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## Stationary Velocity and Charge Distributions of Grains in Dusty Plasmas

A. G. Zagorodny,<sup>1</sup> P. P. J. M. Schram,<sup>2</sup> and S. A. Trigger<sup>3</sup>

<sup>1</sup>*Bogolyubov Institute for Theoretical Physics, National Academy of Sciences of Ukraine,  
14 B, Metrolohichna Street, Kiev 03143, Ukraine*

<sup>2</sup>*Eindhoven University of Technology, P.O. Box 513, MB 5600 Eindhoven, The Netherlands*

<sup>3</sup>*Institute for High Temperatures, Russian Academy of Sciences, 13/19, Izhorskaia Street, Moscow 127412, Russia*  
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Within the kinetic approach the velocity and the charge distributions of grains in stationary dusty plasmas are calculated and the relations between the effective temperatures of such distributions and plasma parameters are established. It is found that the effective temperature which determines the velocity grain distribution could be anomalously large due to the action of accelerating ionic bombarding force. The possibility to apply the results obtained to the explanation of the increasing grain temperature in the course of Coulomb-crystal melting by reduction of the gas pressure is discussed.

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Recently much attention has been paid to theoretical studies of various problems of dusty plasma physics associated with grain dynamics and grain charging. In such studies, it is convenient to treat the grain charge as a new variable (as was done for the first time in Ref. [1]). This makes it possible to statistically describe the grain charge distribution on equal footing with the spatial and velocity grain distributions. Obviously, it is very important to know the stationary (quasiequilibrium) grain distributions and the relation of these distributions to plasma parameters. In spite of the fact that statistical descriptions of dusty plasmas have been already used in many papers, as far as the authors of this Letter know, neither grain charge nor velocity distributions for grains were studied within a consistent kinetic approach. Usually, the problem is avoided by neglecting the thermal dispersion of grain velocity and charge.

The purpose of the present paper is to describe stationary velocity and charge distributions of grains in dusty plasmas in the case of grain charging by plasma currents and to determine the dependences of effective temperatures on plasma parameters. We study dusty plasma consisting of electrons, ions, neutral molecules, and monodispersed dust particles (grains) assuming that every grain absorbs all encountered electrons and ions. Such collisions we define as *charging collisions*. Collisions in which plasma particles do not touch the grain surface we call *Coulomb elastic collisions*.

Using the microscopic equations for dusty plasmas and the relevant Bogoliubov-Born-Green-Kirkwood-Yvon hierarchy [2], it is possible to show that in the case of dominant charging collisions (Coulomb elastic collisions and collisions with neutrals are neglected; the condition of such approximation will be given below) the kinetic equation for the grain distribution function  $f_g(X, t) \equiv f_g(\mathbf{r}, \mathbf{v}, q, t)$  ( $q$  is the charge of the grain) can be written as

$$\left\{ \frac{\partial}{\partial t} + \mathbf{v} \cdot \frac{\partial}{\partial \mathbf{r}} + \frac{q}{m_g} \mathbf{E} \cdot \frac{\partial}{\partial \mathbf{v}} \right\} f_g(X, t) = - \sum_{\sigma=e,i} \int d\mathbf{v}' [\sigma_{g\sigma}(q, \mathbf{v} - \mathbf{v}') |\mathbf{v} - \mathbf{v}'| f_g(X, t) - \sigma_{g\sigma}(q - e_\sigma, \mathbf{v} - \mathbf{v}' - \delta\mathbf{v}_\sigma) |\mathbf{v} - \mathbf{v}' - \delta\mathbf{v}_\sigma| \times f_g(\mathbf{r}, \mathbf{v} - \delta\mathbf{v}_\sigma, q - e_\sigma, t)] f_\sigma(\mathbf{r}, \mathbf{v}', t), \quad (1)$$

where  $\sigma_{g\sigma}(q, \mathbf{v})$  is the cross section for charging:

$$\sigma_{g,\sigma}(q, \mathbf{v}) = \pi a^2 \left( 1 - \frac{2e_\sigma q}{m_\sigma v^2 a} \right) \theta \left( 1 - \frac{2e_\sigma q}{m_\sigma v^2 a} \right), \quad (2)$$

$\theta(x)$  is the Heaviside step function,  $a$  is the grain radius,  $f_\sigma(\mathbf{r}, \mathbf{v}, t)$  is the plasma particle distribution function normalized by the particle density  $n_\sigma$ ,  $\delta\mathbf{v}_\sigma \equiv (m_\sigma/m_g)\mathbf{v}'$  is the grain velocity change due to the collision with a plasma particle, subscript  $\sigma$  labels plasma particle species, and the rest of the notation is traditional.

Equation (1) could be introduced also on the basis of physical arguments as was done in Refs. [3,4]. In fact, the right-hand part of Eq. (1) describes the balance between the grains outgoing from the phase volume element and those incoming to the same element due to charging collisions.

Taking into account the smallness of  $e_\sigma$  and  $\delta\mathbf{v}_\sigma$ , it is possible to expand the right-hand part of Eq. (1) into a power series of these quantities. With the accuracy up to the second order Eq. (1) in the stationary isotropic and homogeneous case is reduced to

$$\frac{\partial}{\partial \mathbf{v}} \left[ \frac{\partial}{\partial \mathbf{v}} [D_{||} f_g(\mathbf{v}, q)] + \beta \mathbf{v} f_g(\mathbf{v}, q) + \frac{\partial}{\partial q} [q \gamma \mathbf{v} f_g(\mathbf{v}, q)] \right] + \frac{\partial}{\partial q} \left[ \frac{\partial}{\partial q} [Q f(\mathbf{v}, q)] - I f_g(\mathbf{v}, q) \right] = 0, \quad (3)$$

where  $D_{\parallel}$ ,  $\beta$ ,  $Q$ ,  $\gamma$ , and  $I$  are the Fokker-Planck kinetic coefficients dependent on velocity and charge and given by

$$\begin{aligned} D_{\parallel} &= \sum_{\sigma} \frac{1}{2} \left( \frac{m_{\sigma}}{m_g} \right)^2 \int d\mathbf{v}' \frac{(\mathbf{v} \cdot \mathbf{v}')^2}{v^2} \\ &\quad \times w(q, \mathbf{v} - \mathbf{v}') f_{\sigma}(\mathbf{v}'), \\ \beta &= -\sum_{\sigma} \frac{m_{\sigma}}{m_g} \int d\mathbf{v}' \frac{\mathbf{v} \cdot \mathbf{v}'}{v^2} \\ &\quad \times w(q, \mathbf{v} - \mathbf{v}') f_{\sigma}(\mathbf{v}'), \\ \gamma &= \sum_{\sigma} \frac{m_{\sigma}}{m_g} \frac{e_{\sigma}}{q} \int d\mathbf{v}' \frac{\mathbf{v} \cdot \mathbf{v}'}{v^2} \\ &\quad \times w(q, \mathbf{v} - \mathbf{v}') f_{\sigma}(\mathbf{v}'), \\ Q &= \sum_{\sigma} \frac{e_{\sigma}^2}{2} \int d\mathbf{v}' w(q, \mathbf{v} - \mathbf{v}') f_{\sigma}(\mathbf{v}'), \\ I &= \sum_{\sigma} e_{\sigma} \int d\mathbf{v}' w(q, \mathbf{v} - \mathbf{v}') f_{\sigma}(\mathbf{v}'), \end{aligned} \quad (4)$$

$$w(q, \mathbf{v} - \mathbf{v}') = |\mathbf{v} - \mathbf{v}'| \sigma_{g\sigma}(q, \mathbf{v} - \mathbf{v}').$$

Deriving the relation for  $\beta$  we omit the terms of higher order in  $(m_{\sigma}/m_g)$  associated with the tensor nature of the diffusion coefficient in velocity space.

With regard for the fact that  $|I(q, v)/Q(q, v)| \rightarrow \infty$  at  $e_{\sigma}/q \rightarrow 0$  and  $|\beta(q, v)/D_{\parallel}(q, v)| \rightarrow \infty$  at  $(m_{\sigma}/m_g) \rightarrow 0$ , the asymptotical solution of Eq. (3) can be written as

$$f_g(\mathbf{v}, q) = n_{0g} Z Q^{-1} e^{-W(q, v) + \lambda v^2} D_{\parallel}^{-1} e^{-V(q, v) + \varepsilon \delta[q - q(v)]^2}, \quad (5)$$

where

$$T_{\text{eff}}(q) = \frac{2T_i(t+z)}{t-z + \frac{(q-q_0)}{q_0} z \left\{ 1 + \frac{t-z}{t+z} \left[ 1 + \frac{2Z_i}{1+Z_i} (1+t+z) \right] \right\}}, \quad (10)$$

$$\tilde{T}_{\text{eff}} = \frac{2}{1+Z_i} \frac{1+t+z}{t+z} T_e, \quad (11)$$

and

$$\begin{aligned} D_{\parallel} &\approx D_0 \left[ 1 + \frac{q-q_0}{q_0} \frac{z}{t+z} \right] \left( 1 + \frac{z}{t} \right), \\ Q &= Q_0 \left[ 1 - \frac{q-q_0}{q_0} \frac{z(t+z-Z_i)}{(t+z)(1+Z_i)} \right] \\ &\quad \times (t+z)(1+Z_i), \\ D_0 &= \frac{4}{3} \sqrt{2\pi} \left( \frac{m_i}{m_g} \right) \left( \frac{T_i}{m_g} \right) a^2 n_i S_i, \\ Q_0 &= \sqrt{2\pi} \left( \frac{T_e}{T_i} \right) e_i^2 a^2 n_i S_i. \end{aligned} \quad (12)$$

$n_{0g}$  is the averaged number density of grains.  $z = e_e^2 Z_g / a T_e$ ,  $t = T_i / Z_i T_e$ ,  $S_{\sigma}^2 = T_{\sigma} / m_{\sigma}$ ,  $Z_g = q_0 / e_e$ ,  $Z_i = |e_i / e_e|$ , and the quantity  $q_0$  is the equilibrium grain charge of stationary particles satisfying the equation

$$\begin{aligned} W(q, v) &= -\int_0^q dq' \left/ \frac{I(q', v)}{Q(q', v)} \right., \\ V(q, v) &= \int_0^v dv' \frac{v'}{D_{\parallel}(q, v')} \\ &\quad \times \left\{ \beta(q, v') + \frac{\partial}{\partial q} [q\gamma(q, v')] - q\gamma(q, v') \right. \\ &\quad \left. \times \left[ \frac{\partial W(q, v')}{\partial q} + Q^{-1}(q, v') \frac{\partial Q(q, v')}{\partial q} \right] \right\}, \end{aligned} \quad (6)$$

$q(v)$  is the stationary charge of the grain moving with the velocity  $v$ , given by the equation

$$I[q(v), v] = 0, \quad (7)$$

$Z$  is a normalization constant,

$$\begin{aligned} \varepsilon &= \frac{1}{D_{\parallel}} \frac{\partial D_{\parallel}}{\partial q} + \frac{\partial V}{\partial q}, \\ \lambda &= \frac{1}{2v} \left\{ \frac{1}{Q} \frac{\partial Q}{\partial v} + \frac{\partial W}{\partial v} + \varepsilon \frac{\partial q(v)}{\partial v} \right\}. \end{aligned} \quad (8)$$

Equations (5)–(8) give the asymptotically exact solution of Eq. (3) at  $(m_g e_{\sigma} / m_{\sigma} q) \rightarrow \infty$ .

Further estimates require the explicit form of the kinetic coefficients. Assuming that plasma particle distributions are Maxwellian, one obtains

$$f_g(\mathbf{v}, q) = n_{0g} Z^{-1} D_{\parallel}^{-1} e^{-\{(m_g v^2)/[2T_{\text{eff}}(q)]\}} \times Q^{-1} e^{-[(q-q_0)^2/(2a\tilde{T}_{\text{eff}})]}, \quad (9)$$

where

$$\begin{aligned} I(q_0, 0) &= 2\sqrt{2\pi} a^2 e_i^2 n_i S_i \\ &\quad \times \left[ 1 + \frac{z}{t} \right. \\ &\quad \left. - \left( \frac{m_i}{m_e} \right)^{1/2} \left( \frac{T_e}{T_i} \right)^{1/2} \frac{n_e}{Z_i n_i} e^{-z} \right] = 0. \end{aligned} \quad (13)$$

For typical values of plasma parameters in dusty plasma experiments ( $t+z > 1$ ) and  $Z_i = 1$ , we have  $\tilde{T}_{\text{eff}} \approx T_e$ . In such a case, the thermal variation of the grain charge  $|q - q_0|^2$  is of the order of  $aT_e$  and  $|q - q_0|z \sim q_0 \sqrt{e_e^2 / aT_e}$ . This means that at weak plasma coupling ( $e_e^2 / aT_e \ll 1$ ) the effective temperature  $T_{\text{eff}}(q) \equiv T_{\text{eff}}$  reduces to

$$T_{\text{eff}} \approx 2T_i \frac{t+z}{t-z}, \quad (14)$$

and

$$D \approx D_0 \left( 1 + \frac{z}{t} \right), \quad Q \approx Q_0 (t+z)(1+Z_i).$$

Thus, in such a case,

$$f_g(\mathbf{v}, q) = \frac{n_{0g}}{\sqrt{2\pi a \tilde{T}_{\text{eff}}}} e^{-(q-q_0)^2/(2a\tilde{T}_{\text{eff}})} \times \left(\frac{m_g}{2\pi T_{\text{eff}}}\right)^{3/2} e^{-m_g v^2/2T_{\text{eff}}}. \quad (15)$$

This distribution describes the equilibrium Maxwellian velocity distribution and the Gibbs grain charge distribution with the temperatures  $T_{\text{eff}}$  and  $\tilde{T}_{\text{eff}}$ , respectively. At  $t < 1$ ,  $z < 1$   $\tilde{T}_{\text{eff}}$  exceeds the electron temperature.

The resulting velocity distribution is described by the effective temperature  $T_{\text{eff}}$ . Even in the case of neutral grains ( $z = 0$ ) this temperature is equal to  $2T_i$ . The presence of the factor 2 is associated with plasma particle absorption by grains. Charging collisions are inelastic, and a part of the kinetic energy of the ions is transformed into additional kinetic energy of the grains. This is the difference between the case under consideration and conventional Brownian motion where the velocity distribution is described by the temperature of the bombarding light particles.

Equation (14) shows that within the approximation of negligible influence of elastic Coulomb collisions the effective temperature could be enhanced at  $z \rightarrow t$ , or even become negative at  $z > t$ . As will be seen later, the condition of such an enhancement is considerably modified by the Coulomb elastic collision, but within the model under consideration this effect is especially well pronounced. Physically it can be explained by the decrease of the friction coefficient with an increase of grain charge:

$$\beta \approx \frac{2}{3} \sqrt{2\pi} \left(\frac{m_\sigma}{m_g}\right) a^2 n_i S_i \left(1 - \frac{z}{t}\right) = \beta_0 \left(1 - \frac{z}{t}\right).$$

The reason is that the difference between the fluxes of ions bombarding the grain surface antiparallel to the grain motion and parallel decreases with the charge increase due to the specific properties of the ionic charging cross section, whose charge-dependent part is larger for ions moving with smaller relative velocities (i.e., in the parallel direction). The condition  $z = t$  corresponds to the zero value of the friction force.

The negative values of the effective temperature  $T_{\text{eff}}$ , if they occur, mean that the system is thermodynamically unstable and the approximation of dominant charging collisions is no longer valid; i.e., elastic Coulomb collisions and collisions with neutrals should be taken into account. In order to involve elastic collisions into consideration, Eqs. (1) and (3) should be supplemented by the appropriate collision terms. We use the Balescu-Lenard collision integral in the Fokker-Planck form:

$$\left(\frac{\partial f_g}{\partial t}\right)^C = \frac{\partial}{\partial \mathbf{v}} \cdot \left[ \frac{\partial}{\partial \mathbf{v}} [D_{\parallel C} f_g(\mathbf{v}, q)] + \mathbf{v} \beta_C f_g(q, v) \right], \quad (16)$$

where  $D_{\parallel C}$  and  $\beta_C$  are the Fokker-Planck coefficients related to Coulomb elastic collisions (see, for example, [5], Chap. 8). With the accuracy up to the dominant logarithmic

terms [in this approximation Eq. (16) is reduced to the Landau collision term],

$$D_{\parallel C} \approx \frac{4}{3} \frac{\sqrt{2\pi} q^2}{m_g^2} \sum_{\sigma=e,i} \frac{n_\sigma e_\sigma^2}{S_\sigma} \ln \Lambda_\sigma \left(1 - \frac{v^2}{5S_\sigma^2}\right),$$

$$\beta_C \approx \frac{4}{3} \frac{\sqrt{2\pi} q^2}{m_g} \sum_{\sigma=e,i} \frac{n_\sigma e_\sigma^2}{S_\sigma^3 m_\sigma} \ln \Lambda_\sigma \left(1 - \frac{v^2}{5S_\sigma^2}\right). \quad (17)$$

In Eqs. (16) and (17) we again neglect the contribution of the transverse part of the diffusion coefficient and disregard the grain-grain Coulomb collisions, assuming the grain density to be small [ $n_g < n_i (Z_i/Z_g)^2 (S_g/S_i)^{1/2} (T_g/T_i)$ ]. We introduced also the Coulomb logarithms  $\ln \Lambda_\sigma$  for each particle species. Usually these quantities are estimated as  $\ln \Lambda_\sigma = \ln(k_{\text{max}}/k_D)$ , where  $k_D = r_D^{-1} = [\sum (4\pi e_\sigma^2 n_\sigma / T_\sigma)]^{1/2}$  and  $k_{\text{max}}$  is the inverse distance of closest approach between colliding particles,

$$k_{\text{max}} \sim \frac{m_\sigma v^2}{|e_\sigma q|} \sim \frac{8T_\sigma}{\pi |e_\sigma q|} = r_{Li}^{-1}. \quad (18)$$

(Here and in what follows, we approximate  $v$  the mean velocity  $\langle v_\sigma \rangle = \sqrt{\frac{8}{\pi} \frac{T_\sigma}{m_\sigma}}$  [6].) However, in the case of plasma particle collisions with finite-size grains this estimate could be invalid, since at  $r_{L\sigma} < a$  the Coulomb logarithm will include the contribution of collisions with particles reaching the grain surface, i.e., charging collisions.

An appropriate modification of  $\Lambda_\sigma$  is achieved by treating  $\ln \Lambda_\sigma$  as a logarithmic factor appearing in the momentum transfer cross section for Coulomb collisions.

$$\ln \Lambda_\sigma = \frac{1}{2} \ln \frac{1 + b_{\text{max}\sigma}^2 \left(\frac{e_\sigma q}{m_\sigma v^2}\right)^2}{1 + b_{\text{min}\sigma}^2 \left(\frac{e_\sigma q}{m_\sigma v^2}\right)^2},$$

where  $b_{\text{min}\sigma}$  and  $b_{\text{max}\sigma}$  are the minimal and maximal impact parameters. Following [6] we find  $b_{\text{min}\sigma}$  from the condition that the distance of closest approach is equal to  $a$  implying

$$b_{\text{min}\sigma} = a \sqrt{1 - \frac{2e_\sigma q}{m_\sigma v^2 a} \theta \left(1 - \frac{2e_\sigma q}{m_\sigma v^2 a}\right)}. \quad (19)$$

Concerning the quantity  $b_{\text{max}\sigma}$ , it is reasonable to put  $b_{\text{max}\sigma} = r_D + a$  instead of  $b_{\text{max}\sigma} = r_D$ , since in the case of a finite-size grain its screened potential is given by the Derjaguin-Landau-Verway-Overbeck potential

$$\Phi(r) = \frac{q}{r} \left(1 + \frac{a}{r_D}\right)^{-1} e^{-(r-a)/r_D}.$$

As a result we have

$$\ln \Lambda_i = \frac{1}{2} \ln \frac{(r_D + a)^2 + r_{Li}^2}{(r_{Li} + a)^2}. \quad (20)$$

In the case  $r_D \gg a$ , Eq. (20) is in agreement with the results of Ref. [6] provided the surface grain potential is much larger than the plasma potential. As is seen, at  $r_{Li} \gg r_D$  the ionic Coulomb logarithm can be a small quantity in contrast to the case of ideal plasmas.

Comparing Eqs. (3) and (16), it is easy to see that in order to take elastic Coulomb collisions into account it

is sufficient to make the following replacements in the obtained solutions:

$$\begin{aligned} D_{\parallel}(q, v) &\rightarrow \tilde{D}_{\parallel}(q, v) = D_{\parallel}(q, v) + D_{\parallel C}(q, v), \\ \beta(q, v) &\rightarrow \tilde{\beta}(q, v) = \beta(q, v) + \beta_C(q, v). \end{aligned} \quad (21)$$

In the case of weak plasma coupling ( $e_e^2/aT_e \ll 1$ ),

$$\begin{aligned} \tilde{D}_{\parallel}(q, v) &\approx D_0 \left( 1 + \frac{z}{t} + \frac{z^2}{t^2} \ln \Lambda_i \right), \\ \tilde{\beta}(q, v) &\approx \beta_0 \left( 1 - \frac{z}{t} + 2 \frac{z^2}{t^2} \ln \Lambda_i \right). \end{aligned} \quad (22)$$

Thus, the correction produced by the elastic collisions could be of the same order as that due to charging collisions. The condition for dominant influence of charging collisions is  $|1 - z/t| > 2z^2/t^2 \ln \Lambda_i$ , which can be realized at small values of  $z/t$ , or at  $z/t \gtrsim r_D^2/a^2 \gg 1$  ( $r_{Li} \gtrsim r_D^2/a \gg a$ ).

Rigorously speaking, Eq. (16) and thus Eq. (22) are definitely valid in the case of weak coupling plasmas ( $r_{Li} \ll r_D$ ) since this is the condition of the derivation of the Balescu-Lenard collision term. However, it is possible to expect that actually the domain of validity of Eqs. (16) and (22) is not too strongly restricted by such a condition. This assumption is in agreement with the direct calculations of the friction coefficient (Coulomb collision frequency) in terms of the binary collision cross sections. Also, as it was shown in Ref. [7], in the case of strong grain-plasma coupling the influence of the Coulomb collision is small and the kinetic equation is reduced again to the Vlasov equation. This means that fluctuation evolution equations, whose solutions determine the explicit form of the Balescu-Lenard collision term, are the same as in the case of weakly coupled plasmas, and thus Eq. (16) continues to be valid.

The new kinetic coefficients give the following effective temperature for thermal grain motion:

$$T_{\text{eff}} = T_i \frac{2(1 + \frac{z}{t} + \frac{z^2}{t^2} \ln \Lambda_i)}{1 - \frac{z}{t} + 2 \frac{z^2}{t^2} \ln \Lambda_i}, \quad (23)$$

i.e., elastic collisions can produce a saturation of the grain temperature. However, in the case of a dominant influence of charging collision  $T_{\text{eff}}$  can be still anomalously large. This fact can be used for a qualitative explanation of the experimentally observed grain temperatures which are usually much higher than the ion temperature,  $T_g \gg T_i$  (see, for example, [8,9],  $T_i \sim 0.1$  eV,  $T_g \sim 4-40$  eV).

Finally, we point out that the obtained results can be modified also for the case of a plasma with a neutral com-

ponent. Since the collision integral describing elastic collisions of neutrals with grains also can be represented in the Fokker-Planck form (it follows from the Boltzmann collision integral), the presence of neutrals results in new additions to  $\tilde{D}_{\parallel}$  and  $\tilde{\beta}$ . As a result,

$$T_{\text{eff}} = 2T_i \frac{[1 + \frac{z}{t} + \frac{z^2}{t^2} \ln \Lambda_i + \frac{n_n}{n_i} (\frac{m_n}{m_i})^{1/2} (\frac{T_n}{T_i})^{3/2}]}{[1 - \frac{z}{t} + 2 \frac{z^2}{t^2} \ln \Lambda_i + 2 \frac{n_n}{n_i} (\frac{m_n}{m_i})^{1/2} (\frac{T_n}{T_i})^{1/2}]}. \quad (24)$$

Thus the effective temperature increases with decreasing neutral density. The influence of neutral density changes on the effective temperature would be especially important at  $1 - \frac{z}{t} + 2 \frac{z^2}{t^2} \ln \Lambda_i \lesssim 0$ . In such a case, a decrease of the neutral gas pressure can produce an anomalous growth of  $T_{\text{eff}}$ . This is in qualitative agreement with the experimental observation of melting by dusty crystals by reduction of the gas pressure [8,9].

The obtained results show that stationary velocity and charge grain distributions are described by effective temperatures different from those of the plasma subsystem. These effective temperatures are determined by the competitive mechanics of collisions: grain-neutral collisions and elastic Coulomb collisions result in the equalization of the effective temperature to the temperature of neutrals, or ions, respectively, while charging collisions can produce anomalous temperature growth. That could be one of the main mechanisms of grain heating.

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