q satisfying (2.5) and (2.6). Thus we obtain

COROLLARY 2.2. Let $\gamma > 0$. Assume D_1 is surjective and D_2 is injective. Then the following statements are equivalent:

(i) There exists a time-varying dynamic compensator Σ_F such that $\|G_{cl}\| < \gamma$.

(ii) There exist differentiable matrix-functions P and Q: $[0,T] \to I\!R^{n\times n}$ such that for all $t \in [0,T]$

$$-\dot{P} = A'P + PA + C_2'C_2 + \frac{1}{\gamma^2}PEE'P - (PB + C_2'D_2)(D_2'D_2)^{-1}(B'P + D_2'C_2),\ P(T) = 0$$

and

$$\dot{Q} = AQ + QA' + EE' + \frac{1}{\gamma^2}QC_2'C_2Q - (QC_1' + ED_1')(D_1D_1')^{-1}(C_1Q + D_1E'),\ Q(0) = 0\ .$$

with, in addition,

$$\gamma^2 I - Q(t)P(t) \text{ invertible for all } t \in [0,T]\ . \qquad \square$$

It can be shown that if the conditions in the statement of Theorem 2.1 (ii) indeed hold, then it is always possible to find a suitable compensator with dynamic order equal to n, the dynamic order of the system to be controlled.

3. COMPLETION OF THE SQUARES

In this section we shall outline the proof of the implication (i) $\Rightarrow$ (ii) of Theorem 2.1. Consider the system Σ. For given u and w, let $z_{u,w}$ denote the output to be controlled, with $x(0) = 0$. Our starting point is the following lemma:

LEMMA 3.1. Let $\gamma > 0$. Assume that for all $0 \neq w \in L_2^l[0,T]$ we have

$$\inf\{\|z_{u,w}\|_2 - \gamma\|w\|_2 \mid u \in L_2^m[0,T]\} < 0\ . \qquad (3.1)$$

Then there exist a differentiable matrix function P: $[0,T] \to I\!R^{n\times n}$ such that $F_\gamma(P)(t) \geq 0$ $\forall t \in [0,T]$, $P(T) = 0$ and $\operatorname{rank} F_\gamma(P)(t) = \operatorname{normrank}(G)$ $\forall t \in [0,T]$.

PROOF. A proof of this can be given by combining the result of [2, Theorem 2.3] with ideas used in the proof of [4, Theorem 5.4]. $\square$

Now, assume that the condition (i) in the statement of Theorem 2.1 holds, i.e. assume there exists a dynamic compensator Σ_F such that $\|G_{cl}\| < \gamma$. Then condition (3.1) holds: let $w \in L_2^l[0,T]$ and $w \neq 0$ and let z be the closed loop output with $x(0) = 0$ and $p(0) = 0$. Then $z = z_{\bar{u},w}$, where $\bar{u}$ is the output of Σ_F. Clearly

$$\frac{\|z\|_2}{\|w\|_2} \leq \|G_{cl}\|$$

and hence $\|z_{\bar{u},w}\| - \gamma \|w\|_2 < 0$. Then also the infimum in (3.1) is less than 0. We may then conclude that, indeed, a differentiable matrix function P exists that satisfies (2.3) and (2.4).

The fact that also (2.5) and (2.6) hold can be proven by the following dualization argument. Consider the dual system

$$\Sigma' \qquad \begin{aligned} \dot{\xi} &= A'\xi + C_1'v + C_2'd \ , \\ \eta &= B'\xi + \qquad\quad D_2'd \ , \\ \zeta &= E'\xi + D_1'v \ , \end{aligned}$$

and apply to Σ' the time-varying compensator

$$\Sigma_F' \qquad \begin{aligned} \dot{q} &= K'(T-t)q + M'(T-t)\eta \ , \\ v &= L'(T-t)q + N'(T-t)\eta \ . \end{aligned}$$

It can be shown that if we denote by $\tilde{G}_{cl}$ the closed loop operator of $\Sigma' \times \Sigma_F'$ (with $\zeta(0) = 0, q(0) = 0$), and it G_{cl}^* denotes the adjoint operator of G_{cl} then the following equality holds:

$$\tilde{G}_{cl} = R \circ G_{cl}^* \circ R \ , \tag{3.2}$$

where R denotes the time-reversal operator $(Rx)(t) := x(T-t)$. Now, if $\|G_{cl}\| < \gamma$ then also $\|G_{cl}^*\| < \gamma$ and therefore, by 3.2, $\|\tilde{G}_{cl}\| < \gamma$. We can therefore conclude that the quadratic differential inequality associated with Σ' has an appropriate solution, say $\tilde{P}(t)$, on $[0,T]$. by defining $Q(t) := \tilde{P}(T-t)$ we obtain a function Q that satisfies (2.5) and (2.6).

Finally, we have to show that condition (2.7) holds. We shall need the following lemma:

LEMMA 3.2. Assume that there exists $P : [0,T] \to \mathbb{R}^{n\times n}$ such that $F_\gamma(P)(t) \geq 0$, $\forall t \in [0,T]$, and rank $F_\gamma(P)(t) = \text{normrank}(G)$, $\forall t \in [0,T]$. Then there exist continuous matrix functions $C_{2,P}$ and D_P such that for all t

$$F_\gamma(P)(t) = \begin{bmatrix} C_{2,P}'(t) \\ D_P'(t) \end{bmatrix} (C_{2,P}(t) \ \ D_P(t)) \ . \tag{3.3}$$

□

Assume $F_\gamma(P)$ is factorized as in (3.3). Introduce a new system, say Σ_P, by

$$\Sigma_P \qquad \begin{aligned} \dot{x}_P &= (A + \frac{1}{\gamma^2} EE'P)x_P + Bu_P + Ew_P \ , \\ y_P &= (C_1 + \frac{1}{\gamma^2} D_1 E'P)x_P + D_1 w_P \ , \\ z_P &= C_{2,P} x_P + D_P u_P \ . \end{aligned} \tag{3.4}$$

We stress that Σ_P is a time-varying system with continuous coefficient matrices. If Σ_F is a dynamic compensator of the form (1.2), let $G_{P,cl}$ denote the operator from w_P to z_P obtained by interconnecting Σ_P and Σ_F.

The crucial observation now is that $\|G_{cl}\| < \gamma$ if and only if $\|G_{P,cl}\| < \gamma$, that is, *a compensator* Σ_F *"works" for* Σ *if and only if it "works" for* Σ_p! A proof of this can be based on the following "completion of the squares" argument:

LEMMA 3.3. Assume that P satisfies (2.3) and (2.4). Assume $x_P(0) = x(0) = 0$, $u_P(t) = u(t)$ for all $t \in [0, T]$ and suppose that w_P and w are related by $w_p(t) = w(t) - \gamma^{-2} E'P(t)x(t)$ for all $t \in [0, T]$. Then for all $t \in [0, T]$ we have

$$\|z(t)\|^2 - \gamma^2 \|w(t)\|^2 = \frac{d}{dt}(x'(t)P(t)x(t)) + \|z_P(t)\|^2 - \gamma^2 \|w_P(t)\|^2 \ .$$

Consequently:

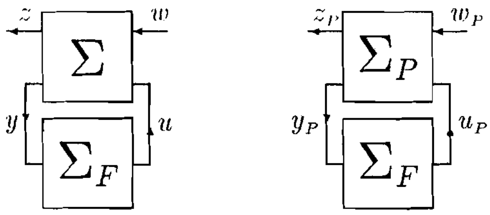
$$\|z\|_2^2 - \gamma^2 \|w\|_2^2 = \|z_P\|_2^2 - \gamma^2 \|w_P\|_2^2 \ . \tag{3.5}$$

PROOF: This can be proven by straightforward calculation, using the factorization (3.3). □

THEOREM 3.4. Let P satisfy (2.3) and (2.4). Let Σ_F be a dynamic compensator of the form (1.2). Then

$$\|G_{cl}\| < \gamma \iff \|G_{P,cl}\| < \gamma \ .$$

PROOF. Assume $\|G_{P,cl}\| < \gamma$ and consider the interconnection of Σ and Σ_F.



Let $0 \neq w \in L_2^l[0, T]$, let x be the corresponding state trajectory of Σ and define $w_P := w - \gamma^{-2} E'Px$. Then clearly $y_P = y$, $x_P = x$ and therefore $u_P = u$. This implies that the equality (3.4) holds. Also, we clearly have

$$\|z_P\|_2^2 - \gamma^2\|w_P\|_2^2 \le (\|G_{P,cl}\|^2 - \gamma^2)\|w_P\|_2^2 \ . \tag{3.6}$$

Next, note that the mapping $w_p \mapsto w_p + \gamma^{-2} E'Px_p$ defines a bounded operator from $L_2^l[0,T]$ to $L_2^l[0,T]$. Hence there exists a constant $\mu > 0$ such that $\|w_P\|_2^2 > \mu\|w\|_2^2$. Define $\delta > 0$ by $\delta^2 := \gamma^2 - \|G_{P,cl}\|^2$. Combining (3.4) and (3.5) then yields

$$\|z\|_2^2 - \gamma^2\|w\|_2^2 \le -\delta^2\mu\|w\|_2^2 \ .$$

Obviously, this implies that $\|G_{cl}\|^2 \le \gamma^2 - \delta^2\mu < \gamma^2$. □

We will now prove that (2.7) holds. Again assume that Σ_F yields $\|G_{cl}\| < \gamma$. By applying a version of Lemma 3.1 for time-varying systems it can then be proven that the *dual* quadratic differential inequality associated with Σ_P:

$$\bar{G}_\gamma(Y) := \begin{bmatrix} -\dot{Y} + A_P Y + Y A_P' + EE' + \frac{1}{\gamma^2} Y C_{2,P}' C_{2,P} Y & Y C_{1,P}' + E D_1' \\ C_{1,P} Y + D_1 E' & D_1 D_1' \end{bmatrix} \ge 0$$

has a solution $Y(t)$ on $[0,T]$, satisfying $Y(0) = 0$ and

$$\text{rank}\, \bar{G}_\gamma(Y)(t) = \text{normrank} \begin{bmatrix} Is - A_P(t) & -E \\ C_{1,P}(t) & D_1 \end{bmatrix} - n \tag{3.7}$$

for all $t \in [0,T]$. Here, we have denoted $A_P = A + \gamma^{-2} EE'P$ and $C_{1,P} = C_1 + \gamma^{-2} D_1 E'P$. Furthermore, it can be shown that Y is *unique* on each interval $[0,t_1]$ ($t_1 \le T$). On the other hand, it can be proven that on each interval $[0,t_1]$ on which $I - QP$ is invertible, the function $\tilde{Y} := (I - QP)^{-1} Q$ satisfies $\bar{G}_\gamma(Y)(t) \ge 0$, $\tilde{Y}(0) = 0$ and the rank condition (3.7). Thus on any such interval $[0,t_1]$ we must have $Y(t) = \tilde{Y}(t)$. Clearly, since $Q(0) = 0$, there *exists* $0 < t_1 \le T$ such that $I - QP$ is invertible on $[0,t_1)$. assume now that $t_1 > 0$ is the smallest real number such that $I - Q(t_1)P(t_1)$ is not invertible. Then on $[0,t_1)$ we have

$$Q(t) = (I - Q(t)P(t))\, \tilde{Y}(t)$$

and hence, by continuity

$$Q(t_1) = (I - Q(t_1)P(t_1))\, \tilde{Y}(t_1) \ . \tag{3.8}$$

There exists $x \ne 0$ such that $x'(I - Q(t_1)P(t_1)) = 0$. By (3.8) this yields $x'Q(t_1) = 0$ whence $x' = 0$, which is a contradiction. We must conclude that $I - Q(t)P(t)$ is invertible for all $t \in [0,T]$.

This completes our proof of the implication (i) ⇒ (ii) of Theorem 2.1.

4. EXISTENCE OF COMPENSATORS

In the present section we will sketch the main ideas of our proof of the implication (ii) $\Rightarrow$ (i) of Theorem 2.1. The main idea is as follows: starting from the original system Σ we shall define a new system, $\Sigma_{P,Q}$, which has the following important properties:
(1) Let Σ_F be any compensator. The closed loop operator G_{cl} of the interconnection $\Sigma \times \Sigma_F$ satisfies $\|G_{cl}\| < \gamma$ *if and only if* the closed loop operator of $\Sigma_{P,Q} \times \Sigma_F$, say $G_{P,Q,cl}$, satisfies $\|G_{P,Q,cl}\| < \gamma$.
(2) The system $\Sigma_{P,Q}$ is almost disturbance decoupable by dynamic measurement feedback, i.e. for *all* $\varepsilon > 0$ there exists Σ_F such that $\|G_{P,Q,cl}\| < \varepsilon$.
Property (1) states that a compensator Σ_F "works" for Σ if and only if it "works" for $\Sigma_{P,Q}$. On the other hand, property (2) states that, *indeed*, there exists a compensator Σ_F that "works" for $\Sigma_{P,Q}$: take any $\varepsilon \leq \gamma$ and take a compensator Σ_F such that $\|G_{P,Q,cl}\| < \varepsilon$. Then by, property (1), $\|G_{cl}\| < \gamma$ so Σ_F works for Σ. This would clearly establish a proof of the implication (ii) $\Rightarrow$ (i) in Theorem 2.1.

We shall now describe how the new system $\Sigma_{P,Q}$ is defined. Assume that there exist P and Q satisfying (2.3) to (2.7). Apply Lemma 3.2 to obtain a continuous factorization (3.3) of $F_\gamma(P)$ and let the system Σ_P be defined by (3.4). Next, consider the dual quadratic differential inequality $\bar{G}_\gamma(Y) \geq 0$ associated with the system Σ_P, together with the conditions $Y(0) = 0$ and the rank condition (3.7). As was already noted in the previous section, the conditions (2.5), (2.6) and (2.7) assure that there exists a unique solution Y on $[0,T]$. (In fact, $Y(t) = (\gamma^2 I - Q(t)P(t))^{-1} Q(t)$.) Now, it can be shown that there exists a factorization

$$\bar{G}_\gamma(Y)\,(t) = \begin{pmatrix} E_{P,Q}(t) \\ D_{P,Q}(t) \end{pmatrix} (E_{P,Q}'(t) \quad D_{P,Q}'(t)),$$

with $E_{P,Q}$ and $D_{P,Q}$ continuous on $[0,T]$. Denote

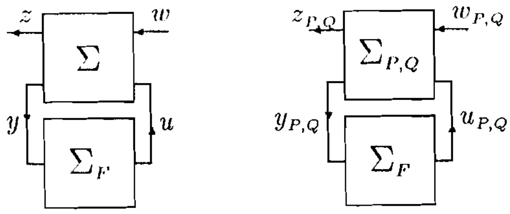
$$A_{P,Q}(t) := A_p(t) + Y(t)\, C_{2,P}'(t)\, C_{2,P}(t),$$

$$B_{P,Q}(t) := B + Y(t) C_{2,P}'(t) D_P(t).$$

Then, introduce the new system $\Sigma_{P,Q}$ by:

$$\Sigma \qquad \begin{aligned} \dot{x}_{P,Q} &= A_{P,Q}\, x_{P,Q} + B_{P,Q}\, u_{P,Q} + E_{P,Q}\, w_{P,Q}, \\ y_{P,Q} &= C_{1,P}\, x_{P,Q} + D_{P,Q}\, w_{P,Q}, \\ z_{P,Q} &= C_{2,P}\, x_{P,Q} + D_P\, u_{P,Q}. \end{aligned}$$

Again, $\Sigma_{P,Q}$ is a time-varying system with continuous coefficient matrices. We note that $\Sigma_{P,Q}$ is in fact obtained by first transforming Σ into Σ_P and by subsequently applying the dual of this transformation to Σ_P. We shall now first show that property (1) above holds. If Σ_F is a dynamic compensator, then let $G_{P,Q,cl}$ be the closed loop operator from $w_{P,Q}$ to $z_{P,Q}$ in the interconnection of $\Sigma_{P,Q}$ with Σ_F:



Recall that G_{cl} denotes the closed loop operator from w to z in the interconnection of Σ and Σ_F. We have the following:

THEOREM 4.1.

$$\|G_{cl}\| < \gamma \iff \|G_{P,Q,cl}\| < \gamma.$$

PROOF. Assume Σ_F yields $\|G_{cl}\| < \gamma$. By Theorem 3.4 then also $\|G_{P,cl}\| < \gamma$, i.e., Σ_F interconnected with Σ_P (given by 3.4) also yields a closed loop operator with norm less than γ. It is easily seen that the dual compensator Σ_F' (see section 3), interconnected with the dual of Σ_P:

$$\Sigma_P' \qquad \begin{aligned} \dot{\xi} &= A_P'(T-t)\xi + C_{1,P}'(T-t)\nu + C_{2,P}'(T-t)d \\ \eta &= B'\xi + D_P'(T-t)d \\ \zeta &= E'\xi + D_1'\nu \end{aligned}$$

yields a closed loop operator $\tilde{G}_{P,cl}$ (from d to ζ) with $\|\tilde{G}_{P,cl}\| < \gamma$. Now, the quadratic differential inequality associated with Σ_P' is the transposed, time-reversed version of the inequality $\bar{G}_\gamma(Y) \geq 0$ and therefore has a unique solution $\tilde{Y}(t) = Y(T-t)$ such that $\tilde{Y}(T) = 0$ and the corresponding rank condition (3.7) holds. By applying Theorem 3.4 to the system Σ_P' we may then conclude that the interconnection of Σ_F' with the dual $\Sigma_{P,Q}'$ of $\Sigma_{P,Q}$ yields a closed loop operator with norm less than γ. Again by dualization we then conclude that $\|G_{P,Q,cl}\| < \gamma$. The converse implication is proven analogously. □

Property (2) is stated formally in the following theorem:

THEOREM 4.2. For all $\varepsilon > 0$ there exists a time-varying dynamic compensator Σ_F such that $\|G_{P,Q,cl}\| < \gamma$. □

Due to space limitations, for a proof of the latter theorem we refer to [5]. By combining theorems 4.1 and 4.2 we immediately obtain a proof of the implication (ii) $\Rightarrow$ (i) in Theorem 4.2.

5. CONCLUDING REMARKS

In this paper we have studied the finite horizon H^∞ control problem by dynamic measurement feedback. We have noted that the results obtained can be specialized to re-obtain results that were obtained before [6] and [2]. The development of our theory runs analogously to the theory developed in [4] and [3] around the standard H^∞ control problem (the *infinite* horizon version of the problem studied in the present paper). In the latter references the main tools are the so-called *quadratic matrix inequalities*, the algebraic versions of the differential inequalities used in the present paper. For the special case that D_1 is surjective and D_2 is injective these quadratic matrix inequalities reduce to the algebraic Riccati equations that were also obtained in [6] and [1].

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