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# Discrete vapour cavity model with efficient and accurate convolution type unsteady friction term

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## Abstract

This paper presents experimental data and numerical simulations of transient vaporous cavitation events generated by a downstream fast valve closure in a pipeline. The experimental apparatus comprises a 37.2 m long constant-sloping pipe of 22.1 mm internal diameter connecting two pressurized tanks. The simulation results show that improper selection of the weighting function in the convolution-based unsteady friction term in the discrete vapour cavity model (DVCM) may significantly attenuate some measured high-frequency pressure pulses and speed-up the timing of the main pressure pulses.

## Introduction

Transient vaporous cavitation (including column separation) occurs in piping systems when the liquid pressure falls to the vapour pressure. Cavitation may occur as a localized vapour cavity (large void fraction) or as distributed vaporous cavitation (small void fraction). A number of vaporous cavitation models have been developed including discrete cavity and interface models (Ref 1). The discrete vapour cavity model (DVCM) with steady pipe flow friction term is widely used in standard water hammer software packages (Ref 2). The DVCM may generate unrealistic pressure pulses (spikes) due to the collapse of multi-cavities, but the model gives reasonably accurate results when the number of reaches is restricted. It is recommended that the maximum volume of the discrete cavity at a section is less than 10 % of the reach volume (Ref 3).

The steady pipe flow friction assumption is satisfactory for slow transients where the wall shear stress has a quasi-steady behaviour. Experimental validation of the steady friction model applied to rapid liquid transients has shown significant discrepancies in attenuation, shape and timing of pressure waves when the computational results are compared to the results of measurements (Ref 4). Researchers have attempted to incorporate a number of unsteady friction models into DVCM. Shuy and Apelt (Ref 5) performed numerical analyses with a number of friction models including steady, quasi-steady and unsteady friction models (instantaneous-mean velocity, instantaneous-acceleration and convolution-based models). They studied 'slow' transients in two long pipelines (2.3 km and 9 km). For the case of water hammer (no cavitation) they found little differences in the results of the five models, but for the case with column-separation (two-phase flow) large discrepancies occurred. Brunone et al. (Ref 6) used the DVCM in combination with an instantaneous-acceleration unsteady friction model. Numerical results were compared with measurements of rapid water hammer and column separation. Significant discrepancies between experiment and theory were found for all runs when using a quasi-steady friction term. Results obtained with the unsteady friction model showed an improved agreement between the computed and measured results. The agreement was better for the water-hammer case than for the column-separation case. Bergant and Simpson (Ref 7) investigated the performance of quasi-steady and unsteady friction models similar to those used by Shuy and Apelt (Ref 5). The instantaneous-acceleration and convolution-based unsteady friction models gave the best fit with experimental data for the case of water hammer. Bughazem and Anderson (Ref 8) developed DVCM with an instantaneous-acceleration unsteady friction term and found good agreement between theory and experiment. Numerical studies by Bergant and Tijsseling (Ref 9) have shown that unsteady friction may cause a significant damping of the pressure spikes observed in measurements. This paper further explores these effects in transient vaporous cavitating pipe flow. The paper presents DVCM with computationally efficient and accurate weighting function type unsteady friction term (Ref 10).

The paper presents a number of experimental data and numerical simulations of transient vaporous cavitation events generated by a fast downstream-valve closure. The experiments were performed in a 37.2 m long constant-sloping pipe of 22.1 mm internal diameter connecting two pressurized tanks. DVCM simulations using computationally efficient and accurate weighting function type unsteady friction term (convolution-based model) are compared with measured data.

## **Mathematical model**

The discrete vapour cavity model (DVCM) allows cavities to form at computational sections in the standard water hammer method of characteristics (MOC) numerical grid when the pressure falls to the liquid's vapour pressure. The standard water hammer solution is no longer valid at a vapour-pressure section. The head at this section is set to the vapour pressure head. Pure liquid with a constant water hammer wave speed is assumed to occupy the reach in between two computational sections. Each discrete vapour cavity is fully governed by two

water hammer compatibility equations, one continuity equation for the vapour cavity volume and the constant vapour head. The numerical solution within the diamond grid of the MOC, when written for a computational section  $i$  at time  $t$ , is (Ref 11):

- water hammer compatibility equation along the  $C^+$  characteristic line ( $\Delta x/\Delta t = a$ )

$$H_{i,t} - H_{i-1,t-\Delta t} + \frac{a}{gA} \left( (Q_u)_{i,t} - (Q_d)_{i-1,t-\Delta t} \right) + \frac{f_0 \Delta x}{2gDA^2} (Q_u)_{i,t} \left| (Q_d)_{i-1,t-\Delta t} \right| = 0 \quad (1)$$

- water hammer compatibility equation along the  $C^-$  characteristic line ( $\Delta x/\Delta t = -a$ )

$$H_{i,t} - H_{i+1,t-\Delta t} - \frac{a}{gA} \left( (Q_d)_{i,t} - (Q_u)_{i+1,t-\Delta t} \right) - \frac{f_0 \Delta x}{2gDA^2} (Q_d)_{i,t} \left| (Q_u)_{i+1,t-\Delta t} \right| = 0 \quad (2)$$

- continuity equation for the vapour cavity volume

$$(\nabla_{vap})_{i,t} = (\nabla_{vap})_{i,t-2\Delta t} + ((1-\psi)((Q_d)_{i,t-2\Delta t} - (Q_u)_{i,t-2\Delta t}) + \psi((Q_d)_{i,t} - (Q_u)_{i,t}))2\Delta t \quad (3)$$

in which  $\Delta x$  = reach length,  $\Delta t$  = time step,  $a$  = water hammer wave speed,  $H$  = piezometric head (head),  $g$  = gravitational acceleration,  $A$  = pipe area,  $Q_u$  = discharge at the upstream side of computational section  $i$ ,  $Q_d$  = discharge at the downstream side of the section  $i$ ,  $f_0$  = Darcy-Weisbach friction factor,  $D$  = pipe diameter,  $\nabla_{vap}$  = discrete vapour cavity volume and  $\psi$  = weighting factor. A single diamond MOC grid is used to eliminate grid-separation instability (Ref 3). The weighting factor  $\psi$  has a value between 0.5 and 1.0. The cavity collapses when its calculated volume becomes less than zero. The liquid phase is re-established and the water hammer solution using Eqs. (1) and (2) only (with  $(Q_u)_i \equiv (Q_d)_i$ ) is valid again. At boundaries (reservoir, valve), a device-specific equation replaces the missing water hammer compatibility equation.

### Convolution-based unsteady pipe flow friction model

This paper investigates the influence of convolution-based unsteady friction modelling on the performance of the DVCM. The first-order numerical approximation of the friction term is used in Eqs. (1) and (2). The steady friction factor  $f_0$  is replaced by the unsteady friction factor  $f$ , expressed as the sum of the steady part  $f_0$  and the unsteady part  $f_u$

$$f = f_0 + f_u \quad (4)$$

The unsteady friction part  $f_u$  depends on past temporal accelerations. Zielke (Ref 12) analytically developed the convolution-based model of unsteady friction for transient laminar flow. The unsteady part of the friction factor is defined by the convolution of a weighting function with past discharge variations

$$f_u = \frac{32\nu A}{DQ|Q|} \int_0^t \frac{\partial Q}{\partial t^*} W_0(t-t^*) dt^* \quad (5)$$

in which  $\nu$  = kinematic viscosity and  $W_0$  = weighting function based on initial flow conditions, i.e. Reynolds number and relative roughness of the pipe wall. Zielke evaluated Eq. (5) using the

full convolution scheme, which is computationally expensive because it requires convolution with a complete history of past discharges. The most recent approach, which is computationally efficient and accurate, makes an approximation of the weighting function by a finite sum of  $N_W$  exponential terms (Ref 10)

$$W_{app}(\tau) = \sum_{k=1}^{N_W} m_k e^{-n_k \tau} \quad (6)$$

in which  $\tau =$  dimensionless time ( $\tau = 4vt/D^2$ ) and  $m_k, n_k =$  coefficients in  $W_{app}$  (= approximation of  $W_0$ ). The coefficients  $m_k$  and  $n_k$  have been developed for Zielke's weighting function for transient laminar flow (Ref 12) and for Vardy-Brown's weighting functions for transient turbulent flow in hydraulically smooth (Ref 13) and fully rough (Ref 14) pipes. The coefficients are available in Vítkovský et al. (Ref 10). The unsteady part of the friction factor is defined now as

$$f_u = \frac{32vA}{DQ|Q|} \sum_{k=1}^{N_W} y_k(t) \quad (7)$$

where the component of the weighting function  $y_k(t)$  is expressed as follows

$$y_k(t) = \int_0^t \frac{\partial Q}{\partial t^*} m_k e^{-n_k K_W (t-t^*)} dt^* \quad (8)$$

in which the factor  $K_W$  ( $K_W = 4v/D^2$ ) converts the time  $t$  into the dimensionless time  $\tau$ . At time  $t + 2\Delta t$  the component  $y_k$  is

$$y_k(t + 2\Delta t) = \int_0^{t+2\Delta t} \frac{\partial Q}{\partial t^*} m_k e^{-n_k K_W (t+2\Delta t-t^*)} dt^* \quad (9)$$

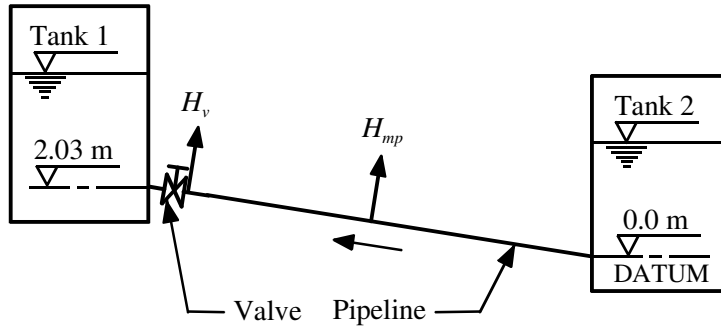
Solving the integral numerically gives an efficient recursive expression for the component  $y_k$  and hence for  $f_u$

$$y_k(t + 2\Delta t) = e^{-n_k K_W \Delta t} (e^{-n_k K_W \Delta t} y_k(t) + m_k (Q(t + 2\Delta t) - Q(t))) \quad (10)$$

The component  $y_k(t)$  has been calculated at a previous time step and is known at time  $t + 2\Delta t$ . There is *no convolution* with the complete history of discharges required at each time step.

## Experimental apparatus

An adjustable experimental apparatus for investigating water hammer and column separation events in pipelines has been designed and constructed (Ref 15). The apparatus comprises a straight  $L = 37.2$  m long sloping copper pipe of  $D = 22.1$  mm internal diameter and  $e = 1.63$  mm wall thickness connecting two pressurized tanks (see Fig. 1). The estimated relative roughness of the copper pipe wall is  $\epsilon/D = 0.00009$ . The wave speed was experimentally determined as  $a = 1319$  m/s.



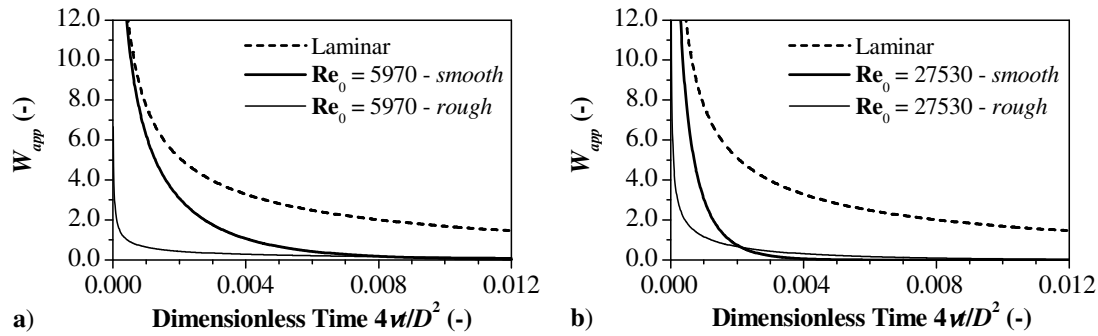
**Figure 1 Experimental apparatus layout.**

Five pressure transducers are located at equidistant points along the pipeline including as close as possible to the end points. Pressures measured at the valve ( $H_v$ ) and at the midpoint ( $H_{mp}$ ) are presented in this paper. The water temperature in Tank 1 is continuously monitored and the valve position during closure is recorded using optical sensors. The initial steady-state velocity of the flow was estimate from the rate of the volume change in Tank 1. The uncertainties in a measurement are fully described by Bergant and Simpson (Ref 15). Specified pressures in tanks are maintained by a computerized pressure control system. A transient event in the apparatus is initiated by the rapid closure of the ball valve.

## Case studies

Results from the DVCM with convolution-based unsteady friction model are compared with results of measurements. The effect of Vardy-Brown's weighting functions developed for transient turbulent flow in hydraulically smooth (Ref 13; DVCM-*smooth*) and fully rough (Ref 14; DVCM-*rough*) pipelines on pressure waves is investigated. Computational and experimental runs were performed for a rapid closure of the valve positioned at the downstream end of the upward sloping pipe at pressurized tank T1 (see Fig. 1).

Computational and measured results of the transient events in experimental apparatus are presented for two different initial flow velocities  $V_0 = \{0.30; 1.40\}$  m/s at a constant static head in pressurized reservoir T2 of  $H_{T2} = 22$  m (Ref 1). The measured water temperature for the low-initial velocity case was  $\nu_w = 16$  °C and for the high-initial velocity case was  $\nu_w = 15.5$  °C. The gauge vapour heads are  $H_{vap} = \{-10.25$  m;  $-10.29$  m} and the initial Reynolds numbers are  $\mathbf{Re}_0 = \{5970; 27530\}$  respectively. The respective Vardy-Brown weighting functions ( $W_{app}$ ) are shown in Fig. 2. The weighting functions for fully rough pipe flow for relative roughness of the pipe wall of  $\epsilon/D = 0.00009$  decay much more rapidly than the weighting functions for smooth pipe flow.



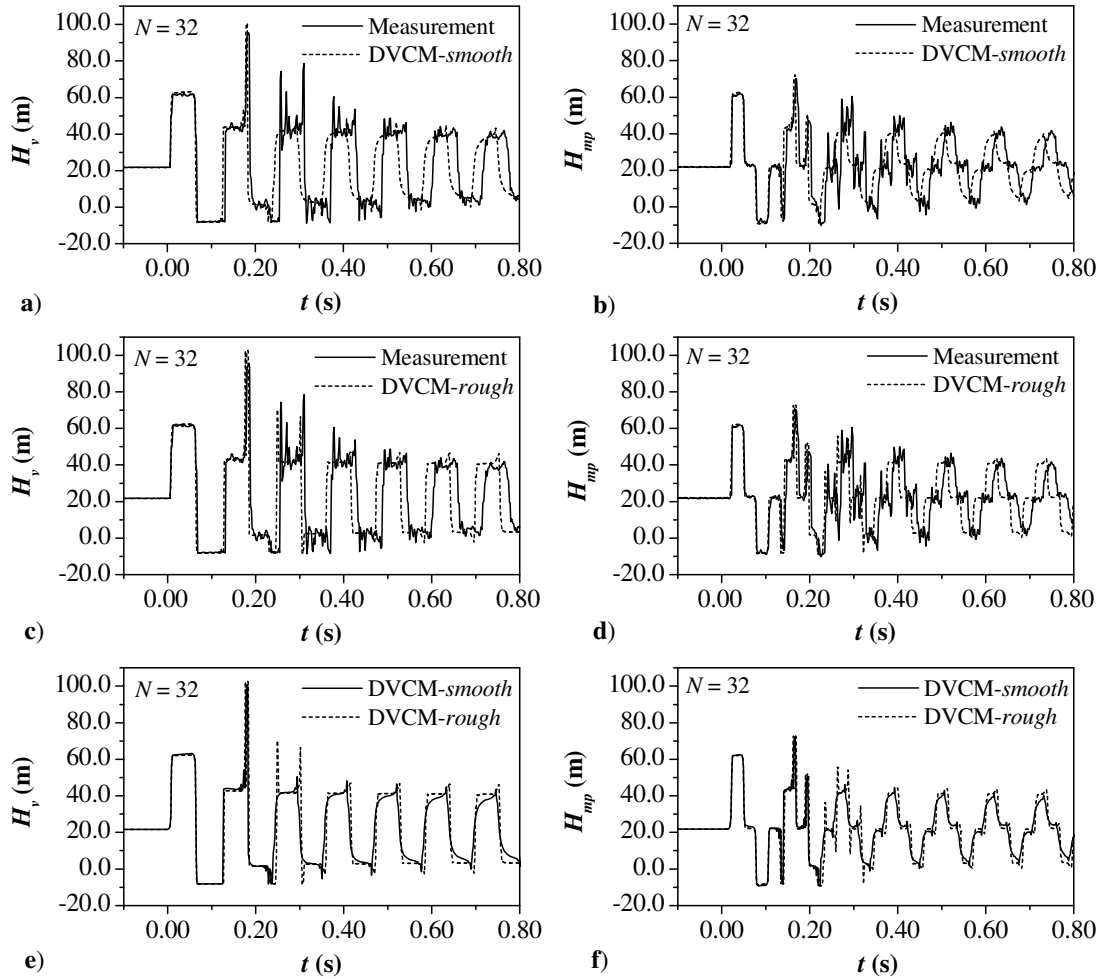
**Figure 2 Weighting functions for transient laminar and turbulent flow (smooth and rough wall pipe flow; relative roughness of the pipe wall of  $\epsilon/D = 0.00009$ ).**

The valve closure time for the two runs was identical,  $t_c = 0.009$  s, which is significantly shorter than the water hammer wave reflection time of  $2L/a = 0.056$  s. The rapid valve closure begins at time  $t = 0$  s. The value of the weighting factor  $\psi = 1.0$  was used in Eq. (3). Different numbers of reaches were selected for each computational run,  $N = \{16, 32, 64, 128, 256\}$ , to examine numerical convergence and robustness of the model.

### Case study 1: $Re_0 = 5970$

Comparison of the numerical and experimental results for an initial flow velocity  $V_0 = 0.30$  m/s and different numbers of computational reaches,  $N = \{32, 128\}$ , is presented in Figs. 3 and 4 respectively. A limited number of reaches is commonly used in engineering water hammer analysis. A larger number of reaches should in principle give more accurate results.

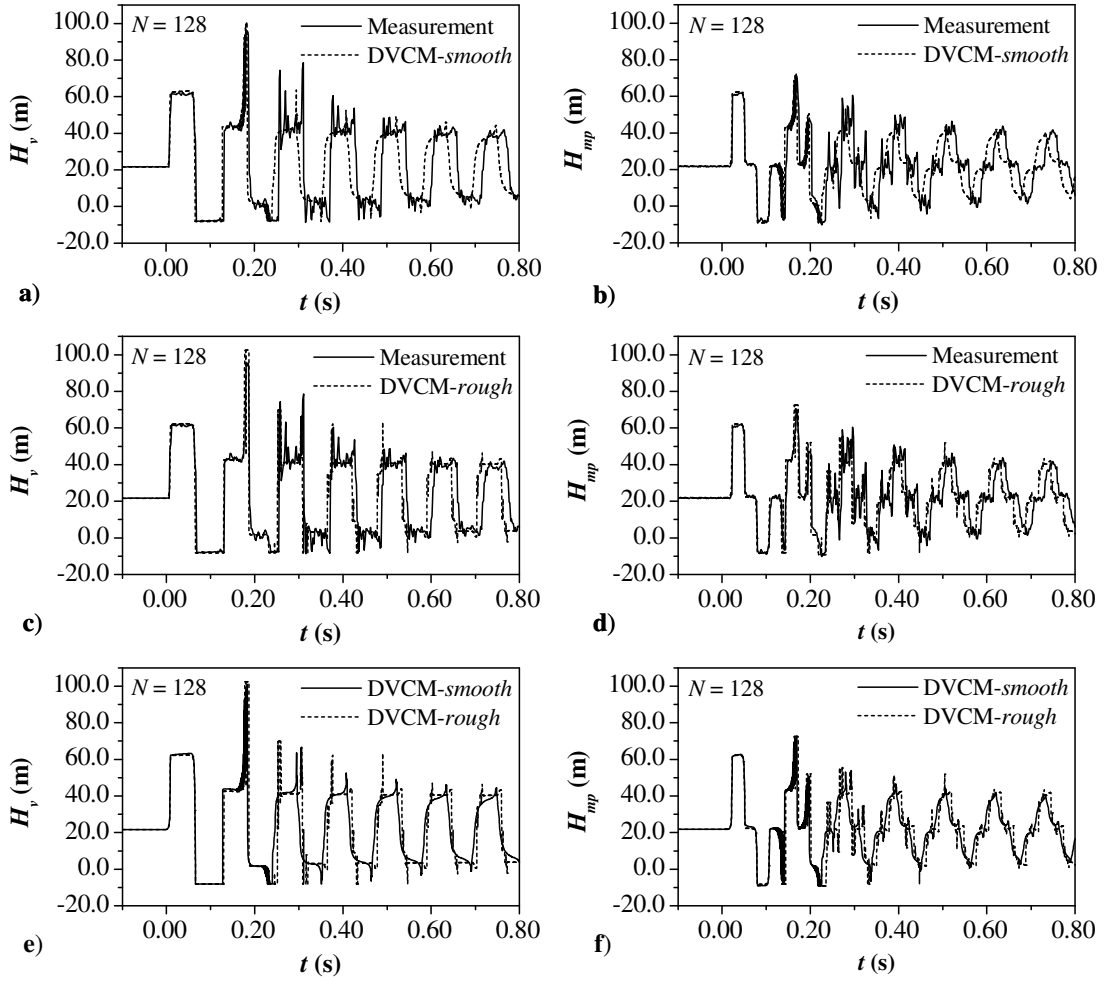
A rapid valve closure for the low-initial flow velocity case generates a water hammer event with *moderate* liquid column separation. The location and intensity of discrete vapour cavities is governed by the type of transient regime, layout of the piping system and hydraulic characteristics (Ref 1). The maximum measured head at the valve  $H_{v, max} = 95.6$  m occurs as a short-duration (narrow) pressure pulse about  $2L/a$  (seconds) after the first cavity collapsed. The maximum computed heads predicted by DVCM-*smooth* and DVCM-*rough* are  $H_{v, max} = \{N = 32: 100.8 \text{ m}; N = 128: 101. \text{ m}\}$  and  $H_{v, max} = \{N = 32: 102.6 \text{ m}; N = 128: 102.5 \text{ m}\}$  respectively. The computed maximum discrete cavity volume is about  $10^{-6} \text{ m}^3$  at the valve (large localized vapour cavity) and  $10^{-9}$  to  $10^{-7} \text{ m}^3$  along the pipeline (distributed vaporous cavitation zones). Measurement of the discrete vapour cavity volume is complex. A reasonable measure of the cavity size is the time of existence of an actual discrete cavity. The time of existence of the first cavity at the valve is  $t_{\forall vap, max} = \{\text{Measurement: } 0.064 \text{ s}; \text{DVCM-} \textit{smooth}: N = 32: 0.062 \text{ s and } N = 128: 0.063 \text{ s}; \text{DVCM-} \textit{rough}: N = 32: 0.063 \text{ s and } N = 128: 0.064 \text{ s}\}$ .



**Figure 3 Comparison of heads at the valve ( $H_v$ ) and at the midpoint ( $H_{mp}$ ):  $V_0 = 0.30$  m/s,  $H_{T2} = 22$  m,  $Re_0 = 5970$ ,  $N = 32$ .**

Computational results obtained by DVCM-*smooth* and DVCM-*rough* agree well with experimental results for the first and the second pressure head pulse. Results obtained by DVCM-*smooth* show strong attenuation of some measured high-frequency pressure spikes and slightly faster timing of the main pressure pulses during vaporous cavitation phase than the results obtained by DVCM-*rough*.

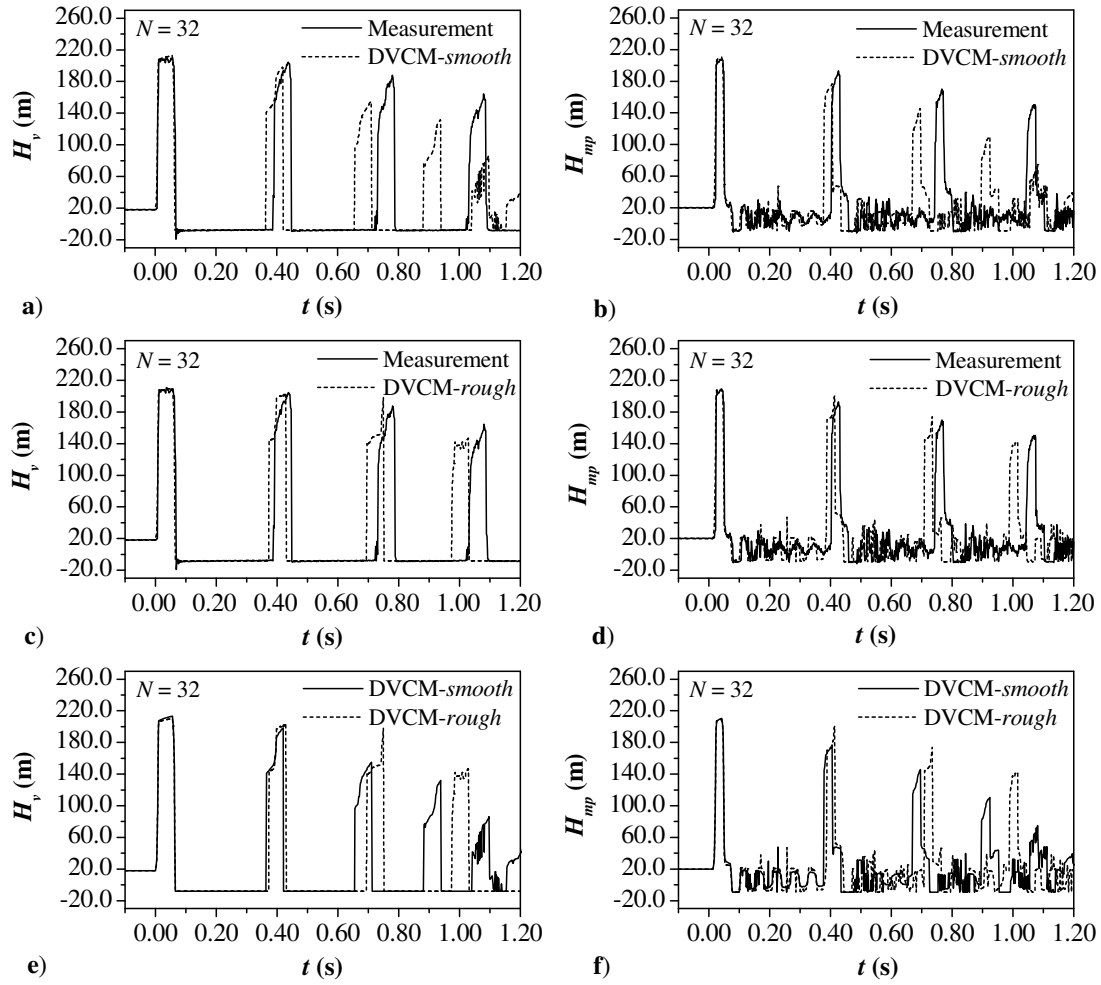




**Figure 4 Comparison of heads at the valve ( $H_v$ ) and at the midpoint ( $H_{mp}$ ):  $V_0 = 0.30$  m/s,  $H_{T2} = 22$  m,  $Re_0 = 5970$ ,  $N = 128$ .**

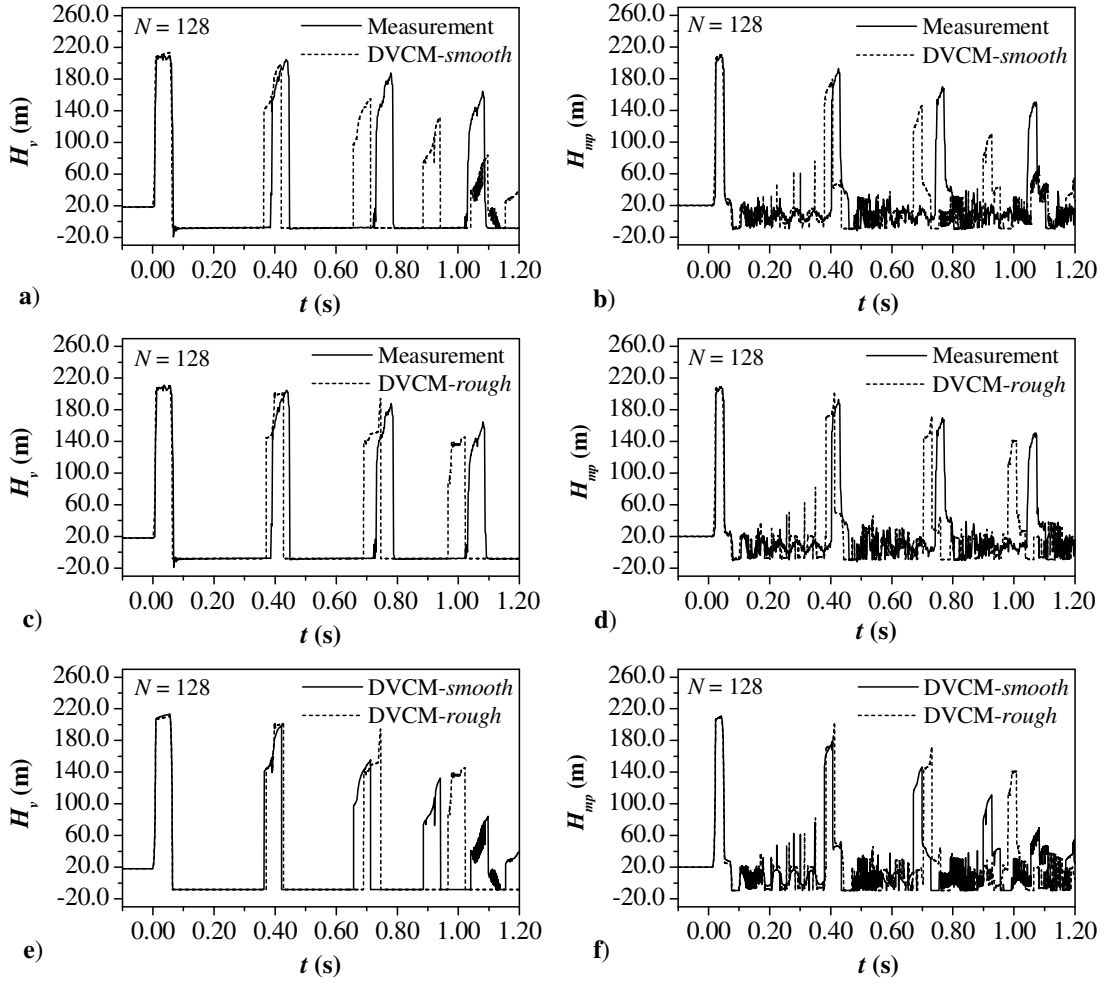
### Case study 2: $Re_0 = 27530$

Comparison of the numerical and experimental results for an initial flow velocity  $V_0 = 1.40$  m/s and different numbers of computational reaches,  $N = \{32, 128\}$ , is presented in Figs. 5 and 6 respectively. A rapid valve closure for this case generates a water hammer event with *severe* liquid column separation. The maximum head at the valve for this case is the water hammer head generated at a time of  $2L/a$  after the valve closure. The value of the maximum measured head is  $H_{v,max} = 210.9$  m. The maximum head predicted by DVCM-*smooth* and DVCM-*rough* matches the measured maximum head. Again, the timing of the main pressure pulses during vaporous cavitation is predicted better by DVCM-*rough* than by DVCM-*smooth*. Results obtained by DVCM-*smooth* show strong attenuation of the main pressure pulses at later times. Similar behaviour has been observed previously with the DVCM using convolution-based unsteady friction model with Zielke's weighting function for transient laminar flow (Ref 7).



**Figure 5 Comparison of heads at the valve ( $H_v$ ) and at the midpoint ( $H_{mp}$ ):  $V_0 = 1.40$  m/s,  $H_{T2} = 22$  m,  $Re_0 = 27530$ ,  $N = 32$ .**

*Severe* cavitation along the pipeline forms distributed vaporous cavitation zones and intermediate cavities that have been recorded by measurements and only approximately accounted for in the DVCM (Figs. 5 and 6). The computed maximum discrete cavity volume is about  $10^{-4}$  m<sup>3</sup> at the valve and  $10^{-8}$  to  $10^{-5}$  m<sup>3</sup> along the pipeline. The computed and measured times of existence of the first cavity at the valve agree to lesser extent i.e.  $t_{\nabla vap, max} = \{\text{Measurement: } 0.318 \text{ s; DVCM-smooth: } N = 32: 0.300 \text{ s and } N = 128: 0.301 \text{ s; DVCM-rough: } N = 32: 0.308 \text{ s and } N = 128: 0.306 \text{ s}\}$ .



**Figure 6 Comparison of heads at the valve ( $H_v$ ) and at the midpoint ( $H_{mp}$ ):  $V_0 = 1.40$  m/s,  $H_{T2} = 22$  m,  $Re_0 = 27530$ ,  $N = 128$ .**

Detailed analysis of numerical results obtained with different numbers of computational reaches,  $N = \{16, 32, 64, 128, 256\}$ , reveals that the magnitude and timing of the main pressure pulses predicted by DVCM-*smooth* and DVCM-*rough* converge to different solutions. Convergence relates to behaviour of the solution as  $\Delta x$  and  $\Delta t$  approach zero, whereas stability is concerned with round-off error growth. There still remain some high-frequency pressure spikes in the experimental measurements that are not reproduced by the DVCM (Figs. 3 to 6). However, there are some numerical spikes too that do not exist in the measurements. This behaviour is attributed to the random nature of the transient vaporous cavitating flow along the pipe. It is clear that *small-scale* cavitation events take place in slightly different ways for different numbers of computational reaches. The vaporous cavitation zones (small void fraction) and intermediate cavities (large void fraction) along the pipeline are not distributed homogeneously. The simulation results clearly show that improper selection of the weighting function in the convolution-based unsteady friction approximation in the DVCM may significantly attenuate some measured high-frequency pressure pulses and speed-up the timing of the main pressure

pulses. It should be noted that assumptions made in the derivation of the weighting function might be violated during violent cavitation events. Moreover, it is not clear what form the weighting function would take, or even if it is applicable, in this situation. Further work on implementation of convolution-based unsteady friction model is being carried out by the authors.

## Conclusions

Results from the discrete vapour cavity model (DVCM) with convolution-based unsteady friction term are compared with results of column separation measurements. Computational and experimental runs were performed for a rapid closure of the valve positioned at the downstream end of the upward sloping pipe. The effect of Vardy-Brown's weighting functions developed for transient turbulent flow in hydraulically smooth and fully rough pipes on pressure waves is investigated. The simulation results show that improper selection of the weighting function in the convolution-based unsteady friction term in the DVCM may significantly attenuate some measured high-frequency pressure pulses and speed-up the timing of the main pressure pulses for serious cavitation. The following recommendations for the further study of the DVCM are suggested: advanced treatment of the growth and collapse of cavities and variable weighting functions induced by the collapse of cavities.

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