Space-time modulation of turbulence in co-flow jets

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ARTICLE INFO

Keywords:
Modulated turbulence
Turbulent mixing
Turbulent jets

ABSTRACT

This paper focuses on the effects of a space-time dependent periodic stirring of a moderately turbulent planar co-flow jet configuration. The baseline flow is agitated in time and in space by small-scale turbulent perturbations in combination with large-scale modulation imposed at the inflow plane of a rectangular domain of size $L \times L \times 2L$ in the $x$, $y$, and $z$ directions respectively. The prescribed large-scale modulation is characterized by a single modulation frequency $\omega$ and modulation wave-number $K$. A parametric study at different modulation frequencies and wave-numbers is performed. We evaluate the system response to the external agitation in terms of key dynamic properties of the flow, e.g., the total kinetic energy $E_T$, the global averaged dissipation rate and additional flow mixing properties. For low modulation frequencies, e.g., $\omega = 0.5\omega_0$, where $\omega_0$ is the large scale turnover frequency, $\omega_0 = U/L$, with $U$ and $L$ being relevant velocity and length scales, and at given wave-number $K$, we observe that $E_T$ follows the imposed oscillation with a periodic amplitude response that is sustained at locations further from the inflow plane, whereas for higher frequencies, the response amplitude rapidly decays. Results of the global dissipation rate show the development of a definite maximum value of the response amplitude at frequencies on the order of $\omega_0$ for any modulation wave-number $K$. To investigate in more detail the effects of modulated turbulence on the jet mixing properties, a passive scalar was injected at the inflow plane. The spreading of the scalar surface in the agitated jet was monitored for a wide range of modulation frequencies. In general, results show enhanced mixing efficiency when the main jet is modulated at frequencies near $\omega_0$ and low $K$ values.

1. Introduction

In this paper, we investigate the response of a moderately turbulent co-flow plane jet subject to a combination of a small-scale random perturbation and a large-scale time-dependent spatially periodic disturbance. We evaluate the dynamic response of the flow in relation to the frequency and length-scale used to stir the turbulent jet flow. Direct Numerical Simulations (DNS) of the Navier-Stokes equations are performed to investigate the role that variation of the frequency and the length-scale associated with the applied spatial modulation have on the downstream developing flow. We focus on the enhancement of mixing properties as a result of variation of frequency and wave-number of the large-scale inflow modulation. Research on modulated turbulence (Von der Heydt et al., 2003a; Kuczaj et al., 2006) has previously focused on the case of homogenous isotropic turbulence (HIT). Here, our analysis is based on a case of inhomogeneous flow, such as the spatially developing jet, and consider its global response to an external agitation. As will be shown, in the co-flow jet configuration an enhancement of the dissipation rate, $\varepsilon$, can be induced by controlling the frequency and the wave-number of the flow stirring mode. This suggests that time-dependent modulation of turbulence could be used to enhance various properties that are in general often associated with the flow mixing capabilities.

Turbulent flows are characterized by a broad range of length and time-scales. Recent studies suggest the additional existence of preferential frequency modes in turbulence (Von der Heydt et al., 2003a; Cekli et al., 2010; Kuczaj et al., 2006) connected to large-scale modulation. By considering the energy cascade process, these investigations have proposed that such preferential time-scales are intimately associated with the time required for the turbulent kinetic energy, largely present at the large scales of the flow, to cascade down to the small flow scales. Wind tunnel experiments with an active stirring, cycled at
preferred frequencies, have shown an enhancement of around 50% in the mean turbulent dissipation rate when the main flow was periodically modulated at these preferred frequencies (Cekli et al., 2010). This suggests that an enhancement in the flow mixing properties could be reached, at given fixed energy input, provided the flow is modulated at the ‘right’ scales. For example, if the dissipation rate could be increased, due to an externally applied disturbance, this would in turn indicate intensified small scales in the flow and an increase in the associated micro-scale mixing. The fact that an increasing in the mixing properties can occur at suitable modulation frequency, offers the possibility to apply the concept of modulated turbulence in generating efficient flow control strategies focusing on mixing applications, e.g., combustion. Despite such investigations, it is also interesting to note that results somewhat opposite to what has been documented so far, namely the enhancement of dissipation and it connection to mixing enhancement, were recently published for a homogenous turbulent flow (Bos and Rubinstein, 2017).

The use of a co-flow jet in this paper relies on its similarity with flow configurations often used in research of premixed combustion of Bunsen-type flames (Vreman et al., 2009; de Souza et al., 2017). In this paper, we rather concentrates on the general characterization of the response of the co-flow planar jet, e.g. mixing properties, due to the imposed large-scale modulation, without entering in details on the role that the interface dynamics between both stream, as investigated by several groups (Da Silva et al., 2014; Da Silva and Taveira, 2010; Gaskin et al., 2004; Geurts, 2001; Hunt et al., 2006; Tordella and Iovieno, 2011; Veeravalli and Warhaft, 1989), has in this context.

By extending the agitation approach used in the reference (Cardoso de Souza et al., 2014; de Souza et al., 2017), we investigate the case of a turbulent co-flow jet subject to a space-time modulation pattern. In this paper, by imposing a large-scale external disturbance, that is periodic in space and time, to the turbulent co-flow jet, we address the question whether there are optimum frequencies to stir the flow via inflow perturbations. For instance, such ‘optimum’ frequencies and scales could be beneficial for applications involving turbulent premixed combustion (de Souza et al., 2017; Verbeek et al., 2012).

The application of a steady modulation via large scale deflection of the flow was shown to lead to a substantially increased response of the co-flow jet configuration (Cardoso de Souza et al., 2014). In that case, it was found that the flow response, characterized for instance in terms of the dissipation rate ε and the jet thickness δε, depends strongly on the imposed length-scales, i.e., the wave-number K. For example, a large response in ε was found near the inflow plane when the flow was disturbed with relatively small length scales, i.e., rather large values of K, while further downstream a response maximum occurs for much larger length-scales of the inflow deflection pattern. Additionally, the width of the mixed region of the jet, δε, was observed to increase faster when the flow was spatially modulated by structures of size comparable to the integral scales of the jet. In general, the results suggest that for such flows ‘resonance’ conditions occur at every length scale of the inflow modulation. At a given modulation wave-number K the largest effect is observed at an associated K-dependent distance from the inflow.

In this paper, we focus on the flow response associated with the introduction of a temporal variation in the amplitude of the large-scale upstream modulation. The response of a flow to an externally imposed disturbance can be characterized in terms of instantaneous flow structures but also by monitoring time-averaged properties of the flow (Cekli et al., 2010; Kuczaj et al., 2006). Here, we incorporate both types of information and evaluate the response at different modulation frequencies and different perturbations length scales. For instance, the response based on a time-averaged property is evaluated in terms of the dissipation rate, ε, while the instantaneous response of the flow is measured in terms of the total kinetic energy of the flow, ET. In case of ET, we apply a phase-averaging procedure to determine the response during a period of forcing. This analysis is performed at different downstream locations. Results for the conditionally averaged response, ET, show a periodic response amplitude when the main jet is agitated by low modulation frequencies, i.e., ω < ω0, where ω0 is the large-eddy turnover frequency, whereas for higher frequencies, ω > ω0, the response amplitude decays. This is found for any modulation wave-number K, i.e., both at large response amplitudes in case K is sufficiently low and at much reduced amplitudes in case K is large. These results are in accordance with previous investigations on the application of time-modulated turbulence in other flow configurations (Von der Heydt et al., 2003a; Cekli et al., 2010; Kuczaj et al., 2006; Cadot et al., 2003). Additionally, the effects of the temporal modulation on the flow mixing properties were investigated considering the spreading of a passive scalar injected at the inflow plane. The results show that the mixing of the scalar is affected by the time-dependent spatial modulation. In comparison with a reference unmodulated jet, for example, we observe that a larger scalar surface area tends to be developed when the flow is also modulated in time, specially at frequencies near ω0.

The organization of this paper is as follows. In Section 2, the computational flow model is outlined. The time-dependent modulation strategy and the methods to extract the flow response are also addressed in this section together with an investigation on the sensitivity of relevant flow properties to the grid spatial resolution. Section 3 starts with the results of the unmodulated reference co-flow jet, after which the results of the time-modulated co-flow jet involving a wide range of modulation frequencies are discussed in Section 4. Results of the scalar mixing are discussed in Section 5. Concluding remarks are contained in Section 6.

2. Layout of the computational model

Firstly, we outline the system of equations and associated boundary conditions, and introduce the computational domain used for the simulations.

The system of Navier-Stokes equations is solved using an incompressible formulation (Cardoso de Souza et al., 2014) yielding the pressure p and the velocity u in the xi directions as a function of time t:}

\[
\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_i^2} \tag{1a}
\]

\[
\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_i^2} \tag{1b}
\]

where p and ν are, respectively, the density and kinematic viscosity of air at room temperature conditions and atmospheric pressure, and Sij is the rate of strain tensor:

\[
S_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \tag{2}
\]

Following reference (Vreman et al., 2009), (1a) and (1b) are solved on a staggered Cartesian grid using a second order finite volume method with the momentum equations integrated in time using an explicit multi-step method for the convective term (second-order Adams-Bashforth) and one-step forward Euler for the viscous term. This hybrid time-stepping approach is preferable over pure Adams-Bashforth or pure forward Euler, since it leads to better linear stability properties (Vreman et al., 2009). The pressure-velocity coupling is treated by solving a Poisson equation for the pressure correction using a multigrid method.

In the x-direction, periodic boundary conditions are applied for the velocity and pressure. To assure that the implications of using periodic boundary conditions are kept to a minimum, thus reducing their impact on the flow statistics, we closely follow (Cardoso de Souza et al., 2014) such that the total length along the x-direction is about four times larger than the size of the inflow integral scales in the periodic direction. At the outflow planes along the y- and z-directions, Neumann boundary...
conditions are used for each velocity component, \( u_i \). For the pressure field, \( p \), Neumann boundary conditions are considered only at the outflow planes in the \( z \)-direction, since in the \( y \)-direction pressure was held constant and equal to the atmospheric pressure. At the inflow plane, the velocity field is prescribed using a mean inflow profile, \( U_0 \), in the streamwise \( x \)-direction in terms of a hyperbolic window function, \( \Xi \), according to:

\[
U_x = (U_k - U_i) \left( 1 - \frac{\Xi}{2} \right) + U_i, \\
\Xi = 1 - \tanh \left( \frac{2(y_D - \frac{1}{2}D)}{\frac{1}{2}} \right)
\]  

(3a)

(3b)

where the centerline velocity \( U_1 = 3 \, m/s \) corresponds to the low speed stream confined to a region of width \( D = 2.4 \, mm \). In the outer region, a faster co-flow surrounds the inner stream with velocity \( U_0 = 7 \, m/s \). In \( y_D \), the thickness of the inflow profile is set to \( \theta = 0.55 \, mm \). The Reynolds number of the centerline stream \( Re = U_1D/\nu \) is equal to 493. The domain with the imposed profile is shown in Fig. 1 thereby completing the set-up.

Turbulence at the inflow plane is generated with the method of filtered random perturbations (Vreman et al., 2009). In this case, the turbulent agitation is imposed at the inflow region of size \( L \times D \), with \( D = L/3 \), and generated at each time step from a uniform stochastic field with random numbers varying between \(-1 \) and \(1\). The associated length-scales are controlled using a top-hat filter of width \( \Delta = 0.65D \), such that the largest scales have a size comparable to the width \( D \). Subsequently, the spatially filtered fluctuations at the inflow \( \mathbf{\Pi} \) drive the temporal evolution of the actually imposed velocity fluctuations \( \mathbf{\Psi} \) according to the following first order stochastic equation:

\[
\frac{d\mathbf{\Pi}(t)}{dt} = \mathbf{\Pi}(t)
\]  

(4)

in these simulations the time-step \( \delta t \) is set to 2 \( 10^{-3} \). For all cases, the inflow small-scale fluctuations are confined mainly to the low speed region by multiplying the space-time filtered velocity \( \mathbf{\Pi} \) with a window function, \( \Xi \), defined as:

\[
\Xi = \frac{1}{4} (1 + 3\Xi)
\]  

(5)

Thus, we also add random perturbations in the high-speed stream, although with much smaller amplitudes in comparison with the fluctuations confined on the low-speed jet region. At the end of this procedure, we multiply the turbulent fluctuations with an amplitude factor of 500, in order to have a turbulent intensity level of \( I = 15\% \) of \( U_1 \) at the inflow region limited by the width \( D \). Notice that the imposed inflow turbulence is not entirely based on a pure set of random noise perturbations that we simply add at the inflow, but rather it is a spatially filtered stochastic field evolving through time according to (4). This strategy leads to the set of inflow length-scales characterized by a locally defined auto-correlation function that gives integral length-scales of size related to the width of the spatial filter, \( \Delta \), which we apply at the inflow plane. Based on this approach we generate turbulent structures with pre-specified size which are coherent over the inflow region. As the flow further develops these structures are stretched by the mean flow thereby generating the subsequent smaller scales. This approach resembles strategies already extensively published on algorithms of generation of inflow turbulence (Kempf et al., 2005).

The computational domain is taken as a rectangular box of size 7.2 \( mm \times 7.2 \, mm \times 14.4 \, mm \). The z-component corresponds to the streamwise direction of the flow, the \( x \)-component to the spanwise direction and the \( y \)-component to the normal direction across the flow. The \( x \)- and \( z \)-directions are discretized with a uniform grid, whereas for the \( y \)-direction a stretch factor is applied at the boundaries to minimize far field effects on the developing co-flow.

In this paper, most of the results are obtained on the grid discretized with \( 72 \times 72 \times 144 \) points. The motivation for such low resolution level is based on the fact that it can provide a systematic investigation on a broad range of modulation frequencies and scales at reduced computing times. Obviously, before investigating the results of this parameter study, confidence in the general reliability of the simulations is established through systematic grid refinement steps. It is shown that the general features of the resonance, i.e., the presence of a maximum response at a certain frequency, can be well identified using this relatively coarse grid. Moreover, in a number of cases we separately verify the conclusions by repeating simulations at increased resolutions. This is sufficient to identify the general trend of the response with respect to the frequency of the temporal modulation and the length-scale of the spatial deflection.

### 2.1. Large-scale space-time forcing

The prescribed baseline flow is modulated considering the inclusion of a time-dependent amplitude variation in the large-scale perturbation acting on the lower speed jet. The stirring strategy is taken according to so-called Beltrami flow (Majda and Bertozzi, 2000; Geurts and Cardoso de Souza, 2018) as follows:

\[
u_x(x, y, t) = A \sin(kx) \sin(ky) (1 + A_F \sin(\omega t)) \\
u_y(x, y, t) = A \cos(kx) \cos(ky) (1 + A_F \sin(\omega t)) \\
u_z(x, y, t) = A \sin(kx) \cos(ky) (1 + A_F \sin(\omega t))
\]  

(6a)

(6b)

(6c)

where for each component of the velocity field the subscript \( B \) refers to Beltrami. The parameters \( A \) and \( A_F \) set respectively the amplitude of the large-scale perturbation and the modulation depth. In this paper, we keep \( A = 1.5 \, m/s \) in order to have a modulation amplitude of 50% of...
the magnitude \( U_1 \) which is nevertheless large compared to the level of the imposed turbulent fluctuations, \( \bar{u}_{rms} = 0.45 \text{ m/s} \). This general setting complies with the investigation on forced turbulent jet splitting using deterministic inflow perturbations (Tyliszczak and Geurts, 2020).

In this reference it was also shown that the magnitude of the deterministic perturbations should be at least similar or higher than the turbulent agitations at the inflow. Additionally, in these simulations the modulation amplitude varies periodically in time with \( A_p = 1/2 \), i.e., also allowing for a 50% variation.

The possibility to vary the length-scales of the spatial modulation defined by (6) allows one to investigate the dependence of the flow response on the stirring pattern mode. Here, we select the modulation wave-number, \( K \), based on the results discussed for the static spatially periodic case (Cardoso de Souza et al., 2014), namely \( K = 2\pi/L_1 \), and \( K = 6\pi/L_2 \). The spatial modulation is confined to the region \( L \times D \) by multiplying the corresponding Beltrami velocity with the window function defined according to (3b). Thus, the superimposed periodic modulation of pre-defined length-scale act only on the region limited by the width \( D \). As an example of the modulation imposed, the velocity contours of the streamwise component \( w_p \) are shown in Fig. 2 for two spatial modes. Regions with \( w_p < 0 \) indicate anti-clockwise motion with positive streamwise vorticity and vice-versa. For the case \( K = 2\pi/L_1 \), two large-scale vortices with opposite vorticity are imposed at the inflow. In case \( K = 6\pi/L_2 \), six vortices with alternating vorticity are imposed.

From the definition of the periodic flow given by (6), the total kinetic energy per unit mass related with the inflow modulation, i.e., \( E_0 = (u_0^2 + v_0^2 + w_0^2)/2 \), integrated over the inflow plane will not depend on the wave-number, \( K \), but only on the amplitude \( A \) of the spatial modulation. Accordingly, for all cases the same energy input is introduced at the inflow plane. Hence, we conclude that any observed enhanced mixing will be only due to flow structuring associated with the large-scale temporal agitation.

In order to have a broad range of agitation frequencies, the modulation range is varied over \( 0 \leq \omega \leq 100\omega_0 \), where \( \omega_0 = U_1/D \) is the low speed jet, \( U_1 \), and the width \( D \) of the inflow region to which the structured modulation is confined. This frequency is also characteristic for the large-eddy turnover time for eddies of size \( D \). Next we detail the procedure to extract the flow response used to characterize the effect of the time-dependent large-scale perturbation.

### 2.2. Flow response extraction

As previously mentioned, the flow response analyzed is primarily based on the total kinetic energy per unit mass \( E_T \) extracted at a given location and instant, \( E_T = (u_x^2 + u_y^2 + u_z^2)/2 \), and the time-averaged global dissipation rate \( \langle \dot{\epsilon}_{\text{xy}} \rangle \), while the modulation effects on the mixing are directly quantified in terms of the time-averaged scalar surface density \( \langle \Sigma \rangle \) and the associated surface wrinkling, \( \langle W \rangle \).

In terms of the response based on the kinetic energy \( E_T \), a phase averaging procedure conditioned on the external modulation frequency is used to show the response variation during a period of forcing. This quantity is extracted from time-series locally recorded at points distributed along the streamwise \( x \)- and the normal \( y \)-direction such that we extract at a certain location of the flow the values of \( E_T \). As a first step, we compute the reference unmodulated co-flow jet. This simulation is performed for approximately 30 flow-through times, where one flow-through corresponds to the time spent for the low speed stream to go through the domain along the streamwise direction, i.e., \( 2L/U_1 \). This data will be used as initial condition for all cases where the spatial and temporal modulation, set for a particular wave number \( K \) and frequency \( \omega \), is imposed. To avoid the transient from the initial state of the unmodulated case to the proper response of the modulated case, the monitored response signal \( E_T \) starts to be collected after four flow-through times.

As a second step, for each simulation a given sample of \( E_T(x_p, y_p, z_p, t) \), where the subscript \( p \) refers to the point of the sample collection, is recorded for a period of time of 4\( T \), where \( T = 2\pi/\omega \) is the period of the modulation and \( \omega \) is the corresponding frequency of the applied forcing for that case. In all cases the period \( T \) corresponds to the extraction of 40 values of \( E_T \), implying that a given sample of \( E_T \) is discretized with 160 points in time, thus any sample of \( E_T \) will form a collection of values obtained for a 4\( T \) period of modulation. In this paper, we extracted in total \( N_p = 15 \) samples of \( E_T \) at various locations in the flow. Therefore, since the modulated case is characterized in terms of the frequency, \( \omega \), and the modulation wave-number, \( K \), each simulation reported in this paper was performed for a total time of 60\( T \). To check the effect of the number of recorded signals to be stored on the evaluation of the phase-averaging response, a separate analysis, not shown in this paper, was performed. It establishes a representative indication of temporal convergence when the number of samples is already set to \( N_p = 15 \).

In case of the response based on the global averaged dissipation rate, \( \langle \dot{\epsilon}_{\text{xy}} \rangle \), initially we evaluate locally the dissipation rate \( \dot{\epsilon} \) using an ‘on-fly’ procedure which determines the values of \( \dot{\epsilon} \) at each instant \( t_i \) according to:

\[
\dot{\epsilon}(x, y, z, t_i) = 2\mu S_0(x, y, z, t_i)S_0(x, y, z, t_i)
\]

Subsequently, we define the volume averaged \( \langle \dot{\epsilon} \rangle \) by considering the volume integral of \( \dot{\epsilon}(x, y, z, t) \) on the whole domain:

\[
\langle \dot{\epsilon} \rangle = \frac{1}{V} \int_0^L \int_0^L \int_0^L \dot{\epsilon}(x, y, z, t) \, dx \, dy \, dz
\]

where \( V \) is the domain volume \( L \times L \times 2L \), with \( L = 3D \).

Eq. (8) is evaluated at each time-step, after which the instantaneous volume average property \( \langle \dot{\epsilon}(t) \rangle \) is averaged in time according to:

\[
\langle \dot{\epsilon}(t) \rangle = \frac{1}{t} \int_0^t \dot{\epsilon}(t) \, dt
\]

where the quantity between the angle brackets \( \langle \cdot \rangle \) denotes a time-averaged quantity.

To evaluate the direct consequences of the space-time modulation on the flow mixing potentiality, we quantify the temporal evolution of the surface of a passive scalar injected into the region associated with the low-speed jet, \( U_1 \). To evaluate explicitly the overall mixing capability of the co-flow jet, we monitor the temporal evolution of the
where $c$ represents the scaled passive scalar concentration varying between 0 and 1. The diffusion coefficient is assumed to be constant and defined as $\mathcal{D} = w/Sc$, with Schmidt number set to $Sc = 0.7$, following reference (Kuczaj and Geurts, 2006). For the scalar equation, the Van Leer third-order MUSCL scheme is applied for the advective term and forward Euler for the time integration (Vreman et al., 2009). Viscous terms are treated as in (1b). Boundary conditions for the scalar $c$ correspond to Neumann boundary conditions at the outflow planes along the $y$- and $z$-directions, while periodic boundary conditions are applied in the $x$-direction. At the inflow plane, a hyperbolic tangent profile is prescribed as follows:

$$W_c = \frac{1}{2} \left[ 1 + \tanh \left( \frac{(y - \frac{1}{2}L)}{\theta} \right) \right]$$

where the thickness $\theta$ is equal to that of the inflow velocity profile, see 3b. We inject the passive scalar only into the region of the low-speed stream, while in the co-flow region associated with the faster stream its concentration is set to zero.

Closely following Bos and Rubinstein (2017), the mixing of the passive scalar could be quantified using a direct approach based on the amount of scalar transferred to the small scales by nonlinear transfer processes. This approach was applied to quantify the amount of mixed scalar on a homogeneous turbulent flow, where the rate of mixing was directly quantified by measuring the injection rate divided by the corresponding scalar variance (Bos and Rubinstein, 2017). Despite this method, in this paper we opt for another approach to characterize the mixing since the main flow is a spatially developing inhomogeneous flow and it is expected that the mixing rate will also depend on another parameters, e.g., turbulent diffusion, thereby making difficult a straightforward definition for it. Here, for the mixing of the scalar by the co-flow jet we consider the surface area $S$ corresponding to a scalar level-set evaluated using a specialized integration procedure (Geurts, 2001). In this approach, the Laplace transform is considered to compute an integral over an arbitrary volume enclosing an iso-surface of the scalar at a value $c^*$. Through this integration procedure, an accurate evaluation of geometric surface properties, e.g., surface area, total curvature and global wrinkling, can be obtained. In this paper, an integral of a density function $f$ over an instantaneous scalar surface associated with the level set at $c^*$ is obtained using the following relation (Geurts, 2001):

$$I_f(c^*, t) = \int_{S(c^*, t)} dS f(x, y, z, t) = \int_V dV \left( \delta(F(x, y, z, t) - c^*) \nabla W(x, y, z, t) \right)$$

where $S(c^*, t) = \{(x, y, z) \in \mathbb{R}^3 : F(x, y, z, t) = c^*\}$ with $F$ denoting the scalar quantity. Thereby, $S(c^*, t)$ corresponds to the scalar level set surface defined by the iso-level function $F(x, y, z, t) = c^*$, over which we integrate. Closely following the reference (Kuczaj and Geurts, 2006), we set for the iso-surface value $c^* = 1/4$. The volume $V$ is an arbitrary volume enclosing the scalar surface $S(c^*, t)$. In case $f(x, y, z, t) = 1$ in (12), the total surface area of the level-set, $A_v$, is computed. In case the density function $f(x, y, z, t) \equiv |V|$, $n_t$ the total wrinkling, $W$, of the surface is determined (Geurts, 2001; 2002). The unitary vector, $n = \nabla c/|\nabla c|$, is defined as the vector normal to the scalar variable surface, with the corresponding divergence, $\nabla \cdot n$, defining the local surface curvature. Details on the accuracy of this procedure are given in the reference (Kuczaj and Geurts, 2006). In this paper, we evaluate the integral in (12) at each time-step, as in (8), such that both level-set properties $S$ and $W$ are evaluated at each instant $t$. The scalar surface density $\Sigma$ is defined as follows:

$$\Sigma(c^*, t) = \frac{S(c^*, t)}{V}$$

Combined, both properties $\Sigma$ and $W$ define quantitatively the mixing response for the co-flow jet. Next we evaluate the variability of the flow properties introduced above to different grid resolutions. Based on a systematic grid refinement analysis, we select a spatial resolution which shows to be sufficiently reliable to set our main conclusions for the modulated co-flow jet.

2.3. Grid sensitivity analysis

In this section, the sensitivity to the spatial resolution of relevant flow properties used to characterize the response of the co-flow configuration, e.g., the global dissipation rate $\langle \epsilon \rangle$, the averaged surface spread $\Sigma$ and corresponding surface wrinkling $W$ of the injected passive scalar, is assessed based on a grid refinement analysis performed on four grid resolutions: $18 \times 18 \times 36$, $36 \times 36 \times 72$, $72 \times 72 \times 144$ and $144 \times 144 \times 288$. This analysis focus rather on the reference unmodulated co-flow jet, where additional properties such as the jet thickness, and time-averaged velocity profiles are also discussed.

Initially, the variability of time averaged cross-section profiles of the streamwise velocity to different spatial resolutions is considered. Following the reference (Cardoso de Souza et al., 2014), the level of turbulent intensity generated at the inflow plane using (4) is kept the same for all resolutions. Fig. 3 shows for the assessed grid resolutions the results of the velocity profiles obtained for the unmodulated reference jet.

As can be seen, no appreciable changes in both mean and r.m.s profiles are observed when the results obtained with the spatial resolution $72 \times 72 \times 144$ are compared with those obtained for the finest grid, i.e., $144 \times 144 \times 288$. For both profiles, a direct indication of convergence becomes noticeable as the spatial resolution is increased. These profiles are obtained by time-averaging a collection of samples corresponding to $yz$-planes of the streamwise velocity extracted at $x = 0$. Clearly, the mean profiles are symmetric with respect to $y = 0$ for most of the resolutions, while a slight asymmetry in the r.m.s. profiles can be noticed. A more detailed analysis to check this minor asymmetry reveals that it is not due to the limited number of samples used to obtain the time-averaged profiles, in fact this minor effect arises from the relatively limited size of the physical domain in the $y$-direction. A separate set of simulations for the unmodulated reference case (not shown), using the same conditions described, were carried out in a slightly larger domain for the spatial resolutions analyzed in this section and for such cases full symmetric r.m.s profiles were obtained even for the coarser resolutions. Thus, considering that for a parametric study a relatively shorter domain is preferable and that most of the variables used to quantify the response of the flow are quantities determined using first order statistics we proceed with the selected domain size presented in Section 2.

In the sequence, we consider the sensitivity of the jet thickness, $\delta_z$, to several spatial resolutions. This property denotes the interrelatin of momentum transferring processes taking place at the large and small flow scales, which in turn determine its subsequent development as the result of vortex merging processes associated with the growth of shear layer instabilities formed at the intersection region between the low- and high-speed streams. Here, the thickness $\delta_z$ refers to the width of the mixing layer across one interface obtained at a certain downstream $z$-location. Closely following the reference (Pope, 2006), this width is quantified by the distance between two locations in the $y$-direction for a plane at height $z$ as follows:

$$\delta_z = y_{\delta_x}(z) - y_{\delta_y}(z)$$

where $y_{\delta_x}(z)$ is the cross-stream location such that:
Results in Fig. 4 show the spatial evolution of $\delta_z$ for grid resolutions. As can be observed, a non-linear growth of the mixing layer, formed between the low- and the high-speed flow region, is captured for most of the resolutions, although with a lower spreading rate occurring for the coarsest case.

Next we evaluate the variability of the averaged dissipation rate, $\langle \varepsilon \rangle_{xyz}$, obtained at different grid resolutions in order to establish the accuracy of the simulations as far as this average quantity is concerned. Fig. 5 shows the results of the grid sensitivity study of the bulk of dissipation rate associated with the unmodulated jet. To analyze the spatial convergence of volume averaged properties, in this paper we define the quantitative deviation, $\sigma_{\langle \varepsilon \rangle_{xyz}}$, of corresponding property relatively to the finest grid resolution, i.e., $144 \times 144 \times 288$. As an example, for the averaged dissipation $\langle \varepsilon \rangle_{xyz}$ the deviation is defined as follows:

$$\sigma_{\langle \varepsilon \rangle_{xyz}} = \frac{|\langle \varepsilon \rangle_{xyz} - \langle \varepsilon \rangle_{xyz}^{288}|}{\langle \varepsilon \rangle_{xyz}^{288}}$$

where the upper-index $n_z$ correspond to the resolution, in the $z$-direction, of the corresponding grid.

As can be seen, for the averaged dissipation $\langle \varepsilon \rangle_{xyz}$ a representative indication of convergence at the spatial resolution $72 \times 72 \times 144$ is already established when compared to the finest resolution.

Additionally, the cumulative time-average scalar surface density $\langle \Sigma \rangle$ and the associated surface wrinkling $\langle W \rangle$ were obtained for the range of grid resolutions considered. Fig. 6 shows the sensitivity of both mixing properties $\langle \Sigma \rangle$ and $\langle W \rangle$ to resolutions assessed. The deviations are especially large for the global surface wrinkling as the grid resolution decreases, while the totality of the surface of the passive scalar is in overall well predicted even for the coarsest resolution, since the differences for all cases are smaller than 2% relatively to the finest resolution.

The results show a significant underestimation of the quantity $\langle W \rangle$ as the grid resolution decreases, such that the spatial resolutions $18 \times 18 \times 36$ and $36 \times 36 \times 72$ are clearly not enough. For increasing resolution, a visual indication of convergence can be appreciated since the distance between the symbols rapidly decreases and already a proper convergence sets in at the grid $72 \times 72 \times 144$, with differences smaller than 1% when compared to the finest grid resolution. Thus, for our purposes, the resolution $72 \times 72 \times 144$ is sufficient to formulate the main conclusions and it will be predominantly used to address the
general features of the flow global response to the externally imposed agitation for the modulated case.

3. Reference unmodulated jet

In this section, we present additional results for the reference unmodulated co-flow jet such as the spatial evolution of the dissipation rate and characteristic flow length-scales. These results provide a reference basis for a comparison with the modulated cases.

Fig. 7 shows a snapshot of the vorticity magnitude on a cross-section $yz$-plane at $x=0$, where the evolution and subsequent growth of the inherent instabilities present in the shear layer are directly observed. As can be seen, in the reference jet vortex shedding and associated vortex merging processes tend to develop further downstream. The combination of these effects lead to the observed growth of the shear layer thickness, where for the co-flow configuration a self-similar region does not fully develop due to the limited axial length of the computational domain.

To estimate the size of the characteristics flow scales, we compute the centerline time-averaged statistics based on the streamwise velocity component as shown in Fig. 8. This estimation gives the indication of a relatively fast spatial development of the flow length-scales for the unmodulated jet, since already at $z = D$ the imposed filtered fluctuations already lead to integral scales of size $\ell \approx 1 \, \text{mm}$ and Taylor micro-scales of size $\lambda \approx 0.7 \, \text{mm}$. The small differences between $\ell$ and $\lambda$ indicate the occurrence of a relatively short inertial range for this case. The Kolmogorov length-scale at such location is $\eta \approx 0.1 \, \text{mm}$ which indicate that it was properly resolved by the DNS reference grid.

Clearly, for the co-flow jet configuration analyzed in this paper the size of integral scales are barely larger than the Taylor microscales, thereby indicating a relatively short inertial range for this case. Veritally, the Reynolds number of turbulent jet flows are frequently much higher than the actual value contemplated in this paper and it is very likely that the dynamics of the flow response to the imposed large-scale agitation may change, as a consequence, for instance, of the intermittency in the mixing layer affecting the high Reynolds jet turbulence. This is the main reason why the amplitude of the inflow modulation has a maximum value larger than the turbulent intensity at the inflow, as described in Section 2.1. A previous analysis (not shown in this paper) of such co-flow configuration indicates that at higher turbulent intensity the modulation effects on the flow tend to be ‘washed’ out by the small-scale filtered fluctuations. While the inertial range is indeed limited in these simulations, it is well documented that jet instability modes, i.e., the shear layer instability mode, are intimately related to scales with size of order of the integral scales of the shear region (Bejan, 2013), thus, as we will show, we decided to balance these levels such that a recognizable effect of the modulation on the flow could be noticeable in this paper. We intend to examine the issues of the flow response variability to a broad range of agitation scales on a similar co-flow configuration with higher inertial range elsewhere.

Fig. 6. Analysis of the sensitivity of (a) the surface density $\langle \Sigma \rangle$ and (b) the global surface wrinkling $\langle W \rangle$ to the spatial resolution for the unmodulated reference jet. The deviation for $\sigma_\Sigma$ and $\sigma_W$ is obtained following (16).

Fig. 7. Snapshot of the vorticity magnitude field, $\omega = \omega_x + \omega_y + \omega_z$ of the unmodulated jet obtained after 6 flow through-times. Contours are shown for the plane $yz$ at $x=0$.

Fig. 8. Results of centerline Kolmogorov length-scale (□ symbols), Taylor micro-scale (+) and integral length-scales (○) based on the streamwise velocity component of the unmodulated jet.
In the next section we quantify the effects due to the structured space-time inflow perturbation, where the flow properties discussed for the unmodulated jet will be used as a reference basis to investigate the direct consequences of the imposed large-scale modulation on the co-flow configuration.

4. Periodically modulated co-flow jet

In this section, we present the flow response to the imposed space-time modulation considering a broad range of cases. Firstly, we discuss the phase-averaged results of the response based on the kinetic energy, $E_T$, after which the results of the averaged dissipation rate, $\langle \varepsilon \rangle_{xyz}$, and the modulation effects on the mixing are presented and discussed.

Figs. 9 and 10 show the time dependence of the response signal $E_T(\omega, t)$ to some selected modulation frequencies for the wave-number $K = 2\pi/L$ at distinct $z$- and $y$- locations. In all cases, we observe that for low modulation frequencies, $\omega < \omega_0$, the flow follows an oscillating pattern in connection with the temporal component of the imposed modulation, i.e., $A \sin(\omega t)$, whereas for higher modulation frequencies, $\omega > \omega_0$, the response follows a similar pattern at smaller amplitudes. The effect of the dependence of the response amplitude to the modulation frequency remains at distances further from the inflow, although, as expected, the amplitude of the response tends to decay. The fact that the results for the co-flow show a time-dependent response with amplitude behaviour depending on the modulation frequency $\omega$ is in agreement with previous investigations of time modulation of turbulence (Von der Heydt et al., 2003a; 2003b; Cekli et al., 2010; Kuczaj et al., 2006; Cadot et al., 2003; Verbeek et al., 2012).

The spatial variation of the large-scale perturbation introduces slight differences depending on the monitored $y$-location. For example, results in Fig. 10 show that for a fix $z$-location, at given modulation frequency, the amplitude of the oscillating response in general decreases for a $y$-location within the mixing layer, i.e., $y = 0.5D$, when compared with the response obtained on a location far from the shear, as shown in Fig. 9. The decrease of $E_T$ is due to the increase of the turbulent intensity which is reportedly higher in the shear layer than in other flow regions. These results show that the phase averaged response is sensitive to the local turbulent fluctuations which tend to damp the

Fig. 9. Phase-averaged response of $E_T(\omega, t)$ for the spatial mode $K = 2\pi/L$, as a function of $t/T_m$ with $T_m$ representing the corresponding period of each driving frequency, $\omega$. In this case, the flow response was monitored at $(x, y, z) = (0.75D, 0, z)$ for several streamwise $z$-locations. Lines correspond to the response signal $E_T(\omega, t)$ obtained for $\omega=0.5\omega_0$ (solid line), $\omega = \omega_0$ (dashed line) and $\omega=4\omega_0$ (dashed-dotted line).

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Fig. 10. Phase-averaged response of $E_T(\omega, t)$ for the spatial mode $K = 2\pi/L$, monitored at $(x, y, z) = (0.75D, 0.5D, z)$, which location corresponds to be in the interface shear region. Lines correspond to the response signal $E_T(\omega, t)$ obtained for $\omega=0.5\omega_0$ (solid line), $\omega = \omega_0$ (dashed line) and $\omega=4\omega_0$ (dashed-dotted line).
imposed oscillatory effects on the flow due to the space-time modulation. Although not shown in this paper, similar effects are observed when the modulation wave-number is set to \( K = 6\pi/L \). The differences are due to the faster decay further downstream of the amplitude of \( E_T \) at higher \( K \) in comparison with the case modulated with \( K = 2\pi/L \).

In fact, such findings are in qualitative agreement with the linear response obtained from Kraichnan’s direct interaction approximation (DIA) theory (Kraichnan, 1964) which shows that in isotropic turbulence the flow response tends to decay faster in case the local intensity of turbulence increases. Noting that the co-flow planar jet configuration can be characterized only as moderately turbulent with a relatively short inertial range, and thereby far from the domain of application of Kraichnan’s theory since it concerns situations involving isotropic turbulence at high Reynolds number, \( Re \), we observe that a spatially inhomogeneous developing flow at moderate \( Re \) still exhibits similar features when compared to the case of high \( Re \) isotropic forced turbulence. This is observed in the results of \( E_T(t) \) which shows that the response to the applied modulation decays when the local turbulent intensity is higher as in the mixing interface between the high and the low speed jet. The modulation effect of the inflow space-time forcing on \( E_T(t) \) tend to be smaller when the local turbulent intensity is relatively high, leading to the decay of the amplitude of the flow response in such regions. At distances further from the inflow plane, the decay of the response is even faster, especially when the frequency of the modulation increases. In addition, we also noticed that similar effects of the modulation of the response were observed in Parezzanovi (2015), which reported experiments with modulated mixing layers that shows similar time-periodic effects on the response of the flow.

The modulation effects on the growth of the thickness of the jet are shown in Fig. 11. As can be seen, the introduction of the space-time upstream modulation results in the development of a thicker jet, when compared to the unmodulated reference jet, already at distances relatively closer to the inflow plane, such that at \( z = 3D \), for instance, the co-flow jet modulated with \( K = 6\pi/L \) and \( \omega = \omega_0 \) is 1.6 times thicker than the unmodulated jet, while for a modulation based on \( K = 2\pi/L \) the resulting jet is almost a factor of two times thicker than the reference case. These results show enhanced momentum transfer between both streams and further stimulation of vortex shedding and merging processes when the co-flow is subjected to the structured perturbation.

So far we characterized the modulation effects based on quantities mainly determined by the large scales of the flow, e.g., \( E_T \), next we investigate how the modulation affects the flow structure at its small scales, we initially investigate the behaviour of the second order structure functions, \( G_2(r) \) where \( r \) is the spatial separation in the streamwise direction. Here, the \( z \)-velocity increments, \( u_z (z + r, t) - u_z (z, t) \), are obtained from a snapshot of the DNS field. Fig. 12 shows results of \( G_2 \) for the space-time modulated cases and the reference jet up to a separation distance of \( r = D \). As can be seen, the imposed modulation does yield appreciable changes at the small scales when compared to the reference jet whereas between the modulated cases smaller differences occur, especially at scales of size comparable to \( D \). These results suggest that the imposed space-time modulation introduces substantial changes in the dynamics of the co-flow configuration such that the main jet tends to develop faster, when compared to the reference jet, in the presence of the structured perturbation. Despite the fact that the time periodicity in the signal can affect the entire structure function, we would like to mention that the total energy of the imposed modulation acting at the inflow has the same amplitude for all cases analyzed, such that the differences in this aspect are only due to the time periodic variation of this amplitude for different cases. Once excluding periodic effects due to boundary conditions and imposing the same time dependent amplitude of the forcing then we expect that any effects on second order statistics shows an indication of substantial changes in the flow structuring.

The volume averaged dissipation rate \( \langle \omega \rangle_{xyz} \) computed using (8) is shown in Fig. 13 for a broad range of modulation frequencies. For both \( K \), the development of a maximum value at frequencies \( \omega \approx \omega_0 \) occurs for the property analyzed. The spatial modulation \( K = 6\pi/L \) is more effective in the enhancement of \( \langle \omega \rangle_{xyz} \) than in case \( K = 2\pi/L \). This response definition gives an indirect indication of the enhancement of the overall small-scale mixing, i.e., \( \langle \omega \rangle \), when the flow is modulated at frequencies close to \( \omega_0 \).

In the next section we consider the effects of the temporal

**Fig. 11.** Results of the normalized jet thickness, defined according to (14), for the reference unmodulated jet (thin solid line), and space-time modulated cases \( K = 2\pi/L \) with \( \omega = \omega_0 \) (thick solid line) and \( K = 6\pi/L \) with \( \omega = \omega_0 \) (dashed line). The jet thickness \( \delta_j \) is normalized by the thickness \( \theta \) of the initial imposed profile.

**Fig. 12.** Second order structure function, \( G_2(r) \), as a function of the axial separation distance \( r \). Results are shown for the unmodulated reference case (thin solid line), cases modulated at wave-number \( K = 2\pi/L \) and frequency \( \omega = \omega_0 \) (thick line) and \( K = 6\pi/L \) and frequency \( \omega = \omega_0 \) (dashed line).

**Fig. 13.** Global averaged dissipation rate, \( \langle \omega \rangle_{xyz} \), as a function of the modulation frequency, \( \omega \), for cases (a) \( K = 6\pi/L \) and (b) \( K = 2\pi/L \). In both figures, \( \langle \omega \rangle_{xyz} \) is normalized by the average dissipation rate of the unmodulated reference case, i.e., \( \langle \omega \rangle_{xyz} \).
modulation on the mixing properties more explicitly through an investigation of the spreading of a passive scalar by the modulated jet. In particular, we are interested to know whether an enhancement of the scalar mixing also occurs when the applied perturbation, at given $K$, temporally modulates with $\omega_0$.

5. Mixing Efficiency in time-modulated turbulence

In this section, the effects of the applied modulation on the mixing capability of the co-flow jet are discussed.

Firstly, in Fig. 14 we show snapshots of the surface spreading of the injected scalar for the reference jet and the modulated cases $K = K_6/L$ and $K = K_2/L$. In all cases, the snapshots correspond to the scalar instantaneous iso-surface $c^* = 1/4$ extracted after 5 flow-through times. For a better qualitative impression on the relation between the dissipation rate and the passively spread surface, these snapshots are coloured with the local values of the dissipation rate $\epsilon(x, y, z, t)$ obtained for each case. As can be seen, the dynamics of the scalar surface is substantially changed in the presence of the applied inflow modulation with more intense small-scale corrugations developing on the scalar surface when the flow is forced at relatively small length-scales, i.e., $K = K_6/L$, when compared with the modulation $K = 2\pi/L$ and the unmodulated reference case. The strong corrugations on the surface of the scalar develops for both wave-numbers when the flow is periodically modulated at frequency $\omega_0$. A direct connection between flow regions with high values of the dissipation rate and local surface distortions can be qualitatively inferred.

For a proper quantification on the modulation effects on the mixing, the scalar surface was determined using equations (12)-(13). Figures 15-16 show, for $K = 6\pi/L$ and $K = 2\pi/L$, respectively, the cumulative time-average scalar surface and the associated surface wrinkling for a broad range of modulation frequencies. As can be seen, a maximum in the global mixing area, $\langle \Sigma \rangle$, is achieved at frequencies around $\omega = \omega_0$ for both modulation wave-numbers. Moreover, at a low $K$ modulation a larger scalar surface develops in comparison with an agitation based on small-scales, i.e., $K = 6\pi/L$. In addition, we observe for both $K$ a similar trend of the surface wrinkling, $\langle W \rangle$, relative to the modulation frequency of the driving forcing. This result is expected (Kuczaj and Geurts, 2006; Geurts, 2002), since a larger area will be developed as the surface wrinkling increases. It is striking that the results of Figs. 15 and 16 show almost a direct correlation with results observed for the averaged dissipation rate $\langle \epsilon \rangle_{xyz}$ since for both quantities, $\langle \Sigma \rangle$ and $\langle W \rangle$, a maximum value develops at similar modulation frequencies, i.e., $\omega = \omega_0$. A plausible explanation connecting this fact with the mixing of a passive scalar, as it is transported by the surrounding jet, emerges. For example, as the injected scalar is passively spread by the modulated jet, the irregular turbulent motions at the large

Fig. 14. Snapshots of the scalar surface, corresponding to the level-set $c^* = 1/4$, obtained after five flow-through times. The surface of the scalar is coloured by the dissipation rate contours, $\epsilon(x, y, z, t)$. Results are shown for (a) the reference unmodulated jet and (b) space-time modulated case $K = 2\pi/L$ and (c) $K = 6\pi/L$. For both wave-numbers, the modulation frequency is $\omega = \omega_0$.

Fig. 15. (a) Time-averaged scalar surface density $\langle \Sigma \rangle$ and (b) wrinkling $\langle W \rangle$ as a function of the modulation frequency $\omega$ for case $K = 6\pi/L$. Both properties are normalized by corresponding values obtained for the unmodulated reference jet, i.e., $\langle \Sigma \rangle_0$ and $\langle W \rangle_0$.

Fig. 16. (a) Time-averaged scalar surface density $\langle \Sigma \rangle$ and (b) wrinkling $\langle W \rangle$ as a function of the modulation frequency $\omega$ for case $K = 2\pi/L$. Both properties are normalized by corresponding values obtained for the unmodulated reference jet, i.e., $\langle \Sigma \rangle_0$ and $\langle W \rangle_0$. 
scales will merely ‘pull out’ the scalar surface, while the small scales in turn will generate strong distortions on it. The latter effect, here globally quantified by the wrinkling ⟨\omega⟩, is the mechanism responsible for the increase of the iso-scalar surface. In fact, when the dissipation rate is enhanced the structure of the flow at the small scales becomes finer grained, such that these scales will lead to higher distortions on the surface of the scalar. It should be noted, however, that such an increase is sensitive to the length-scale of the spatial modulation K. A large-scale perturbation with low wave-number, K, shows to be a little more effective in generating a larger mixing surface than a modulation based on high wave-numbers, as indicated by the ‘height’ of the peak visible in ⟨\omega⟩.

Although the connection between dissipation and mixing is justified only qualitatively in this paper, that is by the presence of a maximum peak for both quantities at a specific frequency of the imposed modulation, based on such results we consider that if one is capable of promoting the presence of finer grain flow structures then any inhomogeneities in the flow will become more rapidly susceptible, in comparison to the situation where the presence of such scales is not further stimulated, to the effects of molecular diffusion, thereby enabling a faster mixing. This is due to the fact that such scales are capable of generating an increase of the scalar surface area and, additionally, larger local gradients in the scalar concentration field. Clearly, when compared to isotropic turbulence, the case considered in this paper is more complicated since it consists of a spatially developing co-flow jet with configuration analogous to the case of incompressible subsonic wake turbulence, despite such differences we note that similar findings relating dissipation and mixing enhancement were also observed in experimental (Cekli et al., 2010) and numerical investigations of isotropic periodically forced turbulence (Kuczaj and Geurts, 2006). On the other hand, recent investigations (Bos and Rubinstein, 2017; Yang et al., 2019) on isotropic turbulence concerning the relation between dissipation and mixing has shown that dissipation enhancement does not necessarily imply always in mixing enhancement.

The existence of a maximum response of the flow properties in the presence of a time-dependent modulation can be connected to the existence of optimized time-scales for energy input in turbulent flows. According to early investigations (Von der Heydt et al., 2003a; 2003b), the response of a turbulent flow subjected to a time-periodic external perturbation depends on the difference between the modulation period, T, of the energy input and the time, τ, needed for the kinetic energy to cascade down from the large to the small flow scales. The latter time scale, τ, is often associated with the time that turbulent kinetic energy injected at the large scales takes to be transported in spectral space towards the small scales, where dissipation eventually becomes the dominant mechanism. These investigations show that at low modulation frequencies the amplitude of the flow response tend to be constant due to the relatively large time scales associated with the energy input rate compared to the time-scale τ, whereas for high modulation frequencies the energy input rate is faster than τ, and for such frequencies a decay of the flow response proportional to 1/ω is expected (Von der Heydt et al., 2003b). Therefore, τ would be an optimum time scale for energy injection and associated mixing enhancement. For the reference jet, we estimate such time scale as τ = ε/U/ν which value is of order of 1/ων. The interrelation between 1/τ and ων would plausibly explain the occurrence of the maximum response on the investigated flow properties of the co-flow jet.

6. Conclusions

In this paper, we applied Direct Numerical Simulations (DNS) to investigate the effects of a time-dependent large-scale modulation on a co-flow planar jet. In these simulations, the jet confined at the inflow region of width D is agitated by a set of specific scales with amplitude varying periodically in space and time. The effects of the imposed time-modulation were assessed based on a parametric study involving a range of modulation frequencies, ω, for two different imposed length scales. The flow response was characterized based on different properties, such as the flow kinetic energy ET, the averaged dissipation rate, ⟨\epsilon⟩, and characteristic mixing properties. In general terms, the modulation effects on the co-flow jet are as follows:

I Results for the conditionally averaged signal, ET, reveal the sensitivity of the response of the spatially inhomogeneous jet to the frequency of the imposed modulation. At given K, an oscillatory behavior of the flow is found. For instance, a steady oscillating pattern of the flow is retained at locations near the inflow plane when the flow is stirred at low frequencies, ω < ων where ων = U/ν, such that the response follows the imposed modulation, whereas in case of high modulation frequencies, i.e., ω > ων, the response quickly decays.

For the flow response based on time-averaged quantities:

II For global properties, such as the volume and time averaged dissipation rate ⟨\epsilon⟩, the response trend presents a similar frequency dependence for both wave-numbers, K. For example, at given K a maximum response of the dissipation rate was observed when the flow was modulated at frequencies near ων.

III To investigate in more detail the modulation effects on the overall mixing of the jet, we consider the spreading of an injected scalar. We observe a direct correlation between the enhancement of ⟨\epsilon⟩ at ω = ων and the maximum area of an iso-surface of the scalar, such that at given modulation wave-number K a large scalar surface generally develops when the flow is modulated at frequencies of order of ω = ων.

IV For both wave-numbers the time dependent modulation yields stronger effects on the agitated flow than a pure spatially periodic modulation.

Combined, these results indicate the occurrence of ‘resonance’ mixing conditions, e.g., mixing enhancement, in a spatially inhomogeneous flow configuration stirred by a time-modulated large-scale perturbation.

Declaration of Competing Interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

CRediT authorship contribution statement

T. Cardoso de Souza: Conceptualization, Methodology, Software, Formal analysis, Investigation, Data curation, Writing - original draft, Writing - review & editing, Visualization. R.J.M. Bastiaans: Project administration, Funding acquisition, Resources. L.P.H. De Goey: Project administration. B.J. Geurts: Conceptualization, Writing - original draft, Writing - review & editing, Supervision, Funding acquisition, Resources.

Acknowledgements

The authors would like to acknowledge the Dutch Technology Foundation STW for the financial support, The Netherlands Computing Facilities (NCF) for grant: SH-061, and the computing facilities of the High Performance Computing Center-NP of the Federal University of Rio Grande do Norte - UFRN.

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