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Vorticity dynamics of a dipole colliding with a no-slip wall

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The active role of vorticity in the collision of a Lamb-like dipole with a no-slip wall is studied for Re values ranging between 625 and 20000. The initial approach of the dipole does not differ from the stress-free case or from a point-vortex model that incorporates the diffusive growth of the dipole core. When closer to the wall, the detachment and subsequent roll-up of the boundary layer leads to a viscous rebound, as was observed by Orlandi [Phys. Fluids A 2, 1429 (1990)] in numerical simulations with Re up to 3200. The net translation of the vortex core along the wall is strongly reduced due to the cycloid-like trajectory. For Re ≤ 2500 wall-generated vorticity is wrapped around the separate dipole halves, which hence become (partially) shielded monopoles. For Re ≥ O(104), however, a shear instability causes the roll-up of the boundary layer before it is detached from the wall. This leads to the formation of a number of small-scale vortices, between which intensive, narrow eruptions of boundary-generated vorticity occur. Quantitative measures are given for the influx of vorticity at the wall and the consequent increase of boundary layer vorticity and enstrophy. © 2007 American Institute of Physics. [DOI: 10.1063/1.2814345]

I. INTRODUCTION

One of the key phenomena in two-dimensional bounded turbulence is the interaction between the vorticity structures inside the domain and the viscous boundary layers at the no-slip walls. For example, in freely evolving turbulence it is often observed that these dipoles collide with the lateral no-slip walls. Typically, under the influence of large-scale vorticity structures the boundary layers are detached from the wall and injected into the domain in the form of small-scale vortices and filaments. This mechanism is nicely illustrated by Wells in laboratory experiments of a quasi-2D flow in a square container where the background rotation is being modulated. Boundary layers are created and detached from the wall during the stages of wall acceleration and deceleration. Subsequently, the detached vorticity filaments roll-up into vortex structures and enter the interior of the domain. In freely evolving 2D turbulence interior vortices may form dipole pairs, which can translate throughout the flow domain.

To study the vortex-wall interactions in more detail we investigate the collision of a single dipolar vortex with a no-slip wall. Numerical simulations of the dipole-wall collision were first performed by Orlandi, who revealed that the creation of vorticity at the wall drives a vigorous rebound of the dipolar vortex. Each dipole half forms a new pair with a secondary vortex consisting of boundary layer vorticity. The new dipoles travel along circular trajectories away from the wall. The production of the secondary vorticity at the wall during the dipole-wall collision was later studied by Coutsias and Lynov and by Clercx and van Heijst. The Reynolds number in the reported simulations is limited to Re=5000 as accurate representations of the small-scale structures near the wall requires a high resolution. Walker11 and Perdrier et al. investigated the behavior of the boundary layer at a no-slip wall affected by a nearby single (point) vortex. This setup permits the use of boundary-layer equations, i.e., a reduced set of the Navier–Stokes equations, on a specially tailored grid to obtain results for large Re values. More recently Obabko and Cassel reported results on this problem based on the full Navier–Stokes equations. Their findings will be discussed in more detail later on.

The present paper aims at giving a clear insight in the dipole-wall collision while concentrating on the vorticity dynamics close to the no-slip wall. We focus mainly on the features of the dipole-wall collisions that are relevant to bounded two-dimensional turbulence. Simulations are performed for several values of the Reynolds number Re, based on the size and translation speed of the dipole. One of the goals is to provide accurate results for the dipole-wall collision for Reynolds numbers up to Re=20000, and the dynamics of the small-scale vortices and vorticity filaments which emerge during this process. Furthermore, attention is given to the vorticity flux at the wall during the approach and the rebound of the dipole halves and the consequent increase of boundary layer vorticity and enstrophy. Finally, for different Re values we investigate the formation of the small-scale vortices that originate from the (detached) boundary layers.

II. PROBLEM DESCRIPTION AND NUMERICAL SETUP

The two-dimensional flow of an incompressible fluid in the x,y plane is described by the vorticity equation

\[ \frac{\partial \omega}{\partial t} + \mathbf{u} \cdot \nabla \omega = \nu \nabla^2 \omega, \]  

(1)
with \( \nu \) the kinematic viscosity and the vorticity \( \omega \) defined as the curl of the velocity field \( u \),
\[
\omega = e_z \cdot \nabla \times u,
\]
where \( e_z \) is the unit vector perpendicular to the plane of flow. In this paper we study the flow in a periodic channel domain \([0, 2) \times [-1, 1] \). The periodic direction of the channel is aligned along the \( x \)-axis. On the lateral walls of the channel at \( y = \pm 1 \) no-slip boundary conditions are applied. The flow is initialized in the form of two shielded Gaussian monopolar vortices, their centers placed at a distance \( d = 0.2 \) apart. The vorticity distribution in each monopole is given by
\[
\omega(0, x) = \omega_0 (1 - r^2/r_0^2) \exp(-r^2/r_0^2),
\]
where \( r_0 \) is the core radius, and \( r = |x - x_0| \) with \( x_0 \) the position of the vortex center. The two isolated monopoles are located at \( x_0 = (1.1, 0) \) and \( x_0 = (0.9, 0) \), respectively. Demanding that the root mean square velocity (rms) velocity is initially equal to unity yields the amplitude of the isolated monopole, \( \omega_0 = 300 \). The core radius of the shielded monopoles is set to \( r_0 = 0.1 \). The vorticity amplitude in the surrounding rings decreases exponentially with \( r \). As a result, the circulation of one isolated monopole calculated over a circular contour around the vortex origin decreases exponentially towards zero for increasing contour radius. Hence, no boundary layers are required at the no-slip walls when constructing the initial flow field.

To solve Eq. (1) numerically with a pseudospectral method, both the velocity and vorticity are expanded in a truncated series of Fourier polynomials for the \( x \)-direction and in a truncated series of Chebyshev polynomials for the nonperiodic \( y \)-direction. A semi-implicit time discretization scheme is applied to the vorticity equation. In most of the cases the nonlinear term in the vorticity Eq. (1) is treated by the explicit second-order Adams–Bashforth (AB) scheme and the diffusion term on the right-hand side of Eq. (1) is treated by the implicit Crank–Nicolson scheme (CN). In some specific cases a third-order accurate semi-implicit scheme (BFD3) is applied, which is based on the backwards differentiating formula. Applying the spatial and time discretizations yields a system of equations for the spectral coefficients for the velocity and vorticity. This system can be resolved using a specially tailored Gaussian elimination technique. The vorticity values at the walls are not known \( a \) priori. Enforcing the vorticity definition (2) with an influence matrix yields the correct boundary values for the vorticity at the walls.\(^ {15} \) A more detailed description of the numerical method is given in Ref. \(^ {16} \).

After releasing the initial set of shielded monopolar vortices [Fig. 1(a)], one observes how the cores combine into a dipole, while the surrounding shields are substantially deformed [Fig. 1(b)]. Somewhat later, the dipolar vortex has travelled away in the negative \( y \)-direction, and the shields left behind combine into another, much weaker dipolar structure that translates in the opposite direction [Fig. 1(c)]. This weak dipole has no considerable impact on the primary dipole and is not discussed further. Previous studies have revealed that the primary dipole vortex closely resembles the Lamb dipole,\(^ {17} \) i.e., a compact dipolar vorticity structure in a circular area with a linear vorticity-stream-function relationship.\(^ {18},^{19} \) For more details on the formation of a dipole from two interacting monopoles the reader is referred to Beckers \(^ {20} \) and Schmidt \(^ {21} \).

The time evolution of the dipole is calculated by the spectral code for several values of the viscosity parameter. The integral-scale Reynolds number for the initial field is given by
\[
Re = \frac{U_{\text{rms}} H}{\nu},
\]
where the characteristic length scale is set to the half-height of the domain \( H = 1 \) and the characteristic velocity to the initial rms velocity \( U_{\text{rms}} = 1 \). This integral Reynolds number differs slightly from the Reynolds number \( Re_d = 0.8 \) based on the dipole translation speed \( U_d \) and dipole radius \( R \). The required number of Fourier polynomials \( N \) and Chebyshev polynomials \( M \) depends on the Reynolds number; the resolution is increased for higher \( Re \) values in order to cope with

\begin{table}[h]
\centering
\begin{tabular}{cccc}
\hline
\( \nu \) & \( Re \) & \( N \) & \( M \) & \( \Delta t \) \\
\hline
\( \times 10^4 \) & & & & \( \times 10^{-5} \) \\
16.0 & 625 & 512 & 512 & 2.5 \\
8.0 & 1250 & 512 & 512 & 2.5 \\
4.0 & 2500 & 1024 & 768 & 1.25 \\
2.0 & 5000 & 1536 & 1024 & 0.625 \\
1.0 & 10000 & 1536 & 1024 & 0.625 \\
\hline
\end{tabular}
\caption{The number of Fourier polynomials \( N \), the number of Chebyshev polynomials \( M \), and the time step \( \Delta t \) used to solve the dipole-wall collision for a number of initial Reynolds numbers. (See Table II for the simulations with \( Re = 20000 \).)\(^ {22} \)}
\end{table}

FIG. 1. The evolution of the initial dipole consisting of two isolated vortices: (a) The initial vorticity distribution; (b) shedding of the surrounding vorticity shields; and (c) formation of a downward-travelling Lamb-type dipole and a weaker upward-travelling dipole. The contour levels are drawn for ...,−150,−90,−30,30,90,150,....
the smaller scales that appear throughout the domain (Table I). Special effort is required to obtain reliable results for the dipole-wall collision with an initial value of Re=20000. The spatial resolution in the x-direction is increased after \( t=0.25 \) and the applied time discretization scheme is changed from AB/CN to BDF3 at \( t=0.31 \).

To verify the accuracy of the numerical method the results are compared to the benchmark results reported by Clercx and Bruneau. For this purpose we have calculated the position of the maximum vorticity, \( x_d \), in the positive dipole half. The position is determined using a second order polynomial fit of the vorticity around its maximum. In Table III the location of the maximum vorticity, \( x_d \), in the positive dipole half is compared with the benchmark results of Clercx and Bruneau obtained with two different numerical methods. One set of numerical simulations was performed with a finite differences code while the other set concerns simulations with large Reynolds numbers. The specific settings used in the simulation are given in Table II.

### Table II. The number of Fourier polynomials \( N \), the number of Chebyshev polynomials \( M \), and time step \( \Delta t \) used to solve the dipole-wall collision for an initial Reynolds number of Re=20000. The spatial resolution in the x-direction is increased after \( t=0.25 \) and the applied time discretization scheme is changed from AB/CN to BDF3 at \( t=0.31 \).

<table>
<thead>
<tr>
<th>( t )</th>
<th>( \nu )</th>
<th>( N )</th>
<th>( M )</th>
<th>( \Delta t )</th>
<th>Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00-0.25</td>
<td>5.0</td>
<td>2048</td>
<td>1024</td>
<td>6.25</td>
<td>AB/CN</td>
</tr>
<tr>
<td>0.25-0.31</td>
<td>5.0</td>
<td>4096</td>
<td>1024</td>
<td>6.25</td>
<td>AB/CN</td>
</tr>
<tr>
<td>0.31-0.54</td>
<td>5.0</td>
<td>4096</td>
<td>1024</td>
<td>6.25</td>
<td>BDF3</td>
</tr>
</tbody>
</table>

### Table III. Numerical results for the maximum vorticity in the positive dipole half \( \omega_d \) and its location \( x_d = (x_d, y_d) \). The location is given relative to the point where the dipole axis crosses the no-slip wall. The \( x_d \)-coordinate measures the distance along the wall and the \( y_d \)-coordinate is the distance normal to the wall. The results are compared to the available benchmark results for a square domain enclosed by no-slip walls as reported in Clercx and Bruneau (Ref. 17).

<table>
<thead>
<tr>
<th>Re</th>
<th>( t )</th>
<th>( x_d )</th>
<th>( y_d )</th>
<th>( \omega_d )</th>
<th>( x_d )</th>
<th>( y_d )</th>
<th>( \omega_d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>⋯</td>
<td>0.0</td>
<td>0.1</td>
<td>1.0</td>
<td>316.84</td>
<td>0.1</td>
<td>1.0</td>
<td>316.84</td>
</tr>
<tr>
<td>625</td>
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<td>0.1656</td>
<td>0.1827</td>
<td>158.70</td>
<td>0.165</td>
<td>0.182</td>
<td>158.9</td>
</tr>
<tr>
<td>625</td>
<td>0.625</td>
<td>0.1655</td>
<td>0.1674</td>
<td>154.22</td>
<td>0.166</td>
<td>0.168</td>
<td>154.2</td>
</tr>
<tr>
<td>625</td>
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<td>0.2543</td>
<td>0.1949</td>
<td>102.64</td>
<td>0.254</td>
<td>0.195</td>
<td>102.6</td>
</tr>
<tr>
<td>625</td>
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<td>0.3068</td>
<td>0.2314</td>
<td>71.03</td>
<td>0.307</td>
<td>0.231</td>
<td>71.0</td>
</tr>
<tr>
<td>1250</td>
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<td>0.1506</td>
<td>0.1260</td>
<td>219.29</td>
<td>0.151</td>
<td>0.126</td>
<td>219.4</td>
</tr>
<tr>
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<td>0.174</td>
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<tr>
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<td>0.292</td>
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<tr>
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<td>261.9</td>
</tr>
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<td>0.1982</td>
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<td>259.59</td>
<td>0.199</td>
<td>0.104</td>
<td>260.0</td>
</tr>
<tr>
<td>2500</td>
<td>1.0</td>
<td>0.2195</td>
<td>0.1738</td>
<td>231.40</td>
<td>0.219</td>
<td>0.174</td>
<td>231.4</td>
</tr>
<tr>
<td>2500</td>
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<td>0.1946</td>
<td>0.2017</td>
<td>201.59</td>
<td>0.195</td>
<td>0.202</td>
<td>201.6</td>
</tr>
<tr>
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<td>0.0948</td>
<td>286.93</td>
<td>0.244</td>
<td>0.097</td>
<td>286.9</td>
</tr>
<tr>
<td>5000</td>
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<td>285.89</td>
<td>0.275</td>
<td>0.116</td>
<td>285.9</td>
</tr>
<tr>
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<td>0.1948</td>
<td>269.01</td>
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<td>269.1</td>
</tr>
<tr>
<td>5000</td>
<td>1.4</td>
<td>0.5141</td>
<td>0.2052</td>
<td>252.08</td>
<td>⋯</td>
<td>⋯</td>
<td>⋯</td>
</tr>
<tr>
<td>10000</td>
<td>0.6</td>
<td>0.2562</td>
<td>0.0961</td>
<td>301.57</td>
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</tr>
<tr>
<td>10000</td>
<td>0.625</td>
<td>0.3032</td>
<td>0.1036</td>
<td>300.59</td>
<td>⋯</td>
<td>⋯</td>
<td>⋯</td>
</tr>
<tr>
<td>10000</td>
<td>1.0</td>
<td>0.6439</td>
<td>0.2738</td>
<td>⋯</td>
<td>⋯</td>
<td>⋯</td>
<td>⋯</td>
</tr>
<tr>
<td>10000</td>
<td>1.4</td>
<td>0.6865</td>
<td>0.3568</td>
<td>⋯</td>
<td>⋯</td>
<td>⋯</td>
<td>⋯</td>
</tr>
</tbody>
</table>
tions that were conducted with a Chebyshev pseudospectral code. These benchmark simulations were performed for a square domain $[-1,1] \times [-1,1]$, enclosed by no-slip walls at all sides. There is a good agreement for the position of the positive dipole half $x_d$ (absolute error smaller than $10^{-3}$) as well as the maximum vorticity $\omega_d$ (better than 0.2% accuracy), confirming the accuracy of the simulations.

The influence of the periodic boundaries at $x=0$ and $x=2$ is small. A validation run has been performed for Re =1250 for a domain with a doubled channel length. The difference in measured positions of the vorticity maximum, $x_d$, does not exceed $1.5 \times 10^{-3}$. This indicates that these boundaries are located sufficiently far from the dipole and thus do not have a substantial impact on the evolution of the dipole for the time interval considered.

### III. GENERAL OVERVIEW

The collision of a dipole with a no-slip wall and the subsequent rebound was first simulated by Orlandi,\textsuperscript{8} for Re values up to 3200. In order to be able to properly compare with the behavior for higher Re values, let us briefly summarize the sequence of events for Re =2500 as is plotted in Fig. 2. In Fig. 3 a schematic representation is given and the different features are labeled using terminology for the right half of the domain. As the dipole approaches the wall, the vorticity in the boundary layers increases in amplitude. As the dipole reaches the wall, the dipole halves separate from each other and subsequently move along the wall in opposite directions. Boundary layer vorticity is advected by the dipole halves and subsequently detached from the wall forming secondary vortices ($\nu-\nu$). The newly formed asymmetric dipoles travel along circular trajectories, leading to another collision with the wall at $t=0.65$. For the special case of Re =2500 the secondary vortices of the left and right dipole match up to form a new dipole at the center line of the domain $x=1$. It is orientated in such a way that it translates away from the boundary along a straight path.

For any two-dimensional viscous flow the total energy $E(t)$ decays according to

$$\frac{dE}{dt} = -\nu \int_D \omega^2 dA = -2\nu \Omega(t).$$

Note that the decay rate is proportional to the total enstrophy, $\Omega(t)$, which is a measure of the (squared) vorticity in the domain. Understanding the evolution of the total enstrophy is therefore of crucial importance for explaining the energy decay. For a domain with no-slip boundaries the change in total enstrophy is governed by

$$\frac{d\Omega}{dt} = -\nu \int_D |\nabla \omega|^2 dA + \nu \int_{\partial D} \omega (\mathbf{n} \cdot \nabla \omega) ds.$$
the flow. The second term represents the vorticity production at the no-slip boundaries. Note that the vorticity influx at the no-slip boundaries is equal to \[ \int_\Gamma \mathbf{n} \cdot \nabla \mathbf{\omega} \, ds. \] In the case of a square domain with stress-free or periodic boundary conditions the second term on the right-hand side of Eq. (6) vanishes. As a result, the total enstrophy cannot increase for a domain with stress-free or periodic boundary conditions and is thus always bounded by its initial value.\(^{24}\)

In Fig. 4 the evolution of the total kinetic energy and total enstrophy are plotted for \( \text{Re}=2500 \). Note that the energy steadily decreases from its normalized initial value of \( E=2 \) towards \( E=1 \) for \( t=2 \). At \( t=0.32 \) the kinetic energy decays faster, which is due to the increased enstrophy on the domain. The peak in the enstrophy coincides with the first collision of the dipole with the wall. During this first collision the boundary layers build up a large amount of vorticity. The enstrophy in the boundary layers is then the main contribution to the total enstrophy. At \( t=0.65 \) another smaller peak is visible, which is due to the second collision. In Sec. IV we will investigate the enstrophy change due to the vorticity flux at the no-slip boundaries, governed by the term \( \int_\Gamma \mathbf{n} \cdot \nabla \mathbf{\omega} \, ds \) in Eq. (6), in more detail.

For Reynolds numbers larger than \( \text{Re}=2500 \), the dynamics is more interesting, in particular after the first collision of the dipole with the no-slip wall. The first differences are visible immediately after the first collision: The vorticity filaments that are detached from the wall are thinner but stronger in amplitude (Fig. 5). The filament rolls up, forming a vortex \( (\nu-) \), and together with one of the primary vortices \( (V+) \) it forms the secondary dipole. This boundary-layer induced vortex has a higher vorticity amplitude than the primary vortex. As a result of the high amplitude, some outer vorticity of the primary vortex is advected around the secondary vortex. After the secondary vortex is created, even more vorticity is detached from the wall, which leads to the creation of multiple small vortices. The halves of the dipole \( (V+) \) separate faster from each other for higher Reynolds number. During the second collision both the primary \( (V+) \) and the secondary vortex \( (\nu-) \) advect boundary layer vorticity away from the wall, resulting in the production of a number of both positive and negative vortices. This is in contrast to the case \( \text{Re}=2500 \), for which the detached boundary-layer vorticity is wrapped around the primary vortex.

### IV. TRAJECTORY OF THE DIPOLE

The trajectory of a dipole approaching a free-slip wall perpendicularly is usually modeled using a set of two point vortices of opposite circulation and two mirror point vortices.\(^{18}\) Each point vortex moves along a hyperbolic path given by

\[
1/x^2 + 1/y^2 = 1/a^2, \tag{7}
\]

---

**FIG. 4.** Time evolution of the total energy \( E(t) \) and the total enstrophy \( \Omega(t) \) for the dipole-wall collision at \( \text{Re} = 2500 \). The first maximum in the enstrophy, \( \Omega_{\text{max}} \), appears during the first collision.

**FIG. 5.** Sequence of vorticity contour plots showing the flow evolution of a dipole colliding with the wall for \( \text{Re} = 5000 \) and 10000. Only the right-hand side symmetrical part of the domain is shown, \( [1.0,1.5] \times [-1,-0.5] \). The contour levels are drawn for ..., \(-100,-60,-20,20,60,100,...\).

---
where \((x, y)\) gives the position with the origin at the intersection of the wall and the symmetry axis. When the point-vortex pair approaches the free-slip wall the vortex separation first slowly increases, but close to the wall the separation speeds up dramatically under the influence of the mirror vortices. Saffman\textsuperscript{25} investigated the effect of a finite core size using elliptically shaped vortex patches in an inviscid fluid. He concluded that the finite size itself does not cause a rigorous departure from the point vortex model, and that deformations of the vortex cores when colliding with a wall cannot explain the rebound.

Following Saffman’s conclusion\textsuperscript{25} it can safely be conjectured that the trajectories of the Lamb-dipole halves for a normal collision with a free-slip wall (i.e., inviscid flow) are properly described by Eq. (7). Here, the point \((x, y)\) represents the location of either the maximum or the minimum vorticity in the Lamb dipole. The presence of finite viscosity will cause the gradual growth of the vortices. We will now briefly describe how the effects of viscosity can be included in Eq. (7). Nielsen and Rasmussen\textsuperscript{26} found that a freely moving Lamb dipole keeps its shape in a viscous fluid, but it gradually expands due to diffusion of vorticity. For the change in radius, \(R\), they found

\[
R(t) = R(t_0) \sqrt{\left(\nu J_1(R_0^2) \right)(t - t_0) + 1}.
\]

with \(R(t_0)\), the radius at \(t_0\), and \(J_1 = 3.8317\), the first zero of the Bessel function, \(J_1\). The distance between the vorticity extrema changes accordingly; thus for the point-vortex model we set

\[
a(t) = a(t_0) \sqrt{\left(\nu \gamma_1^2(R_0^2) \right)(t - t_0) + 1}.
\]

This means that \(a = a(t)\) is now a function of time, i.e., the viscous growth of the Lamb dipole is modeled by a gradual increase of the vortex separation in the point-vortex model. Choosing \(t_0 = 0.15\), i.e., when the shape of the dipole in the simulations is the Lamb-type, we can calculate the value for \(a(t_0)\) by inserting the position of maximum vorticity \(x_m(t=0.15)\) in Eq. (7). If one coordinate of the vortex center, e.g., \(y_d(t)\), is specified, the model provides the other coordinate, \(x_m(t)\), according to

\[
\frac{1}{x_m(t)} = \frac{1}{a^2(t)} - \frac{1}{y_d^2(t)}.
\]

To investigate this model for the trajectory of a Lamb dipole approaching a stress-free wall, we performed a simulation for an equivalent problem of two dipoles colliding head-on for \(Re=1250\). On an expanded domain \([0,4] \times [-1,1]\) a right-traveling dipole is initially located at \((1,0)\) and a left-traveling dipole is initially located at \((3,0)\) (Fig. 6).

The two dipoles will collide at the line \(x=2\), which can be considered to be a stress-free boundary. At this line both the vorticity and the velocity component \(u\) are by definition equal to zero, thus satisfying the stress-free boundary condition. The trajectory of the center of one of the dipole halves is shown in Fig. 7. For convenience the axis is shifted and rotated, so the wall is located at \(y=0\) and the symmetry line at \(x=0\). The hyperbolic trajectory found for the inviscid point vortex model [i.e., with \(a(t)\) constant] already describes the

![Figure 6](image1.png)

**FIG. 6.** A dipole collision with a stress-free wall is studied considering the equivalent problem of two dipoles colliding frontally. At the symmetry line \(x=2\) the stress-free boundary condition is satisfied. Vorticity contour plots are given for \(t=0\) (a) and \(t=0.4\) (b).

![Figure 7](image2.png)

**FIG. 7.** The trajectory of a dipole approaching a stress-free or no-slip wall for \(Re=1250\). (a) For the stress-free case, the position of the maximum vorticity in the core (drawn) is compared to the path of two point vortices (dotted) and the point-vortex path corrected for viscous diffusion (dashed). (b) The path of a dipole colliding with a no-slip wall (black) compared to the path in the stress-free case (gray). Both figures also give the position of the maximum core vorticity at specific times (+○), where the time is denoted by the labels.
path of the Lamb dipole quite well. If the viscous correction for the increase of the vortex cores is used to correct the hyperbola, we obtain a good prediction of the path of a Lamb dipole approaching a stress-free wall. When the dipole halves change orientation in proximity of the wall, the shape does not resemble a Lamb dipole. The viscous correction is thus not valid from $t=0.3$ to $t=0.35$. This does not have a large effect as for $Re=1250$ viscous effects seem to be relatively small [see inset Fig. 7(a)].

In Fig. 7(b) the path of a dipole colliding with a no-slip wall is compared to the stress-free case. In the initial stages both trajectories overlap and the translation speed is equal, which indicates that the effect of the boundary layers is still rather weak. From $t=0.3$ the trajectory of the dipole colliding with the no-slip wall departs from the stress-free case. The dipole comes less close to the no-slip wall than to the stress-free wall at $t=0.34$ as it is hindered by (detached) boundary layer vorticity. Thereafter an asymmetric dipole is formed when the primary vortices $(V^+)$ pair with secondary vortices $(V^-)$. Without any influence of other vortex structures, the asymmetric dipole would follow a circular trajectory. However, mirror vortices (mimicking the influence of the wall) induce a substantial velocity at both the primary and secondary vortex directed along the wall away from the symmetry line. Consequently, the trajectory of the primary vortex core is cycloid-like.

For $Re=1250$ one observes two rebounds [Fig. 8(a)], thereafter the primary vortex core moves slowly along and away from the boundary. Then the primary vortex is surrounded by a ring of opposite vorticity. This shielding causes the very slow translation along the wall. The movement away from the boundary could be caused by the growth in size of the vortex by diffusion, but it is more likely due to the asymmetry of the shielding. When $Re$ is increased to 2500 the primary vortex core follows a cycloidal trajectory with a larger radius after the first collision [Fig. 8(b)]. This implies that the ratio, $\Gamma_1=[\Gamma_{V^-}/||\Gamma_{V+}||]$, between the circulation in the secondary $(V^-)$ and primary vortex $(V^+)$ has increased. In our view the increased relative strength of the secondary vortex stems from the larger amount of vorticity that is created at the no-slip wall for increased Reynolds numbers. In this particular case, the primary vortices remain close to each other near the symmetry line $x=1$. Recall that for $Re=2500$ the secondary vortices form a small dipole at the symmetry line $x=1$, which subsequently translates away from the wall (Fig. 2). The secondary vortex does not re-enter the region.

FIG. 8. Trajectories followed by the positive half of the dipole when colliding with the no-slip boundary for $Re=1250$ (a), 2500 (b), and 5000 (c). The origin of the axes is shifted to coincide with the wall and symmetry axis.

This illustrates that the vorticity flux from the boundaries depends on the tangential pressure gradient at the wall and the boundary acceleration. For a detailed discussion on both these mechanisms, the reader is referred to Morton.27

V. BOUNDARY LAYER FORMATION AND DETACHMENT

Lighthill25 discussed the existence of a diffusive flux of vorticity at a no-slip wall into the flow domain. For a wall moving with velocity $V$ in its own plane we define the $x,y$-coordinate system with the origin at the wall, $x$ along the wall and $y$ normal to it. For the vorticity flux at the wall, $-\nu\partial\omega/\partial y|_{y=0}$, the $x$-component of the Navier–Stokes equation yields

$$
-\nu \frac{\partial \omega}{\partial y} \bigg|_{y=0} = \rho^{-1} \frac{\partial \rho}{\partial x} \bigg|_{y=0} + \frac{dV}{dt}. \tag{11}
$$

This illustrates that the vorticity flux from the boundaries depends on the tangential pressure gradient at the wall and the boundary acceleration. For a detailed discussion on both these mechanisms, the reader is referred to Morton.27

In the early stages the evolution of the dipole did not differ strongly from the stress-free solution. In the interior domain viscous effects are negligible and thus at the edge of the boundary layer the velocity (free-stream) is assumed to be equal to the velocity $u_{sf}$ at a stress-free wall. Before the impact of the dipole the vorticity at the wall can be approximated by $\omega|_{y=0} \sim -u_{sf}/\delta$ with $\delta$ the local thickness of the boundary layer. In Fig. 9 both the vorticity profile and the...
vorticity flux Eq. (11) at the wall are shown for the right half of the domain. The profile of the stress-free velocity is similar to the vorticity at the wall. The exact vorticity profile at the wall is due to the cumulative effect of vorticity production at the wall and of inward diffusion of vorticity. For \( t=0.3 \), the nearby dipole half \((V+)\) causes the production of opposite-signed vorticity at the wall for \( x \gtrsim 0.1 \), as is illustrated by the gray line in Fig. 9. The vorticity at the wall is still of single sign.

Points on the wall where the vorticity changes sign \((\omega|_{y=0}=0)\) are dynamically very important. In these points streamlines start or end, indicating that the flow separates from or attaches to the wall. Thus a small region of opposite boundary layer vorticity (for example, \( b^+ \) in Fig. 3) at the wall indicates the presence of a recirculation cell between the two points on the boundary where \( \omega|_{y=0}=0 \). To understand the formation of a recirculation region we investigate the vorticity fields at the time \( t_{\text{min}} \) when the primary vortex is closest to the wall (Fig. 10). For \( \text{Re}=20000 \) a detailed view of the boundary region is given in Fig. 11 for times around \( t_{\text{min}} \).

At \( t=t_{\text{min}} \) the boundary layer \((B^-)\) is already separating from the wall, although the dipole half \((V+)\) hardly moved along the boundary. For all \( \text{Re} \) values the point of detachment is found about one dipole radius \( R \) from the symmetry line \( x=1 \). The translation speed of the dipole is smaller than

![FIG. 9. Distribution of vorticity \( \omega|_{y=0} \) (black) and vorticity flux, \(-\nu \partial \omega / \partial y|_{y=0} \) (gray), along the wall \((y=0)\) for \( \text{Re}=2500 \) at \( t=0.2,0.3 \). For \( t=0.2 \) the dashed line gives the velocity at the wall corresponding to the stress-free case.](image)

![FIG. 10. (Color) Graphs showing the vorticity field for the time \( t=t_{\text{min}} \) at which the dipole halves are closest to the no-slip wall for different Reynolds numbers. The recirculation cell is represented by the zero contour of the stream function.](image)
the velocity at its edge where it touches the boundary layer. This causes a fast advection of boundary layer vorticity from the rear of the primary vortex to the front. The amount of vorticity in the boundary layer \( \frac{B}{H^{20849}} \) at the front of the dipole is then larger than required to satisfy the no-slip boundary constraint. Oppositely signed vorticity, thus now with the same sign as the vorticity of the primary vortex \( \frac{V}{H^{20849}} + \frac{B}{H^{20850}} \), is produced to balance the situation. Eventually, a secondary boundary layer \( \frac{b}{H^{20849}} + \frac{B}{H^{20850}} \) is formed in front of the primary vortex.

Between the points where \( \frac{y}{H^{20841}} = 0 \), a recirculation cell is formed which decreases in size for larger Re values. This picture is confirmed by investigating the vorticity and vorticity flux at the wall for \( t = t_{\text{min}} \). Recall that \( -\frac{\partial \omega}{\partial y} = \rho^{-1} \frac{\partial p}{\partial x} \) at the wall in the absence of boundary acceleration Eq. (11). The pressure on the wall is in first approximation induced by the primary vortex. In the rear part of the vortex the pressure gradient is negative, and thus the flow is accelerated. In the front part the pressure gradient is positive, and the flow decelerates. This is in agreement with an accelerated advection of boundary-layer vorticity \( B^- \) below the rear part of the dipole half and the pile-up of this vorticity below the front half. The part of the boundary layer that is already detached from the wall also induces a pressure gradient on the wall. This causes the two smaller peaks in the pressure gradient for \( \text{Re} = 2500 \), visible in Fig. 12 located near \( x = 0.2 \).

The head of the detached boundary layer \( B^- \) tends to roll up, which leads to the formation of the secondary vortex \( (v^-) \). For larger Re values this roll-up is much faster as the vorticity in the boundary layer is of substantially larger amplitude. If \( \text{Re} > \mathcal{O}(10^4) \) the roll-up does not only occur at the head of the detached boundary layer, but also in the boundary layer upstream of the point where the detachment occurs. This leads to the formation of multiple small-scale vortices, which are situated between the wall and the primary vortex.

**VI. SHEAR INSTABILITY IN THE BOUNDARY LAYER**

For \( \text{Re} = 20000 \), and to a lesser extent \( \text{Re} = 10000 \), we observe narrow regions of strong eruptions of vorticity (vorticity spikes) that originate at the wall (Fig. 10). A shear instability causes the boundary layer \( (B^-) \) to roll-up even...
before it detaches from the wall (Fig. 11). A necessary condition for the instability to occur is that there is an inflection point in the velocity profile (Rayleigh criterion), which is the case for a boundary layer in an adverse pressure gradient (see e.g., Ref. 28). Furthermore, for the instability to appear the Reynolds number, \( \text{Re}^* = \frac{\bar{u} \delta}{\nu} \), based on the displacement thickness of the boundary layer (\( \delta_B \)) and the free-stream velocity (\( \bar{u} \)), has to reach a critical value. The critical Reynolds number for the instability to occur is about \( \text{Re}^* = 600 \) for a boundary layer in zero pressure gradient.29 For our setup the critical Reynolds number is hard to determine as the instability occurs at a certain time and place. For the run with domain-based Reynolds number \( \text{Re}_D = 20000 \) the velocity induced by the vortex core at the point where the instability occurs measures about \( u^* = 8 \) and the boundary layer thickness is about \( 1.0 \times 10^{-2} \) (measured as the distance from the wall where the vorticity changes sign). If the displacement thickness is estimated to be one-third of the boundary layer thickness (see, for example, Ref. 28), we obtain \( \text{Re}^* = 530 \) (where we used \( \nu = 5 \times 10^{-5} \)) thus supporting the possibility of shear instabilities.

Note that the vortex core induces an adverse pressure gradient along the wall for all investigated Reynolds numbers. Shear instabilities are suppressed by viscosity for \( \text{Re} \approx 5000 \), but for \( \text{Re} \approx 10000 \) the stabilizing effect of viscosity is too small.

In the region where the roll-up of the primary boundary layer (\( B^- \)) occurs a thin secondary boundary layer of opposite-signed vorticity (\( b+ \)) is present between the wall and the primary boundary layer (\( B^- \)) (Fig. 11). The roll-up in the primary vorticity boundary layer and subsequent formation of small vortices causes a deformation of the secondary boundary layer. A little bulge emerges between two vortices. Upstream of the first vorticity spike the primary boundary layer again rolls up leading to a new eruption of vorticity. In this way a series of small-scale vortices is created. As these vortices are formed in the boundary layer itself, a complicated distribution of vorticity is required to keep the velocity zero at the wall. The vorticity distribution and the vorticity production at the wall becomes highly peaked as can be seen in Fig. 12. A number of these small vortices merge to form the secondary vortex, which pairs with the primary vortex.

The occurrence of vorticity spikes at the boundary was first observed by van Dommelen30 and by van Dommelen and Shen31,32 in recirculation cells in the wake of a cylinder. A little bulge emerges between two vortices. Upstream of the first vorticity spike the primary boundary layer again rolls up leading to a new eruption of vorticity. In this way a series of small-scale vortices is created. As these vortices are formed in the boundary layer itself, a complicated distribution of vorticity is required to keep the velocity zero at the wall. The vorticity distribution and the vorticity production at the wall becomes highly peaked as can be seen in Fig. 12. A number of these small vortices merge to form the secondary vortex, which pairs with the primary vortex.

The size and strength of the vortices formed from wall-generated vorticity depend on the thickness of the boundary layer and the vorticity amplitude. If the shear inside the boundary layer is strong, vorticity eruptions occur, creating small vortices with a large vorticity amplitude. For weaker boundary layers, a vortex is formed from the detached boundary layer. The strength of these secondary vortices depends on whether they originate from the small-scale vorticity eruptions or from the large-scale boundary-layer detachment and thus on the strength of the primary boundary layer.

We have determined the circulation of the primary boundary layer, \( \Gamma_B^- \), and the secondary boundary layer, \( \Gamma_{b+} \), by numerically integrating the vorticity in these regions. Separate contiguous regions can be distinguished where the absolute vorticity is larger than a certain level (\( \omega = \omega_s/50 \)). A factor 2 reduction of the selection level leads only to a small increase of the circulation by 0.5% or less. The circulation of the different regions are given in Table IV. Note that when the dipole reaches the wall, it is considerably weaker for lower Reynolds numbers, e.g., the circulation \( \Gamma_{v_d} \) of a dipole half is 2.76 for \( \text{Re} = 625 \), while it is 3.71 for \( \text{Re} = 20000 \). To partially counteract this problem we also give the ratio between the circulation in the primary vortex (\( V^+ \)) and the circulation related to the opposite vorticity that is created at the wall \( V^- \), \( \Gamma_{b-} \), i.e., a combination of the circulation of the secondary vortex (\( v^- \)) and the circulation of the (partially) detached boundary layer (\( B^- \)). For \( \text{Re} \approx 10000 \) the circulation in the boundary layer seems to become virtually
TABLE IV. Specification of the circulation $\Gamma_{V_k}$ of the positive dipole half, circulations $\Gamma_{V_{-\kappa}}$ and $\Gamma_{V_\kappa}$ in the primary and secondary boundary layers, respectively. At $t=0.5$ the circulation $\Gamma_{V_{-\kappa}}$ of the secondary vortex is also given. In Table V the values for $t_{\text{min}}$ are specified.

<table>
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<th>Re</th>
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<th>$\Gamma_{V_k}$</th>
<th>$\Gamma_{V_{-\kappa}}$</th>
<th>$\Gamma_{\bar{V}_{-\kappa}}$</th>
<th>$\Gamma_{\bar{V}_k}$</th>
<th>$\Gamma_{V_{-\kappa}}+\Gamma_{V_k}$</th>
<th>$\Gamma_{V_{-\kappa}}+\Gamma_{\bar{V}_k}$</th>
<th>$\Gamma_{V_{-\kappa}}$</th>
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<td>···</td>
<td>0.003</td>
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<td>···</td>
<td>0.047</td>
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<td>0.787</td>
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-independent of the Reynolds number, as it only changes from $\Gamma_{\bar{V}_{-\kappa}}=-2.80$ to $-2.92$ when we increase from Re=10000 to 20000. This can be expected, as the pressure gradient along the wall Eq. (11) determines the vorticity production. Note that the ratio $|\Gamma_{\bar{V}_{-\kappa}}+\Gamma_{V_k}|/|\Gamma_{V_k}|$ increases even if the primary vortex core moves away from the wall. Thus vorticity is still being produced during this stage of the evolution. A part of the vorticity that is created at the wall will end up in the secondary vortex. The strength of this vortex is highly dependent on the strength of the boundary layers, and thus increases for higher Reynolds numbers.

- From $t=t_{\text{min}}$ to $t=0.5$ the circulation in the primary vortex decreases due to cross-diffusion with boundary-generated vorticity. An equal amount of positive and negative vorticity is lost, thus the circulation related to boundary-produced vorticity has changed by an amount $\Gamma_{V_k}(t=0.5)-\Gamma_{V_k}(t_{\text{min}})$. If we add this number to $\Gamma_{\bar{V}_{-\kappa}}+\Gamma_{V_k}$, we can correct for the loss due to cross-diffusion. The resulting circulation related to boundary-produced vorticity is $-3.22$ for Re=10000. Compare this to the circulation in the primary vortex at $t_{\text{min}}$, $\Gamma_{V_k}=+3.64$. Thus a nearly equal amount of circulation is generated at the wall, as is present in the primary vortex. For lower Reynolds numbers the difference is larger, e.g., $-2.18$ generated against $+2.76$ in the primary vortex for Re=625. After the first collision multiple small vortices are generated if Re=20000, and the vorticity distribution becomes too complex to measure the circulation of the various structures. Clercx and van Heijst found that the circulation in the boundary layer becomes independent of the Reynolds number for Re $\approx$ 20000. It is thus likely that the amount of vorticity generated during the first collision does not exceed the circulation of the primary vortex.

- The secondary vortices are stronger for larger Reynolds numbers. This also leads to an increased ratio between the strength of the secondary and primary vortex, $|\Gamma_{\bar{V}_{-\kappa}}|/|\Gamma_{V_k}|$, if the Reynolds number is increased to 5000. For Re=10000 secondary vortices are also created by the shear instability in the boundary layer. Some of the smaller vortices merge to form one secondary vortex. The relative strength of this vortex is higher than for Re=5000, as reflected by the higher ratio $|\Gamma_{\bar{V}_{-\kappa}}|/|\Gamma_{V_k}|$.

It is interesting to see how the enstrophy changes due to the production of vorticity at the wall. This change is governed by Eq. (6), where the term

$$\int_{\partial p} \omega (n \cdot \nabla \omega) ds$$

represents the enstrophy increase or decrease by vorticity flux at a straight wall. For the dipole-wall collision, the enstrophy is always increased by the production of vorticity at the wall (Fig. 13). The integrand in Eq. (12) is not necessarily positive for all points on the wall, e.g., in Fig. 9 for $t=0.3$ there is a region where the vorticity $\omega$ at the wall is positive while there is an influx of negative vorticity. The peaks in the enstrophy production coincide with the time the primary vortices are close to the wall. If they move away from the wall the enstrophy production drops to zero, reflecting that there is minimal vorticity production at the wall.

![Graphs showing the change in total enstrophy due to vorticity production at the wall for Re=2500 and 10000.](image-url)
The enstrophy that is located in the boundary layers for the time the primary vortex is closest to the wall is specified in Table V. Clercx and van Heijst\textsuperscript{10} found that the peak enstrophy scales like $\Omega_{\text{max}} \sim R \Omega_{\text{h}}^{0.5}$ for $R > 20000$. The peak in the enstrophy occurs when the primary vortex is closest to the wall. For lower Reynolds numbers they found that the peak enstrophy decreases much faster with decreasing Re, consistent with our observation of a decrease faster than $R^{-1.0}$ between $R=1250$ and 625. The scaling relation, $\Omega_{\text{max}} \sim R^{0.5}$, is based on the assumption that the circulation in the boundary layer is independent of the Reynolds number. An assumption that seems to be reasonable, in agreement with the minor change of the circulation of the boundary layer as observed in the previous section when $R$ is changed from 10000 to 20000. The deviation at lower Reynolds numbers is not surprising, as viscous diffusion weakens the primary vortex considerably before it reaches the wall. The approach of the dipole to the wall for $R \approx 20000$ depends less on the Reynolds number; thus the interior flow can be considered to be the same. However, the observed scaling behavior at large Reynolds numbers is quite surprising as the boundary layer itself is very active at $R=20000$ and does not resemble a laminar single-signed vorticity boundary layer as assumed for the model. We also observe a strong increase of enstrophy contained by the secondary boundary layer, changing from 0.5% to 20%, relative to the enstrophy in the primary boundary layer, when Reynolds is increased from 625 to 20000.

<table>
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<th>$t_{\text{min}}$</th>
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<th>$\Omega_{\text{h}}^{-}$</th>
<th>$\Omega_{\text{h}}^{0}$</th>
<th>$t_{\text{max}}$</th>
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<tr>
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<td>3559</td>
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<td>3544</td>
<td>1102</td>
<td>0.3129</td>
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TABLE V. For time $t_{\text{min}}$ the dipole halves are closest to the wall, the total enstrophy is subdivided in the enstrophy of the dipole half $\Omega_{\text{h}}^{+}$, of the primary boundary layer $\Omega_{\text{h}}^{-}$, and of the secondary boundary layer $\Omega_{\text{h}}^{0}$. As a comparison, the time $t_{\text{max}}$ is given when the total enstrophy $\Omega_{\text{max}}$ is maximum.

Secondary vortices are created from the vorticity generated at the wall by two different mechanisms. For Reynolds numbers smaller than $O(10^4)$, the detached boundary layer rolls up and forms a single vortex. If $R > O(10^4)$ a shear instability occurs in the boundary layer. As a result strong vorticity eruptions occur in the secondary boundary layer that cut off parts of the primary boundary layer. Multiple small-scale vortices are created between a dipole half and the no-slip wall, which later merge to one secondary vortex.

The secondary vortex forms together with the primary vortex and an asymmetric dipole that translates along a cycloidal trajectory. For $R \approx 2500$ the dipole halves are shielded by opposite vorticity in the later stages, slowing down the translation along the wall. For larger Reynolds numbers, the separation between the two dipole halves increases much faster, though still not as fast as observed when the stress-free conditions are applied at the wall.

The increased separation speed is of some interest to the problem of airplane trailing vortices that interact with the ground during touchdown. It is known that these vortices can hover (in the absence of a strong cross-wind) for a considerable time over the landing strip, thereby hindering the landing of following airplane. The $R$ value of the present results is smaller than typical for trailing vortices, i.e., $R=5 \times 10^6$. In the results of our simulations we observe that the separation between the two dipole halves increases faster for higher Reynolds number. The path that the dipole half follows becomes irregular for $R=5000$. This is due to the multiple interactions with secondary and tertiary wall-generated vortices. Spalart\textsuperscript{35} uses simulations with different turbulence models to obtain results for $R=5 \times 10^6$. At this Reynolds number they also observe the viscous rebound of the dipole. It depends on the used turbulence model whether the typical cycloidal trajectory is retrieved. This emphasizes the importance to resolve the fine-scale structures near the wall. The narrow vorticity eruptions that we observe for $R \approx 10000$ might have an influence on the trajectory of the separated dipole halves. Three-dimensional instabilities might also have a considerable effect on the dipole-wall collision and in particular on the lifetime of the vortices.

Of interest to two-dimensional turbulence is the detachment of wall-generated vorticity. Clercx and van Heijst\textsuperscript{1} argue that this injection of small-scale vorticity acts as a forcing of the interior flow at a forcing scale comparable to the boundary-layer thickness. In their numerical simulations of
decaying two-dimensional turbulence in a bounded domain they observed an inverse energy cascade starting at the boundary-layer scale. This is opposed to numerical simulations on a periodic domain, where in the same range a direct enstrophy cascade was observed. Later, Wells et al.\(^9\) provided more proof with a numerical simulation in which the injection of wall-generated vorticity was the only mechanism of forcing the interior flow. The size and strength of the vortices that interact with the wall in the turbulence simulations correspond to the lower Reynolds number dipole-wall collisions (typically Re \(\leq 2500\)). For higher Reynolds number it can be stated that the vortices injected into the flow become smaller but have a higher vorticity amplitude. This is of importance, as boundary-generated vortices can destabilize even domain-sized vortices.\(^{36,37}\)

The question arises if the stronger wall-generated vortices can even prevent the formation of a domain-sized structure. If the Reynolds number is increased even further, e.g., Re \(> 10000\), the eruption of vorticity spikes at the boundary leads to the formation of multiple vortices. The small-scale vorticity spikes can bring the forcing effects of the wall to even larger wave numbers.

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