Prohibiting cherry-picking: Regulating vehicle sharing services who determine fleet and service structure

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A R T I C L E   I N F O

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A B S T R A C T

Vehicle sharing services make travel more convenient, but increase traffic, pollute the environment, and negatively impact existing markets. To achieve a high societal welfare, public authorities regulate vehicle sharing. However, it remains unclear whether these regulations work efficiently without unexpected influences on seemingly unrelated aspects of the fleet and service structure. We study impact of regulations on decisions of vehicle sharing operators, and measure efficiency and effectiveness of these regulations. By investigating the influences of these interactions on societal welfare, we gain insights on how operators should adjust their decisions and regulators should adjust their regulations. Operators adapt fleet size, open stations, availability, and rebalancing operations to given regulations. We formalize the decision on the optimal fleet and service structure as a Mixed Integer Second-Order Cone Program. Using examples, we show that the inter-dependencies between different regulations and societal welfare indicators are non-trivial, possibly even counter-intuitive, suggesting a numerical approach to determine the direction and relevance of influencing factors. We, thus, conduct a large numerical study on artificial instances and a case study using data from New York City. Regulating the number of open stations, fleet size, or total distance reduces empty vehicle distance to a similar extent (by more than 30% of empty vehicles in the NYC case study), and more substantially than regulating the empty vehicle distance directly. Among all studied regulations, enforcing equal service availability results in the least profit loss (10% in the NYC case study). Unlike other regulations, imposing a tax predominantly affects the societal welfare indicators linearly, and can thus be enforced and controlled more easily.

1. Introduction

Currently, vehicle sharing operators optimize their operations as well as their fleet and service structure to maximize profitability which may harm customer service, i.e., they may “cherry-pick” served customers: (i) If a part of the service region is unprofitable, operators will not continue service there. (ii) During demand peaks or substantial demand imbalances, operators reduce the fraction of customers they serve. On the other hand, operators provide excessively large fleets to increase their coverage. Thus, the societal benefits attributed to vehicle sharing services such as reducing the number of vehicles in major cities (Clewlow, 2019), providing mobility to currently under-served populations (TechCrunch, 2018), and complementing public transit (Stiglic et al., 2018) may revert back (Tirachini, 2019; Agarwal et al., 2021). Unregulated vehicle sharing services cannibalize public transit (Lanzetti et al.,...
2020) and taxis (Forbes, 2018) and increase traffic on roads (Li et al., 2016). Thus, public authorities aim at regulating vehicle sharing services. Such regulations include fleet size (e.g., in New York City), minimum availability, and equal availability (as suggested in the theoretical contributions of Zhang and Pavone (2016)). However, most public authorities do not consider the interplay of the operator’s decisions on service region, fleet size, and availability when regulating vehicle sharing services. To sustain a large service region, operators either reduce their service level, increase the fleet size, or increase their rebalancing operations resulting in additional traffic. To reach a high service level target, operators can either reduce the service region, increase the fleet size, or increase empty mileage. If the maximum fleet size is regulated, operators may downsize their service region, or reduce the service level. Too strong regulations may force operators to forgo service altogether, and thus, positive impacts of vehicle sharing services cannot take. As a result, regulations may not reach their goals and harm the regulator’s target of providing a high societal welfare.

So far, it is unknown how regulations affect the profitability of vehicle sharing operators and the welfare of society, i.e., if policymakers reach their goals. This paper addresses this gap, and specifically answers the following research questions:

RQ1 How do operators adjust their fleet and service structure as well as rebalancing operations for maximal profitability as a reaction to a given regulation?
RQ2 How does a given regulation and optimal reaction of operators impact societal welfare, given by fleet and service structure, traffic congestion, and service level?

The research questions address the impact of regulations, and whether these regulations are beneficial or detrimental to the goals of the regulatory authority. The three strategic/tactical decisions, i.e., fleet size, service region, and service availability, and the operational rebalancing decision are closely linked. This research contributes both to the methodological understanding of vehicle sharing systems, and provides managerial and regulatory insights. We exemplarily outline unexpected inter-dependencies between the four decisions, and numerically quantify their impacts. Methodologically, we model the decision on the fleet and service structure in a vehicle sharing system as an open queuing network, and formulate it as a Mixed Integer Second-Order Cone Program. Prior research ignored the possibility to strategically set the service level, and thus model the safety stock by a linear constraint. Managerially, given the optimal fleet and service structure in the system under different regulations, we study societal impacts of the regulated vehicle sharing system and indicate how the operators and regulators should adjust their decisions to improve societal welfare.

The remainder of this paper is organized as follows. Section 2 presents related literature on operations and regulations of vehicle sharing systems. Section 3 specifies the high-level problem and formulates the model. We introduce measures of societal welfare as well as regulations in Section 4. In Section 5, we discuss observations from the numerical analysis, the sensitivity analysis, and the case study. We summarize and conclude this paper in Section 6. We refer to the online appendix for proofs.

2. Literature review

Herein, we review literature on operational and strategic decisions in vehicle sharing systems and regulations thereof.

2.1. Operations of vehicle sharing systems

The research into Operations Research methods for vehicle sharing systems often focuses on fleet operations, including vehicle rebalancing (Laporte et al., 2018). Most literature addresses static rebalancing problems during the night (e.g. Huang et al., 2020; Li and Liu, 2021; Martin and Minner, 2021), whereas we consider dynamic rebalancing during the day using models similar to Braverman et al. (2019), Zhang and Pavone (2015) and George and Xia (2011). Several papers extend these (and other similar) models to study different objectives which are similar to the societal welfare indicators studied in this paper. Zhang and Pavone (2016) adjust routing policies in autonomous vehicle sharing systems, alleviating congestion. Chang et al. (2017) optimize vehicle location and rebalancing operations in a mixed-fleet vehicle sharing system but do not strategically set the availability. Under an environmental constraint, their optimal solutions have a high vehicle utilization rate and service quality. Wallar et al. (2018) optimize rebalancing in vehicle sharing systems minimizing the waiting time and travel delay. Integrating vehicle sharing with public transit services, Ma et al. (2019) structure a non-myopic queuing model that optimizes vehicle assignment and rebalancing for minimal service costs.

The above literature shows that vehicle rebalancing has been studied extensively. However, the integration of vehicle operations with other tactical and strategic decisions, i.e., fleet size and service area, is not yet well developed for planning vehicle sharing systems.

2.2. Strategic decisions in vehicle sharing systems

Strategic decisions in any vehicle sharing system mainly cover the decisions on fleet size and service area/open stations. In the following, we review papers in which the multiple decisions are optimized simultaneously, or explicitly address the interplay of decisions.

Nair and Miller-Hooks (2014) propose a bi-level problem to decide upon station locations and their capacity including an initial vehicle assignment. Reformulating the problem into an integer program, they propose optimal service structure designs
given different cost settings. He et al. (2017) optimize the service region with vehicle rebalancing in electric vehicle sharing systems. Since the operator prefers areas with high demand over areas with low demand, an optimized vehicle sharing system can serve more demand and results in lower carbon emissions. Basciftci et al. (2021) present a decision-dependent distributionally robust optimization model that determines facility locations in vehicle sharing systems. The model addresses the interplay between customer demand and facility locations; considering this results in higher profits and service quality than other stochastic programming and distributionally robust methods. Fu et al. (2022) optimize station location and rebalancing operations in a bike-sharing system. Their two-stage robust optimization model is more computationally efficient than a scenario-based method. For integrated mobility services that allow vehicle sharing, Luo et al. (2021) design a stochastic program that determines transfer locations and fleet operations for maximal profits. In the obtained solution, the operator serves more customers with higher profits than by optimizing a standard location problem which maximizes coverage/requests. Liu and Ouyang (2021) design a model that optimizes service station, fleet size, and vehicle rebalancing operations to minimize costs of an integrated mobility services. The optimal solutions have higher service quality and less costs than fixed-route bus feeder systems.

Most existing literature focuses on planning and operating vehicle sharing systems, while the inter-dependencies among these decisions in regulated vehicle sharing systems remains unclear and existing literature does not address the full breadth of decisions considered in this paper. We combine the decisions on fleet and service structure with rebalancing operations to study a vehicle sharing system. Moreover, we investigate applicability of aforementioned methodologies to solve the research questions.

2.3. Regulating and subsidizing vehicle sharing services

We review recent literature that focuses on the impacts of vehicle sharing systems on societal welfare, such as emissions and congestion, consumer benefits, and current transportation markets.

The impact of vehicle sharing services on congestion and emissions is mainly studied for ride-hailing systems, and there, research is inconclusive: Li et al. (2016) present empirical evidence that ride-hailing service operators, such as Uber, decrease traffic congestion and carbon emissions. In contrast, Ruch et al. (2020) argue that the efficiency gains in ride-hailing services cannot balance lower convenience, privacy issues, and increased travel time. Alisoltani et al. (2021) find that ride-hailing services reduce traffic congestion in large-scale networks but not in smaller ones while Li et al. (2022) argue that congestion increases in dense areas while it may decrease in suburban regions. Dhanorkar and Burtch (2021) find that ride-hailing services reduce traffic due to pooling effects on weekdays but increase traffic on weekends. Naumov et al. (2020) argue that sharing of automated vehicles can even trigger a death spiral for public transportation.

Due to the not yet well understood impact of vehicle sharing services on societal welfare, both regulations and subsidizations can be necessary to control and support this business model. Various regulations of vehicle sharing systems have been studied in literature, including parking reservation policies (Kaspi et al., 2016), environmental regulations (Bellos et al., 2017), and toll charging rules in presence of carpooling (Ostrovsky and Schwarz, 2019), also outlining the necessity for regulations. Some studies focus on the various participants in the systems and discuss regulations and subsidization of drivers and riders (e.g., Wang et al., 2018; Feng et al., 2021; Jacob and Roet-Green, 2021). Specifically, Hyland and Mahmassani (2020) study shared-ride policies in an autonomous mobility on demand system and suggest that public authorities shall incentivize the system and its riders and system operators may also incentivize riders to improve the system performance and societal welfare.

The literature on regulation and subsidization of vehicle sharing services developed both from the side of regulators and the side of operators. Luo et al. (2019) introduce a dynamic subsidy policy maximizing efficiency of autonomous vehicles by accelerating their adoption process. Yu et al. (2020) suggest that the government should reduce taxi fares and should not set too strict regulations to balance gains among stakeholders. Benjaafar et al. (2022) propose labor pool size and wage-floor regulations which benefit both drivers and riders to maximize labor welfare and societal welfare. Abouee-Mehrizi et al. (2021) suggest that governments shall subsidize the vehicle sharing operators using electric vehicles or improve electric vehicles’ performances to increase societal welfare. Song et al. (2021) study subsidization in ridesharing services and find that subsidizing ridesharing users increases societal welfare, while increasing the service booking fee decreases societal welfare and travel time.

The aforementioned works study vehicle sharing systems under a few specific regulations, but they are lack of discussing the relative effectiveness and actual efficiency of distinct regulations on the services if a large set of regulations is given. Considering that different regulations may have similar or opposite effects on the system, we apply multiple regulations on the vehicle sharing system for observing unexpected influences of the regulations and comparing effects of each regulation on the system and societal welfare.

3. Model

In this section, we introduce the problem, and subsequently formalize the model as a Mixed Integer Second-Order Cone Program.

3.1. Problem statement

A vehicle sharing operator considers to start business in a new city. The operator must adhere to regulations from the city’s authorities. The service can span a set of stations $I$; these stations may approximate districts in a free-floating system. In literature, it is common to assume that every (virtual) station attracts customers in its vicinity, for example up to a maximum walking distance (e.g., Kabra et al., 2020). Similar to most models in literature (e.g., He et al., 2017; Martin et al., 2021), our model does
not account for inter-dependent arrival rates at neighboring stations, and does not strategically set those (virtual) stations. Future research may address the impact of size and shape of virtual stations beyond customers’ willingness to walk. To maximize the total contribution margin per trip served between stations \(i\) and \(j\), \(e_{ij} \in \mathbb{R}, \forall i,j \in I\) (0 otherwise), what fraction of customers to serve at which station, i.e., the service level, \(\tilde{\alpha}_{ij} \forall i \in I\), how many vehicles to rebalance from station \(i\) to \(j\), \(e_{ij} \forall i,j \in I\), and empty vehicles idling at an open station, \(e_{ii} \forall i \in I\). Obviously, the procured vehicles also include full serving vehicles moving from station \(i\) to \(j\), \(f_{ij} \forall i,j \in I\).

Customers arrive at an open station \(i\) following a Poisson process with arrival rate \(\lambda_i\). The customer is served by any idle vehicle (if any are available at station \(i\)) and travels from station \(i\) to \(j\) with probability \(p_{ij}\). For ease of notation, we use \(\lambda_{ij} = \lambda_i p_{ij}\) henceforth. Clearly, customers can only travel to an open station \(j \in I\). Customers then arrive at their destination, station \(j\), on average after the expected travel time \(\frac{1}{\mu_j}\) has passed, i.e., return rates follow a generic distribution around the return rate \(\mu_j\). The contribution margin per trip served between stations \(i\) and \(j\) is \(r_{ij}\). Vehicles rebalanced from station \(i\) to \(j\) incur rebalancing cost \(c_{ij}\). Every vehicle in the fleet incurs a fixed investment cost that is periodized as \(h\). Due to the mesoscopic nature of the fleet size and station location decisions, we assume that arrival rates, travel times, contribution margins, and rebalancing costs are constant. If no idle vehicle is available at station \(i\), the customer leaves the system immediately due to the availability of alternative transportation modes, and does not result in a payoff for the operator (cf. Braverman et al., 2019). Literature indicates that demand may increase substantially following the introduction of a vehicle sharing service (e.g., Henao and Marshall, 2019); we assume that \(\lambda\) already accounts for this induced demand. Similar to Li et al. (2021) and Basciftci et al. (2021), this model could be extended to endogenous demand. We further ignore inter-dependent demand patterns at adjacent stations, i.e., that passengers may rent a vehicle from a nearby station if the station closest to their starting location is closed.

For fixed strategic decisions, i.e., fleet size, open stations, and service level, the operator manages vehicles within the system. Fig. 1 (left) shows a three-station network in which all stations open. There, the fleet comprises of empty (rebalancing) vehicles and full (customer-serving) vehicles traveling between any two stations. Other empty vehicles idle at every station. For a system in which the operator decides to close station 2 and opens only stations 1 and 3, Fig. 1 (right) shows that no vehicle travels to or from the closed station 2. All vehicles move between open station 1 and 3. Analogous to the setting on vehicle flows in Braverman et al. (2019), we assume that at the beginning, every vehicle is empty and assigned at station \(i \in I\).

### 3.2. Mathematical formulation

To model the integrated decisions on fleet and service structure as well as rebalancing, we adapt models from He et al. (2017) and Braverman et al. (2019) using a fixed-population-mean approximation (Whitt, 1984). For a vehicle sharing system with a fixed number of vehicles, we assume that empty vehicles idling at station \(i\), \(e_{ii}\), arrive from outside the system at a rate endogenous to the system (see Fig. 1). We refer the interested reader to He et al. (2017) and Whitt (1984).

The quadratic programming formulation of the operator’s problem is

\[
\begin{align*}
\max_{n,\alpha,\mu,\lambda} & \quad \pi = \left( \sum_{i \in I} \sum_{j \in I} \lambda_{ij} \tilde{\alpha}_j x_j r_{ij} - \sum_{i \in I} \sum_{j \in I, j \neq i} c_{ij} \mu_{ij} e_{ij} - h \cdot n \right) \quad (1a) \\
\text{subject to} & \quad \lambda_{ij} \tilde{\alpha}_j x_j = \mu_{ij} f_{ij} \quad \forall i,j \in I \quad (1b) \\
& \quad \sum_{j \in I} \lambda_{ij} \tilde{\alpha}_j x_j + \sum_{j \in I, j \neq i} \mu_{ij} e_{ij} = \sum_{k \in I, k \neq i} \mu_{ki} e_{ki} + \sum_{k \in I} \mu_{ki} f_{ki} \quad \forall i \in I \quad (1c) \\
& \quad \tilde{\alpha}_i \leq x_j \quad \forall i \in I \quad (1d) \\
& \quad \frac{\tilde{\alpha}_i}{1 - \tilde{\alpha}_i} x_i \leq e_{ii} \quad \forall i \in I \quad (1e)
\end{align*}
\]
where the objective function (1a) maximizes the total profit, i.e., contribution margins for served customers minus rebalancing and periodized investment costs. Constraints (1b) represent Little’s Law for full vehicles, and ensure that actual service of each origin–destination pair is fulfilled by all full-in-service vehicles traveling on that arc. Constraints (1c) balance the vehicle flow at each station \( i \). Constraints (1d)–(1e) link availability \( \tilde{a}_i \) of an open station \( i \) with the presence of empty vehicles using Whitt (1984)’s fixed-population-mean approximation. Constraint (1f) links the number of empty and full vehicles in the system to the fleet size. Constraints (1g)–(1i) define the variable domains.

Problem (1) formalizes the rebalancing problem in the vehicle sharing system as a queueing network, similar to He et al. (2017) and Braverman et al. (2019), among others. Analogous to He et al. (2017), we give the model full control over open stations and fleet size, and additionally permit the choice of the service level. Thus, our model is bi-linear in constraint (1e), while He et al. (2017) set the minimum number of vehicles idling at a station using a linear constraint. In analogy to Braverman et al. (2019), we consider vehicle flows as units rather than rebalancing rates. This improves the readability and interpretability of the model since the number of vehicles at stations, or between stations, is easy to infer, but does not affect feasibility or optimality as flows and rates are directly convertible for fixed travel times.

3.3. Reformulation as a second-order cone program

Problem (1) is a quadratically-constrained quadratic problem and cannot be solved with off-the-shelf solvers as-is; we reformulate it as a Second-Order Cone Program in two steps. First, we linearize the objective function (1a) and Constraints (1b)–(1c) using auxiliary variables \( a_{ij} \) that represent the availability between stations \( i \) and \( j \).

Corollary 1. Replacing \( \tilde{a}_ix_j = a_{ij} \forall i, j \in I \) in the objective function (1a) and Constraints (1b)–(1c) is equivalent to the original constraints when adding constraints

\[
\begin{align*}
& a_{ij} \leq x_i & & \forall i, j \in I \quad (2a) \\
& a_{ij} \leq x_j & & \forall i, j \in I \quad (2b) \\
& \tilde{a}_i - a_{ij} \leq 1 - x_j & & \forall i, j \in I \quad (2c) \\
& a_{ij} \leq \tilde{a}_i & & \forall i, j \in I \quad (2d) \\
& 0 \leq a_{ij} \leq 1 & & \forall i, j \in I \quad (2e)
\end{align*}
\]

Since the operator cannot strategically reject customers based on their destinations because destinations are unknown a-priori, the fraction of served customers must be the same for all trips starting from station \( i \), as long as their destination \( j \) is opened as well. Constraints (2c)–(2d) ensure this.

Following from Corollary 1, the objective (1a) and Constraints (1b)–(1c) become

\[
\begin{align*}
& \max_{\pi, x, n, \lambda, \mu} \pi = \left( \sum_{i \in I} \sum_{j \in J} \lambda_{ij}a_{ij}r_{ij} - \sum_{i \in I} \sum_{j \in J, j \neq i} c_{ij}\mu_{ij}e_{ij} - h \cdot n \right) \quad (3a) \\
& \sum_{j \in J} \lambda_{ij}a_{ij} = \mu_{ij}f_{ij} & & \forall i, j \in I \quad (3b) \\
& \sum_{j \in J} \lambda_{ij}a_{ij} + \sum_{j \in J, j \neq i} \mu_{ij}e_{ij} = \sum_{k \in K, k \neq i} \mu_{ki}e_{ki} + \sum_{k \in K} \mu_{ki}f_{ki} & & \forall i \in I. \quad (3c)
\end{align*}
\]

Second, Constraints (1e) are not positive semi-definite on \( \tilde{a}_i, x_j, i \in I \), and thus cannot be handled by standard solvers. We reformulate this constraint using several sets of auxiliary variables.

Lemma 1. Constraints (1e) can be replaced by

\[
\begin{align*}
& u_iu_i - v_iw_i \leq 0 & & \forall i \in I \quad (4a) \\
& u_i \leq 1 & & \forall i \in I \quad (4b) \\
& v_i + \tilde{a}_i \leq 1 & & \forall i \in I \quad (4c) \\
& w_i - e_{ii} \leq 1 & & \forall i \in I \quad (4d) \\
& u_i, v_i, w_i \geq 0 & & \forall i \in I \quad (4e)
\end{align*}
\]

where \( u_i, v_i, \text{and} w_i \) are auxiliary continuous, non-negative variables.
Using Corollary 1 and Lemma 1, we restate the problem.

**Corollary 2.** Problem (1) is equivalent to

\begin{align*}
(3a) \\
(1d) \rightarrow (1i) \\
(2a) \rightarrow (2e) \\
(3b) \rightarrow (3c) \\
(4a) \rightarrow (4e).
\end{align*}

Problem (5) is a Mixed Integer Second-Order Cone Program, and thus positive semi-definite and convex.

Preliminary computational studies show that this reformulation reduces the runtime from hours to seconds on realistically sized instances compared to directly solving the original formulation with Gurobi’s non-linear functionality.

4. Societal welfare and regulations

To increase fairness, reduce congestion and pollution, and not harm existing transport services, public authorities regulate vehicle sharing services. In this section, we introduce measures of societal welfare and outline various regulations. We also analyze potentially unexpected impacts of the regulations on the optimal fleet and service structure in vehicle sharing services.

4.1. Indicators of societal welfare

**Open Stations.** Vehicle sharing services can serve customers only if their origin and destination stations are covered. Thus, opening stations at locations with low service rate from other transportation modes could fulfill travel demands via vehicle sharing services (while closing specific stations restricts operators from market share in the covered locations).

**Fleet Size.** In analogy to the case of Moia in Hamburg, Germany (Automotive News, 2019), the traditional taxi business is less affected by the market entrance of vehicle sharing services if the services’ fleet size is low. Further, the urban image changes less if the number of additional vehicles is low averaging over the entire city.

**Raised Taxes.** For a public authority, collecting a contribution from vehicle sharing services can be beneficial. Such a tax can for example support the public transit system.

**Total Traffic.** Every trip or rebalancing operation in vehicle sharing services increases congestion in urban areas. As such, the total traffic induced by vehicle sharing services should be little.

**Deadheading.** Every rebalancing trip increases total traffic, and thus negatively impacts the environment. Therefore, deadheading should be low in vehicle sharing services.

4.2. Regulations

We consider the following regulations which address one of the aforementioned indicators of societal welfare.

**Maximum Fleet Size.** City governments can restrict the fleet size $\bar{N}$, with the aim of reducing the amount of additional traffic on the road. Such a fleet size limit can easily be implemented by adding the constraint

$$
n \leq \bar{N}.
$$

(6)

**Maximum Number of Open Stations.** Another way for traffic control is limiting the number of stations $\bar{X}$ which can be implemented by adding the constraint

$$
\sum_{i \in I} x_i \leq \bar{X}.
$$

(7)

**Minimum Service Availability.** To ensure a high service quality, governments can regulate the minimum availability at all or specific open stations. These regulations can be implemented by adding constraint

$$
\tilde{a}_i \geq x_i - (1 - \tilde{A}_i)
$$

(8)

where $\tilde{A}_i$ is the regulated minimum availability for station $i$.

**Comparable/Equal Service Availability.** As Zhang et al. (2018) suggest, public authorities may require the operator to offer a comparable (or equal) service level in the entire city. To adhere to a comparable or equal service level regulation among open stations, the operator imposes

$$
\tilde{a}_i \leq \tilde{a}_j \cdot (1 + \epsilon) + 2 - x_i - x_j \quad \forall \ i, j \in I
$$

(9)

where $\epsilon \geq 0$ is the maximum relative deviation. For equal service levels, the operator sets $\epsilon = 0$. 

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Maximum Total Distance. Every kilometer driven by the vehicle sharing service increases congestion. As such, city authorities may impose
\[
\sum_{i \in I} \sum_{j \in J, j \neq i} d_{ij} \mu_{ij} \epsilon_{ij} + \sum_{i \in I} \sum_{j \in I} d_{ij} \mu_{ij} f_{ij} \leq \bar{D}
\]
where the left-hand side of Constraint (10) refers to the expected total distance, given by the number of vehicles entering arc \( ij \) per period of time (\( \epsilon_{ij} \mu_{ij} \)), \( f_{ij} \)) and the distance per vehicle (\( d_{ij} \)). \( \bar{D} \) refers to the maximum distance the operator is permitted to drive.

Maximum Absolute Deadheading (Empty Vehicle Distance). One of the issues arising with vehicle sharing services is empty mileage which eventually results in additional congestion and pollution. A public authority may thus restrict the empty mileage. To comply with a regulation on the maximum absolute deadheading, the operator must introduce the constraint
\[
\sum_{i \in I} \sum_{j \in J, j \neq i} d_{ij} \mu_{ij} \epsilon_{ij} \leq \bar{E}
\]
into Problem (5) where the left-hand side of Constraint (11) refers to the expected empty vehicle distance, and \( \bar{E} \) refers to the maximum permitted amount of deadheading.

Maximum Relative Deadheading. With the previously described restriction on deadheading, the empty mileage is not set into relation to the served customer demand. A city may accept some more empty mileage if more customers can be served by the service. Thus, a public authority may regulate the relative deadheading, given by
\[
\sum_{i \in I} \sum_{j \in J, j \neq i} d_{ij} \mu_{ij} \epsilon_{ij} \leq E' \sum_{i \in I} \sum_{j \in I} d_{ij} \mu_{ij} f_{ij}
\]
where \( E' \geq 0 \) refers to relative empty mileage.

Taxation. Many cities now consider to impose a tax on vehicle sharing services (Governing, 2018). Minimum wages for drivers in a non-autonomous setting translate to a special case for a taxation. We assume that the operator increases prices accordingly, resulting in a lower demand. If prices increase by some value \( t \), customer demand decreases according to some function \( f(t) \). The input must be adapted accordingly as
\[
\hat{\lambda}_{ij} = \lambda_{ij} \cdot f(t)
\]
where \( \hat{\lambda}_{ij} \) is the unregulated arrival rate.

4.3. Exemplary impact of regulations on a vehicle sharing system

The impact of regulations is not always linear, or even monotone. In the following, we give four examples for cases in which a regulation has an unexpected inter-dependency with another, seemingly unrelated metric of societal welfare. Each stylized example shows a different inter-dependency. We must take these inter-dependencies into consideration in the numerical study, even though they only seldomly occur.

Counter-intuitively, increasing the permitted number of open stations can result in the operator optimally closing individual stations, instead of adding stations to the previously optimal decision. Formally, an open station \( i \) subject to the regulation \( \sum_{i \in I} x_i \leq \bar{X} \) may not be included in the optimal solution subject to \( \sum_{i \in I} x_i \leq \bar{X} + 1 \).

Example 1. Assume that in a five-station system, customers travel between stations 1 and 2, and between stations 3 – 5, but not between those two sets, and only in the direction of the arrows in Fig. 2. All stations have an arrival rate of 1, and contribution margins and rebalancing costs equal to 1$ for all pairs of stations. If the periodized investment cost is 0.2$, and if the regulation of the maximum number of open stations increases from \( \sum_{i=1}^5 x_i \leq 2 \) to \( \sum_{i=1}^5 x_i \leq 3 \), the operator switches from opening stations 1 and 2 in Fig. 2 (left) to opening stations 3 – 5 in Fig. 2 (right).

Analogous to if the number of open stations is regulated, the operator may change her optimal station decisions that impact the average availability if equal service availability is enforced. Formally, enforcing equal service availability, i.e., \( \bar{a}_i = \bar{a}_i \forall i, j \in I, x_i = x_j = 1 \) can increase the availability of all stations which remain open, i.e., \( \bar{a}_i \geq \bar{a}_i' \forall i \in I, x_i = x'_i = 1 \) where \( \bar{a}_i \) and \( x'_i \) refer to the optimal solution without equal service availability (Example 2), or decrease the availability of all stations, i.e., \( \bar{a}_i \leq \bar{a}_i' \forall i \in I, x_i = x'_i = 1 \) (Example 3).
Example 2. We study the five-station system presented in the top-left corner of Fig. 3, with distances according to the table in the top-right corner of Fig. 3. The rebalancing costs are 60$/km, the contribution margins are 0.6$/km, and the periodized investment cost is 4$ per vehicle. Customers travel to each of the other stations with equal probability, i.e., \( p_{ij} = 0.25 \forall i, j \in I, i \neq j \). If no regulation applies, the system opens all stations; the profit and values of some societal welfare indicators can be found in the bottom-left corner of Fig. 3. If equal service availability is enforced, the operator optimally closes station 4, and the other stations remain open. As the optimal profit and values of some societal welfare indicators in the bottom-right of Fig. 3 show, the availability of all stations which remain open increases from consistently below 0.40 to 0.92, and the average availability of open stations \( \bar{\alpha} \) increases from 0.46 to 0.92.

Example 3. Consider a five-station system presented in the top-left corner of Fig. 4 with distances according to the table in the top-right corner of Fig. 4. The rebalancing costs are 30$/km, the contribution margins are 0.6$/km, and the periodized investment cost equals 4$. Customers travel to each of the other stations with equal probability, i.e., \( p_{ij} = 0.25 \forall i, j \in I, i \neq j \). If no regulation applies, the operator opens all stations; the profit and values of some societal welfare indicators can be found in the bottom-left corner of Fig. 4. The operator optimally closes station 4 if equal service availability is enforced, and the other stations remain open. As the optimal profit and values of some societal welfare indicators in the bottom-right of Fig. 4 show, the availability of the stations that remain open decreases from 0.92 to 0.89 each which results in a higher average availability, since the closed station had the lowest availability. All stations in the regulated system decrease their availability.

The availability of each open station can decrease if the minimum service availability is regulated. Formally, increasing the minimum service availability \( \bar{A} \) can result in all open stations, \( x_i = 1 \), having a lower availability, i.e., \( \bar{\alpha}_i \geq \bar{\alpha}'_i \) \( \forall i \in I, x_i = 1 \) where \( \bar{\alpha}'_i \) is the availability with a stricter regulation.

Example 4. We study the five-station system presented in the top-left corner of Fig. 5 with distances according to the table in the top-right corner of Fig. 5. The rebalancing costs are 60$/km, the contribution margins are 0.6$/km, and the periodized investment cost equals 1$. The transition probability is proportional to distance between each two stations. If no regulation applies, all stations open, and the average availability of open stations, \( \bar{\alpha}_{\text{free}} \), is 0.82. The bottom-left corner of Fig. 5 shows the optimal solutions of the system with regulation 85.7% (i.e., 20% closer to full service) in which all stations open. If the regulation increases to 87.4% (i.e., 30% closer to full service), however, station 2 and 3 close, and the bottom-right corner of Fig. 5 shows that the stations which remain open decrease their availability. Notably, in this example the operator does not only decrease the availability of open stations on average, but also the availability of each station which remains open, individually.

These examples suggest that (i) public authorities should consider the impact of their regulations in detail, rather than carrying over results from similar instances, (ii) operators may have to completely re-optimize their fleet if a regulation changes, and (iii) stylized examples for investigating the magnitude of regulations’ impact would either ignore substantial relevant inter-dependencies, or become rather complex. Instead, we opt for a numerical analysis of the impact of regulations.

5. Numerical analysis

In this section, we introduce the experimental design and study the impact of regulations on the system.
5.1. Experimental design

We conduct experiments on a large set of randomly generated data. In brief, the instances differ in the number of stations with their spatial distributions, average arrival rates, transition probabilities, contribution margins, rebalancing costs, and periodized investment costs, similar to Martin et al. (2022). To model free-floating vehicle sharing demand, we use equi-distant districts, i.e., “virtual stations”, on an either quadratic (Q) or hexagonal (H) plane. The quadratic instances contain 9, 16, or 25 stations (9 BAL, 16 BAL, 25 BAL); the hexagonal instances contain 7 or 19 stations (7 BAL, 19 BAL). Fig. 6 shows the spatial distribution of the stations.

The diameter of the plane is 20 km in all instances, and the distances between the stations differ accordingly. The expected travel time in hours between any pair of stations \( i \in I \) and \( j \in I \) is given by their distance divided by an average velocity of 25 km/h. To account for preparation time of each vehicle serving an order or starting to rebalance, we estimate the travel time in hours between any pair of stations \( i, j \in I \), \( t_{ij} \), as the sum of preparation time and expected travel time, such that \( t_{ij} = \frac{1}{\mu_{ij}} + \frac{d_{ij}}{25} \). The return rate is \( \mu_{ij} = \frac{1}{t_{ij}} \). The average arrival rate of the above instances is sampled from a uniform distribution \( \mathcal{U}[80, 120] \).
Fig. 6. Experimental design: Distribution of stations (From left to right: instances with 7/9/16/19/25 stations).

### Table 1
Experimental design: Parameters.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>BAL</td>
<td>Instances with balanced arrival rates</td>
</tr>
<tr>
<td>IMB</td>
<td>Instances with imbalanced arrival rates</td>
</tr>
<tr>
<td>$\lambda_i$</td>
<td>Arrival rate</td>
</tr>
<tr>
<td>$p_{ij}$</td>
<td>Transition probability</td>
</tr>
<tr>
<td>$r_{ij}$</td>
<td>Contribution margins per served trip</td>
</tr>
<tr>
<td>$c_{ij}$</td>
<td>Rebalancing costs per vehicle</td>
</tr>
<tr>
<td>$h$</td>
<td>Periodized investment costs</td>
</tr>
<tr>
<td>No. of stations: 7, 9, 16, 19, 25</td>
<td></td>
</tr>
<tr>
<td>No. of stations: 16, 19, 25</td>
<td></td>
</tr>
<tr>
<td>High ($\lambda \in [80, 120]$), low ($\lambda \in [40, 60]$)</td>
<td></td>
</tr>
<tr>
<td>Equal ((\tfrac{1}{n-1}) to all other stations), Prop. (proportional to distance)</td>
<td></td>
</tr>
<tr>
<td>High (1.5$/km), low (0.6$/km)</td>
<td></td>
</tr>
<tr>
<td>High (17.5$/h), low (7.5$/h)</td>
<td></td>
</tr>
<tr>
<td>high (4$), low (18)</td>
<td></td>
</tr>
</tbody>
</table>

For the larger instances (with at least 16 stations), we further investigate the instance with decreasing arrival rates towards the outer boroughs (16 IMB, 19 IMB, 25 IMB). Then, arrival rates follow a uniform distribution sampled from $\lambda \in [80, 120]$ at more central stations, and $\lambda \in [40, 60]$ for more remote stations. To obtain instances with low arrival rates, we divide the previously sampled rates by 2. Customers arriving at a station $i \in I$ either have equal transition probability traveling to another station $j \in I$, or travel to station $j$ with a probability proportional to the distance between station $i$ and $j$. The contribution margins per served user are directly proportional to the distance between stations $i$ and $j$, and amount to 0.6$ (low) or 1.5$ (high) per km. The former contains revenues of approx. 1$ per km, and direct costs (fuel, wear and tear) amounting to roughly 0.4$. The latter accounts for a higher valuation of customer retention and service level, or some incentives from city or parent company. We set the periodized investment cost per hour to 1$ (low) or 4$ (high). The costs for rebalancing a car is either 7.5$ (low) or 17.5$ (high) per hour, depending on whether the car rebalances itself autonomously, or whether an employee performs the relocation operation. Table 1 lists all parameters.

Studying all combinations of station distribution, arrival rates, transition probabilities, contribution margins, rebalancing costs, and periodized investment costs, we investigate 256 instances. For each regulation except on availability and tax, we regulate the related parameter to have at most 50% of its non-regulated value in every instance. For example, if the optimal non-regulated fleet size in some instance is 100, its maximum fleet size must not exceed 50 under the regulation on the maximum fleet size. For the regulation of the minimum service availability, we regulate the availability of open stations in every instance to be 50% closer to full service than its non-regulated average availability of open stations, i.e., $\overline{A}_i = (1 - \overline{\alpha}) \cdot 0.5 + \overline{\alpha} \forall i \in I$. These strong regulations permit us to observe substantial effects. We consider a tax of 20% of all contribution margins, and a proportional reduction in customer demand. Similar to the other regulations, this high tax permits substantial effects, while roughly mimicking direct taxes plus VAT in various US cities, for further insight we refer to Kim and Puentes (2018).

### 5.2. Comparison of regulations

We first compute profits and values of societal welfare indicators without regulations (see Table 2), and then consecutively study the impact of the regulations. The operator serves customers in all instances if no regulation applies. We compare all regulations to investigate which regulation has the strongest positive or negative effect on the indicators of societal welfare. For each comparison, we omit the regulation that is directly associated with the societal welfare indicator.

No regulation forces the operator out of service using realistic instances, even though the regulations are drastic. Regarding the impact of each regulation on the system and associated societal welfare indicators, Table 3 shows that regulating the number of open stations decreases fleet size the most, and regulating the minimum service availability increases fleet size the most, on average over all instances. The average availability of open stations increases most if equal service availability is enforced, and decreases most if the total distance is regulated. The number of open stations does not vary substantially under all regulations. With the increase in fleet size, regulating the minimum service availability increases the total distance the most, and regulating the number of open stations decreases the total distance the most. Enforcing equal service availability increases the empty vehicle distance the most, while regulating the total distance decreases the empty vehicle distance the most. For the operator, profits reduce most if the number of open stations is regulated and reduce least if equal service availability is enforced.
Regulating the number of open stations, maximum fleet size, or maximum total distance decreases the empty vehicle distance, Result 1.

Comparison of all regulations, Averaged over all artificial instances (bold: lowest value per column, italic: highest value per column except directly related measures).

<table>
<thead>
<tr>
<th>Setting</th>
<th>Fleet size</th>
<th>Avg. Avail.</th>
<th># Open stations</th>
<th>Total empty Dist.</th>
<th>Total full Dist.</th>
<th>Rel. DH ratio (%)</th>
<th>Profits</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 BAL</td>
<td>2.94E+2</td>
<td>0.908</td>
<td>7</td>
<td>1.51E+2</td>
<td>3.89E+3</td>
<td>3.74</td>
<td>3.41E+3</td>
</tr>
<tr>
<td>9 BAL</td>
<td>5.10E+2</td>
<td>0.927</td>
<td>9</td>
<td>2.11E+2</td>
<td>7.41E+3</td>
<td>2.76</td>
<td>6.59E+3</td>
</tr>
<tr>
<td>16 BAL</td>
<td>8.97E+2</td>
<td>0.925</td>
<td>16</td>
<td>4.52E+2</td>
<td>1.29E+4</td>
<td>3.41</td>
<td>1.15E+4</td>
</tr>
<tr>
<td>Instances</td>
<td>16 IMB</td>
<td>5.89E+2</td>
<td>0.884</td>
<td>16</td>
<td>7.19E+2</td>
<td>7.41E+3</td>
<td>9.35</td>
</tr>
<tr>
<td>19 BAL</td>
<td>6.12E+2</td>
<td>0.884</td>
<td>19</td>
<td>3.52E+2</td>
<td>7.29E+3</td>
<td>4.52</td>
<td>6.23E+3</td>
</tr>
<tr>
<td>19 IMB</td>
<td>4.10E+2</td>
<td>0.807</td>
<td>19</td>
<td>5.94E+2</td>
<td>4.16E+3</td>
<td>13.1</td>
<td>3.33E+3</td>
</tr>
<tr>
<td>25 BAL</td>
<td>1.39E+3</td>
<td>0.929</td>
<td>25</td>
<td>6.17E+2</td>
<td>2.01E+4</td>
<td>3.01</td>
<td>1.78E+4</td>
</tr>
<tr>
<td>25 IMB</td>
<td>1.00E+3</td>
<td>0.889</td>
<td>25</td>
<td>1.38E+3</td>
<td>1.28E+4</td>
<td>10.5</td>
<td>1.06E+4</td>
</tr>
</tbody>
</table>

\[ \lambda_i \]

<table>
<thead>
<tr>
<th>Setting</th>
<th>Fleet size</th>
<th>Avg. Avail.</th>
<th># Open stations</th>
<th>Total empty Dist.</th>
<th>Total full Dist.</th>
<th>Rel. DH ratio (%)</th>
<th>Profits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>5.08E+2</td>
<td>0.880</td>
<td>17</td>
<td>3.58E+2</td>
<td>6.21E+3</td>
<td>6.17</td>
<td>5.32E+3</td>
</tr>
<tr>
<td>High</td>
<td>9.19E+2</td>
<td>0.909</td>
<td>17</td>
<td>7.60E+2</td>
<td>1.28E+4</td>
<td>6.42</td>
<td>1.11E+4</td>
</tr>
</tbody>
</table>

\[ \rho_j \]

<table>
<thead>
<tr>
<th>Setting</th>
<th>Fleet size</th>
<th>Avg. Avail.</th>
<th># Open stations</th>
<th>Total empty Dist.</th>
<th>Total full Dist.</th>
<th>Rel. DH ratio (%)</th>
<th>Profits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equal Prop.</td>
<td>6.76E+2</td>
<td>0.897</td>
<td>17</td>
<td>4.37E+2</td>
<td>8.87E+3</td>
<td>5.17</td>
<td>7.69E+3</td>
</tr>
<tr>
<td>High</td>
<td>8.51E+2</td>
<td>0.940</td>
<td>17</td>
<td>6.81E+2</td>
<td>1.01E+4</td>
<td>7.42</td>
<td>8.73E+3</td>
</tr>
</tbody>
</table>

\[ \tau_{ij} \]

<table>
<thead>
<tr>
<th>Setting</th>
<th>Fleet size</th>
<th>Avg. Avail.</th>
<th># Open stations</th>
<th>Total empty Dist.</th>
<th>Total full Dist.</th>
<th>Rel. DH ratio (%)</th>
<th>Profits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>7.28E+2</td>
<td>0.909</td>
<td>17</td>
<td>6.21E+2</td>
<td>9.61E+3</td>
<td>7.06</td>
<td>8.35E+3</td>
</tr>
<tr>
<td>High</td>
<td>6.99E+2</td>
<td>0.879</td>
<td>17</td>
<td>4.97E+2</td>
<td>9.38E+3</td>
<td>5.53</td>
<td>8.07E+3</td>
</tr>
</tbody>
</table>

\[ c_{ij} \]

<table>
<thead>
<tr>
<th>Setting</th>
<th>Fleet size</th>
<th>Avg. Avail.</th>
<th># Open stations</th>
<th>Total empty Dist.</th>
<th>Total full Dist.</th>
<th>Rel. DH ratio (%)</th>
<th>Profits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>8.38E+2</td>
<td>0.937</td>
<td>17</td>
<td>6.42E+2</td>
<td>9.87E+3</td>
<td>7.13</td>
<td>9.23E+3</td>
</tr>
<tr>
<td>High</td>
<td>5.89E+2</td>
<td>0.851</td>
<td>17</td>
<td>4.76E+2</td>
<td>9.12E+3</td>
<td>5.46</td>
<td>7.19E+3</td>
</tr>
</tbody>
</table>

\[ h \]

<table>
<thead>
<tr>
<th>Setting</th>
<th>Fleet size</th>
<th>Avg. Avail.</th>
<th># Open stations</th>
<th>Total empty Dist.</th>
<th>Total full Dist.</th>
<th>Rel. DH ratio (%)</th>
<th>Profits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg./Sum</td>
<td>7.14E+2</td>
<td>0.894</td>
<td>17</td>
<td>5.59E+2</td>
<td>9.49E+3</td>
<td>6.29</td>
<td>8.21E+3</td>
</tr>
</tbody>
</table>

Table 3
Comparison of all regulations, Averaged over all artificial instances (bold: lowest value per column, italic: highest value per column except directly related measures).

<table>
<thead>
<tr>
<th>Regulation</th>
<th>Fleet size</th>
<th>Avg. Avail. (Open)</th>
<th>Avg. Avail. (All)</th>
<th># Open stations</th>
<th># Empty Veh.</th>
<th># Full Veh.</th>
<th>Total empty Dist.</th>
<th>Total full Dist.</th>
<th>Rel. DH ratio (%)</th>
<th>Total Dist.</th>
<th>Profits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max fleet size</td>
<td>−50.0</td>
<td>−29.9</td>
<td>−29.9</td>
<td>0.00</td>
<td>−81.2</td>
<td>−29.0</td>
<td>−83.0</td>
<td>−28.8</td>
<td>−78.7</td>
<td>−31.5</td>
<td>−21.5</td>
</tr>
<tr>
<td>Max No. Stations</td>
<td>−70.0</td>
<td>−1.13</td>
<td>−77.7</td>
<td>−52.7</td>
<td>−65.0</td>
<td>−73.2</td>
<td>−77.4</td>
<td>−72.6</td>
<td>−19.0</td>
<td>−72.7</td>
<td>−72.8</td>
</tr>
<tr>
<td>Min service Avail.</td>
<td>37.7</td>
<td>6.66</td>
<td>6.66</td>
<td>0.07</td>
<td>84.6</td>
<td>6.10</td>
<td>174</td>
<td>60.0</td>
<td>139</td>
<td>7.73</td>
<td>−4.80</td>
</tr>
<tr>
<td>Equ. Service Avail.</td>
<td>4.01</td>
<td>3.18</td>
<td>2.97</td>
<td>0.07</td>
<td>7.82</td>
<td>2.40</td>
<td>176</td>
<td>2.30</td>
<td>151</td>
<td>4.04</td>
<td>−2.06</td>
</tr>
<tr>
<td>Max total Dist.</td>
<td>−64.2</td>
<td>−48.2</td>
<td>−48.3</td>
<td>−0.04</td>
<td>−90.1</td>
<td>−47.1</td>
<td>−98.6</td>
<td>−46.9</td>
<td>−97.2</td>
<td>−50.0</td>
<td>−40.0</td>
</tr>
<tr>
<td>Max empty Dist.</td>
<td>−10.0</td>
<td>−6.82</td>
<td>−6.82</td>
<td>0.00</td>
<td>−15.7</td>
<td>−6.00</td>
<td>−50.2</td>
<td>−5.88</td>
<td>−47.0</td>
<td>−8.25</td>
<td>−2.83</td>
</tr>
<tr>
<td>Rel. DH Ratio</td>
<td>−10.0</td>
<td>−7.52</td>
<td>−7.52</td>
<td>0.00</td>
<td>−15.0</td>
<td>−6.60</td>
<td>−53.4</td>
<td>−6.46</td>
<td>−50.2</td>
<td>−9.05</td>
<td>−3.31</td>
</tr>
<tr>
<td>Tax (20%)</td>
<td>−17.6</td>
<td>−1.16</td>
<td>−20.9</td>
<td>0.00</td>
<td>−12.4</td>
<td>−20.9</td>
<td>−22.9</td>
<td>−20.9</td>
<td>−25.1</td>
<td>−21.0</td>
<td>−21.6</td>
</tr>
</tbody>
</table>

Table 4
Number of instances in which the operator stops rebalancing under a Three Regulations (BAL: balanced, IMB: imbalanced instances, \( \lambda_i \): arrival rate, \( \rho_j \): transition probability, \( r_{ij} \): contribution margins, \( c_{ij} \): rebalancing cost, \( h \): periodized investment cost). “No Reb.(I)” additional number of instances in which the operator stops rebalancing if the total distance is regulated. “No Reb.(II)” total number of instances in which the operator stops rebalancing if the total distance is regulated.

<table>
<thead>
<tr>
<th>Setting</th>
<th>Instances</th>
<th>( \lambda_i )</th>
<th>( \rho_j )</th>
<th>( r_{ij} )</th>
<th>( c_{ij} )</th>
<th>( h )</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 BAL</td>
<td>9 BAL</td>
<td>16 BAL</td>
<td>16 IMB</td>
<td>19 BAL</td>
<td>19 IMB</td>
<td>25 BAL</td>
<td>25 IMB</td>
</tr>
<tr>
<td>Low</td>
<td>High</td>
<td>Equal Prop.</td>
<td>Low</td>
<td>High</td>
<td>Low</td>
<td>Low</td>
<td>High</td>
</tr>
<tr>
<td>No Reb.</td>
<td>12</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>No Reb.</td>
<td>24</td>
<td>22</td>
<td>11</td>
<td>5</td>
<td>12</td>
<td>9</td>
<td>11</td>
</tr>
<tr>
<td>Max (Total Dist.)</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
</tbody>
</table>

Result 1. Regulating the number of open stations, maximum fleet size, or maximum total distance decreases the empty vehicle distance substantially, and to a similar extent. This suggests that empty vehicle distance may not be a suitable indicator of societal welfare to compare the efficacy of regulations.

As visible from Table 3, on average over all artificial instances the empty vehicle distance in the system decreases more than 70% if any of these regulations applies. Table 4 shows that the instances in which the operator stops rebalancing due to a regulation on the number of open stations also force the operator to stop rebalancing under a regulation on the maximum fleet size, and the...
instances in which the operator stops rebalancing under a regulation on the maximum fleet size also force the operator to stop rebalancing under a regulation on the maximum total distance. The instances in which the operator stops rebalancing under any of these regulations commonly have low contribution margins, high periodized investment costs, equal transition probabilities, and high rebalancing costs. Many of the additional instances which force the operator to stop rebalancing if the maximum total distance is regulated have high contribution margins, low periodized investment costs, and low rebalancing costs. If regulators consider to apply only one of these regulations, regulators should use other societal welfare indicators rather than empty vehicle distance to evaluate performances of each regulation.

Most of the instances that decrease empty vehicle distance under any aforementioned regulation have balanced arrival rates and low non-regulated empty vehicle distance. Regulating the number of open stations, fleet size, or total distance forces the operator to maintain a more balanced system by decreasing empty vehicle distance for profitable services. Also, though the operator reduces the fleet size and total distance the most if the number of open stations is regulated, the regulation forces the operator to close more stations with a significant loss in the average availability of the overall demand than the other two regulations. Therefore, for regulators attempting to reduce both the fleet size and rebalancing operations while opening more stations, applying either regulation on the fleet size or total distance is sufficient.

Result 2. Regulating the fleet size or total distance reduces rebalancing more than regulating the empty vehicle distance. Regulating the empty vehicle distance, however, is the most precise regulation, and is easy to adjust.

As visible from Table 3, regulating the empty vehicle distance reduces the associated societal welfare indicator by half on average, while regulating the fleet size or total distance reduces the empty vehicle distance by more than 80%. However, such a significant reduction in the empty vehicle distance also decreases availability and, thus, full vehicle distance. Regulating the fleet size or total distance incurs more profit losses than regulating the empty vehicle distance. Therefore, for regulators aiming to sustain the service while reducing the traffic load, regulating the empty vehicle distance is more efficient.

Furthermore, Table 3 shows that regulating the minimum service availability or enforcing equal service availability on average increases the average availability of open stations. Though regulating the minimum service availability is more efficient to increase the average availability of open stations than enforcing equal service availability, the latter regulation needs comparably fewer vehicles, incurs less profit loss, and ensures equal availability of all open stations. Regulators need to make a trade-off between increasing average availability and inducing traffic loads with decreasing profits of operators while considering which regulation on the availability to apply.

5.3. Influence of individual regulations

We investigate the influences of every individual regulation on societal welfare indicators. For each regulation, we report the percentage change of non-regulated value in fleet size, average availability of open stations, average availability of all stations, number of open stations, empty vehicle distance, full vehicle distance, and profits. In the following, we report the regulations on the number of open stations, fleet size, and tax, since these proved to be most relevant in comparison. We refer to the online appendix for a tabular overview of the other regulations.

Maximum number of stations.

Result 3. Regulating the maximum number of open stations decreases the average availability of those stations which remain open. The average availability increases only in a few instances, and this is more frequent in instances with low contribution margins, high rebalancing costs, proportional transition probabilities, and high periodized investment costs. Regulators considering to apply a regulation on the maximum number of stations should thus carefully measure unwanted side effects.

As Table 5 shows, the average availability of the open stations in instances with increased availability increases 12.1% on average. In more detail, the average availability of open stations increases in 24.2% of instances with low contribution margins, 20.3% of instances with high rebalancing costs, 17.2% of instances with proportional transit probabilities, and 14.8% of instances with high periodized investment costs.

The average availability of open stations increases in these instances because their empty vehicle distance decreases. Since low contribution margins, high rebalancing and periodized investment costs, and proportional transition probabilities make rebalancing more expensive, the operator opens stations that have high non-regulated availability and need less rebalancing activities rather than shouldering the costs if the number of open station is regulated. Though the average availability of open stations does increase in a few instances under the regulation, due to the result that the average availability of open stations decreases in most instances if the number of open stations is regulated, regulators should carefully evaluate the side effects of such a regulation.

Fleet size.

Result 4. Regulating the fleet size reduces empty vehicle distance by stopping rebalancing operations. This occurs more frequently in instances with low contribution margins, high periodized investment costs, high rebalancing costs, and equal transition probabilities. Regulators should thus ensure that their regulation does not harm fairness of service as a result of discontinued rebalancing.
Regulators could regulate either the fleet size or empty vehicle distance to reduce vehicle operator decreases rebalancing distance to zero in instances with equal transition probability, accepting a loss of availability, but regulated availability with less empty vehicle distance. Since regulating the fleet size reduces the possible vehicle distance, the fleet size is regulated.

As visible from Table 6, the operator stops rebalancing in 43.0% of all instances with equal transition probabilities if the maximum margins, high periodized investment costs, or high rebalancing costs incur high operating costs which enforce the operator to reduce 

\[ \lambda_i \] periodized investment cost. "No Rebalance": percentage of regulated instances that stop rebalancing).
rebalancing, and operators could strategically select stations with approximately balanced transition rate to keep more stations open if either of these regulations is applied.

Tax.

Result 5. Imposing a tax more commonly decreases the fleet size in instances with high periodized investment costs and low contribution margins. Since taxation does not show any surprising effects on other societal welfare indicators, it poses a comparably precise, and easy-to-implement regulation.

As visible from the left-most column in Table 7, the fleet size reduces to similar extents under all regulations, ranging between 16.4% and 18.8%. These extremal values are associated with low and high periodized investment costs, respectively. Also, with low contribution margins, the fleet size decreases by 10.2% more than with high contribution margins. The differences in the fleet size reduction occur because of the low non-regulated availability and low non-regulated profits in instances with high periodized investment costs and low contribution margins (see Table 2). While imposing a tax decreases the demand, maintaining the same fleet size reduction occur because of the low non-regulated availability and low non-regulated profits in instances with high periodized investment costs, respectively. Also, with low contribution margins, the fleet size decreases by 10.2% more than with high contribution margins. The differences in the fleet size reduction occur because of the low non-regulated availability and low non-regulated profits in instances with high periodized investment costs and low contribution margins (see Table 2). While imposing a tax decreases the demand, maintaining the same fleet size as before with high periodized investment costs or low contribution margins is undesirable for the operator to have profitable services. Therefore, the operator in instances with high periodized investment costs or low contribution margins prefers to reduce the fleet size more with the decreased availability for less profit loss if a tax is imposed. Nevertheless, because imposing a tax has similar influences on instances in decreasing the fleet size, vehicle distances, and profits, it is a relatively stable regulation for regulators to apply.

Table 7

<table>
<thead>
<tr>
<th>Setting</th>
<th>Fleet size</th>
<th>Avg. Avail. (Open)</th>
<th>Avg. Avail. (All)</th>
<th># Open stations</th>
<th>Total empty Dist.</th>
<th>Total full Dist.</th>
<th>Rel. DH Ratio</th>
<th>Profits</th>
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<td>Instances</td>
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<tr>
<td>7 BAL</td>
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<td>-5.92</td>
<td>-21.5</td>
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<tr>
<td>9 BAL</td>
<td>-17.5</td>
<td>-0.83</td>
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<td>-2.59</td>
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<tr>
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<td>-0.85</td>
<td>-20.7</td>
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<td>-1.22</td>
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<td>-20.9</td>
<td>-1.09</td>
<td>-21.7</td>
</tr>
<tr>
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<td>-1.30</td>
<td>-21.0</td>
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<tr>
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<td>-0.85</td>
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<td>-21.8</td>
<td>-20.7</td>
<td>-1.43</td>
<td>-21.3</td>
</tr>
<tr>
<td>25 IMB</td>
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<td>-1.17</td>
<td>-20.9</td>
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<table>
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</tr>
<tr>
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<td>2.19E+3</td>
</tr>
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<td>1.26E+3</td>
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<td>1.24E+3</td>
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<td>25 IMB</td>
<td>2.17E+3</td>
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5.4. Sensitivity analysis: Impact of the magnitude of the regulations

We conduct a sensitivity analysis to examine the efficiency of regulations on fleet size, availability, and tax. For the maximum fleet size constraint, i.e., \( n \leq \beta \cdot N^{\text{free}} \), and the minimum service availability constraint, i.e., \( \alpha_i \geq a^{\text{free}} + \gamma \cdot (1 - a^{\text{free}}) \), we increase the thresholds \( \beta \) and \( \gamma \) each from 50% to 100% with a step size of 2.5%. For the comparable/equal service availability constraint, \( \alpha_i \leq a_j \cdot (1 + \epsilon) + 2 - x_j - x_i \forall i, j \in I \), we increase the value of \( \epsilon \) from 0 to 0.2 with a step size of 0.02. For the taxation, we increase the tax as \( \tilde{\tau} \) of the contribution margins from 0% to 30% with a step size of 1.5%. We separate the instances into those with balanced arrival rates (BAL) and those with imbalanced arrival rates (IMB).

Result 6. The average availability of open stations increases sub-linearly if the threshold \( \beta \) increases in the regulation of the maximum fleet size. Thus, even comparably small fleet sizes can be sufficient for customers, allowing regulators to impose a comparatively strict regulation.

If the threshold \( \beta \) in the maximum fleet size constraint increases, the average availability of open stations, traveled distances, and profits increase sub-linearly as Fig. 7 shows. Regulators can thus impose a relatively small fleet size, without hurting the (marginally increasing) profits of the operators too much, and without hurting service availability for their citizens too badly.
Result 7. If the threshold $\gamma$ in the regulation of the minimum service availability is large, the operator increases the fleet size super-linearly to open all stations until closing all stations becomes more profitable due to the “safety stock” of $\frac{\alpha}{1-\alpha}$ vehicles at each open station. This suggests that the availability enforced by a regulating authority should not be too high.

As visible from the left graph in Fig. 8, if the threshold of the minimum service availability constraint increases, the fleet size increases super-linearly and then decreases substantially as the threshold reaches 95%. Such significant change in the fleet size implies that the operator needs a comparably large fleet size to keep services or she forgoes service or closes stations if the threshold is high. The middle graph in Fig. 8 shows the counts of the number of instances in which the operator closes some stations that were open without the regulation for an increasing threshold $\gamma$. The operator starts to close stations in imbalanced instances if the threshold exceeds 85%, and the operator starts to close stations in balanced instances if the threshold exceeds 87.5%.

As visible from Table 2, all stations are open and have high service availability close to full service if no regulation applies. Thus, if the minimum service availability is regulated, the operator prefers opening all stations for more profits than closing some until the threshold in the regulation is too high such that the contribution margins cannot recover the operating costs. Consequently, the right graph in Fig. 8 shows that the profits on average decrease substantially if the threshold increases. Therefore, regulators should not set the threshold in the regulation of the minimum service availability too high such that the operator gains no profits and closes service. The value of the threshold which ensures positive profits can trivially be obtained by reformulating Problem (5).
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Fig. 9. Sensitivity analysis: Tax (“ALL”: all, “BAL”: balanced, “IMB”: imbalanced instances).

Result 8. If the imposed tax $t$ increases, the values of the societal welfare indicators in fleet size, availability, and distances, as well as profits decrease linearly.

Thus, compared to the other regulations which have non-linear effects on the societal welfare indicators, imposing a tax is the simplest regulation for regulators to introduce at a new location, since it does not cause uncontrollable and unforeseeable side effects on other indicators of societal welfare. Thus, Result 8 aligns with Result 5 which shows that imposing a tax has similar influences on instances in decreasing the fleet size, distances, and profits, as supported by Fig. 9.

5.5. Sensitivity analysis: Impact of costs while waiting for customers

We assume that empty vehicles idling at a station do not incur costs. However, parking or moving while idling can be costly. For example, Kaspi et al. (2016) and Lin and Kuo (2021) explicitly analyze the impact of parking. In ride-hailing systems, idling vehicles can incur costs, since drivers search for customers. We measure the impact of cost $\tilde{c}_i$ in a sensitivity experiment: We increase costs from $\tilde{c}_i = 0$, i.e., the current value, to $\tilde{c}_i = 9$, i.e., each empty vehicle incurs a cost of 9 per period of time. These costs can refer to parking costs, or driving at a velocity between 0 and 30 km/h at rebalancing costs of 0.3$ per kilometer in line with the autonomous case in the numerical design (velocity · duration gives distance). In the latter case, idling vehicles increase the empty, and thus also the total, distance. To account for the increases costs, we adapt the objective function to

$$\max_{n,x,\hat{\alpha},e} \left( \sum_{i,j \in I} \lambda_{ij} \tilde{a}_{ij} x_{ij} - \sum_{i \in I} \sum_{j \in I, j \neq i} c_{ij} \mu_{ij} e_{ij} - \sum_{i \in I} \tilde{c}_i e_{ii} - h \cdot n \right).$$

Result 9. If the cost per empty vehicle, $\tilde{c}_i$, increases, the operator decreases the availability which decreases both the length of the customer and the rebalancing trips. The total driven distance continues to increase to account for movement of idling vehicles. To balance decreasing profits, the operator decreases the fleet size sub-linearly. Thus, no movement while idling is advantageous for the operator, and according to most indicators of societal welfare, also for society.

Result 9, supported by Fig. 10, unsurprisingly suggests that costs for idling vehicles such as parking fees or due to moving while idling is disadvantageous for operators while also worsening the service offered to customers. Firstly, in line with Kaspi et al. (2016) and Lin and Kuo (2021) this suggests that regulatory authorities should carefully consider parking fees for vehicle sharing operators. Secondly, we observe that any idling distance due to searching for a next customer is disadvantageous for society and operators. To study whether moving when idling is advantageous for independent drivers is beyond the scope of this paper.
If the costs refer to parking costs, the costs may reasonably differ between remote and central locations. To account for this difference, we replicate the above analysis with half the costs for remote locations in the imbalanced instances. Then, in Fig. 11, we observe the same trend as reported in Result 9.

5.6. Case study

We quantify the performance of the regulations with respect to societal welfare in a case study. We use the NYC taxi cab data of 18 quadratic locations in Downtown Manhattan starting between 9AM–10AM between Monday and Friday in January 2015 that correspond to a maximum walking distance of 700 m. Fig. 12 visualizes the locations studied in the case study where the surface area is proportional to the number of rentals (left) and returns (right), and the location corresponds to the centroid of both rentals and returns in this virtual station. The periodized investment cost is 1.15$, and the tax is 16.7% of the contribution margins, approximately in line with the NYC congestion tax which amounts to 2.5$ per trip (New York State, 2021). Since travel times are known, we do not require distance as a proxy. However, we cannot directly regulate distances (total, empty, full), but use the number of vehicles rebalancing at a given time as proxy.

Table 8 shows the values of the societal welfare indicators under the regulations.
Table 8
Case study values of societal welfare indicators ("# Empty Veh. (I)"; number of empty vehicles en-route, "# Empty Veh. (II)"; number of empty vehicles idling at open stations, "# Full Veh.": number of full vehicles en-route. bold: lowest value, italic: highest value per column except directly related measures).

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>None</td>
<td>578</td>
<td>0.956</td>
<td>0.956</td>
<td>0.974</td>
<td>0.762</td>
<td>18</td>
<td>30.7</td>
<td>321.9</td>
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<td>8.89E+3</td>
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<tr>
<td>Max fleet size</td>
<td>289</td>
<td>0.826</td>
<td>0.824</td>
<td>0.916</td>
<td>0.245</td>
<td>17</td>
<td>22.4</td>
<td>73</td>
<td>193</td>
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<tr>
<td>Max No. Stations</td>
<td>414</td>
<td>0.960</td>
<td>0.717</td>
<td>0.971</td>
<td>0.947</td>
<td>9</td>
<td>22.8</td>
<td>223.1</td>
<td>168</td>
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<tr>
<td>Min service Avail.</td>
<td>1019</td>
<td>0.978</td>
<td>0.975</td>
<td>0.978</td>
<td>0.978</td>
<td>17</td>
<td>31.9</td>
<td>757.1</td>
<td>230</td>
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<td>Equ. Service Avail.</td>
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<td>0.954</td>
<td>0.954</td>
<td>0.954</td>
<td>18</td>
<td>31.5</td>
<td>369.8</td>
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<td>8.77E+3</td>
</tr>
<tr>
<td>Max total Veh</td>
<td>215</td>
<td>0.552</td>
<td>0.550</td>
<td>0.975</td>
<td>0.175</td>
<td>17</td>
<td>1.10</td>
<td>84.6</td>
<td>129</td>
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</tr>
<tr>
<td>Max empty Veh</td>
<td>380</td>
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<td>0.786</td>
<td>0.981</td>
<td>0.394</td>
<td>17</td>
<td>15.3</td>
<td>179.4</td>
<td>185</td>
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<tr>
<td>Rel. DH (Veh)</td>
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<td>0.731</td>
<td>0.983</td>
<td>0.366</td>
<td>17</td>
<td>11.7</td>
<td>177.6</td>
<td>172</td>
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<tr>
<td>Tax</td>
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<td>0.952</td>
<td>0.972</td>
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<td>25.4</td>
<td>293.1</td>
<td>187</td>
<td>7.34E+3</td>
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</table>

As visible from Table 8, the relevant effects of every regulation on the measured societal welfare indicators are consistent with the experimental results in Section 5.2. Except for the directly related measures of fleet size and number of empty or total vehicles, regulating the number of open stations decreases fleet size the most, by 28.4%. Enforcing equal service availability decreases the profits the least, by 1.3%. We further observe that regulations can increase the fairness with respect to availability within the system, since the minimum availability increases and the difference between minimum and maximum availability decreases. In this regard, regulating the maximum number of stations is most efficient with exception of the regulations directly affecting the service availability. On the other hand, regulations that directly or indirectly affect the number of vehicles decrease the fairness between different stations. Some differences between case study and artificial instances exist: as regulating the number of open stations forces the operator to close more stations than other regulations, and the operator often starts by closing stations with low availability, the average availability of open stations increases by 0.4% if the number of open stations is regulated. Further, in the case study regulating the number of open stations has less impact than in the artificial instances. This is because in the case study, arrival rates differ more between stations. As consequence, the maximum number of vehicles in transit (“Max Total Veh”) becomes the regulation that reduces many societal welfare indicators most.

Since the assumptions of exponentially distributed inter-arrival times and generally distributed travel times are restrictive, we also use the case study to verify our findings in a simulation, comparing three scenarios: (i) exponentially distributed inter-arrival and travel times ($s_1$); (ii) data-driven inter-arrival and travel times for full vehicles, and exponentially distributed inter-arrival and
travel times for empty vehicles ($s_2$); (iii) data-driven inter-arrival and travel times for full vehicles, and constant inter-arrival and travel times for empty vehicles ($s_3$).

Table 9 shows that all three scenarios result in similar indicators of societal welfare under no regulation. The societal welfare indicators of all three scenarios in the simulation are similar to each other and the fluid approximation’s societal welfare indicators. The small differences stem from randomness in the simulation. In particular, stations with low arrival rates might randomly observe full availability.

6. Conclusion

This paper studies the influence of regulations on the profitability and societal impact of vehicle sharing services. Extending upon existing literature, we model fleet size, number of open stations, service availability, and rebalancing operations as the decisions of the operator. To that end, we approximate the problem as an open queueing network which we model as a Mixed Integer Second-Order Cone Program to find the profit-maximizing fleet and service structure for an operator subject to given regulations. By means of a numerical analysis on a large artificial data set and a case study, we study the decisions of the operator under regulations and measure influences of these regulations on vehicle sharing services and associated societal welfare. Our results can be summarized as follows:

Regulating the number of open stations, fleet size, or total distance decreases the empty vehicle distance substantially. On average over the artificial instances, the empty vehicle distance decreases under any of the aforementioned regulations, among which regulating the total distance forces the operator to stop rebalancing in the most instances. In the case study, these regulations force the operator to decrease the number of empty vehicles by more than 30% which also reduces the amounts of empty vehicles driving. Surprisingly, regulating the number of open stations, fleet size, or total distance reduces the empty vehicle distance more significantly than directly regulating the empty vehicle distance. For regulators who primarily aim to reduce vehicle rebalancing, regulating either the number of open stations, fleet size, or total distance is sufficient.

The average availability of open stations decreases under all studied regulations except for the ones on availability and the number of open stations. In the case study, the average availability of open stations decreases by at least 0.4% if the fleet size is regulated, if distance-related regulations are enforced, or if a tax is imposed. The average availability of open stations increases by 2% if the minimum service availability is regulated. Also, because the system is more imbalanced in the artificial instances, enforcing equal service availability increases the average availability of open stations in artificial instances but decreases it by 0.2% in the case study. For the same reason, if the number of open stations is regulated, the average availability of open stations decreases in artificial instances but increases by 0.4% in the case study due to the operator closing more stations with low availability. Regulators should consider carefully before applying any of these regulations to ensure that the service could meet specified service availability while gaining other societal welfare benefits.

Enforcing equal service availability causes the least profit loss of all regulations. Compared to other regulations that decrease profits more on artificial instances, enforcing equal service availability reduces less than 1.5% of the profits in the case study. The operator is least impacted if equal service availability is enforced. Regulators should measure other societal welfare indicators for system performances if equal service availability is enforced.

Imposing a tax has the least unpredictable and unforeseen effects on societal welfare indicators. In a sensitivity analysis on artificial instances, increasing the tax as a percentage of the contribution margins decreases the fleet size, average availability of open stations, distances, and profits linearly, while changing the regulation thresholds in the fleet size constraint or availability constraint has non-linear effects on these measures of societal welfare. Since the effects of taxation are predominantly linear, imposing a tax is the most straight-forward regulation, and regulators can implement tax without excessive analysis of the specific system.

Future research may extend towards endogenous demand patterns, and modal choice. In practice, customers choose their mode of transport, such as private vehicle, public transportation, or vehicle sharing, based on cost and service quality. Some customers may choose a nearby station instead of leaving the system, while others only choose vehicle sharing if the chance of finding a vehicle is sufficiently high. Secondly, future research may investigate optimal decision-making of the regulatory authority, e.g., setting the optimal tax, or optimal minimal service level.

CRediT authorship contribution statement

Wu Hao: Formal analysis, Funding acquisition, Investigation, Methodology, Project administration, Resources, Software, Validation, Visualization, Writing – original draft, Writing – review & editing. Layla Martin: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Project administration, Resources, Supervision, Validation, Writing – original draft, Writing – review & editing.

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Appendix A. Proofs and additional tables

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References


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