Towards reference-grade multi-mode fiber connectors — Impact of fiber geometry on attenuation and encircled flux

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ABSTRACT

The connection attenuation for perfectly aligned graded-index multi-mode fiber connections with dissimilar core diameter (CD) and numerical aperture (NA) is derived in analytical form for spatially large intensity patterns that include target encircled-flux (EF) compliant launches and overfilled (OF) launches. The launches are spatially stable in that it is possible to describe the ray density distribution (and hence the EF), the intrinsic connection attenuation and the redistribution of the rays in the receiving fiber by means of the ray turning points. This has enabled us to derive the analytical formulation. Although imposing an EF compliant launch significantly reduces the sensitivity of the attenuation due to fiber geometry mismatch by about a factor of three for the NA and a factor of six for the CD as compared to an OF launch, we show that tightening the tolerance of the CD to 50 ± 2 μm (instead of ±2.5 μm) and the tolerance of the NA to 0.2 ± 0.002 (instead of ±0.015) is necessary to ensure < 0.1 dB attenuation among reference-grade connectors (instead of ≤ 1.2 dB).

1. Introduction

The attenuation grade of a multi-mode fiber connector is determined through a connection attenuation measurement with respect to a light-emitting connector. The attenuation must be as low as 0.1 dB for pairs of reference-grade connectors \cite{1}. The longitudinal, lateral and angular alignment precision of the two fibers in a connector system depends on the specific end-face geometry of each connector, which are specified on component level in optical interface standards such as \cite{2}. Attenuation due to stress and strain on the fiber or due to imperfections that lie hidden inside a connector are usually revealed during environmental testing, in which the connection attenuation is monitored during temperature and humidity cycling \cite{3}.

Fiber-intrinsic attenuation effects are often attributed to the performance of the connector, however, the minimum attainable attenuation also crucially depends on the core diameter (CD) and numerical aperture (NA) tolerances that are defined in the fiber geometry standard \cite{4}. A typical multi-mode fiber has a 50 μm CD with a tolerance of ±2.5 μm (5%) and a NA of 0.2 with a tolerance of ±0.015 (7.5%). For connection attenuation measurements and simulations to be meaningful and repeatable, the intensity pattern in the transmitting fiber needs to be standards compliant, which means that the normalized cumulative near-field power distribution, also known as encircled flux (EF), should stay within tight bounds, and ideally passes through the four prescribed targets that define the accumulated power inside four concentric discs that are [20, 30, 40, 44] μm in diameter \cite{5}. With the repeatability ensured in the EF setting, now is the time to focus on the question whether the current CD and NA bounds are tight enough to guarantee the 0.1 dB connection attenuation for reference-grade connectors.

To identify appropriate CD and NA bounds, we adopt a geometrical-optics (GO) approach, and use it to derive the intrinsic attenuation in analytical form. A GO approach has been demonstrated to be efficient and sufficiently accurate to evaluate the attenuation due to physical contact connections \cite{6,7}. In this paper, we map out the iso-attenuation curves for the worst-case connection attenuation. The overall worst-case attenuation can be as high as 1.2 dB, which means that tighter tolerances on CD and NA are needed for reference-grade connectors. To appropriately limit the fiber-intrinsic attenuation and achieve a targeted connector specification, we provide general iso-attenuation curves due to imposed limits on the CD and NA variability for otherwise perfectly aligned fiber connections. Particularly for reference-grade connectors, a random connection attenuation can only be kept below

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0.1 dB with a CD tolerance of ±0.5μm (instead of ±2.5μm) and a NA tolerance of ±0.002 (instead of ±0.015). Although these are respectively a factor of six and three less restrictive than they would be for an overfilled launch case, these are still very tight from a measurement perspective [8]. Possibly, the measurement of the change of the EF distribution after coupling into the receiving fiber could provide additional information of the fiber geometries.

First, we briefly review a spatially stable launch GO launch construction in Section 2, and use it to derive the intrinsic attenuation in analytical form in Section 3. For an overfilled launch, a tolerance analysis on CD and NA mismatch gives a worst-case attenuation of 2 dB which occurs when the receiving fiber is at its smallest geometry and the transmitting fiber at its largest.

The tolerance analysis for a target EF launch in Section 4 is a bit more involved. We discuss the required re-shaping of the ray-equivalent modal power distribution that is needed to properly model a target EF-compliant launch in the transmitting fiber for its specific CD and NA, which effect the shapes of the iso-attenuation curves compared to its OFL counterpart. The overall worst-case attenuation reduces to 1.2 dB, and for small NA mismatch, the tuning of the launch renders the connection relatively insensitive to CD mismatch.

In Section 5, we clarify the ray physics for the redistribution of the EF targets that are defined at the four fixed radial coordinates that we subsequently configure for either an EF compliant launch or the overfilled launch case, these are still very tight from a measurement perspective [8]. Possibly, the measurement of the change of the EF distribution as the rays have propagated to a new equilibrium turning point, which is used in Section 6 to discuss the observed change in the receiving fiber. Even though the EF is usually not measured at the end of the receiving fiber, the observed change in EF due to a connection might prove instrumental for measuring small-scale fiber geometry mismatches that are nigh impossible to measure otherwise.

2. Encircled-flux launch and overfilled launch

Consider an axisymmetric lossless multi-mode fiber that is aligned with the longitudinal z-axis with a refractive index profile described by

\[ n(r) = \begin{cases} \sqrt{n_0^2 + NA^2 (1 - (r/R)^2)} & \text{for } r \leq R \\ n_0 & \text{elsewhere,} \end{cases} \]

(1)

where \( r \) is the radial coordinate, \( R \) is the core radius and \( NA = \sqrt{n_0^2 - n_c^2} \) is the numerical aperture, which is a measure for the contrast between the core and cladding refractive indices that are denoted by \( n_0 \) and \( n_c \) respectively. The dimensions of the physical core diameter \( D = 2R \) and the NA of all multi-mode fibers are standardized and should satisfy CD = \( \mu_{CD} = 2.5 \mu m \) with respect to the mean diameter \( D = 50 \mu m \), and \( NA = \mu_{NA} = 0.015 \) with respect to the mean numerical aperture \( \mu_{NA} = 0.2 \) [4].

From a geometrical-optics perspective, given a ray launch coordinate and direction, and the entire spiral path in the core is implicitly described by two ray invariants [9],

\[ \vec{p} = n(r) \cos \theta_1(r), \]

(2)

and

\[ \vec{T} = (r/R)n(r) \sin \theta_1(r) \cos \theta_2(r), \]

(3)

where \( \theta_2(r) \) is the ray direction with respect to the optical z-axis and \( \theta_1(r) \) is the angle between the ray direction in the cross-sectional plane and the azimuthal unit vector \( \hat{\mathbf{u}}_x \times \hat{\mathbf{u}}_z \). Even without conducting actual ray tracing, the invariants provide valuable information. For instance, the smallest and largest radial coordinates that a ray path can reach are respectively known as the \textit{inner caustic} \( r_{caustic} \) and the \textit{turning point} \( r_{tp} \), and are roots of the equation [9],

\[ n^2(r) - \vec{T}^2 (R/r)^2 = 0. \]

(4)

The ray invariants allow us to classify the propagation behavior. For instance, a guided ray propagates unattenuated along the core of the fiber when \( n_0 < n_c \). However, when \( \beta < n_c \), the ray either refracts into the cladding if \( \vec{p}^2 + \vec{T}^2 < n_c^2 \), or if \( \vec{p}^2 + \vec{T}^2 > n_c^2 \), it maintains a spiral path inside the core that gradually leaks energy into the cladding as a \textit{tunneling ray} [9].

To generate a ray-based target encircled-flux compliant launch, we follow [6] and introduce the distribution of ray invariants

\[ \vec{p} = \sqrt{n_0^2 + NA^2 (1 - \delta)}, \quad \delta \in (0, 1), \]

(5)

according to a scaled counterpart of the modal power distribution for linearly polarized modal electromagnetic fields. By imposing the common condition that all modes in a mode group are equally strongly excited, a unique modal power distribution MPD(\( \delta \)) exists and is proportional to the desired intensity distribution \( I(\rho) \) through the equation [10]

\[ \text{MPD}(\delta) \propto \frac{\delta I(\rho)}{\sqrt{\rho}} \]

(6)

where \( \rho = r/R \) is the normalized radial coordinate.

Inspired by familiar intensity pattern shapes, we choose the intensity expansion

\[ I(\rho) = (1 - \rho^2)^3 \sum_{j=0}^{4} c_j e^{-2j} \]

(7)

that shapes the tail of the polynomial \( 1 - \rho^2 \) by a weighted sum of Gaussian functions [6]. The polynomial \( 1 - \rho^2 \) ensures that \( I(\rho) \) vanishes as \( \rho \rightarrow 1 \) so that the ray-based normalized cumulative power distribution EF(\( \rho \)) = \( \int_{\rho}^{\infty} I(\rho')\rho' d\rho' \) reaches unity at the core-cladding interface [6].

For an overfilled launch (OFL) described by \( I(\rho) = 1 - \rho^2 \), we only use the first coefficient \( c_1 \) and let \( w_1 \rightarrow \infty \).

For a target EF compliant launch, the amplitudes \( c_j \) and Gaussian widths \( w_j \) are chosen for \( j \in (1, 4) \) to make the EF curve pass through the four EF targets that are defined at the four fixed radial coordinates \( r = (10, 15, 20, 22) \mu m \) as standardized in [2]. For every launch fiber with its unique CD, the Gaussian weights need to be appropriately configured to ensure that the ray-based cumulative distribution function EF(\( \rho \)) is target EF compliant. In Section (4), we show that the NA is only associated with a scaling of the ray-angular distribution.

By subsequently distributing all rays with equal \( \vec{p} \) uniformly in the cross-sectional plane, the launch becomes spatially stable in the sense that the ray density distribution does not fluctuate along the length of the fiber, even though all individual rays propagate along their unique (mostly elliptical) spiral paths [11].

We proceed with deriving an analytical ray approximation for the connection attenuation for two perfectly aligned fibers with dissimilar CD and NA, that we subsequently configure for either an EF compliant launch or an OFL.

3. Connection attenuation in analytical form

Consider two perfectly aligned fibers with parabolic refractive index profiles \( n_1(r) \) and \( n_2(r) \), each with their own CD and NA as described by Eq. (1), where the subscripts 1 and 2 respectively denote the transmitting and receiving fiber. By combining Eq. (2) and Snell’s law \( n_1(\theta) \sin \theta_1 = n_2(\theta) \sin \theta_2 \) at the connection interface, the ray invariant \( \vec{p}_1 \) in the transmitting fiber excites the ray invariant \( \vec{p}_2 \) in the second fiber while preserving

\[ \vec{p}_2^2 = \vec{p}_1^2 + n_2^2(r) - n_1^2(r). \]

(8)

The transition \( \vec{p}_1 \rightarrow \vec{p}_2 \) across the interface induces a shift of the
turning points of the rays, and thus effect a change of the ray density distribution in the receiving fiber. Moreover, the connection attenuation due to the quantity of rays that satisfy \( n_2 < n_1 \) can be determined analytically by virtue of the analytical constituents in Eq. (6) that shape the stable launch distribution of \( \beta_1 \).

The challenge is to properly incorporate the ray position radius \( r \) in Eq. (8). To achieve a uniform distribution of \( \beta \) for a spatially stable launch \([6]\), we let \( r \to r_{\alpha,0} \beta \) with \( p \in \mathbb{R}(0,1) \), which distributes the ray positions uniformly in discs that are bounded by the largest turning points \( r_{\alpha,0} \) that can only be reached by meridional rays \( (\beta = 0) \) \([9,11]\).

At these bounds, Eq. (2) reduces to \( n(r_{\alpha,0}) = \beta_1 \), and for the parabolic−core profile in Eq. (1), the turning point in the transmitting fiber satisfies

\[
\zeta_{\alpha,0}^2 = \frac{R^2}{NA^2_1} \left( n^2_1 + NA^2_1 - \beta^2_1 \right).
\]

By combining Eqs. (8) and (9), we obtain

\[
\beta_{\alpha,1}^2 = \beta_1^2 + NA^2_1 - NA^2_1 + p \left( \frac{NA^2_1}{R^2_1} - \frac{NA^2_2}{R^2_2} \right) \zeta_{\alpha,0}^2,
\]

and the parameter \( p \in (0,1) \) spreads all rays that have a fixed \( \beta_1 \) in the launch fiber to rays with a range of \( \beta_{\alpha,1} \) in the receiving fiber. By combining Eqs. (9) and (10), and applying the transformations \( \beta_1 \to \delta_1 \) and \( \beta_{\alpha,1} \to \delta_2 \) according to Eq. (5), we obtain

\[
\delta_2 = \delta_1 \left( 1 - p \frac{NA^2_1}{NA^2_2} + \frac{R^2_1}{R^2_2} \right),
\]

which furnishes the linear spread of the relative mode-group numbers \( \delta_2 \) in the receiving fiber in terms of the relative ray radius \( p \) and the relative mode-group numbers \( \delta_1 \) in the transmitting fiber.

To determine the connection attenuation, we compute the portion of the optical power of the launch that does not couple to the guided regime of the receiving fiber, for which \( \delta_2 > 1 \). The attenuation function in integral form

\[
\text{Att} [\text{dB}] = -10 \log_{10} \left( 1 - \int_{\delta_1(p), \beta_1} \text{MPD}(\delta_1) d\delta_1 \right),
\]

can be solved analytically for the modal power distribution associated with our choice of the modal power distribution in Eq. (7), which satisfies

\[
\text{MPD}(\delta_1) = \sum_{j=1}^{\Delta} \epsilon_j e^{-\frac{\Delta \delta_1}{\omega^2_{j}}}(2\delta_1 + \delta_1^2 - \delta_1^2 w^2_j).
\]

We normalize the coefficients \( c_j \to \epsilon_j / P_j \) to ensure unit power in the launch fiber \( P_j = \int_0^{\beta_1} \text{MPD}(\beta_1) d\beta_1 \). Further, we introduce the ratios \( q_{\text{NA}} = NA^2_1 / NA^2_2 \) and \( q_0 = R^2_1 / R^2_2 \) to compact notation, and for the double integral in Eq. (12) we obtain

\[
\int_{\delta_1(\alpha,0) > 1} \text{MPD}(\delta_1) d\delta_1 dp = \int_{\delta_1(\alpha,0) > 1} \text{MPD}(\delta_1) d\delta_1 dp.
\]

Let us defer the discussion about the integration bounds \( p_1 \) and \( p_0 \) to the subsections below. In Eq. (14), the lower bound

\[
\delta_1(\alpha,0) = \frac{1}{1 - p q_{\text{NA}} + p q_0},
\]

is determined from Eq. (11) by letting \( \delta_1 \to \delta_{1,\text{rb}}(p) \) for \( \delta_2 = 1 \), the cut-off of the guided rays in the receiving fiber. We assume an upper bound \( \delta_{1,\text{ub}} = 1 \), even though it should be noted that the modal power distribution function can become negative when imposing a target EF launch in a transmitting fiber that has a CD > 51.5 \( \mu \)m, which would clearly be unphysical. The reason is that in that case the interpolation function in Eq. (7) reaches \( I(r) = 0 \) at \( r = 25.75 \mu \)m, instead of \( r = R \). In the geometrical-optics approach \([6]\) that can only construct positive modal power distribution functions, this would result in a thin dark ring without rays between \( r = 25.75 \mu \)m and the core radius at \( r = R \). It might seem unreasonable to have an up to \( 1 \mu \)m dark ring close to the cladding, but (real) rays can simply not be added in this region without sacrificing the overall shape of the EF curve. Similarly, classic geometrical optics (real rays) can not describe an evanescent tail in the cladding either.

Nevertheless, for multi-mode connection attenuation modeling, the ray-based approach shows good agreement with modal electromagnetic field approaches \([12]\). To reproduce the dark ring behavior in the analytical form also, we have also determined the upper bound \( \delta_{1,\text{ub}} \) from \( \text{MPD}(\delta_{1,\text{ub}}) = 0 \) in Eq. (6). However, we found that this correction has no noticeable impact on the results presented in Section (4), indicating that in this pathological case, the tail is not significant.

By combining Eqs. (13)−(15), the integral in Eq. (12) is solved analytically as

\[
\int_{\beta_1}^{\beta_{\alpha,1}} \int_{\delta_1(\alpha,0) > 1} \text{MPD}(\delta_1) d\delta_1 dp = \sum_{j=1}^{\Delta} c_j \int_{\beta_1}^{\beta_{\alpha,1}} \int_{\delta_1(\alpha,0) > 1} \text{MPD}(\delta_1) d\delta_1 dp,
\]

with

\[
I(p) = \left( \left( 1 - 2w^2_j \right)/\left( 1 - p \right) q_{\text{NA}} + p q_0 - 1 \right) \times \exp \left( 2w^2_j (1 - p) q_{\text{NA}} - p q_0 \right)^{-1}.
\]

There are five distinct cases to consider when using Eqs. (12)−(16) and (17). Depending on the quantities \( q_{\text{NA}} \) and \( q_0 \), the integration bounds \( p_1 \) and \( p_0 \) need to ensure that \( \delta_{1,\text{ub}}(p) \) is in the valid range

\[
0 \leq \delta_{1,\text{ub}}(p) \leq 1.
\]

3.1. Case \( q_{\text{NA}} < 1 \) and \( q_0 < 1 \)

When the receiving fiber has both a larger CD and larger NA than the launch fiber due to \( q_{\text{NA}} < 1 \) and \( q_0 < 1 \), there is no attenuation because \( \delta_{1,\text{ub}}(p) \) in Eq. (15) exceeds the upper limit \( \delta_1 = 1 \) for all \( p \).

3.2. Case \( q_{\text{NA}} > 1 \) and \( q_0 < 1 \)

When the receiving fiber has only a smaller NA than the launch fiber due to \( q_{\text{NA}} > 1 \) and \( q_0 < 1 \), the denominator in Eq. (15) is only larger than one for \( p \in (p_1, p_0) \), with \( p_1 = 0 \) and \( p_0 = p_c \), where the threshold

\[
p_c = \frac{1 - q_{\text{NA}}}{q_0 - q_{\text{NA}}}.
\]

is associated with \( \delta_{1,\text{ub}}(p_c) = 1 \) in Eq. (15).

3.3. Case \( q_{\text{NA}} < 1 \) and \( q_0 > 1 \)

When the receiving fiber has only a smaller CD than the launch fiber due to \( q_0 > 1 \) and \( q_{\text{NA}} < 1 \), the denominator in Eq. (15) is only larger than one for \( p \in (p_1, p_0) \), with \( p_1 = 0 \) and \( p_0 = p_c \), with \( p_c \) in Eq. (19).

3.4. Case \( q_{\text{NA}} = q_0 > 1 \)

When the receiving fiber has both a smaller CD and smaller NA for the specific case that also \( q_{\text{NA}} = q_0 > 1 \), the parameter \( \delta_2 \) in Eq. (10) becomes independent of \( p \). The integration bound \( \delta_{1,\text{ub}}(p) \) reduces to

\[
\delta_{1,\text{ub}}(p) = \frac{1}{1 - p q_{\text{NA}} + p q_0}.
\]
\( \delta_{1,\beta} = q_{\beta}^{-1} \) and the double integral in Eq. (16) is replaced by the single integral
\[
\int_{0}^{\bar{\delta}_{\beta}} \text{MPD}(\bar{\delta}) \, d\bar{\delta} = \sum_{j=1}^{K} p^2 \left( \frac{2w_j^2}{(2w_j^2 - 1)(2w_j^2 + \bar{\delta}_j + \bar{\delta}^2)} \right) \int_{\bar{\delta}}^{\bar{\delta}_{\beta}} \, d\bar{\delta}.
\]

(20)

3.5. Case \( q_{\text{NA}} > 1 \) and \( q_{\beta} > 1 \), \( q_{\beta} \neq q_{\text{R}} \)

When the receiving fiber has both a smaller CD and a smaller NA than the launch fiber due to \( q_{\text{NA}} > 1 \) and \( q_{\beta} > 1 \), and \( q_{\beta} \neq q_{\text{R}} \), the requirement in Eq. (18) is met for all normalized ray radii \( p \in (p_l, p_u) \) with \( p_l = 0 \) and \( p_u = 1 \), because everywhere in that range, the denominator of Eq. (15) is larger than one.

As an example for an overfilled launch by letting \( c_1 = 1 \) and \( w_1 \to \infty \), Eq. (16) reduces to \( 1 - q_{\text{NA}}^{-1} \), so that the connection attenuation in Eq. (12) reduces to,
\[
\text{Att}[dB] = -10\log_{10} \left( \frac{\text{NA}_{2}^2 \beta_{1}^2}{\text{NA}_{1}^2 \beta_{2}^2} \right).
\]

(21)

which coincides with the textbook equation for the worst-case connection attenuation for a uniform modal power distribution \([13,14]\).

As seen in Eq. (21), the worst-case connection attenuation for an OFL occurs when the receiving fiber has the largest CD and NA, while the receiving fiber has the smallest CD and NA. We confirmed that these combination also produce the worst-case iso-attenuation curves in a full 4-dimensional parameter space that includes \( \text{CD}_{1}, \text{NA}_{1}, \text{CD}_{2}, \) and \( \text{NA}_{2} \).

Fig. 1 shows the worst-case iso-attenuation curves in a full 4-dimensional parameter space that includes \( \text{CD}_{1}, \text{NA}_{1}, \text{CD}_{2}, \) and \( \text{NA}_{2} \).

For an OFL, the worst-case intrinsic attenuation exceeds 2 dB. To meet the \(<0.1\) dB attenuation requirement for reference-grade connectors \([1]\), an overfilled launch would render the required fiber geometry specification exceptionally tight: \( \pm0.15\mu m \) for the CD and \( \pm0.0005\) for the NA.

We proceed with the tolerance analysis for an EF target launch.

4. Worst-case attenuation due to CD and NA mismatches for EF target launch

The mapping of the (fixed) modal power distribution in Eq. (6) to a launch fiber with an arbitrary NA amounts to distributing \( \beta \in (n_{\beta}, n_{\text{R}}) \) according to Eq. (5). The ray-invariant based power density distribution for a target-EF compliant launch in a nominal fiber is shown in the right subfigure of Fig. 2 as a black dashed curve. In a small NA = 0.185 fiber (blue solid curve), a target EF launch has a much narrower \( \beta \)-distribution, whereas in a large NA = 0.215 fiber (red solid curve), the \( \beta \) distribution is much wider. Among the pertaining launch fibers, the EF curves overlap nicely in the left subfigure, showing that all near-field patterns are identical and are thus compliant to the IEC standard \([2]\). However, the near-field measurements do not unveil that the large NA fiber excites rays with a relatively wide ray direction distribution \( \theta_{\beta}(r) \), whereas a small NA fiber excites rays with a relatively narrow ray direction distribution \( \theta_{\beta}(r) \). The consequence is an additional penalty on the connection attenuation. For example, the attenuation measurement of a nominal receiving fiber amounts to 0.24 dB when the large NA launch fiber is used, whereas the connection is lossless when measured with nominal or small NA launch fibers. To meet the worst-case attenuation requirement of 0.1 dB among reference-grade connectors, the NA specification for launch fibers must thus be restricted significantly. To select an appropriate bound, we first need to consider the impact of core diameter variations among launch fibers.

To impose a target EF launch on a fiber with a core diameter that deviates from 50 \( \mu m \), a re-evaluation of the coefficients \( c_1 \) and \( w_1 \) that shape \( f(\rho) \) in Eq. (7) is needed because the radial coordinate \( R \) denoting the core-cladding interface in Eq. (4) changes relative to the four fixed EF target coordinates. For a nominal fiber with a \( \text{CD}_{1} = 50\mu m \) core diameter, the target EF compliant launch is denoted by the black dashed lines in Fig. 3, and for comparison, an overfilled launch is represented by a magenta dotted straight line in the figure on the right.

The EF curve associated with the overfilled launch is shown in the left subfigure with the magenta dotted curve with \( \square \) markers, and relative to the target EF curve (horizontal black dashed curve), the near-field pattern carries too much power in the outer region to keep within the EF bounds. Interestingly, an overfilled launch imposed on a smaller \( \text{CD}_{1} = 47.5 \mu m \) fiber (magenta dotted curve with \( \square \) markers) is much narrower and does keep within the EF bounds. With a few adjustments to the \( \beta \)-distribution (blue solid curve with \( \square \) markers), the EF curve crosses all four EF targets, albeit with a large contribution of near-meridional rays that can reach up to the core-cladding interface.

In a large core fiber, \( \text{CD}_{1} = 52.5 \mu m \), the MPD (red solid curve) looks

![Fig. 1](image1.png)

Fig. 1. Worst-case iso-attenuation curves for an overfilled launch due to CD and NA mismatch in terms of the variation bounds \( \delta_{\text{CD}} \) and \( \delta_{\text{NA}} \). These variation bounds are also expressed in terms of percentage of the nominal fiber geometry dimension \( (\text{CD}_{1} = 50\mu m, \text{NA}_{1} = 0.2) \).

![Fig. 2](image2.png)

Fig. 2. A target EF compliant launch in launch fibers with an arbitrary NA is attained by appropriate scaling of the \( \beta \) ray-invariant distribution (rightmost figure). The required ray-angle distribution is relatively narrow (wide) for fibers with a small (large) NA.
very similar to the nominal case, albeit with a steep drop in power close to the core-cladding interface. As mentioned in Section 3, the MPD in Fig. 3 (red solid curve) becomes negative near $\bar{\beta} \mu_{\text{NA}}$ for a target EF launch in large core launch fibers and produces a dark ring between $r = 25.75 \mu m$ and $r = R$. However, from a reconstruction of Fig. 4 with the analytical approach with and without truncation of the negative MPD region, the results are hardly noticeable.

We evaluated the connection attenuation between two perfectly aligned multi-mode fibers with a 4-dimensional parameter sweep including $CD$, $\mu_{\text{NA}}$, $\mu_{\text{CD}}$, and $\beta$, while imposing a target EF compliant launch in the transmitting fiber. Fig. 4 shows the worst-case attenuation as it occurred in the set of all fiber pairs that are bound by the variation bounds $\delta_{\text{CD}}$ (horizontal axis) and $\delta_{\text{NA}}$, limiting the transmitting fiber to geometries that satisfy $|CD_1 - \mu_{\text{CD}}| \leq \delta_{\text{CD}}$, and $|\beta_1 - \mu_{\beta}| \leq \delta_{\beta}$, and limiting the receiving fibers to the geometries that satisfy $(CD_2 - \mu_{\text{CD}}) \leq \delta_{\text{CD}}$, and $|\beta_2 - \mu_{\beta}| \leq \delta_{\beta}$. The full sweep is necessary because unlike the OFL attenuation case in Eq. (21), the worst case does not occur at the largest $CD$ difference, because the launch is now adjusted for every different launch fiber to ensure target EF compliance.

Compared to Fig. 1, the 0.1 dB iso-attenuation curve has shifted significantly by using an EF target launch instead of an overfilled launch, easing the $CD$ tolerance by a factor six, and easing the $\beta$ tolerance by a factor three. The larger the $\mu_{\text{NA}}$ variation bound $\delta_{\text{NA}}$, the steeper the slope of the iso-attenuation curves, meaning that the relative insensitivity of the attenuation due to $CD$ mismatches wears off rapidly. The tightened $CD$ tolerance of $\pm 1 \mu m$ and a $\beta$ tolerance of $\pm 0.002$ should suffice for a reference-grade connector specification [1]. Of course, other attenuation contributors such as a finite lateral and angular misalignment would need budget in that specification too.

From a practical perspective, connection attenuation measurements are typically in the low tens of a dB range, and the worst-case 1.2 dB in the top-right corner of Fig. 4 would consume almost the entire connector power budget of 40GbE, 100GbE and faster Ethernet networks [15]. One can conclude that modern multi-mode fibers geometries are more accurately and consistently manufactured than that they can be measured. As an alternative to geometry measurements, we would like to explore the possibility to extract geometry information from EF measurements in the receiving fiber. To this end, we propose to leverage the observation that rays settle to a new spatially stable distribution in the receiving fiber in about 0.3m [6], where the intensity (viz. the EF pattern) shape is governed by the redistribution of the turning points discussed below.

5. Turning point shift due to fiber geometry mismatch

As shown in Fig. 5, consider a receiving fiber core (solid orange profile) that is larger than the transmitting fiber (solid blue profile). The five red arrows (A-E) show the motion of the pertaining turning points to a larger radius for all five characteristic rays that cross the connection interface at radial coordinates that are labeled $a$-$e$.

A ray is meridional when its path intersects the optical axis. Consider three rays that propagate along the same but longitudinally displaced trajectories (thin horizontal line at $\bar{\beta} = 1.453$ in Fig. 5) and cross the connection interface at the radial coordinate labeled $a$, $b$ and $c$. For the ray crossing at $r = a$, the right-hand side of Eq. (8) vanishes and the ray continues as a meridional ray in the receiving fiber that has a larger turning point (shift indicated by arrow A). For the ray crossing at $r = b$, the relatively small contrast on the right-hand side of Eq. (8) leads to the excitation of a meridional ray in the receiving fiber with $\bar{\beta}_2$ on a trajectory (dashed horizontal line) that also has a larger turning point (arrow B). When the ray crossing occurs at the turning point itself at $r = c$, the ray path in the receiving fiber (dotted horizontal line) is meridional with the same turning point because of the normal incidence of the ray.

![Fig. 3. Left: EF relative to a target launch when excited in a small fiber with $CD = 47.5 \mu m$ (blue solid curve with diamond markers), a nominal fiber with $CD = 50 \mu m$ (black dashed curve) and a larger fiber with $CD = 52.5 \mu m$ (red solid curve). Right: The distribution of the $\beta$ ray-invariant that shows the small (large) fiber needs a relatively overfilled (underfilled) ray distribution to achieve a spatially stable target EF-compliant launch.](image)

![Fig. 4. Worst-case iso-attenuation curves for a target EF launch due to CD and NA mismatch expressed with variation bounds $\delta_{\text{CD}}$ and $\delta_{\text{NA}}$ in the 4-dimensional set of all fiber connections with transmitting and receiving fibers with a CD in the range $\{CD_1, CD_2\} = \mu_{\text{CD}} \pm \delta_{\text{CD}}$ and NA in the range $\{\mu_{\text{NA}}, \mu_{\text{NA}}\} = \mu_{\text{NA}} \pm \delta_{\text{NA}}$. The variation bounds are also expressed in terms of percentage of the nominal fiber geometry dimension ($\mu_{\text{CD}} = 50 \mu m, \mu_{\text{NA}} = 0.2$).](image)

![Fig. 5. Visualization of the change in the $\beta$ ray-invariant and the turning point (thick red arrows) depending on the radial coordinate (red dots) for five rays that cross the connection interface from a launch fiber to a larger receive fiber.](image)
Rays on skew trajectories have an inner caustic \( r_u > 0 \), and excite skew rays in the receiving fiber that either have the same or larger turning points in the receiving fiber. The skew ray that crosses the interface at \( r = d \) (thin horizontal line at \( \vec{p} = 1.458 \) in Fig. 5), continues as a skew ray with a larger turning point (arrow D). When the ray crosses the interface at the inner caustic (left \( \times \) marker), the right-hand side of Eq. (8) is non-zero and the shift of the turning point can no longer be horizontal (right grey arrow near arrow D). When the ray crosses the interface at the turning pint (right \( \times \) marker), the ray continues as a skew ray with the same turning point (vertical grey arrow).

As a last example, we show that the circular skew rays at \( r = e \) in Fig. 5 excites skew rays in the receiving fiber with a larger turning point (arrow E).

In case the receiving fiber has a smaller NA and the same CD as the launch fiber due to a higher cladding level (horizontal orange dashed line with cut-off at \( \vec{p} = 1.4562 \)), the ray paths would change in the same way, although ray A and B would be refracting into the cladding. From a practical perspective, an equivalent NA mismatch situation would be better represented by the situation in Fig. 6 where the cladding levels of both fibers are the same. Although the entire profile \( r_\mu \) shifts downwards compared to the situation in Fig. 5, the fixed reduction on the right-hand side of Eq. (8) turns out to hardly impact how the turning points shift towards the higher radii in Fig. 6. A meridional ray that crosses the interface at \( r = a = 0 \) still excites a meridional ray in the receiving fiber with a larger turning point (shift indicated by arrow A), even though \( \vec{p}_2 \) is now smaller than \( \vec{p}_1 \). A meridional ray that crosses the interface at the turning point \( r = b \) remains meridional with the same turning point (arrow B), a skew ray that crosses the interface at \( r = c \) remains skew with a larger turning point (arrow C), and circular skew rays become skew with a larger turning point (arrow D).

The shifts of the turning points towards the higher radii for all rays leads to a widened intensity pattern in the receiving fiber due to the larger CD or smaller NA compared to the launch fiber. Vice versa, when the receiving fiber has a smaller CD or a larger NA than the launch fiber, the intensity pattern becomes smaller.

6. EF change due to fiber geometry mismatch

Consider a perfectly aligned connection of two identical fibers. Because the ray paths do not change across the interface, the spatially stable EF distribution in the receiving fiber also does not change. This is shown by the horizontal blue zero baseline curve in the right subfigure in Fig. 7, which coincides with the target EF launch that crosses all four EF targets (four green \( \bullet \) markers). This right subfigure shows the EF curves relative to the target EF curve in the left subfigure (blue curve). It is necessary to zoom in on the tightest two upper bounds (red markers) at \( r = \{20, 22\} \) \( \mu \)m, because the EF curves in the left subfigure can hardly be distinguished. The EF bounds for \( r = \{10, 15\} \) \( \mu \)m are at \( \pm 0.057 \), and fall outside the range of the right subfigure.

Cases A show connections with receiving fibers that either have a 1\( \mu \)m smaller core with \( CD_1 = \mu_{CD} + 1 \) \( \mu \)m (solid green curve) or a slightly larger NA with \( NA_1 = \mu_{NA} + 0.005 \) (purple dashed line). The ray density distribution narrows, and the pertaining EF curves closely approach the upper bounds. Conversely, cases B show connections to receiving fibers that either have a 1 \( \mu \)m larger core with \( CD_2 = \mu_{CD} + 1 \) \( \mu \)m (green solid curve) or a slightly smaller NA with \( NA_2 = \mu_{NA} - 0.005 \) (purple dashed line), which have EF curves that approach the lower bounds. The worst-case attenuation for these receiving fibers is about 0.07 dB.

For receiving fibers that are at the extremes of the fiber geometry tolerance field [4], cases C and D concern \( CD_3 = \mu_{CD} - 2.5 \mu \)m (grey solid curve) or \( NA_3 = \mu_{NA} + 0.015 \) (black dashed line), while case D shows the opposite with \( CD_4 = \mu_{CD} + 2.5 \mu \)m (grey solid curve) or \( NA_4 = \mu_{NA} - 0.015 \) (black dashed line). The worst-case attenuation for these receiving fibers is about 0.26 dB.

Even though the EF is normally only measured at the end-face of the launch fiber prior to conducting attenuation measurements, the examples in Fig. 7 show that fiber geometry mismatches across an otherwise perfectly aligned fiber interface impact the shape of the EF curve significantly. Even for relatively small mismatches, the EF curves can already cross the standardized EF bounds, and can therefore form the basis for more accurate fiber geometry measurements, albeit that it may be challenging to identify the CD and NA mismatch for the combinations that counteract each others impact on the EF curve.

7. Discussion and conclusions

The analytical constituents in the geometrical-optics based modal power distribution and the spatial stability of the spatially large launch allowed us to model the fiber-intrinsic connection attenuation in analytical form. Depending on the simultaneous CD and NA mismatch, there are five cases to distinguish to appropriately set the integration bounds. For an EF-compliant launch, the standard industry specified tolerances are too crude for use in reference-grade connectors, because the worst-case attenuation of 1.2 dB exceeds the \( < 0.1 \) dB limit. A specification with \( \delta_{CD} = 0.5 \mu \)m, and \( \delta_{NA} = 0.002 \) should suffice.

From a practical perspective, connection attenuation values as high
as 1.2 dB are unacceptably high from modern networks, thus suggesting that fiber geometries are generally much tighter than the standardized tolerance. However, accurately measuring the fiber geometry within the tolerance of reference-grade connectors is nigh impossible. To provide more insight, the change in the near-field pattern due to fiber geometry mismatches across a connection might be a sensible path forward to determine fiber geometry mismatches in an indirect manner. That is, if the characteristic widening (or contracting) of the near-field pattern in the receiving fiber due to a larger (or smaller) CD, or a smaller (or larger) NA, compared to the launch fiber can be accurately measured with near-field scanners.

CRediT authorship contribution statement

Sander J. Floris: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Data curation, Writing - original draft, Writing - review & editing, Visualization. Bastiaan P. de Hon: Conceptualization, Methodology, Formal analysis, Investigation, Resources, Data curation, Writing - original draft, Writing - review & editing, Supervision. Ton Bolhaar: Conceptualization, Validation, Resources, Data curation, Supervision, Funding acquisition. Martijn C. van Beurden: Investigation, Methodology, Resources, Writing - original draft.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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