Frequency Coupling Suppression Control Strategy for Single-Phase Grid-Tied Inverters in Weak Grid

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Abstract—In single-phase voltage source inverters under a weak grid, the frequency coupling, caused by the asymmetrical system structure, poses a challenge to system modeling and controller design. In this article, a frequency coupling suppression control strategy is presented, where all the frequency coupling components in the current reference are eliminated. As a result, the single-phase grid-tied inverter is directly modeled as a simple single-input single-output (SISO) admittance rather than a complicated multiple-input multiple-output admittance matrix. The SISO admittance model is simple, compact, and accurate, which facilitates design-oriented analysis. The admittance characteristic analysis shows that the Symmetrical phase-locked loop (Sym-PLL) has a critical impact on the system stability under weak grid conditions. To improve the system stability, a prefilter-based impedance-shaping control strategy is proposed by mitigating the negative resistor behavior of the Sym-PLL. Finally, simulations and experimental results validate the effectiveness of the frequency-coupling suppression control strategy.

Index Terms—Frequency coupling, phase-locked loop (PLL), single-phase grid-tied inverters, weak grid.

I. INTRODUCTION

In recent years, single-phase voltage source converters have been increasingly used in power systems due to high conversion efficiency and high controllability [1]. However, various new stability problems are also introduced by the interaction between the converters and the grid. Particularly, when the inverters are connected to a weak grid, harmonics may be excited at the point of the common coupling (PCC) on a wide frequency range from several hundreds to several kilohertz [2], [3].

To analyze the system stability of the single-phase grid-connected converters, an effective method is the impedance-based analytical approach [4]. Generally, the small-signal impedance modeling methods are mainly categorized into two kinds [2], [5]: the linear-time-invariant (LTI) framework-based and the linear-time-periodic (LTP) framework-based methods. The former one applies to the system operating at dc steady-state points (e.g., three-phase system in the dq-frame). Based on the complex vector notation [6] or algebraic calculation [7], corresponding impedance models are obtained. For the system with no available dc operating point, the impedance modeling has to be directly conducted on the original frame, which is an LTP system in general. In this regard, the harmonic linearization [8], [9] and the harmonic transfer function [10], [11] can be adopted. For single-phase systems, the LTI modeling is not suitable because of the time-periodic operating trajectory [12]. Thus, the LTP framework-based modeling approach is a promising choice for the impedance modeling of such systems.

Based on the analysis of the impedance model, it has been reported that the PLL may have a significant impact on the stability of the grid-connected converter, especially under weak grid conditions [13]–[15]. In the synchronous reference frame (SRF)-PLL, only the q-axis component of the PCC voltage is controlled for the phase tracking. The asymmetrical control dynamics between d and q axes result in frequency couplings [16], [17]. As the grid impedance increases, the coupling between the control dynamics and the power grid becomes stronger, and the oscillation frequency decreases. Consequently, due to the frequency coupling components, the large grid impedance can result in subsynchronous oscillations and even destabilize the power system [18].

In a weak grid, additional sideband loops are formed by the frequency coupling effect (FCE) through the grid impedance,
which adds significant complexity to the modeling and controller design of the single-phase converter. To capture the frequency coupling dynamics in the presence of the grid impedance, the single-phase converter impedance model should be represented by an infinite-dimensional matrix in theory [19]. As a result, the generalized Nyquist criterion is required for the stability assessment of the multi-input multi-output (MIMO) system. To facilitate the stability analysis, some research efforts have been made to approximate the MIMO system to a single-input single-output (SISO) system [13], [20]. However, the truncation of high-order harmonics is inevitable, and obtained impedance expression is very complicated. Although the magnitudes of high-order coupling components in the MIMO impedance matrix are small in some cases, ignoring them can still lead to inaccurate stability implications [21], which makes it difficult to determine the harmonic order for the PLL. Besides, the complicated impedance expression makes it difficult to design controllers analytically. In summary, the elimination of such a frequency coupling dynamic is of significance in suppressing the associated low-frequency oscillation and facilitating the design-oriented analysis.

At present, existing studies are mostly focused on suppressing the frequency couplings of the three-phase balanced systems [22]–[25]. The key to suppressing the frequency coupling in the three-phase balanced system is to eliminate the conjugate operator in the dq-frame model [16]. In order to counteract the FCE introduced by the structural asymmetry of SRF-PLL, the concept of asymmetric current controller is proposed in [22]. In [23], the Sym-PLL structure is proposed, which tracks both the amplitude and phase of PCC voltage. Therefore, the frequency coupling phenomenon caused by the PLL is eliminated. Furthermore, to suppress the FCE in the three-phase rectifiers based on conventional control, the symmetrical control strategy and the asymmetric feedforward compensation strategy are presented in [24] and [25], respectively. The effectiveness of the symmetrical control is achieved by improving the symmetry of the control structure, while the asymmetric feedforward compensation lies in canceling the asymmetric loop.

The frequency coupling suppression control strategies for the three-phase balanced systems are developed in the dq-frame [23]–[25]. Due to the lack of an autonomous β signal, the dq frame-based frequency coupling suppression control strategies cannot directly extend to the single-phase systems. In addition, the frequency-domain dynamics of the single-phase systems are complex because of the inherent time-periodicity and lack of symmetry. Specifically, the number of coupled frequencies in a single-phase system is conceptually infinite. To the best knowledge of the authors, the effective frequency coupling suppression control strategy for the single-phase grid-tied converter is still missing here; therefore, this article aims to bridge this gap.

To suppress the FCEs in the single-phase grid-tied inverter system, a frequency coupling suppression control strategy is proposed in this article. The main contributions of this article are summarized as follows.

1) The frequency coupling dynamics in the single-phase grid-tied inverter system are fully eliminated with the proposed method. Consequently, the low-order harmonic couplings between the inverter and the grid admittance are eliminated, which greatly facilitates the stability analysis.

2) A compact and accurate SISO inverter admittance model without cumbersome calculation is established. It is illustrated that the negative resistor effect introduced by the PLL is the major origin of system instability under weak grid conditions.

3) A simple but effective impedance-shaping control strategy is proposed, with which the inverter system stability under weak grid conditions is enhanced.

II. FREQUENCY COUPLING EFFECT IN SRF-PLL-BASED GRID SYNCHRONIZATION

Due to the time-periodic operating trajectory, the FCE is widespread in single-phase grid-tied systems, e.g., the harmonic at the frequency $\omega$ is coupled with that at $\omega \pm \omega_1$ [13]. Recently, it has been reported that neglecting the coupling dynamics will inevitably lead to inaccurate model and stability analysis. Therefore, many studies have been devoted to analyzing the frequency coupling dynamics in single-phase systems [13], [20], [26]–[28]. From these works, it is obtained that the frequency couplings comprise not only the stability analysis but also the physical measurement. In this section, the analysis of the FCE in the single-phase grid-tied inverter based on SRF-PLL is carried out.

Fig. 1 shows the topology and control block of a single-phase LCL-filtered grid-tied voltage source inverter. The LCL filter consists of the converter-side inductor $L_1$, the filter capacitor $C$, and the grid-side inductor $L_2$. Considering the large dc-link capacitor $C_{dc}$ and the low control bandwidth of the dc-link voltage, $u_{dc}$ is assumed to be constant. A synchronization unit is used to synchronize the PCC voltage $u_{pcc}$. An ideal grid voltage $u_g$ in series with the grid impedance $Z_g$ (i.e., $sZ_g+R_g$) is used to emulate the weak grid.

Without loss of generality, the classical grid-current control is taken as the analytical example in this article. The grid current reference $i_{ref}$ is generated from the synchronization unit. The synchronization unit in Fig. 1 is commonly realized by the conventional SRF-PLL, and its control block diagram is depicted in Fig. 2, where $\omega_1$ is the fundamental angular frequency of the grid voltage and $I_m$ is the magnitude of the current reference $i_{ref}$. The PLL can also be the filtered SRF-PLLs, while the same conclusions are achieved in the same way. The used orthogonal signal generator (OSG) can be the second-order generalized integrator (SOGI), the T/4-delay, inverse Park transformation.
As shown in Fig. 3, the FCE is illustrated by the following example: assuming that the grid’s fundamental frequency components of the estimated phase are generated at frequencies 60 Hz. Due to the FCE of the SRF-PLL, two oscillation components (10 and 210 Hz) are all identified. Also, it is found that an oscillation below the fundamental frequency is generated from the FCE. When there is an impedance \( Z_g \) in the grid side, the number of couplings extends to infinite, as shown in Fig. 5, where \( G_{\text{plant}}(s) \) is the transfer function from \( i_{\text{ref}} \) to \( i_g \) and \( Y_{\text{inv}}(s) \) is the output admittance without the PLL effect. In practical analysis, model truncation is necessary and the accuracy of the truncated model increases as the considered harmonic order increases.

The frequency coupling phenomenon is clearly observed from (2) and (3). As shown in Fig. 3, the FCE is illustrated by the following example: assuming that the grid’s fundamental frequency is 50 Hz and the grid voltage \( u_g \) contains an undamped oscillation at 110 Hz. Due to the FCE of the SRF-PLL, two oscillation components of the estimated phase are generated at frequencies 60 and 160 Hz. Consequently, applying the trigonometric function to the estimated phase in the current reference generation, three coupled frequency components would be observed at 10, 110, and 210 Hz.

To better visualize the FCE resulting from the conventional control scheme, the typical waveform is illustrated in Fig. 4, where a perturbation voltage (110 Hz, 10% magnitude of the fundamental) is intentionally injected into the PCC voltage (main parameters are listed in Table I) and no grid impedance is inserted. By analyzing the frequency spectrum of grid current \( i_g \), the corresponding component (110 Hz) and two coupling components (10 and 210 Hz) are all identified. Also, it is found that an oscillation below the fundamental frequency is generated from the FCE.

When there is an impedance \( Z_g \) in the grid side, the number of couplings extends to infinite, as shown in Fig. 5, where \( G_{\text{plant}}(s) \) is the transfer function from \( i_{\text{ref}} \) to \( i_g \) and \( Y_{\text{inv}}(s) \) is the output admittance without the PLL effect. In practical analysis, model truncation is necessary and the accuracy of the truncated model increases as the considered harmonic order increases. The admittance model with a truncation order of three is given in (4).
TABLE I
PARAMETERS OF THE SINGLE-PHASE GRID-TIED INVERTER SYSTEM

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_{dc}$</td>
<td>Grid voltage</td>
<td>110 V (rms)</td>
</tr>
<tr>
<td>$\omega_1$</td>
<td>Input angular frequency</td>
<td>100\pi rad/s</td>
</tr>
<tr>
<td>$L_1$</td>
<td>Converter-side inductance</td>
<td>1 mH</td>
</tr>
<tr>
<td>$C$</td>
<td>Filter capacitance</td>
<td>5 \mu F</td>
</tr>
<tr>
<td>$L_2$</td>
<td>Grid-side inductance</td>
<td>0.6 mH</td>
</tr>
<tr>
<td>$u_{	ext{dc}}$</td>
<td>Input de-link voltage</td>
<td>200 V</td>
</tr>
<tr>
<td>$k_p$</td>
<td>Proportional gain of the current regulator</td>
<td>6.9</td>
</tr>
<tr>
<td>$k_v$</td>
<td>Resonant gain of the current regulator</td>
<td>11941</td>
</tr>
<tr>
<td>$k_{sp}$</td>
<td>Proportional gain of the PLL</td>
<td>2.4</td>
</tr>
<tr>
<td>$k_g$</td>
<td>Integral gain of the PLL</td>
<td>158.6</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Damping ratio of the SOGI</td>
<td>0.707</td>
</tr>
<tr>
<td>$L_n$</td>
<td>Grid current amplitude reference</td>
<td>20 A</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Power factor angle</td>
<td>0 rad</td>
</tr>
<tr>
<td>$T_c$</td>
<td>Control period</td>
<td>50 \mu s</td>
</tr>
</tbody>
</table>

Fig. 5. Multifrequency diagram of the single-phase grid-tied inverter in the conventional control scheme.

It is emphasized that the admittance model $Y_{\text{con}}(s)$ is coupled with the grid admittance $Y_g(s) = 1/Z_g(s)$, posing a challenge to the controller design. Moreover, the admittance model expression is quite complicated, increasing the complexity of stability analysis.

$$Y_{\text{con}}(s) = Y_{\text{inv}}_o(s) - G_{\text{plant}}(s) H_{\text{PLL}_0}(s) - G_{\text{plant}}(s) H_{\text{PLL}_n}(s)$$

$$- G_{\text{plant}}(s) H_{\text{PLL}_0}(s) G_{\text{plant}}(s-j\omega_1) h_{\text{PLL}_0}(s+j\omega_1)$$

$$- G_{\text{plant}}(s) H_{\text{PLL}_n}(s) G_{\text{plant}}(s-j\omega_1) h_{\text{PLL}_n}(s+j\omega_1)$$

$$Y_g(s) = Y_{\text{inv}}_o(s) - G_{\text{plant}}(s) H_{\text{PLL}_0}(s)$$

$$- G_{\text{plant}}(s) H_{\text{PLL}_n}(s) G_{\text{plant}}(s-j\omega_1) h_{\text{PLL}_0}(s+j\omega_1)$$

$$- G_{\text{plant}}(s) H_{\text{PLL}_n}(s) G_{\text{plant}}(s-j\omega_1) h_{\text{PLL}_n}(s+j\omega_1)$$

(4)

According to Fig. 5, it is found that the greater the grid impedance is, the stronger the coupling between the converter and the power grid will be. To illustrate this coupling effect, Fig. 6 shows the Bode diagram of $Y_{\text{con}}(s)$ under different short-circuit ratios (SCRs). Some observations are given as follows.

1) There is a low-order harmonic coupling between the converter and the grid, i.e., the inverter admittance varies along with the grid admittance in the low-frequency regions.
2) The negative resistor effect increases as the grid impedance increases. Consequently, the inverter tends to be unstable in a weak grid.

Based on the above analysis, the problems caused by the FCE in the single-phase inverter system are summarized as follows.

1) The FCE tends to introduce a sideband oscillation below the fundamental frequency, which can easily trigger the subsynchronous oscillations.
2) Truncation is necessary to obtain a model that is suitable for analysis, laying a potential risk to model accuracy and increasing the modeling complexity.
3) The converter admittance model expression is complex and is coupled with the grid impedance, which makes it difficult to design controllers analytically.

III. FREQUENCY COUPLING SUPPRESSION CONTROL SCHEME

To suppress the frequency-coupling effects in the single-phase grid-tied inverter, it is necessary to cancel $H_{\text{PLL}_n}(s)$ and $H_{\text{PLL}_p}(s)$ in the current reference model. In light of this observation, a frequency coupling suppression control scheme for a single-phase grid-tied inverter is proposed, as shown in Fig. 7. The modification of the current reference $i_{\text{ref}}$ is the key to the proposed method.

Fig. 8 depicts the simplified control diagram of the grid-current control.

1) The FCE tends to introduce a sideband oscillation below the fundamental frequency, which can easily trigger the subsynchronous oscillations.
2) Truncation is necessary to obtain a model that is suitable for analysis, laying a potential risk to model accuracy and increasing the modeling complexity.
3) The converter admittance model expression is complex and is coupled with the grid impedance, which makes it difficult to design controllers analytically.
Fig. 9. Control block diagram of the Sym-PLL.

is the proportional–resonant (PR) current regulator, where \( k_{pc} \) and \( k_{ic} \) are respectively the proportional and resonant gains of PR controller. And \( G_d(s) \) is the digital control delay resulted by zero-order hold effect and the computation, which is expressed as

\[
G_d(s) = e^{-sT_s} \frac{1 - e^{-sT_s}}{sT_s}
\]

(5)

where \( T_s \) is the control period.

Fig. 9 draws the control block of the Sym-PLL [23]. The \( d \)-axis component \( \theta_d \) is the same as the conventional phase angle \( \theta \), while the \( q \)-axis component \( \theta_q \) defines the deviation of the PCC voltage magnitude from its nominal value \( V_1 \). And \( G_{PLL}(s) = k_{pp} + k_{ip}/s \) is the PI controller of the PLL, in which \( k_{pp} \) and \( k_{ip} \) are the proportional and integral gains, respectively.

The conventional SRF-PLL only tracks the phase angle of \( u_{pcc} \). In the control scheme using the SRF-PLL, the current reference generation ignores the voltage amplitude information, which results in frequency couplings. Different from the conventional SRF-PLL, the Sym-PLL considers both the phase and amplitude of \( u_{pcc} \), which are reflected by \( \theta_d \) and \( \theta_q \), respectively. In the frequency suppression control scheme, both \( \theta_d \) and \( \theta_q \) are used for the current reference generation.

Remark: In the frequency coupling suppression control scheme, the current controller and OSG greatly affect the system performance under different grid perturbations (e.g., background harmonics, frequency jump, and voltage sag). For example, under grid frequency perturbation, the use of fixed-frequency SOGI and the T/4-delay would lead to Sym-PLL output oscillation. In this case, the IPT-based OSG can solve the frequency perturbation issue while retaining the frequency coupling suppression capability. A brief discussion is given in Appendix A. In addition, many other OSGs as overviewed in [29] and [30] and current controllers as introduced in [31] and [32] are also candidates. Therefore, the proposed control scheme has a great potential of being promoted to other scenarios.

IV. SISO ADMITTANCE MODEL OF THE GRID-TIED INVERTER

A. Symmetrical PLL Modeling

According to the Sym-PLL structure, the \( d \)-axis voltage \( u_d \) and \( q \)-axis voltage \( u_q \) are obtained as

\[
\begin{bmatrix}
  u_d \\
  u_q
\end{bmatrix} = e^{\theta_q} \begin{bmatrix}
  \cos (\theta_d) & \sin (\theta_d) \\
  -\sin (\theta_d) & \cos (\theta_d)
\end{bmatrix} \begin{bmatrix}
  u_{\alpha} \\
  u_{\beta}
\end{bmatrix} \cdot
\]

(6)

Perturbing the variables in the time-domain equations, the linearized small-signal model of the Sym-PLL can be derived as

\[
\begin{align*}
\ddot{u}_d &= e^{\theta_q} \ddot{u}_q + e^{\theta_q} \left[ \cos (\theta_d) \ddot{u}_\alpha + \sin (\theta_d) \ddot{u}_\beta + \ddot{u}_\theta \right] \\
\ddot{u}_q &= e^{\theta_q} \ddot{u}_q + e^{\theta_q} \left[ -\sin (\theta_d) \ddot{u}_\alpha + \cos (\theta_d) \ddot{u}_\beta - \ddot{u}_\theta \right]
\end{align*}
\]

(7)

where the superscript “\( \dagger \)” represents the steady-state quantity. The steady-state quantities are given by \( \ddot{u}_d = \omega_1 t, \ddot{u}_q = 0, \ddot{u}_\alpha = V_1, \) and \( \ddot{u}_\beta = 0. \)

Substituting the steady-state quantities into (7) gives

\[
\begin{align*}
\ddot{u}_d &= V_1 \ddot{u}_q + \cos (\omega_1 t) \ddot{u}_\alpha + \sin (\omega_1 t) \ddot{u}_\beta \\
\ddot{u}_q &= -V_1 \ddot{u}_d - \sin (\omega_1 t) \ddot{u}_\alpha + \cos (\omega_1 t) \ddot{u}_\beta
\end{align*}
\]

(8)

Then, applying the LTP modeling technique [26], the frequency-domain expression of (8) can be obtained as

\[
\begin{align*}
\ddot{u}_d(s) &= V_1 \ddot{u}_q(s) + \frac{1}{s} \left[ \dddot{u}_\alpha(s - j\omega_1) + \dddot{u}_\alpha(s + j\omega_1) \right] \\
& \quad - \frac{1}{2} \left[ \dddot{u}_\beta(s - j\omega_1) - \dddot{u}_\beta(s + j\omega_1) \right] \\
\ddot{u}_q(s) &= -V_1 \ddot{u}_d(s) + \frac{1}{s} \left[ \dddot{u}_\alpha(s - j\omega_1) - \dddot{u}_\alpha(s + j\omega_1) \right] \\
& \quad + \frac{1}{2} \left[ \dddot{u}_\beta(s - j\omega_1) + \dddot{u}_\beta(s + j\omega_1) \right]
\end{align*}
\]

(9)

According to the Sym-PLL diagram, the following relationships hold:

\[
\begin{align*}
\ddot{u}_\alpha(s) &= G_{OSGa}(s) \dddot{u}_{pcc}(s) \\
\ddot{u}_\beta(s) &= G_{OSGb}(s) \dddot{u}_{pcc}(s) \\
\ddot{\theta}_d(s) &= \frac{G_{PLL}(s)}{s} \dddot{u}_q(s) \\
\ddot{\theta}_q(s) &= -\frac{G_{PLL}(s)}{s} \dddot{u}_d(s)
\end{align*}
\]

(10)\(\sim\) (13)

where \( G_{OSGa}(s) \) and \( G_{OSGb}(s) \) are defined as the transfer functions from \( u_{pcc} \) to \( u_\alpha \) and \( u_\beta \), respectively.

In this article, the fixed-frequency SOGI-based OSG is adopted, which can be expressed as

\[
G_{OSGa}(s) = 1
\]

(14)

\[
G_{OSGb}(s) = \frac{2\zeta\omega_1^2}{s^2 + 2\zeta\omega_1 s + \omega_1^2}
\]

(15)

where \( \zeta \) is the damping ratio.

\[
\begin{align*}
\ddot{\theta}_d(s) &= \frac{T_{PLL}(s)}{2} \left\{ [jG_{OSGa}(s - j\omega_1) + G_{OSGb}(s - j\omega_1)] \dddot{u}_{pcc}(s - j\omega_1) \\
& \quad - [jG_{OSGa}(s + j\omega_1) - G_{OSGb}(s + j\omega_1)] \dddot{u}_{pcc}(s + j\omega_1) \right\} \\
\ddot{\theta}_q(s) &= j\frac{T_{PLL}(s)}{2} \left\{ [jG_{OSGa}(s - j\omega_1) + G_{OSGb}(s - j\omega_1)] \dddot{u}_{pcc}(s - j\omega_1) \\
& \quad + [jG_{OSGa}(s + j\omega_1) - G_{OSGb}(s + j\omega_1)] \dddot{u}_{pcc}(s + j\omega_1) \right\}
\end{align*}
\]

(16)
B. SISO Admittance Modeling

The modified grid current reference is

\[ i_{\text{ref}} = I_m e^{-\theta_d} \cos(\theta_d + \varphi). \]  

The corresponding small-signal form of (18) is given as

\[ \tilde{i}_{\text{ref}} = -I_m e^{-\theta_d} \left[ \cos(\theta_d + \varphi) \tilde{\theta}_q + \sin(\theta_d + \varphi) \tilde{\theta}_d \right]. \]  

Substituting the steady-state quantities into (19), it is obtained that

\[ \tilde{i}_{\text{ref}} = -I_m \left[ \cos(\omega_1 t + \varphi) \tilde{\theta}_q + \sin(\omega_1 t + \varphi) \tilde{\theta}_d \right]. \]

In the frequency domain, (20) is rewritten as

\[ \tilde{i}_{\text{ref}}(s) = -I_m \left\{ \frac{e^{j\varphi}}{2} \left[ \tilde{\theta}_q(s - j\omega_1) - j\tilde{\theta}_d(s - j\omega_1) \right] + e^{-j\varphi} \left[ \tilde{\theta}_q(s + j\omega_1) + j\tilde{\theta}_d(s + j\omega_1) \right] \right\}. \]  

Substituting (16) into (21), the transfer function from the PCC voltage \( \tilde{u}_{\text{pcc}} \) to the current reference \( \tilde{i}_{\text{ref}} \) can be readily simplified into SISO transfer function, expressed as

\[ \tilde{i}_{\text{ref}}(s) = H_{\text{PLL}}(s) \tilde{u}_{\text{pcc}}(s). \]  

C. Discussion

The current references in the conventional control scheme and the proposed control scheme are the same in the steady state, which implies that both the two control schemes have the same steady-state performance. Comparing (2) with (22), it is found that the frequency couplings are fully eliminated by the current reference modification. Similarly, by intentionally perturbing the PCC voltage at the frequency 110 Hz (10% magnitude of the fundamental), the waveform under the frequency coupling suppression control scheme is shown in Fig. 12. Only the oscillation component at 110 Hz can be observed in the grid current, indicating that the proposed control method is effective in eliminating the FCE.

Compared with the conventional control schemes based on SRF-PLL, the proposed frequency coupling suppression control method has the following advantages:

1. The subsynchronous oscillation caused by the FCE is eliminated.
2. The single-phase grid-tied inverter model can be directly represented by the SISO admittance without truncation.
3. There is no low-order harmonic coupling between the converter and the grid.

In summary, eliminating the frequency coupling in the single-phase grid-tied inverter allows a straightforward and design-oriented stability analysis.
Fig. 12. Illustration of the FCE in the proposed control scheme when the PCC voltage is perturbed at the frequency 110 Hz. (a) Experimental waveforms. (b) Spectrum of $i_g$.

Fig. 13. Validation of the admittance model $Y(s)$.

V. CASE STUDY OF SINGLE-PHASE GRID-TIED INVERTER

A. Model Validation and Stability Analysis

In this study, the inductive weak grid is considered, which is expressed as

$$Y_g(s) = \frac{1}{L_g s}. \quad (27)$$

The main parameters are listed in Table I. The design procedure of PLL parameters is given in Appendix B. The impact of the grid admittance on the system stability will be analyzed using the impedance-based analysis method.

A comparison of grid-tied single-phase system admittance model $Y(s)$ and simulation results $Y_m(s)$ under an inductive weak grid ($L_g = 9$ mH, SCR = 2.7) is shown in Fig. 13. The red curves represent the theoretical results and the black dots represent the measurement results. As observed, the analytical model $Y(s)$ matches the measurement results $Y_m(s)$ well. Thus, the accuracy of the analytical model $Y(s)$ is validated.

Using the frequency coupling suppression control method, the stability analysis can be greatly simplified without cumbersome calculation. The frequency responses of $Y(s)$ and grid admittance $Y_g(s)$ are shown in Fig. 14. It is noted that $Y(s)$ remains almost the same under different $Y_g(s)$ and the small deviation is caused by the increased steady-state voltage drop of the grid impedance. This verifies that the proposed control method decouples the converter dynamics with the power grid. The elimination of converter-grid interactive coupling is beneficial for system analysis and impedance shaping.

On the other hand, the phase margin (PM) of the system under $L_g = 9$ mH (SCR = 2.7) is positive, meaning a stable system. As the grid inductance increases to $L_g = 11$ mH (SCR = 2.2), the PM of the system becomes negative, indicating that the system is unstable. According to the intersection frequency of $Y(s)$ and $Y_g(s)$, the oscillation frequency should be around 116 Hz.

B. Analysis of PLL Effect on Admittance Model

Based on the SISO admittance analysis, the oscillation phenomenon of the inverter-grid interconnected system will occur as the SCR decreases. According to Fig. 11, the Sym-PLL will introduce one PCC voltage feedforward transfer function $H_{PLL}(s)$, which influences the admittance characteristics of a single-phase grid-tied inverter system. To further analyze the influence factor and design a suitable phase compensator, the admittance $Y(s)$ is rewritten as

$$Y(s) = Y_{inv-o}(s) + Y_{PLL}(s) \quad (28)$$

where $Y_{PLL}(s) = -H_{PLL}(s)G_{plant}(s)$ is the parallel admittance caused by the Sym-PLL. Fig. 15 shows the block diagram of the inverter admittance subsystem.

Fig. 16 depicts the frequency response characteristic curves of the inverter admittance subsystem. As shown, $Y_{PLL}(s)$ dominates the admittance characteristics of $Y(s)$ from 15 to 150 Hz. And $Y_{PLL}(s)$ behaves as a negative resistor within 15 and 250 Hz, which tends to destabilize the system in the weak grid. In addition, $Y_{inv-o}(s)$ plays a critical role in the frequency response characteristics of $Y(s)$ above 150 Hz or less than 15 Hz. It is noted that the phase curve of $Y_{inv-o}(s)$ is always within $[-90^\circ, 90^\circ]$. Thus, the subsystem $Y_{inv-o}(s)$ is always stable under an inductive weak grid when the effect of Sym-PLL is ignored.
By decreasing arg($\omega$), it is intuitive to increase the PM in the low-frequency regions. Then, the PM in the low-frequency regions can be approximated as
\[
PM \approx 180^\circ - [\arg(Y_{PLL}(\omega_i)) - \arg(Y_g(\omega_i))] \tag{29}
\]
where $\omega_i = 2\pi f_i$.

From (29), it is observed that the PM will be increased by decreasing arg($Y_{PLL}(\omega_i)$) or increasing arg($Y_g(\omega_i)$). However, arg($Y_g(\omega_i)$) represents the phase of the grid admittance that is difficult to control. In consequence, it is intuitive to increase the PM by decreasing arg($Y_{PLL}(\omega_i)$).

To decrease the phase of $Y_{PLL}(s)$, the desired admittance model used for the impedance shaping is given by
\[
Y_{PLL\text{-shaping}}(s) = K_{shaping}(s) Y_{PLL}(s) \tag{30}
\]
where $Y_{PLL\text{-shaping}}(s)$ is the synthesized admittance and $K_{shaping}(s)$ is the transfer function that shapes the inverter output admittance phase.

In this study, the transfer function of the typical lag compensator [33] is chosen as $K_{shaping}(s)$, expressed as
\[
K_{shaping}(s) = \frac{1 + \alpha s/\omega_m}{1 + s/\omega_m} \tag{31}
\]
where $\alpha$ ($\alpha < 1$) is the phase compensation coefficient and $\omega_m$ is the compensation frequency coefficient.

Fig. 17 shows the frequency response of the shaping function $K_{shaping}(s)$. It can be seen that the compensation phase will peak at a certain frequency $\omega_m$. The phase–frequency function of (31) can be derived as
\[
\arg(K_{shaping}(j\omega)) = \arctan \left( \frac{\alpha - 1}{\omega_m^2 + \omega_m^2} \right) \tag{32}
\]

The derivative of (32) with respect to $\omega$ is
\[
\frac{d\arg(K_{shaping}(j\omega))}{d\omega} = \frac{\omega_m^2 + (\omega_m - \omega_m^2)}{(\omega_m^2 + \omega_m^2)^2 + (\alpha - 1)^2 \omega_m^2} \tag{33}
\]

By solving $d(\arg(K_{shaping}(j\omega)))/d\omega = 0$, the frequency that has the maximum phase compensation is obtained as
\[
\omega_m = \frac{\omega_m}{\sqrt{\alpha}} \tag{34}
\]

Substituting (34) into (32), the maximum compensation phase $\varphi_m$ can be derived as
\[
\varphi_m = -\arcsin \frac{1 - \alpha}{1 + \alpha} \tag{35}
\]

Therefore, by designing $\alpha$ and $\omega_m$ appropriately, the goal of decreasing the phase arg($Y_i(\omega_i)$) at the frequency of intersection will be realized.

Observing from Fig. 15(b), the impedance-shaping control strategy can be implemented by adding a prefilter for PCC voltage $\hat{u}_{pcc}$. Thus, the final implementation of impedance shaping control is depicted in Fig. 18.

According to Fig. 14, the intersection frequency $\omega_i = 232\pi$ rad/s and PM $= -8^\circ$ when SCR $= 2.2$. Therefore, by choosing the maximum compensation phase $\varphi_m = -20^\circ$ and $\omega_m = \omega_i$, it is derived that $\alpha = 0.49$ and $\omega_m = 162\pi$ rad/s. Fig. 19 shows the admittance Bode diagram of grid-tied inverter system with or without impedance shaping control. It is clear that this
shaping control changes both the magnitude and phase angle of the \( Y(s) \) and thereby the system stability margin. When the SCR of the grid is 2.2, the intersection frequency of \( Y(s) \) and \( Y_g(s) \) moves from 116 to 137 Hz after shaping. The difference of phase decreases from 188° to 167°, which brings the system from unstable to stable due to the sufficient PM.

The proposed impedance shaping control strategy employs a prefilter to act directly on the synchronous signal, which helps mitigate the adverse effect of \( H_{PLL}(s) \). The merits of the prefilter-based impedance shaping control strategy are concluded as follows.

1) The exact \( H_{PLL}(s) \) and circuit parameters are not required, which simplifies the implementation in practical applications.

2) The compensation coefficients of \( \omega_m \) and \( \phi_m \) can be designed analytically without the tedious tuning process.

VI. EXPERIMENTAL VALIDATION

In this section, a prototype of the single-phase grid-tied inverter is built to verify the effectiveness of the proposed control method, as shown in Fig. 20. The system specifications are listed in Table I. The control platform is based on a floating-point digital signal processor TMS320F28335. The grid voltage is generated by a regenerative grid simulator (Chroma 61830). The input voltage is supplied by a dc source (Itech IT6018C).

The experimental waveforms for a step-change in the amplitude and power factor angle of grid current reference, with no grid impedance inserted, are shown in Fig. 21. When there is a step-change in the amplitude or power factor angle of the current reference, the grid current quickly tracks the current reference. It is demonstrated that the proposed control strategy can work well under a step-change in the grid current reference. To improve dynamic performance, other advanced current controllers can be used [31], [32].

Fig. 22 shows the experimental waveforms of the single-phase grid-tied inverter using the frequency coupling suppression control strategy under a weak grid with \( L_g = 9 \) mH (SCR = 2.7). The grid current \( i_g \) and the PCC voltage \( u_{pcc} \) are both sinusoidal, indicating a stable system. In addition, the grid current is in phase with the PCC voltage, and its amplitude tracks the reference.
Fig. 23. Experimental results using the frequency coupling suppression control strategy under the grid condition with $L_g = 11$ mH (SCR = 2.2). (a) Experimental waveforms. (b) Spectrum of $i_g$.

The experimental waveforms of the single-phase grid-tied inverter using the frequency coupling suppression control strategy under a weak grid with $L_g = 11$ mH (SCR = 2.2) are shown in Fig. 23(a). The PCC voltage $u_{pcc}$ and grid current $i_g$ are distorted, indicating an unstable system. The oscillation frequency of the grid current at 113 Hz is identified, as shown in Fig. 23(b), which closely agrees with the analyzed results in Fig. 14. Meanwhile, no sideband oscillation can be found in the harmonic spectra of the grid current, which verifies the effectiveness of the proposed control strategy in eliminating the FCE.

Fig. 24. Experimental results using the impedance shaping strategy under the weak grid condition (SCR = 2.2).

In order to verify the effectiveness of the proposed impedance shaping control method, Fig. 24 shows the experimental waveforms of the single-phase grid-tied inverter system with the impedance shaping under the weak grid condition (SCR = 2.2). When the signal $Trig$ represented by the green line steps into a high level, the impedance shaping control is enabled. As a result, the grid current $i_g$ and the PCC voltage $u_{pcc}$ return to the sinusoidal shapes. The spectrum analysis of the grid current after impedance shaping is shown in Fig. 25. After impedance shaping, the oscillation at 113 Hz is no longer present. Finally, the system remains stable in the steady-state, demonstrating the validity of the impedance shaping method in Section V.

VII. CONCLUSION

In this article, a frequency coupling suppression control strategy was proposed to address the frequency coupling oscillations in the single-phase grid-tied inverter. The small-signal modeling showed that the single-phase grid-tied inverter using the proposed control method could be represented as a SISO admittance without approximation. According to the admittance analysis, the Sym-PLL will introduce a parallel admittance, which plays a critical role in the system stability under the weak grid condition. The theoretical analysis illustrates that the prefilter-based phase compensation method can reduce the adverse effects of the PLL under the weak grid condition. Thus, the synchronization stability of grid-connected converters can be enhanced. Experimental results have confirmed the effectiveness of the proposed control strategy in terms of frequency coupling suppression and stability enhancement.

APPENDIX

A. Sym-PLL Under Grid Frequency Perturbation

The output of the Sym-PLL with fixed-frequency SOGI will inevitably oscillate under grid frequency perturbation. Fig. 26 shows the simulation and experimental results of the fixed-frequency SOGI-based Sym-PLL, where the grid frequency $f_1$ is first at its nominal value (50 Hz) and suddenly undergoes a +2-Hz step change. Experimental results in Fig. 26(b) match well with those in the simulation in Fig. 26(a). It can be observed that the fixed-frequency SOGI-based Sym-PLL suffers from oscillatory ripples under off-nominal grid frequency.

To eliminate the oscillation caused by grid frequency perturbation, the IPT-based OSG can be adopted in the Sym-PLL, as...
Fig. 26. Operational waveforms of the fixed frequency SOGI-based Sym-PLL when the grid frequency $f_1$ jumps from 50 to 52 Hz. (a) Simulation results. (b) Experimental results.

Fig. 27. Control block diagram of the IPT-based Sym-PLL.

Fig. 28. Operational waveforms of the IPT-based Sym-PLL when the grid frequency $f_1$ jumps from 50 to 52 Hz. (a) Simulation results. (b) Experimental results.

Fig. 29. Approximated PLL model using SOGI-based OSG.

Based on the PLL structure, the approximated PLL model can be obtained, as shown in Fig. 29. The open-loop transfer function is derived as

$$T(s) = \frac{V_1}{1 + \tau_{OSG}s} G_{PLL}(s). \quad (B2)$$

Suppose $\omega_c$ is the cutoff angular frequency and PM/deg is the PM of the system, then it gives

$$T(j\omega_c) = -e^{\frac{PM}{180}\pi} := -e^{j\alpha}. \quad (B3)$$

By separating the real part and imaginary part of (B3), the expressions of $k_{pp}$ and $k_{ip}$ are derived as

$$\begin{cases} k_{pp} = \omega_c \sin(\alpha) + \tau_{OSG}\omega_c \cos(\alpha)/V_1 \\ k_{ip} = \omega_c^2 [\cos(\alpha) - \tau_{OSG}\omega_c \sin(\alpha)]/V_1 \end{cases} \quad (B4)$$

Choosing $\omega_c = 80\pi$ rad/s and PM = 27°, the controller parameters are calculated as $k_{pp} = 2.4$ and $k_{ip} = 158.6$. The Bode diagram of the open-loop transfer function of the Sym-PLL under the calculated controller parameters is plotted as Fig. 30. As observed, the controller meets the design objectives.
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