Mixed convection behind a heated cylinder

PROEFSCHRIFT

ter verkrijging van de graad van doctor aan de
Technische Universiteit Eindhoven, op gezag van de
Rector Magnificus, prof.dr. M. Rem, voor een
commissie aangewezen door het College voor
Promoties in het openbaar te verdedigen op
dinsdag 23 mei 2000 om 16.00 uur

door

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geboren te Alkmaar
Dit proefschrift is goedgekeurd door de promotoren:

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Printed by the Eindhoven University Press.

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NUGI: 812
Trefwoorden: verwarmde cilinder; gemengde convectie / verwarmde cilinder; wervelstraat / verwarmde cilinder; zogtransitie / 2D Hoge Resolutie-PV / Super PTV / 3D Particle Tracking Velocimetry / 3D PTV / temperatuurmeting; 2D Laser Induced Fluorescence / 2D LIF / Spectrale-elementenmethode
Keywords: heated cylinder; mixed convection / heated cylinder; vortex wake / heated cylinder; wake transition / 2D High Resolution Particle Velocimetry / Super PTV / 3D Particle Tracking Velocimetry / 3D PTV / temperature measurement; 2D Laser Induced Fluorescence / 2D LIF / Spectral Element Method

This work is part of the research programme of the ‘Stichting voor Fundamenteel Onderzoek der Materie (FOM)’, which is financially supported by the ‘Nederlandse Organisatie voor Wetenschappelijk Onderzoek (NWO)’. 
## Contents

1 Introduction .................................................................................................................. 5  
  1.1 Mixed convection ........................................................................................................ 5  
  1.2 Research objectives ................................................................................................. 7  
  1.3 Scope and outline of this thesis ................................................................................ 10  

2 Theoretical background ................................................................................................. 13  
  2.1 Conservation laws .................................................................................................... 13  
  2.2 Vortex shedding and wake behaviour ...................................................................... 15  
  2.3 Point-vortex model .................................................................................................. 18  

3 Numerical method ......................................................................................................... 23  
  3.1 Temporal discretisation .......................................................................................... 23  
    3.1.1 Operator splitting ............................................................................................... 23  
    3.1.2 Pressure correction ............................................................................................. 25  
  3.2 Spectral Element Method ....................................................................................... 27  
    3.2.1 Fundamentals of SEM ....................................................................................... 27  
    3.2.2 Preconditioning ................................................................................................. 31  
    3.2.3 Spectral convergence ......................................................................................... 32  
  3.3 Calculation of an external flow around a cylinder .................................................. 34  
    3.3.1 Calculation domain and boundary conditions .................................................. 35  
    3.3.2 Grid dependency ............................................................................................... 38  

4 Experimental methods ................................................................................................. 41  
  4.1 Experimental set-up ................................................................................................. 41  
  4.2 Flow visualisation .................................................................................................... 46  
  4.3 Particle Velocimetry ............................................................................................... 47  
    4.3.1 2D High Resolution Particle Velocimetry ......................................................... 48  
    4.3.2 3D Particle Tracking Velocimetry ................................................................... 54  
  4.4 Measurement procedure ......................................................................................... 63  

5 Two-dimensional wake dynamics ............................................................................... 67  
  5.1 Vortex trajectories ................................................................................................. 68  
  5.2 Relative motion of the vortex structures ............................................................... 73
5.3 Structure strength .......................................................... 77
5.4 Point vortex simulations .................................................. 81
5.5 Discussion ............................................................... 83

6 Vortex formation and shedding process 85
6.1 Vorticity distribution ...................................................... 85
6.2 Sources of vorticity ....................................................... 88
6.3 Temperature distribution ............................................... 94
6.4 Discussion ............................................................... 97

7 Wake transition 103
7.1 Global process .......................................................... 103
7.2 2D analysis of the transition process ................................. 104
7.3 3D analysis .............................................................. 112
7.4 Discussion ............................................................... 114

8 Recommendations on experimental techniques 117
8.1 3D PTV ................................................................. 117
  8.1.1 Yield .............................................................. 118
  8.1.2 Processing time ..................................................... 121
8.2 Temperature measurements using LIF .............................. 121
  8.2.1 Temperature sensitivity .......................................... 123
  8.2.2 2D-temperature measurement ................................... 125
  8.2.3 Accuracy analysis ................................................ 126
  8.2.4 Conclusion ......................................................... 130

9 Concluding remarks 131

A Explicit 3rd order Taylor-Galerkin scheme 141

B Grid refinement 143

C Vortex trajectory variation 145

D Vortex strength 147

Nomenclature 149

Summary 153

Samenvatting 155

Nawoord 157
Chapter 1

Introduction

1.1 Mixed convection

Mixed convection is an intermediate type of flow between two extremes of the scale, namely forced and free convection (fig. 1.1). These two different flow types can be distinguished by the source that drives the fluid motion. While for forced convection the motion is created by an externally exposed pressure drop or mass flow, for free convection the motion is driven by density variations. These density variations may arise from concentration or temperature differences. Both types of flows frequently occur in environmental and in engineering flow situations. In a mixed-convection flow, both driving sources play a role. The interaction between these two different sources results in a flow with characteristics that are different from those of purely free- or forced-convection flows. Frequently, the interaction results in a strong mixing between the forced convection and the heat-driven free convection. This mixing process is of interest in many applications where dispersion of heat and mass is a key aspect.

In environmental situations mixed-convection flows can be encountered, for example, during or after strongly exothermic factory-plant accidents with large heat sources. The heat sources, in combination with the presence of a strong wind, may give rise to a situation where the heat-induced flow is as strong as the forced-convection flow. For many situations the strong interaction between the two types of flows leads to an effective mixing behaviour which is responsible for a rapid dispersion of the pollution.

Although in many industrial heat transfer devices the flow rate is chosen rather high, mixed-convection flows can be found in a variety of engineering situations. In several industrial devices the geometric complexity or the desired performance imposes strong restrictions on the flow rate. These restrictions, in combination with a high heat exchange rate, create a situation in which the heat-induced motion is as strong as the cooling flow. A typical example can be found in solar domestic hot water systems where the desired vessel stratification causes a severe restriction on the mass flow rate through the helically coiled heat exchanger (Sillekens (1995)). Also in heat-storage vessels mixed convection occurs during the charging and discharging of the vessel. On the interface of the cold and warm fluid coherent flow structures are generated which are then affected both by free and forced convection (van Berkel (1997)).
Another example can be found in small electronic devices where the high energy dissipation causes a flow as strong as the cooling flow induced by the fans. Components positioned in this cooling flow will influence the heat exchange of downstream positioned components. Especially when coherent vortex structures shed from the first cooled component are involved, the effectiveness of the heat transfer may be strongly influenced.

Due to the occurrence of mixed-convection flows in systems with a high demand on performance and reliability, investigation of these flows is both challenging and useful. The challenge is especially found in the strong and nonlinear coupling between the forced-convection flow and the thermally induced free convection. Interesting phenomena occur when strong coherent vortex structures are involved originating for example from flow separations. These structures may be affected by the induced heat, which can cause their structure cores to become warmer than the surrounding fluid. Once formed, the thermal structures are advected further downstream by the main flow. These hot blobs can be of major importance for the performance and service life of such specific devices. For example, if components to be cooled are positioned in the trajectories of these hot blobs, temporarily the cooling performance of these components will decrease, which may cause severe damage. Knowledge of the interaction between heat and vortex structures can lead to optimisation of such devices. Better prediction of the corresponding heat transfer processes requires fundamental knowledge of the heat effects on the wake structures and on the wake stability.
1.2 Research objectives

In order to investigate the effect of heat addition on the behaviour of vortex structures, a simplified configuration is introduced (fig. 1.2). It consists of a horizontal heated cylinder with constant temperature \( T_1 \) which is exposed to a uniform horizontal cold cross-flow with velocity \( U_0 \) and temperature \( T_0 \). The cylinder has an outer diameter \( D \) and a length \( l \), resulting in an aspect ratio \( \mathcal{A} \) equal to \( l/D \). The flow is determined by the fluid properties (expressed by the \( Pr \) number \( Pr = \nu/\kappa \)), the strength of the forced main flow (expressed by the Reynolds number \( Re_D = U_0 D/\nu \)) and the heat-induced effects (expressed by the Grashof number \( Gr_D = (T_1 - T_0) D^3 g \beta /\nu^2 \)). The relative importance of the forced and heat induced effects is indicated by the Richardson number \( (Re_D)^2 / Pr \). In the definition of the above dimensionless numbers, \( \kappa \) denotes the thermal diffusivity, \( \nu \) the kinematic viscosity, \( g \) the gravity constant and \( \beta \) the thermal expansion coefficient.

For a forced-convection flow with Reynolds numbers between 50 and 190 the flow is 2D and behaves fully periodic. This periodicity is created by the sequential shedding of vortex structures originating in the shear layers at both the upper and lower side of the cylinder. These vortex structures form a street of 2D vortices existing out of two vortex rows: one row of clockwise rotating structures (negative vortices) positioned above a row of counter-clockwise rotating structures (positive vortices) (fig. 1.3). The vortices in both rows form a staggered pattern, in which vortices in the upper row are shifted about half a streamwise vortex spacing \( (a) \) with respect to the vortices in the lower row. For a forced-convection flow, this staggered pattern appears to be very stable and is most frequently referred to as the Von Kármán vortex street, named after one of the first researchers who made notion of it. Also some early drawings of Leonardo Da Vinci showed this staggered pattern of fluid structures behind an obstacle. The alternate arrays of eddies were later sketched by Bénard (1908). Von Kármán discovered that this vortex street could only be stable for a well-defined configuration. To that end, he introduced a point-vortex street in which he considered an eddy as a single point-vortex (von Kármán and Rubach (1912)).

From then on, the dynamics of the vortex street seems to have drawn a lot of attention. Many
CHAPTER 1. INTRODUCTION

Formation region

Upper vortices

a

Wake axis

Lower vortices

Figure 1.3: Terminology for vortex shedding shown in a visualisation result for $Re_D = 73$ and $Ri_D = 0$

researchers investigated the forced-convection vortex street for a wide range of Reynolds numbers by means of visualisation experiments and hot-wire anemometry. For $Re_D < 4$ a creeping flow around the cylinder could be observed in which viscous effects dominate. For this particular situation an analytical solution could be derived. A further increase of $Re_D$ results in a flow in which a stable pair of vortices can be observed in the cylinder near wake. For $Re_D \approx 50$ this twin vortex configuration becomes very sensitive to disturbances, leading to a flow where vortices are shed sequentially at upper and lower sides of the cylinder. This sequential shedding of 2D vortex structures causes the wake flow to behave fully periodic, with a natural frequency $f$. The non-dimensional representation of this frequency, the Strouhal number ($St = fD/U_0$), starts initially at $St \approx 0.11$ and then gradually increases as $Re_D$ increases (fig. 1.4).

The measurements for small $Re_D$ (fig. 1.4a) show some irregularities around $Re_D = 60$ and $Re_D = 100$. It turns out that these irregularities are due to oblique vortex shedding, which implies that the vortex tubes are slanted with respect to the cylinder axis. This oblique shedding is caused by the end effects of the cylinder (Slauoti and Gerrard (1980)). For a very large cylinder aspect ratio, $A > 150$, oblique shedding does almost not occur in the interior region. By installing proper cylinder ends, as for example end plates or diameter thickening, the oblique shedding can be suppressed (Williamson (1989)). By increasing $Re_D$ above 200, 3D modes arise in the cylinder wake (Eisenlohr and Eckelmann (1989), Kariadakis and Triantafyllou (1992)). A further increase of $Re_D$ results in a transition to a fully turbulent wake, in which still coherent vortex structures are shed from the cylinder. For very high Reynolds number, even the boundary layers become turbulent.

Sophisticated measurement devices and techniques made it possible to investigate the stability
of the vortex street. By vibrating the cylinder, it turned out that the vortex street could be disturbed severely, even leading to an early collapse under certain circumstances (Griffin and Ramberg (1976)). Also external objects placed in the cylinder wake appear to have a strong effect on the wake behaviour as for example the presence of small pins or wake splitters (Igarashi (1984)). In all these investigations the attention was focused on the wake behaviour, the vortex formation process is less frequently investigated. Only a few investigations were focused on the vortex shedding process. For example Gerrard (1966) introduced, with the aid of smoke visualisations, a vortex formation model. Here, advection processes were assumed to be the most dominant processes during the formation of a coherent vortex structure out of the boundary layer produced vorticity. New measuring techniques as Particle Image Velocimetry (PIV) and Particle Tracking Velocimetry (PTV) opened new research possibilities. By performing detailed PTV experiments the vortex formation and shedding processes were investigated thoroughly (Green and Gerrard (1993)). It turned out that for $50 < Re_D < 100$ the vortex formation and shedding process is dominated by shear stress phenomena and that for higher Reynolds numbers advection processes take over. These observations suggest that the shedding mechanism is dependent on $Re_D$, especially in the regime where the shedding process is 2D. This can also be seen from the dependency of the Strouhal number on the Reynolds number (fig. 1.4). By increasing $Re_D$, viscous effects become ess important. This decrease of viscous effects then causes the vortex shedding mechanism only to depend on convection, resulting in an almost constant $St$ number. This appears to happen for $Re_D \approx 500$ (fig. 1.4b).

The effect of heat on the cylinder wake is investigated less thoroughly. In the early seventies, this effect was investigated to determine the global effects of the induced heat on the heat exchange coefficient (Hatton et al. (1970)). It turned out that for $Ri_D > 0.2$, the heat transfer coefficient was influenced considerably by the thermal induced flow. These investigations were carried out
for various angles between the gravity vector and the main flow direction, and resulted in a set of critical $Ri_D$ values above which thermal effects influence the heat exchange coefficient. In the investigations the circumstances could be established under which hot-wire velocity measurements could be applied, but almost no attention was paid to the effect of heat on the wake structures and wake behaviour.

More detailed studies on the effect of heat on the vortex wake were carried out slightly later (Noto et al. (1985), Noto and Matsumoto (1987), Badr (1984)) and some remarkable results were obtained. These studies describe the flow around a heated horizontal cylinder exposed to a vertically upward directed flow. By increasing the heat input (increasing $Ri_D$), the Strouhal number first increases. However, above a critical Richardson number, the Strouhal number becomes zero and vortex shedding is suppressed. For this heat input the vortex street converts again to a wake in which two twin vortices are situated in the near wake. A further increase of the heat input causes these vortices to disappear, ending up with a thermal plume. These results were later verified experimentally (Michaux-Leblond and Bélorgey (1997)), and here the dualism between buoyancy and viscous effects was then considered to be the source of the sudden disappearance of the vortex shedding phenomenon.

The influence of the angle between the main flow direction and the gravity vector was numerically investigated by Noto (1991). From this study it was found that the angle has a large influence on the vortex street characteristics. Starting with an angle of $180^\circ$ (an angle of $180^\circ$ corresponds to a vertical upward flow and $90^\circ$ to the configuration sketched in figure 1.2) it was found that for a decreasing angle the wake becomes periodic again. For angles smaller than $90^\circ$, the natural frequency becomes even larger than for the unheated situation. Unfortunately, the processes responsible for the observed change in the wake characteristics were not explained. Besides in Noto’s study only the near-wake effects were evaluated, downstream effects of the induced heat were disregarded.

1.3 Scope and outline of this thesis

The effect of the induced heat on the behaviour of coherent structures shed from a cylinder will be the main subject of this thesis. Revealing the processes and mechanisms involved in the heat-induced change of the structure characteristics and early wake transition will be the main focus. These processes are studied for flows which are initially two-dimensional and for which the shedding mechanism is constant ($Re_D \approx 75$). Therefore the influence of the heat input is fully expressed in the variation of $Ri_D$ ($0 < Ri_D < 2$).

In chapter 2 of the thesis the governing equations will be presented. Also some light will be shed on the vortex formation process and a model for the wake flow is discussed. Chapter 3 deals with the numerical method. A brief overview of the used Spectral Element Method (SEM) and the time discretisation will be given. Furthermore, some of their elementary properties are discussed by means of a simplified problem. Application of the method on the assumed flow configuration (fig. 1.2) will be the subject of the last part of this chapter. Besides numerical simulations, experiments have been carried out as well. For this purpose a towing tank was designed and built. Its design considerations and characteristics will be explained in the first part of chapter 4. In
1.3. SCOPE AND OUTLINE OF THIS THESIS

the experiments some relatively new techniques are applied, like High Resolution Particle Velocimetry (HiRes-PV) and 3D Particle Tracking Velocimetry (3D PTV). A short description of both techniques, their characteristics and accuracy will also be presented in chapter 4. The first results will be presented in chapter 5. This chapter deals with the effect of heat on the still stable vortex street. This effect will be investigated thoroughly for different $Ri_D$ numbers. In chapter 6 the vortex shedding and formation processes for $Ri_D > 0$ will be discussed. It has turned out that most of the phenomena as described in chapter 5, originate during the formation of the vortex structures. A further increase of the heat input will lead to an unstable vortex street which finally breaks down. This process will be the main subject of chapter 7. Also, some parameter variation will be applied to elucidate the proposed mechanism. In chapter 8 recommendations with respect to the measurement techniques used will be presented. Also the possibilities and limitations of a 2D temperature measurement technique (still in progress) will be discussed. Finally, the conclusions of the numerical and experimental work are presented in chapter 9.
Chapter 2

Theoretical background

2.1 Conservation laws

The flow of an incompressible Newtonian fluid around a heated cylinder is described by the conservation laws of mass, momentum and energy. In differential form these equations read

\[ \nabla \cdot \vec{u} = 0, \]  
(2.1)

\[ \frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \vec{u} + \vec{g}, \]  
(2.2)

\[ \frac{\partial T}{\partial t} + \vec{u} \cdot \nabla T = \kappa \nabla^2 T, \]  
(2.3)

with \( \vec{u} = [u_1, u_2, u_3]^T \) the velocity vector, \( p \) the total pressure, \( t \) the time, \( \rho \) the density, \( \nu \) the kinematic viscosity, \( T \) the temperature, \( \kappa \) the thermal diffusivity and \( \vec{g} = [0, -g, 0]^T \) the gravity vector with \( g \) the gravitational constant. In this representation it is assumed that loss of kinetic energy by the action of shear stresses does not contribute to the energy equation while heat conduction obeys Fourier’s law. Furthermore, no internal volumetric heat sources are present.

In the equations 2.1, 2.2 and 2.3 the density variations are assumed to be small. For the problem to be discussed the small density variations still give rise to buoyancy. To take into account these effects, the Boussinesq approximation is used. Here, only small variations of the density, pressure and temperature around their reference-state are assumed. The density can then be written as \( \rho = \rho_0 + \rho' \) with \( \rho_0 \gg \rho' \), the pressure as \( p = p_0 + p' \) with \( p_0 \gg p' \) and the temperature as \( T = T_0 + \Theta \) with \( T_0 \gg \Theta \). For the reference pressure, the hydrostatic pressure \( p_h \) at depth \( h \) is chosen to be written as \( p_h = \rho_0 g h \). Using the relations for the density and pressure in equation 2.2 one obtains

\[ \frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} = -\frac{1}{\rho_0 + \rho'} \nabla p' - \frac{\rho_0}{\rho_0 + \rho'} \vec{g} + \frac{\eta}{\rho_0 + \rho'} \nabla^2 \vec{u} + \frac{\rho_0 + \rho'}{\rho_0 + \rho'} \vec{g}, \]  
(2.4)

with \( \eta \) the dynamic viscosity.

As \( \rho_0 + \rho' \approx \rho_0 \) equation 2.4 can be written as

\[ \frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} = -\frac{1}{\rho_0} \nabla p' + \nu_0 \nabla^2 \vec{u} + \frac{\rho'}{\rho_0} \vec{g}, \]  
(2.5)
with \( \nu_0 = \eta / \rho_0 \).

The energy equation is coupled with the momentum equation by assuming that the density \( \rho \) is linearly dependent on the temperature, the so-called Boussinesq approximation:

\[
\frac{\rho'}{\rho_0} = \frac{\rho(T) - \rho_0}{\rho_0} = -\beta_0 (T - T_0),
\]

with \( \beta_0 = -\frac{1}{\rho_0} \frac{\partial \rho}{\partial T} \big|_{T=T_0} \) the thermal expansion coefficient at constant pressure. By using this approximation the conservation of momentum can be written as

\[
\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} = -\frac{1}{\rho_0} \nabla p + \nu \nabla^2 \vec{u} - \beta_0 (T - T_0) \vec{g},
\]

where now the pressure \( p \) denotes the dynamic part of the stagnation pressure.

The momentum conservation relation as presented in equation 2.7 is based on the assumption of small temperature variations. Especially the assumption of a linear temperature dependency of the density is only valid when these variations remain limited. For larger temperature variations also the other physical parameters as the viscosity \( \nu \) and thermal diffusivity \( \kappa \) can not be considered as constants anymore. As a rule of thumb one can state that for flows of water, temperature differences in the order of a few degrees will not violate the Boussinesq approximation (Gray and Giorgini (1976)).

The non-dimensional form of the conservation equations is obtained by introducing the following dimensionless variables,

\[
\tilde{x} = \frac{x}{D}, \quad \tilde{u} = \frac{u}{U_0}, \quad \tilde{p} = \frac{p}{\rho_0 U_0^2}, \quad \tilde{\rho} = \frac{\rho}{\rho_0}, \quad \tilde{t} = \frac{t}{D/U_0}, \quad \tilde{\Theta} = \frac{T - T_0}{T_1 - T_0}.
\]

Here \( T_1, T_0, U_0 \) and \( D \) are defined according to figure (1.2). Substitution and rearranging then lead to

\[
\nabla^* \cdot \tilde{u}^* = 0,
\]

\[
\frac{\partial \tilde{u}^*}{\partial \tilde{t}^*} + \tilde{u}^* \cdot \nabla^* \tilde{u}^* = -\nabla^* \tilde{p}^* + \frac{1}{R_e D} (\nabla^*)^2 \tilde{u}^* - R_i D \Theta \tilde{g}^*,
\]

\[
\frac{\partial \Theta^*}{\partial \tilde{t}^*} + \tilde{u}^* \cdot \nabla^* \Theta^* = \frac{1}{R_e D \Pr} (\nabla^*)^2 \Theta^*.
\]

with \( R_e D = U_0 D / \nu \) the Reynolds number, \( R_i D \) the Richardson number defined as \( G_r D / R_e D = g \beta \Delta T D / U_0^2 \) with the Grashof number \( G_r D = g \beta \Delta T D^3 / \nu^2 \) and \( \Pr = \nu / \kappa \) the Prandtl number. In the further discussion the star (denoting the dimensionless quantity) will be dropped.

From the conservation laws (eqs. 2.9, 2.10, 2.11) the vorticity transport equation can be derived by taking the curl of the momentum equation and using the incompressibility constraint. This results in

\[
\frac{D \vec{\omega}}{Dt} = \frac{\partial \vec{\omega}}{\partial t} + \vec{u} \cdot \nabla \vec{\omega} = (\vec{\omega} \cdot \nabla) \vec{u} - R_i D \nabla \times \vec{g} + \frac{1}{R_e D} \nabla^2 \vec{\omega},
\]

where the vorticity \( \vec{\omega} \) is defined as the curl of the velocity vector, \( \vec{\omega} = \nabla \times \vec{u} \). From this definition it can be seen that clockwise rotation of fluid elements is considered as negative rotation while anti-clockwise rotation is considered as positive rotation.
For a 2-D flow, the first term on the right hand side is zero. This term represents the effect of vortex tube stretching and tilting, a process which only occurs in 3D flows. Furthermore, for a 2D flow, the vorticity vector contains only one component, the out-of-plane component. The vorticity equation then reduces to a scalar equation which can be written as

\[
\frac{\partial \omega_z}{\partial t} + \mathbf{u} \cdot \nabla \omega_z = \frac{\partial \theta}{\partial x} + \frac{1}{Re_D} \nabla^2 \omega_z, \tag{2.13}
\]

where \(\omega_z\) is the out-of-plane component of the vorticity vector \(\mathbf{\omega}\), which is defined as

\[
\omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}. \tag{2.14}
\]

Although this equation describes the behaviour of \(\omega_z\) as a passive scalar, one should keep in mind that \(\omega_z\) is a derived quantity of \(\mathbf{u}\). Therefore a dynamical interaction exists between the advection of \(\omega_z\) and \(\omega_z\) itself.

Examination of the separate 2D vorticity transport equation terms allows the analysis of the formation and shedding process of a coherent vortex from the boundary layer produced vorticity. By treating the vorticity as a scalar which can be both positive and negative, the different terms can cause an increase or decrease of the local fluid rotation denoted by \(\partial \omega_z / \partial t\). A positive advection term \(\mathbf{u} \cdot \nabla \omega_z\), then causes negative rotation to increase while it has a decaying influence on positive rotation. A similar discussion can be held for the diffusion term \(\nu \nabla^2 \omega_z\) and the baroclinic production term \(\frac{\partial \theta}{\partial x}\), which also influences the local \(\omega_z\). Hence, the advection and diffusion terms can be seen as redistribution terms. These terms will have no net effect on the total vorticity or circulation inside the domain. They describe how vorticity initially present in the domain or introduced at the boundaries, is distributed in the flow. This in contrast to the production term \(\frac{\partial \theta}{\partial x}\), which causes a net change of vorticity depending on the temperature field.

### 2.2 Vortex shedding and wake behaviour

As already observed by Bénard (1908) and von Kármán, the staggered eddies are not directly shed from the cylinder but they are formed somewhat further downstream. Although the phenomenon was already discovered about a century ago, this formation process of vortex structures is still a point of discussion. In this section a brief review will be given of the major findings for a 2D forced convection flow.

The process in which boundary-layer produced vorticity organises into a coherent vortex structure takes place within the so-called formation region. This region is the area downstream of the cylinder and its downstream end is denoted by the formation length \(L_f\) (fig. 2.1). For the formation length different definitions can be found in the literature (Griffin (1995)). Within this work \(L_f\) is based on the position for which the vorticity contours form a closed contour around the structure. A shed vortex is then defined as the area with a local vorticity extreme \(\omega_{ext}\) bounded by the closed vorticity contour of the value 0.1\(\omega_{ext}\). The distance between the cylinder surface and the centre of this vortex structures is then denoted as \(L_f\).
In the formation region other characteristic areas or points can be defined. In the very near-wake, the flow recirculates. Therefore a recirculation region with length $L_r$ can be defined for which the average motion within the area enclosed by $L_r$ shows a double recirculation pattern. Furthermore, on the cylinder surface one can find two stagnation points $St_1$, $St_2$ and two boundary layer separation points $Sp_1$, $Sp_2$ (fig. 2.1).

During the vortex formation process the boundary-layer-produced vorticity is transported from the boundary layers into the cylinder near-wake. Close to the cylinder surface this transport is dominated by diffusion processes, causing the vorticity to enter the main flow. When advection processes take over, boundary-layer vorticity is transported downstream and forms a strand of vorticity in the cylinder near-wake (fig. 2.2a).

Within this near-wake the vortex formation and shedding process takes place. According to Green and Gerrard (1993) this process can be thought to be divided into three distinguishable stages. The first stage concerns the accumulation of the boundary-layer produced vorticity, resulting in the emergence of a coherent blob of vorticity at the tip of the strand (fig. 2.2b). In the second stage, the separation of this blob from its own source, the boundary layer, takes place. At a certain moment, upstream of the strand tip, a constriction process is initiated (fig. 2.2c). This constriction process causes the strand tip to transform into a region where a coherent vortex structure arises. The constriction process for $Re_D \approx 100$ is assumed to be dominated by the shear rate $S_2 = \partial v/\partial x + \partial u/\partial y$ which is relatively large in this constriction area. According to Green this shear rate causes a decay of kinetic energy, which does not take place in the area where vorticity accumulates. For larger $Re_D$ numbers ($Re_D > 200$) Gerrard (1966) stated that this constriction process is dominated by advection and entrainment processes. The final stage of the formation process is the actual shedding (fig. 2.2d). As soon as the constriction has been accomplished the vortex structure is accelerated in the downstream direction and leaves the formation region.

A useful quantity for analysing the vortex-shedding process was introduced by Weiss (1991):

$$Q = \frac{1}{2}(S_1^2 + S_2^2 - \omega_z^2),$$

with $S_1 = \partial u/\partial x - \partial v/\partial y$. This local quantity measures the relative importance of the square of the vorticity with respect to the square of the strain rate. In areas where $Q < 0$ vorticity is
2.2. VORTEX SHEDDING AND WAKE BEHAVIOUR

Figure 2.2: Schematical representation of the vortex-shedding process as suggested by Green and Gerrard (1993)

relatively unaffected by strain, usually indicating the presence of a coherent structure. Because of the shape of the corresponding streamlines (fig. 2.3a) these areas are called ‘elliptic’. In areas where the strain rate dominates ($Q > 0$), the fluid elements are stretched and deformed. In such areas, referred to as ‘hyperbolic’ areas, the strain prevents vorticity to become organised in coherent structures (fig. 2.3b). For these areas it can then be shown that the magnitude of the vorticity gradient will tend to grow in time at an exponential rate and that a transfer of vorticity to the smaller scales take place (Weiss (1991)).

Figure 2.3: Streamline patterns for the two distinguishable flow states: (a) hyperbolic state with $Q > 0$ and (b) elliptic state with $Q < 0$

One could think that the total amount of vorticity produced at the cylinder wall ends up in the formed vortex structures. However, different investigations showed that the formed structures contain less vorticity (Zdravkovich (1997)). An estimation of the vorticity redistribution in the cylinder near-wake (fig. 2.4) showed that due to vortex cancellation (counter-diffusion), spreading and even 3D effects only about 66 % of the total produced vorticity accumulates in the shed structures (fig. 2.4). This percentage increases for increasing $Re_D$, which seems to be caused by the relative decaying importance of the viscosity in the vortex-shedding process. This dependency can also be deduced from the dimensionless vortex strength (as found from the experimental results in Zdravkovich (1997)) which increases for increasing $Re_D$. 
Redistribution of vorticity

- undefined amount due to mixing
- about 4% due to entrainment
- about 4% due to non linear effects
- undefined amount due to 3D effects
- about 8% due to viscous effects

Figure 2.4: Loss of vorticity in the cylinder near-wake Zdravkovich (1989)

After the formation process, the shed structures form a staggered pattern of vortices consisting of two rows of structures with opposite rotation sense. The structures in the upper row are shifted half a streamwise spacing distance \( a \) (fig. 2.5) with respect to structures in the lower row. Once formed, the structures are almost 'frozen' into this staggered pattern, i.e. they are advected collectively in the same (downstream) direction. Although the structures are strongly linked, within this pattern the vortex structures are more or less isolated patches of vorticity, becoming even more isolated due to strong mixing of irrotational fluid from the outer wake into the inner wake.

Further downstream a widening of the vortex street takes place, which is due to the viscous spreading effect of the vortex cores. This spreading effect can eventually lead to a situation where vorticity cross-diffusion takes place between opposite-signed structures. The cross-diffusion then causes the circulation of the individual structures to decay.

### 2.3 Point-vortex model

In the cylinder wake flow, different interactions between the wake structures occur that determine the behaviour of the wake as a whole. A simple approach to describe these interactions is by means of a point-vortex model in which each vortex structure is modelled by a discrete point containing the total strength \( \Gamma \) of the structure. This strength \( \Gamma \) can be calculated from the vorticity field by

\[
\Gamma = \int_A \omega_z(x, y) \, dA,
\]

where the area \( A \) is determined by a yet undefined closed contour \( s \). The position of the structures can also be calculated from the vorticity field. This position \((X, Y)\) is defined as

\[
X = \frac{1}{\Gamma} \int_A \omega_z(x, y) x \, dA,
\]

\[
Y = \frac{1}{\Gamma} \int_A \omega_z(x, y) y \, dA,
\]

which marks the vorticity centroid in \( A \) as the position of the structure.
2.3. POINT-VORTEX MODEL

![Figure 2.5: Vortex street configuration](image)

This discrete point vortex induces a velocity field \( \mathbf{u} = [u, v]^T \) which can be written for \( r > 0 \) as

\[
\begin{align*}
    u(x, y) &= \frac{\Gamma(Y - y)}{2\pi r^2}, \\
v(x, y) &= \frac{\Gamma(x - X)}{2\pi r^2},
\end{align*}
\]

(2.18)

with \( r = \sqrt{(x - X)^2 + (y - Y)^2} \). The effect of an array of point vortices and a superposed potential flow \( \mathbf{U}_0 \) is simply accounted for by a linear summation of the flow field induced by the point vortices separately and the potential flow field. The obtained flow field \( \mathbf{u} \) then reads for \( r_j > 0 \)

\[
\begin{align*}
    u(x, y) &= U_0 + \sum_{j=0}^{j=n_v} \frac{\Gamma_j(Y_j - y)}{2\pi r_j^2}, \\
v(x, y) &= V_0 + \sum_{j=0}^{j=n_v} \frac{\Gamma_j(x - X_j)}{2\pi r_j^2},
\end{align*}
\]

(2.19)

where \( j \) the denotes the effect of the \( j \)th vortex, with \( r_j = \sqrt{(x - X_j)^2 + (y - Y_j)^2} \) and \( n_v \) is the total number of vortices.

This point-vortex approach will be used to model a vortex street by two parallel opposite-signed rows of vortices (fig. 2.5) placed in a uniform horizontal cross-flow \( U_0 \). The strength of the vortices in both rows is considered to be equal, while the position of the vortices in the upper row is shifted half a streamwise distance \( a \) with respect to the lower vortices. The vertical distance \( b \) between the two rows cannot be chosen arbitrarily. Stability analyses show (von Kármán and Rubach (1912)) that for the ratio \( b/a = 0.281 \) no relative displacement between the vortices in the vortex street occurs. For that situation, the vortex street is considered to be stable. For this particular situation, all vortices move with constant horizontal group velocity

\[
\begin{align*}
u_g &= U_0 - \frac{1}{a\sqrt{8}}, \\
v_g &= 0.
\end{align*}
\]

(2.20)
Although in this approach some major simplifications are used the global behaviour of the structures in a vortex street is predicted remarkably well. Especially in the region between 5 and 25 $D$ behind the cylinder, the measured average spacing ratio $b/a$ varies between 0.18 and 0.33 depending on the downstream position, against 0.281 derived by the point-vortex model (Zdravkovich (1997)).

For the mixed-convection flow behind a cylinder the shed structures will be influenced by the induced heat. To model the behaviour of the shed structures inside the wake a numerical simulation code has been written based on the point-vortex model. In a specified domain the vortices are positioned in a potential flow with a horizontal component $\frac{\partial \phi}{\partial y}$. The integration routine that calculates the vortex trajectories between two simulation steps is based on a 4th-order explicit Runge-Kutta scheme. Within the integration routine an iterative solver is used, ensuring that the calculated trajectories are independent of the time step.

Due to the finite size of the domain, the vortex rows are of finite length, which creates disturbances at both ends of the rows. The absence of upstream vortices causes the vortex street to become unstable. To compensate for this absence of upstream vortices a so-called inflow region (fig. 2.6) is defined. This region is nothing more than a region with a constant number of vortices, positioned according to the stability criterion as derived by von Kármán. Between two simulation steps the vortices move only in $x$-direction according to $\Delta x = u_y \Delta t$ with $\Delta t$ the time step between two simulation steps. As soon as a vortex is inside the domain of interest, all restrictions on its position and displacement are released and the trajectory of this vortex is fully determined by the vortices induced flow field and the potential flow.

At the other side vortices will leave the domain of interest. After having left the domain they do not contribute to the flow field anymore. This sequential leaving of positive and negative vortices will cause wrinkles in the trajectories of the upstream vortices. Due to the presence of a main flow $U_0$ these wrinkles remain limited to the outflow region. Especially if the strength of the vortices is assumed to decay during their downstream convection this effect becomes negligible. This decay is assumed to be caused by the viscous spreading of the vortex cores while travelling downstream. Due to this viscous effect, cross diffusion can cause the strength of the vortex cores to decay. An estimation of the decaying vortex strength can be derived assuming a flow with confined circular
paths around a vortex structure. For the vorticity equation in polar coordinates one then can write

$$\frac{\partial \omega_z}{\partial t} = \nu \left( \frac{\partial^2 \omega_z}{\partial \zeta^2} + \frac{1}{\zeta} \frac{\partial \omega_z}{\partial \zeta} \right),$$

(2.21)

with $\zeta$ the coordinate measured from the centre of the structure. For an isolated vortex initially of strength $\Gamma_0$ concentrated around the structure axis $\zeta \to 0$, the vorticity as function of time can then be written as

$$\omega_z = \frac{\Gamma_0}{4\pi \nu t} e^{\zeta^2/(4\nu t)}.$$  

(2.22)

For the total circulation $\Gamma$ inside a closed contour with radius $R$ one can write

$$\Gamma = \Gamma_0 \left[ 1 - e^{-R^2/(4\nu t)} \right].$$

(2.23)

From visualisation it is found that the vortices in the cylinder near-wake have a typical diameter of $2D$ (with $D$ the cylinder diameter). By assuming now that the strength $\Gamma$ of the vortex is determined by the vorticity located within this area ($R = D$), it is finally found that

$$\Gamma = \Gamma_0 \left[ 1 - e^{-D^2/(4\nu t)} \right].$$

(2.24)

This relation will be used in later simulations in order to account for the viscous effects on the vortex strength.
Chapter 3

Numerical method

To solve the conservation relations for mass, heat and momentum, they need to be discretised in time and space. In this chapter the applied discretisation scheme will be explained and the performance of the numerical algorithm with respect to its convergence rate and accuracy will be tested. Furthermore, in this chapter the effect of the boundary conditions as well as the chosen computational domain will be studied for the flow around a heated horizontal cylinder. Finally, the influence of grid refinement is discussed in order to establish the most optimal computational grid which will be used in the investigation.

3.1 Temporal discretisation

The temporally discretised version of the Navier-Stokes equations is obtained by using an approximate projection scheme (Timmermans et al. (1996)). Within this scheme the pressure is treated by a pressure-correction method. As the nonlinear convection operator in combination with the presence of a diffusion operator lead to a severe restriction on the time step, an operator-splitting method is used. This discretisation process, leading to the time-discretised Navier-Stokes equations, will be discussed in this section.

3.1.1 Operator splitting

In order to describe the time-discretisation method, the original conservation relations for momentum and energy are written in terms of operators,

\[
\frac{\partial \vec{u}}{\partial t} = (\mathcal{C} + \mathcal{D}_u)\vec{u} + \vec{f},
\]

\[
\frac{\partial \Theta}{\partial t} = (\mathcal{C} + \mathcal{D}_\Theta)\Theta,
\]

where the source term $\vec{f}$ accounts for the pressure gradient and the buoyancy term. The operators can then be written as

\[
\mathcal{C} \equiv -\vec{u} \cdot \nabla, \quad \mathcal{D}_u \equiv 1/Re_D \nabla^2, \quad \mathcal{D}_\Theta \equiv 1/(Re_D Pr) \nabla^2.
\]
Considering the difference in the characteristics of the convection and diffusion operator’s eigenvalues (Canuto et al. (1988)), the most suitable time discretisation would be an unconditional stable method like, for example, the Euler-implicit method. Especially for problems in which the matrices arising during the spatial discretisation are not time-dependent and can be decomposed by a LU decomposition, this implicit method together with a direct solving technique may be the best approach. Unfortunately, the convection matrix is directly dependent on the flow field and not constant in time. This means that at every time step, the system matrix (which contains the convection matrix) needs to be rebuilt as well as inverted and this is a very time-consuming approach. Therefore, a fully explicit approach seems to fit better, but the difference in eigen-values of the operators causes that this approach can only be used if small time steps are made (the diffusion eigen-values are negative and very large (Canuto et al. (1988)), causing a severe restriction on the time step). If a combination of an implicit and an explicit discretisation method could be applied an efficient method is derived. Such a combination can be achieved by applying a so-called operator-splitting method in which the convection term is split from the convection-diffusion relation. To that end a suitable integration operator (Maday et al. (1990)) is used to rewrite the original convection-diffusion equation. For the momentum equation the integration operator acts on all three velocity components and is therefore a tensor, whereas for the energy equation the operator acts only on one component and is written as an integration factor. By defining the integration operators as (Maday et al. (1990))

\[
\frac{\partial Q^{(\tau, t)}_{\bar{u}}}{\partial t} + \beta q\bar{u}^{n+1} = \Delta t \left[ D_{\bar{u}}\bar{u}^{n+1} + \bar{f}^{n+1} \right],
\]

with \(I\) the identity tensor and \(I\) the identity operator. Applying these operators to the convection diffusion equations (eqs. 3.1, 3.2), the equations can be written as

\[
\frac{\partial \tilde{u}}{\partial t} = Q^{(\tau, t)}_{\bar{u}} \left( D_{\bar{u}}\bar{u} + f \right),
\]

\[
\frac{\partial \tilde{\Theta}}{\partial t} = Q^{(\tau, t)}_{\Theta} \left( D_{\Theta}\Theta \right),
\]

with

\[
\tilde{u} = Q^{(\tau, t)}_{\bar{u}} \bar{u} , \quad \tilde{\Theta} = Q^{(\tau, t)}_{\Theta} \Theta.
\]

Taking \(t^* = t^{n+1}\) and \(\Delta t = t^{n+1} - t^n\) and applying a second-order Backward Difference scheme, the temporal discretised versions of equations 3.5 and 3.6 read

\[
\beta q\bar{u}^{n+1} - \sum_{i=1}^{2} \beta_i \tilde{u}^{n+1-i} = \Delta t \left[ D_{\bar{u}}\bar{u}^{n+1} + \bar{f}^{n+1} \right],
\]

\[
\beta q\Theta^{n+1} - \sum_{i=1}^{2} \beta_i \tilde{\Theta}^{n+1-i} = \Delta t \left[ D_{\Theta}\Theta^{n+1} \right],
\]
3.1. TEMPORAL DISCRETISATION

with \( \tilde{u}^{n+1-i} = Q_{\tilde{u}}^{(n+1,n+1-i)} \tilde{u}^{n+1-i}, \Theta^{n+1-i} = Q_{\Theta}^{(n+1,n+1-i)} \Theta^{n+1-i}, \beta_0 = 3/2, \beta_1 = 2 \) and \( \beta_2 = -1/2 \).

To integrate the derived discretised diffusion equations for the velocity and the temperature, the terms which include the integration factors \( Q_{\tilde{u}}^{(n+1,n+1-i)}, Q_{\Theta}^{(n+1,n+1-i)} \) need to be evaluated. To avoid explicit construction of the integrating factor \( q \), the following initial-value sub-problems are introduced for \( \tilde{u} \) and \( \Theta \) (Maday et al. (1990))

\[
\frac{\partial \tilde{u}}{\partial s} = C_{\tilde{u}} \tilde{u}(s), \quad 0 < s < i\Delta t, \\
\tilde{u}(0) = \tilde{u}^{n+1-i}, \\
\frac{\partial \Theta}{\partial s} = C_{\Theta} \Theta(s), \quad 0 < s < i\Delta t, \\
\Theta(0) = \Theta^{n+1-i}. 
\]

(3.10)

(3.11)

In this initial-value problem the time parameter \( s \) is defined as an intermediate time within an implicit time step \( i\Delta t \). By multiplying equations 3.10 and 3.11 with \( Q_{\tilde{u}}^{(n+1,n+1-i)}, Q_{\Theta}^{(n+1,n+1-i)} \) respectively and by using the equations 3.3, 3.4 and the initial conditions of equations 3.10 and 3.11, it can be derived that

\[
Q_{\tilde{u}}^{(n+1,n+1-i)} \tilde{u}^{n+1-i} = \tilde{u}(i\Delta t), \\
Q_{\Theta}^{(n+1,n+1-i)} \Theta^{n+1-i} = \Theta(i\Delta t).
\]

(3.12)

(3.13)

The convection relations (eq. 3.10 and 3.11) are discretised by applying a three steps explicit 3rd-order accurate Taylor-Galerkin scheme (Appendix A) which enables one to make several explicit time steps \( \Delta s \) within one implicit time step \( \Delta t \).

3.1.2 Pressure correction

In the previous section, the influence of the pressure on the time discretisation was hidden in the external body force \( \tilde{f}^{n+1} \). More explicitly this term can be written as

\[
\tilde{f}^{n+1} = -\nabla p^{n+1} - R i D \Theta^{n+1} \tilde{g}.
\]

(3.14)

From the heat-diffusion equation (eq. 3.6), which is discretised by the BDF scheme, the temperature term \( R i D \Theta^{n+1} \) can be evaluated. The pressure term \( \nabla p^{n+1} \) needs to be treated separately. Within the used algorithm a so-called pressure-correction method is applied (Timmermans et al. (1996)) where first a guess is made for the pressure at \( t = t^{n+1} \) by taking the pressure at \( t = t^n \), the so-called predicted pressure \( \tilde{p}^{n+1} \). In general, the velocity field \( \tilde{u}^{n+1} \) calculated with this predicted pressure is not divergence-free and is used to determine a correction term for the pressure. This correction term on its turn, is used to calculate a corrected divergence-free velocity field \( \tilde{u}^{n+1}_c \). The correction term \( q \) is found by using the relation for the predicted velocity field \( \tilde{u}^{n+1}_p \) and the corrected velocity field \( \tilde{u}^{n+1}_c \). This results in a Poisson equation for the correction term \( q \). The algorithm as described above can be written according to the following sequence of steps
• Determine $\Theta^{n+1}$ by solving the energy equation (3.9).

• Derive a relation for the predicted velocity $\tilde{u}_p^{n+1}$ by replacing $p^{n+1}$ by the known pressure $p^n = p_p^{n+1}$ in the momentum equation (3.8), leading to

\[
(3/2 - \frac{\Delta t}{Re_D} \nabla^2) \tilde{u}_p^{n+1} = 2\tilde{u}^n - 1/2\tilde{u}^{n-1} + \Delta t(-\nabla p_p^{n+1} - Ri_D(\Theta^{n+1}\tilde{g})),
\]

where $\tilde{u}^n$ and $\tilde{u}^{n-1}$ are calculated from the associated initial-value problem (eq. 3.10).

The predicted velocity field $\tilde{u}_p^{n+1}$ satisfies identical boundary conditions as imposed for $\tilde{u}^{n+1}$. For the corrected velocity and pressure one can write

\[
(3/2 - \frac{\Delta t}{Re_D} \nabla^2)\tilde{u}_c^{n+1} = 2\tilde{u}^n - 1/2\tilde{u}^{n-1} + \Delta t(-\nabla p_c^{n+1} - Ri_D(\Theta^{n+1}\tilde{g})).
\]

• Subtraction of the correction equation (eq. 3.16) from the prediction equation (eq. 3.15), taking the divergence of this difference and assuming the divergence of the corrected velocity field equal to zero, a Poisson equation for the correction term $q$ can be found

\[
\nabla^2 q^{n+1} = \frac{3}{2\Delta t} \nabla \cdot \tilde{u}_p^{n+1},
\]

with $q$ defined as

\[
q^{n+1} = p_c^{n+1} - p_p^{n+1} + 1/Re_D \nabla \cdot \tilde{u}_p^{n+1}.
\]

The boundary conditions for $q$ are derived from the global mass conservation (Timmermans et al. (1996)) and read

\[
\frac{\partial q^{n+1}}{\partial n} = 0,
\]

on all boundaries around the interior domain $\Omega$ on which for $\tilde{u}_p^{n+1}$ Dirichlet boundary conditions are described. Here $n$ denotes the normal direction on the boundaries around $\Omega$.

• Correct $\tilde{u}_p^{n+1}$ by employing

\[
\tilde{u}_c^{n+1} = \tilde{u}_p^{n+1} - \frac{2}{3} \Delta t \nabla q .
\]

• Compute the pressure at the present time level by

\[
P_c^{n+1} = p^n + (q - 1/Re_D \nabla \cdot \tilde{u}_p^{n+1}).
\]

About the boundary conditions for the velocity the following remarks can be made. Although the corrected velocity $\tilde{u}_c^{n+1}$ is divergence-free within the interior of the domain $\Omega$, on the boundaries this is not proven (Timmermans et al. (1996)). However, the error made at the boundaries shows to be second-order in time.
3.2 Spectral Element Method

The equations presented in chapter 2, are spatially discretised by applying a Spectral Element Method (SEM). This method, which can be considered as a mixture of a Finite Element Method (FEM) (Cuvelier et al. (1986)) and a Spectral Method, combines the advantages of both methods. The principle is relatively straightforward. Within a small domain inside the fluid, a so-called element, the equations are discretised by using a spectral method. Combining the solutions obtained in all these elements finally results in the solution on the entire domain. Of course, special consideration has to be given to the continuity of the solution over the element boundaries.

The decomposition of the domain in elements gives the method the geometric flexibility which can also be found for the Finite Element Method. By using a spectral approach within such an element a spectral convergence rate and small numerical errors, corresponding to numerical dispersion and diffusion, can be found. Especially when the solution is sufficiently smooth this method shows a high level of accuracy (Timmermans and van de Vosse (1993)). In section 3.2.3 some examples of the properties as described above will be elucidated.

3.2.1 Fundamentals of SEM

The fundamentals of SEM will be described on a level which allows the reader to understand the basic ideas of this method. A more thorough discussion can be found in e.g. Canuto et al. (1988), Timmermans (1994) and Patera (1984).

After the temporal discretisation a set of equations remains of the form

\[ A c - B \nabla^2 c = f, \tag{3.22} \]

where \( c \) denotes the velocity vector \( \vec{u}_{p}^{n+1} \) or the temperature scalar \( \Theta^{n+1} \). Therefore \( f \) can be either a vector or a scalar. For the velocity the coefficients \( A, B \) and \( f \) are defined as (see eq. 3.15),

\[ A = 3/2, \quad B = \frac{\Delta t}{Re_D}, \quad \vec{f} = 2 \vec{u}_p^n - 1/2 \vec{u}_p^{n-1} + \Delta t(- \nabla p_p^{n+1} - R l_D(\Theta^{n+1} \vec{g})), \tag{3.23} \]

for the temperature as (see eq. 3.9)

\[ A = 3/2, \quad B = \frac{\Delta t}{Re_D Pr}, \quad f = 2 \Theta^n - 1/2 \Theta^{n-1}, \tag{3.24} \]

and for the pressure as (see eq. 3.18)

\[ A = 0, \quad B = -1, \quad f = \frac{3}{2\Delta t} \nabla \cdot \vec{u}_p^{n+1}. \tag{3.25} \]

In the discussion of the spatial discretisation the basic equation form (eq. 3.22) will be used in which \( c \) and \( f \) will be considered to be a scalar. At the end of the discussion about the spatial discretisation, the full discretised version will be given again with \( A, B \) and \( f \) written explicitly.
The basic formulation of the Spectral Element Method is equivalent to the formulation of the Finite Element Method. By considering a strong formulation of the partial-differential equation and involving a chosen weighting functions \(w\), a so-called weighted residual formulation is derived. The equation in the weighted residual formulation reads,

\[
\int_{\Omega} [Ac - B \nabla^2 c - f] wd\Omega = 0 ,
\]

where \(\Omega\) denotes the domain in which \(c\) is defined.

A more convenient presentation of this equation, in which the order of the differential equation is lowered by applying Green’s formula followed by integration by parts, is

\[
\int_{\Omega} [Ac - f] wd\Omega + B \int_{\Omega} (\nabla c \cdot \nabla w) d\Omega - \int_{\partial \Omega} w \nabla c \cdot \vec{n} d\partial\Omega = 0 ,
\]

with \(\partial \Omega\) the boundary surface around \(\Omega\) and \(\vec{n}\) the normal vector on \(\partial \Omega\). For the Spectral Element Method, a domain decomposition is applied in which the entire domain is divided in \(n_{el}\) non-overlapping sub-domains \(\Omega_e\) so that

\[
\Omega = \bigcup_{e=1}^{n_{el}} \Omega_e , \quad \bigcap_{e=1}^{n_{el}} \Omega_e = \emptyset .
\]

Applying this decomposition to equation 3.27 one can write

\[
\sum_{e=1}^{n_{el}} \left( \int_{\Omega_e} [Ac - f] wd\Omega_e + B \int_{\Omega_e} (\nabla c \cdot \nabla w) d\Omega_e - \int_{\partial \Omega_e} (w \nabla c \cdot \vec{n} d\partial\Omega_e) \right) = 0 ,
\]

with \(\partial \Omega_e\) the boundary surfaces of \(\partial \Omega_e\). Evaluation of the third term will only result in a contribution for element surfaces coinciding with \(\partial \Omega\).

The sub-integrals over the sub-elements will be evaluated on a standard domain \(e = [-1, 1] \times [-1, 1] \times [-1, 1]\). Therefore the original element \(\Omega_e\) will be mapped on this standard domain by use of a transformation Jacobian \(J\). For the sake of simplicity, this Jacobian is assumed to be the unity transformation Jacobian.

So far the equations are still exact. The final part of the discretisation is obtained by defining a truncated expansion \(c_h\) for \(c\) within each element,

\[
c_h(\vec{x}) = \sum_{i=0}^{N_{el}} c_i \Psi_i(x, y, z) ,
\]

with \(N_{el}\) the total number of nodal points in an element, \(c_i\) the value of \(c\) in point \(i\) and \(\Psi_i(x, y, z)\) an approximation function. By using an \(n_e\)th-order approximation function, convergence of the solution, defined by \(\lim_{N \to \infty} ||c_h - c|| \cdot N^{-k}\) with \(k > 0\) and \(N\) the total number of integration points, can be reached by either increasing \(n_e\) or by increasing the number of elements \(n_{el}\). The former approach, which is more likely to be called a spectral approach, very well fits to simple geometries like cylinders and cubes. For a more complex geometry convergence can be reached by
increasing the number of elements \( n_{el} \) for which the order of the interpolants \( n_o \) remains constant. Setting \( n_o \) equal to 1 or 2 results in a Finite-Element formulation.

For the basis functions of the truncated expansion, \( \Psi_i(x, y, z) \), a combination of the one-dimensional orthogonal Lagrangian interpolants, defined on the Gaus-Lobatto points, are used. These interpolants \( \psi^{n_o} \) are defined within the element and can be written for the \( i \)th nodal point as

\[
\psi^{n_o}_i(x) = -\frac{1}{n_o(n_o+1)} \frac{1 - x^2}{L_{n_o}(x_i)} \frac{dL_{n_o}}{dx},
\]

with \( L_{n_o} \) the \( n_o \)th-order Legendre polynomial defined as

\[
L_{n_o+1}(x) = \frac{2n_o + 1}{n_o + 1} x L_{n_o}(x) - \frac{n_o}{n_o + 1} L_{n_o-1}(x), \quad L_1(x) = x, \quad L_0(x) = 1.
\]

Within an element the basis functions are chosen such that at the nodal point \( i \) (with position \( x_i \)) the function value is equal to one, while in all other nodal points the function value is zero (fig. 3.1).

![Figure 3.1: 7th-order Lagrangian interpolants for \( i = 0, 1, 2 \)](image)

In order to obtain the three-dimensional basis function \( \Psi_i \) multiplication of 3 one-dimensional basis functions is applied. The truncated expansion of \( c \) can then be written as

\[
c_i(x, y, z) = \sum_{p=0}^{n_o} \sum_{q=0}^{n_o} \sum_{r=0}^{n_o} c_{ppq} \psi_p(x) \psi_q(y) \psi_r(z),
\]

(3.33)
with \( \psi_p(x) \), \( \psi_q(y) \) and \( \psi_r(z) \) the chosen one-dimensional interpolants. Here it is assumed that for the momentum as well as the energy equation the same grid and thus the same number of points is used and that the order of the approximation polynomial is equal in all three directions.

Choosing the basis of the trial functions \( \psi \) to be the same as the test or weight functions \( w \) results in the Galerkin weighted residual formulation. For the weighting function \( w \) one can write

\[
w(\vec{x}) = \sum_{q=0}^{n_\alpha} \sum_{r=0}^{n_\alpha} \sum_{p=0}^{n_\alpha} w_{pqr} \psi_p(x) \psi_q(y) \psi_r(z).
\]

The test functions and the basis functions now span the same space. For this choice it can be proven that the residual reaches a minimum and that the error introduced by the approximation will also show a minimum (Minev (1996)). Using the truncated expansion \( c_h \) and \( w \) results in a set of integral equations.

Using high-order polynomials in the truncated expansion of the trial and weighting functions does not necessarily result in a spectral accuracy. Also the integration of the relations (eq. 3.29) over the element domain \( \Omega_e \) needs to be performed with a spectral accuracy. The integration of these equations is done by applying a quadrature for which the introduced integration error is of the same order or smaller than the approximation error. Integration over the elements is carried out by using a Gauss quadrature based on \( n_\alpha + 1 \) Legendre-Gauss-Lobatto points, resulting in a linear set of equations which can be written in matrix notation as

\[
AM\bar{c} + BDu\bar{c} = Mf,
\]

with \( M \) the mass matrix and \( Du \) the diffusion matrix and where \( \bar{c} \) and \( f \) denote the columns filled with the nodal point values of \( c \) and \( f \), respectively.

Unfortunately the integration is only exact for polynomials of order \( 2n_\alpha - 1 \) where as in our case the polynomials to be integrated are of the order \( 2n_\alpha \) (trial function of order \( n_\alpha \) and weighting function too). It can be proven, however, that the integration error still shows a spectral convergence (Maday and Patera (1989)).

Rewriting equation 3.35 in the explicit notation of \( A \), \( B \) and \( f \) one can write for the pressure correction algorithm the following sequence of steps:

- Determine \( \Theta \) by solving

\[
(3/2M_\Theta + \Delta tD_{u_\Theta})\tilde{\Theta}^{n+1} = M_\Theta(2\tilde{\Theta}^n - 1/2\tilde{\Theta}^{n-1}),
\]

with \( D_{u_\Theta} \) the temperature diffusion matrix, \( M_\Theta \) the temperature mass matrix and where \( \tilde{\Theta}^n \) and \( \tilde{\Theta}^{n-1} \) are solved from the discretised associated initial-value problem:

\[
\tilde{\Theta}^{m+\frac{1}{2}} = \tilde{\Theta}^m + M_\Theta^{-1}\Delta s \; C_\Theta^{m+\frac{1}{2}} \tilde{\Theta}^{m+\frac{1}{2}};
\]

\[
\tilde{\Theta}^{m+\frac{1}{2}} = \tilde{\Theta}^m + M_\Theta^{-1}\Delta s \; C_\Theta^{m+\frac{1}{2}} \tilde{\Theta}^{m+\frac{1}{2}};
\]

\[
\tilde{\Theta}^{m+1} = \tilde{\Theta}^m + M_\Theta^{-1}\Delta s \; C_\Theta^{m+\frac{1}{2}} \tilde{\Theta}^{m+\frac{1}{2}},
\]

(3.37)
and initial conditions $\Theta^0 = \Theta^{n+1-i}$ with $C^m_i$, the temperature-convection matrix on the corresponding time level $m_i$. The velocity appearing in $C^m_i$ is extrapolated from the previous time step by using a second-order accurate in time extrapolation scheme (Minev et al. (1994)).

- Determine $u^{n+1}_p$
  
  \[
  (3/2M_u + \Delta t D_u)u^{n+1}_p = M_u (2\tilde{u}^n - 1/2\tilde{u}^{n-1} + \Delta t f^{n+1}),
  \]

  with $D_u$, the velocity diffusion matrix, $M_u$, the velocity mass matrix and where $\tilde{u}^n$ and $\tilde{u}^{n-1}$ follow from

  \[
  \tilde{u}^{m+\frac{1}{3}} = \tilde{u}^m + M^{-1}_u \frac{\Delta s}{3} C^m_u \tilde{u}^m, \\
  \tilde{u}^{m+\frac{1}{2}} = \tilde{u}^m + M^{-1}_u \frac{\Delta s}{2} C^m_u \tilde{u}^{m+\frac{1}{2}}, \\
  \tilde{u}^{m+1} = \tilde{u}^m + M^{-1}_u \Delta s C^m_u \tilde{u}^{m+\frac{1}{2}},
  \]

  with $C^m_u$ the velocity convection matrix on the corresponding time level $m_i$, $f^{n+1} = -RiD\Theta^{n+1}q^T - Q\tilde{u}_p^{n+1}$ and $Q$ the gradient matrix.

- Compute the correction term $q$ from
  
  \[
  Kq = -\frac{3}{2\Delta t} Ldw^{n+1}_p,
  \]

  with $K$ the Laplacian and $Ld$ the divergence matrix.

- Calculate the corrected velocity $w^{n+1}_c$ by
  
  \[
  w^{n+1}_c = w^{n+1}_p - \frac{2}{3}\Delta t M^{-1}_u Q\tilde{u}. 
  \]

- Compute the pressure at time level $n + 1$ by
  
  \[
  p^{n+1} = p^n + q - \frac{1}{Re_D} M^{-1}_p Ld\tilde{u}^{n+1}_p.
  \]

### 3.2.2 Preconditioning

After rearranging and filling in the terms which can be calculated or estimated one finally arrives at a point where the total system can be written in matrix notation

\[
S \xi = M f, 
\]

with $S$ the system matrix of the spectral system. This matrix in general is fairly full populated, especially when the order of the approximation polynomials becomes large. Consequently, a direct
method of solving this system becomes very time consuming, especially for problems in more dimensions. An efficient method therefore would be an iterative solution procedure. Unfortunately, the system matrix is ill-conditioned (Canuto et al. (1988)). Preconditioning of the matrix is therefore applied by using the finite element equivalent of this spectral system matrix (Deville and Mund (1985)). This matrix $F$ is built on the same nodal points as the spectral matrix, so it can be expected that this matrix will be a good preconditioner for the spectral matrix. The iterative method as proposed by Deville and Mund (1985) is a simple Richardson scheme and reads as

\begin{align}
F \xi^0 &= M f, \\
F \xi^k &= F \xi^{k-1} + \gamma_n (M f - S \xi^{k-1}) & k = 1, 2, \ldots.
\end{align}

with $k$ the iterative counter and $\gamma_n$ a relaxation parameter (Deville and Mund (1990)) which varies during the iteration. It is now clear that the costly operation computing the inverse of the large system matrix $S^{-1}$ is omitted in order to find the solution of $\xi$. Since the residuals $M f - S \xi^{k-1}$ are calculated on element level also a severe cutting of the memory storage is obtained.

### 3.2.3 Spectral convergence

As already mentioned in the introduction of this chapter, one of the advantages of the Spectral Element Method is its spectral convergence. This spectral convergence, which reveals itself by a stronger than linear convergence rate, is obtained by choosing high-order basis functions within the Galerkin approach and by taking the Gauss-Lobatto points as integration points. In this section this spectral convergence is elucidated by assuming a simple differential equation which resembles the system equation 3.22. Therefore the following one-dimensional partial differential equation is chosen

\[-\frac{\partial^2 u}{\partial x^2} = K \cos(Kx), \quad x \in [0, \pi],\]

with $K$ a positive integer and with essential boundary conditions

\[
\begin{align*}
u(0) &= 1, \\
u(\pi) &= \cos(K\pi).
\end{align*}
\]

For the exact solution one then can write

\[
u(x) = \cos(Kx).
\]

For the determination of the numerical accuracy the maximum found error in the integration points is used:

\[
\varepsilon = ||u(x) - u_h(x)||_{\infty,gl}.
\]

In this definition the subscript $\infty, gl$ means that the error is evaluated in the Gauss-Lobatto points.

For $K = 1$ the effect of an increasing number of integration points $N$ on the numerical error is analyzed. Increasing the number of integration points can be achieved by increasing the order
of the approximation polynomial \( n_o \) or by increasing the number of elements \( n_{el} \). For a fixed polynomial order and an increasing number of elements, the discrete maximum error \( \varepsilon \) decreases linearly (in a \( \log - \log \) graph, fig. 3.2a). As one can see, the slope of this line is determined by the order of the used polynomial \( n_o \). For calculations on a mesh with a constant number of elements but with an increasing polynomial order, the discrete maximum error decreases faster than linear (fig 3.2b) which illustrates the spectral convergence rate. In conclusion, one can state that grid refinement by increasing the number of elements results in a linear decrease of the integration error, where the slope is defined by the used order of the approximation polynomial. For grid refinement by increasing the order of the polynomials, a spectral convergence can be obtained.

![Figure 3.2: Discretisation error as function of the total number of integration points \( N \) in the domain: (a) for increasing number of elements, (b) for 2 elements and increasing order \( n_o \) (here \( N = 2n_o \)).](image)

By increasing the wavenumber \( K \) of the solution, the performance of the code with respect to the capability to solve the different wave numbers can be analyzed. This performance is elucidated by doing simulation sequences with varying \( K \) and constant number of integration points \((N = 25)\). For every sequence in which \( K \) is varied the number of elements and order of the approximation remained fixed but between sequences the order and the number of elements was varied (where \( N \) remained constant). Again the maximum discrete error \( \varepsilon \) was used as interrogation parameter and it is plotted as function of the wave number \( K \) (fig. 3.3). For the simulation with a low-order approximation polynomial \((n_o < 3)\) \( \varepsilon \) increases already for relatively small \( K \) values \((K < 5)\). For simulations with high-order polynomials \((n_o > 8)\), \( \varepsilon \) remains small up to \( K \approx 12 \), and then increases rapidly. The value of \( K \) for which this increase starts is dependent on the used order, as one can see from the figure.

In a more theoretical approach, the Nyquist criterion shows that the maximum solvable wave
number $K_{max}$ is fully determined by the used grid and can be written as

$$K_{nyq.} = \frac{\pi}{\Delta x},$$

with $\Delta x$ the spacing between nodal points. For the presented simulations where the average $\Delta x$ is equal to $\pi/(N - 1) = \pi/24$, this maximum wave number is $K_{nyq.} = 24$. The results show, however, that for the simulations with the higher-order polynomials ($n_o > 8$) the maximum error $\varepsilon$ remains small up to $K \approx 10$ and then increases rapidly. From this observation it can be concluded that the maximum wave number $K_{max}$ that can be represented is about half of the theoretical maximum wave number as predicted by the Nyquist criterion. For simulations with lower-order approximation polynomials this wave number becomes even smaller.

As one can see, the maximum solvable wave number for which $\varepsilon$ remains acceptable (less than $10^{-2}$) does not increase dramatically by increasing $n_o$ starting from $n_o = 8$. On the other hand, the computational time and storage requirement increase faster than linear (Minev (1996)) with increasing order $n_o$. Considering the convergence rate (fig. 3.2) and the capability of resolving the wave number of the solution (fig. 3.3), an optimal choice for $n_o$ is around 8. Further grid refinement can then be obtained by increasing the number of elements.

### 3.3 Calculation of an external flow around a cylinder

It is obvious that the grid and the boundary conditions chosen for the calculation of an external horizontal flow over a heated cylinder will influence the accuracy of the obtained solution. Therefore the effect of the domain parameters in combination with the chosen boundary conditions (fig. 3.4) are analyzed.
3.3. Calculation of an external flow around a cylinder

3.3.1 Calculation domain and boundary conditions

The influences of the chosen variables $B_1$, $B_2$, $L$ and the used boundary conditions on a forced convection flow is already investigated by other researchers. Especially the results of Karniadakis and Triantafyllou (1992) highlight this influence where the Strouhal number is used as interrogation parameter determined at $(x/D, y/D) = (10, 0)$. These results show that the inflow length $B_1$ only influences the Strouhal number for $B_1/D < 8$. By changing the wake length $B_2$, the Strouhal number was almost not affected for $B_2/D > 25$. The results also show that for an increasing domain width $L$ a constant Strouhal number is reached. This constant value is reached slightly earlier if at the upper and lower boundary ($BC_{upper}$, $BC_{lower}$) potential flow conditions are applied rather than periodic boundary conditions. Although the number of presented values of $L$ is limited, it can be stated that for $L/D > 12$ the influence of these boundaries on the Strouhal number becomes small.

For mixed convection flows an additional problem arises. Due to the buoyant forces, a vertical upward flow occurs. This flow imposes probably a more severe restriction on the value of $L$ than a forced convection flow. The effect of an increasing $L$ on the obtained solution is numerically investigated. The prescribed boundary conditions are presented in table 3.1. At the horizontal walls no outflow is allowed ($v = 0$) and in combination with $\partial u/\partial y = 0$ this implies that at these walls no tangential stresses occur ($\sigma_t = 0$, with $\sigma_t$ the tangential stress). For the boundaries on which $u$ is prescribed by a Dirichlet condition, the boundary conditions for $q$ follow immediately from the global mass conservation (Timmermans et al. (1996)) and read $\partial q/\partial n$. At the outflow
Table 3.2: Interrogation quantities as function of the domain width $L$

| $L/D$ | $St_{x/D=10}$ | $\bar{u}_{x/D=0}/U_0$ | $|v_{max}/U_0| + |v_{min}/U_0|_{x/D=10}$ |
|-------|----------------|-------------------------|-----------------------------------|
| 8     | 0.1543         | 1.067                   | 0.7257                            |
| 12    | 0.1529         | 1.043                   | 0.7127                            |
| 16    | 0.1527         | 1.032                   | 0.6952                            |
| 24    | 0.1527         | 1.021                   | 0.6628                            |

wall where $\bar{u}$ is prescribed by a Neumann boundary condition, $q$ is prescribed by the Dirichlet condition $q = 0$. In combination with the conservation of mass and using $p^D_\rho = 0$ this implicitly states that the pressure $p$ is zero at the outflow wall (eq. 3.18). These boundary conditions for $q$ and $\bar{u}$ can be translated into a normal-stress-free condition at this wall ($\sigma_n = 0$ with $n$ the normal direction to the wall). At the cylinder wall the no-slip conditions are applied, while the dimensionless temperature is set to one. With these boundary conditions the effect of $L$ is investigated for $L/D = 8, 12, 16$ and $24$ with the other geometric parameter values set to $B_1/D = 8$ and $B_2/D = 25$. Here it is ensured that for changing domain parameters no changes are made within the grid-point distribution.

As interrogation quantities the Strouhal number and the velocity profiles at $x/D = 10$ are used. The calculations were carried out for $Re_D = 73$ and $Re_D = 1$ for which the flow is supposed to

![Figure 3.5: Temporal evolution of the velocity component $v$ at $(x/D, y/D) = (10, 0)$ for domain widths $L/D = 8, L/D = 12, L/D = 16$ and $L/D = 24$ for $Re_D = 75$ and $Re_D = 1$: (a) total time calculation, (b) one period](image)

be 2D periodic. By increasing the width $L$ the relative flow-through area at $x/D = 0$ is enlarged, which causes the average velocity $\bar{u}_{x/D=0}/U_0$ at that cross-section to approach this limiting value $\bar{u}_{x/D=0}/U_0 = 1$. The small dependency of $L$ on $\bar{u}_{x/D=0}/U_0$, even for $L/D = 24$, will probably
have an influence on the strength of the shear layers and the produced vorticity. Consequently, the strength of the shed structures and the shedding frequency will be influenced. This can be clearly seen from the difference between the extreme values of the $y$-velocity $|v_{\text{max}}| + |v_{\text{min}}|/U_0$, measured at $(x/D, y/D) = (10, 0)$ and from the Strouhal number, measured at a downstream position $(x/D, y/D) = (10, 0)$ (tab. 3.2). For a decreasing $L$, the largest Strouhal number and largest velocity difference are measured.

Although the Strouhal number becomes independent of the domain width $L$ for $L/D > 16$ (Table 3.2), the velocity remains slightly dependent on $L$. This can also be seen from the time-dependent velocities measured at the point at which the Strouhal number is calculated (fig. 3.5a). Closer examination of the signal reveals that the extreme values of $v/U_0$, respond to the widening of the domain.

The domain width $L$ not only influences the vortex shedding process, it can also influence the expansion of the wake further downstream. The influence of $L$ on the wake expansion becomes evident by considering the velocity profile at the downstream position $x/D = 10$ (fig. 3.6a). These velocities are taken for the different $L$’s at a similar time position in the period. This implies that phase differences occurring between the different results have no influence on the presented velocities. For increasing widths $L$, the velocity component $v/U_0$ in this cross-section decreases in the wake core but increases on both sides of the wake center axis. At $L/D = 24$ it can be concluded that even for this width the velocity component $v$ is still dependent on $L$. Therefore it is assumed that the flow imposes a force on the horizontal boundaries and, by Newton’s first law, the walls impose a force on the internal fluid flow. This influence can be estimated by calculating

\[
\begin{align*}
\text{Figure 3.6: Effect of the domain width } L \text{ on the wake behaviour for } &\text{Re}_D = 75 \text{ and } Ri_D = 1: \text{ (a) the velocity component } v/U_0 \text{ at } x/D = 10 \text{ at similar time moments within the period, (b) period average of the stresses at the wall for } L/D = 24
\end{align*}
\]
the net force exerted by the horizontal walls on the fluid flow, which is equal to

\[ f_{n,\text{boundary}} = - \int_{B_1+B_2} \sigma_n dx = - \int_{B_1+B_2} (1/Re_D \frac{\partial v}{\partial y} - p) dx. \] (3.49)

For \( L/D = 24 \) it turns out that the average stresses at the upper wall are slightly larger than the ones at the lower wall (fig. 3.6b). The observed difference between these stresses gives rise to a net force, which can cause the flow to deflect in negative \( y \)-direction. An estimation of this deflection can be made by conservation of momentum over a total box with inflow velocity \( U_0 \), stress free conditions at the outflow and no outflow through the horizontal walls. When the horizontal walls impose a net force on the flow equal to the one as calculated for \( L/D = 24 \), a negative \( y \)-velocity component will occur at the outflow, equal to \( 0.004 U_0 \). One can conclude, that the force imposed by the horizontal walls on the fluid flow has a negligible effect on the velocity component \( v \).

### 3.3.2 Grid dependency

In order to investigate the grid dependency, grid refinement is applied on a domain with geometric parameters \( B_1/D = 8, B_2/D = 25, L/D = 24 \), boundary conditions as shown in table 3.1 and \( Re_D = 73 \) and \( Ri_D = 1 \).

In appendix B the element-wise subdivisions of the different grids are presented, which are referred to in the text and figures. The calculations are performed by using a 9th-order polynomial. The first calculation is done on grid3, which contains 170 elements and 14031 nodal points. Further refinement is obtained by applying local grid refinement close to the cylinder and in the cylinder near-wake. In these regions the grid is about 1.5 times finer than for grid3, resulting in grid2, containing 328 elements and 26910 nodal points. An additional refining of the grid close around the cylinder and in the wake core results in grid1, which contains 508 elements and 41238 nodal points.

The effect of grid refinement on the solution is investigated by considering the Strouhal number measured at a downstream position \((x/D, y/D) = (10, 0)\) (Table 3.3). The Strouhal number shows how grid refinement influences the strength of the shed structures and the shedding mechanism. For increasing nodal points in the wake, it can be seen that the Strouhal number is different on grid3 and grid2, but remains constant on grid1 and grid2. This implies that the shedding process is independent of the computational grid for grid2 or finer.

For the wake characteristics the influence of grid refinement is analyzed by considering the velocity profiles in a cross-section at \( x/D = 5 \) and \( x/D = 20 \) (figs. 3.7a,b). At the latter position the wake is fully developed and shows all characteristic features. The velocity profiles in the cross-section show that only for grid3 the results are dependent on the used computational grid.

---

<table>
<thead>
<tr>
<th>Grid</th>
<th>1 (fine)</th>
<th>2 (medium)</th>
<th>3 (coarse)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( St )</td>
<td>0.1527</td>
<td>0.1527</td>
<td>0.1540</td>
</tr>
</tbody>
</table>
For further grid refinement (from grid2 to grid1) no influence of the grid on the wake velocity can be detected. This also holds at $x/D = 5$.

![Graph](image)

Figure 3.7: Effect of grid refinement on the velocity component $u$ in a cross-section for $Re_D = 73$, $Re_D = 1$ at $t = 60$: (a) the velocity component $u/U_0$ at $x/D = 5$, (b) the velocity component $u/U_0$ at $x/D = 20$

So far the effect of grid refinement on the wake characteristics is considered. Also interesting is the effect of grid refinement on the boundary-layer characteristics, as they probably influence the shed structures. The effect of grid refinement on the boundary layer is investigated by considering the vorticity along the cylinder upper wall (fig. 3.8a) and the velocity in a cross-section at $x/D = 0$ (fig. 3.8b). As can be concluded from the figures, the influence of the grid is only visible for the coarsest grid (grid3). Further refinement from grid2 to grid1 does not influence the results.

For calculations with $Pr > 1$, the temperature field will impose stronger restrictions on the used grid. This can be seen in the grid refinement calculations where the calculated temperature shows a stronger grid dependency. In the temperature profile along the $x$-axis small spikes of negative temperature coefficients can be observed (fig. 3.9). The results show that due to grid refinement these spikes decrease. Although these small spikes have no direct effect on the obtained velocity field, derived quantities as vorticity production may be influenced. Therefore the calculations are carried out for both the velocity and temperature on the most refined grid, which is grid 1.
Figure 3.8: Effect of grid refinement on the boundary layer characteristics at $t = 50$ for $Re_D = 73$ and $Ri_D = 1$: (a) wall vorticity along the upper cylinder, (b) the velocity component $u/U_0$ through the boundary layer at $x/D = 0$.

Figure 3.9: Calculated temperature in the cylinder wake for $Re_D = 73$ and $Ri_D = 1$ at $t = 50$: (a) dimensionless temperature along the $x$-axis (full scale), (b) dimensionless temperature along the $x$-axis (zoomed scale).
Chapter 4

Experimental methods

For the experimental investigation a water-tank facility had been designed and built in which several measuring techniques are applied. The design of this set-up will be the subject of the first part of this chapter (section 4.1). In the second part, the experimental techniques as flow visualisation (section 4.2) and two novel particle tracking techniques, 2D High Resolution Particle Velocimetry (2D HiRes-PV) and 3D Particle Tracking Velocimetry (3D PTV), are discussed. Especially the development of the latter two methods has been a research topic on its own. In section 4.3 these two methods will be introduced and their performance with respect to the considered flow field will be discussed in detail. In section 4.4 a brief overview is given of the measuring procedure and the post-processing of the results.

4.1 Experimental set-up

The physical phenomena as well as the used measurement techniques impose different specific demands on the test rig. The used measurement techniques (HiRes-PV, 3D PTV and visualisation techniques) create several restrictions on the dimensions and materials used for the test-section. For the particle tracking and visualisation measurements the set-up needs to be optical accessible, for 3D PTV at least from two directions. This requirement led to a design of a fully transparent set-up made out of glass or perspex windows. Special attention had to be given to restrict the thickness variations and bending of the windows, since these may induce optical distortions, hence causing observation errors. The diameter of the cylinder is determined by the demand to optically detect the shed vortex structures and by the demand that the aspect ratio $\mathcal{A} = l/D$ should be set as large as possible. It turns out that the dimensions of the shed vortices are of the order of the cylinder diameter $D$ and those can be easily detected for a typical size of a centimetre or larger. In combination with the restriction on $\mathcal{A}$, this has led to the choice $D = O(1cm)$.

Considering the physical phenomena to be investigated, it is known that boundary layers along the side walls may have a large influence on the vortex shedding process. Especially for relatively low velocities, the boundary layers develop over a short distance. As already mentioned in the introductory chapter, boundary layers can cause oblique vortex shedding (Williamson (1989)). Hence, the relative boundary layer thickness with respect to the spanwise dimension of the cylin-
Chapter 4: Experimental Methods

der needs to be minimised. Also variations in the inflow conditions affect the dynamics and the stability of the vortex street. For example, fast fluctuations in the inflow velocity and temperature can penetrate into the shear layers and may result in a disturbance of the vortex shedding process and the subsequent development of the wake. Consequently, the inflow conditions need to be very stationary and in the worst case, only small variations with a large spatial frequency are allowed.

The above mentioned specific demands have led to the design of a towing tank set-up where a relative velocity of the cylinder with respect to the fluid is realised by towing the cylinder through the water tank. The main advantages of this device are a minimal creation of boundary layers and almost uniform inflow conditions (Anagnostopoulos and Gerrard (1978)).

The specific dimensions of the towing tank, which are length \( \times \) width \( \times \) height = 500 cm \( \times \) 50 cm \( \times \) 75 cm (fig. 4.1a), are chosen by considering their effect on the flow field and measurement. By choosing \( Re_D \approx 75 \), the length of the tank allows to measure about 60 characteristic periods of the specific flow problem. This number of periods is enough to apply reasonable statistics on the acquired data. The width of the tank is chosen such that the relative thickness of the boundary layers occupy less than a few percent of the section width. These boundary layers occur along the carrying plates of the cylinder (fig. 4.1b). For \( Re_D \approx 75 \) and a cylinder diameter of \( D = 8.5 \) mm, it turns out that at the cylinder position the relative thickness is about 1% of the total width and grows to 5% at about 30 \( D \) behind the cylinder. Although the boundary layers can still cause oblique vortex shedding, their direct influence is assumed to be relatively small. The height of the test section is determined by the effect of the tank bottom and the free surface on the flow around the cylinder. The tank bottom can have a blocking effect on the flow. If the distance between the bottom and the cylinder is larger than about 20\( D \), blockage effects become negligible (Nishioka and Sato (1974)). Therefore, in the designed water tank, the cylinder is positioned about 25\( D \) above the bottom surface. The effect of the free surface, especially for heat induced flows, is less investigated. In the present set-up, the distance between the free surface and the cylinder is safely set to 50\( D \).

In the set-up, the cylinder (with diameter \( D = 8.5 \) mm and length \( l = 495 \) mm), is kept in position by two perspex plates (fig. 4.1b) with specific dimensions length \( \times \) width \( \times \) height = 50 cm \( \times \) 0.4 cm \( \times \) 65 cm. This contraption allows to position the cylinder perpendicular to both the main flow and the direction of gravity with an accuracy of 0.05°. Besides, the plates also act as end plates and are assumed to suppress oblique vortex shedding (Williamson (1989)). The plates are connected to a stiff structure carrying the cylinder and measuring equipment. The carrier is translated along two guiding rails, mounted on top of the water tank. For the translation of the cylinder containing structure an electromotor is used. The motor is coupled to the drive-wheel by using a 1 : 100 gear and is corrected for its variation in rotational speed by means of a closed circuit, resulting in a variation in the rotational speed of less than 0.2%. Around the drive and the idle wheel of the translation system an almost inelastic fiber based tape is looped. This fiber is connected to the camera/cylinder structure.

To obtain the desired cylinder wall temperature a rod heater is used with a maximum heat density of 8.0 W/cm² and a diameter of 6.35 mm. Around this heating element two copper shields are mounted (fig. 4.2a) with a total thickness of 1.1 mm. The inner shield has an outer diameter of 8.0 mm and is used to damp out fast temperature fluctuations of the rod heater and to homogenize the spatial temperature variations in spanwise and tangential direction. At the outer edge of the
inner shield three thermocouples are placed (fig. 4.2a). To obtain once more a smooth surface, a second shield is mounted over the thermocouples. This shield has an outer diameter of 8.5 mm and surface roughness of 0.5 μm.

Despite the two copper shields, the thermocouple measurements, averaged over about 500 samples, still show a temperature difference between the front ($T_a$) and rear end ($T_c$) of the cylinder (fig. 4.2b, see + marks). This temperature difference, increasing for increasing $Ri_D$, most likely arises due to a circumferential variation in heat transfer along the cylinder wall. However, the relative value of this temperature difference with respect to the temperature difference $T_1 - T_0$ is found out to be almost constant and equal to 2% (fig. 4.2b, see * marks). Here the cylinder wall temperature $T_1$ is chosen to be $T_b$, the intermediate temperature between $T_a$ and $T_c$.

Also spanwise temperature variations can arise due to spanwise variations in the heat transfer coefficient from the cylinder wall to the main flow or due to spanwise variations in the heat source. The spanwise variations of the heat transfer coefficient are estimated to be a few percent of the variations in circumferential direction while the spanwise heat source variations are, according to the specifications of the heating element, less than 1%. Therefore, it is assumed that spanwise temperature variations are at least an order of magnitude smaller than circumferential temperature variations.

The time dependent signals of the thermocouples show that variations in time are very small. From a typical temperature signal (fig. 4.3a), it can be concluded that temporal temperature variations are limited to ±0.05 K. These variations are mainly caused by the random noise on the temperature couples and by the resolution of the thermocouple reading device.

During the experiments the camera can be translated along the test section windows. The thickness variations of these windows can cause an error in the measurements. By taking test section windows with a maximum thickness variation of 50 μm (according to the manufacturer’s speci-
Figure 4.2: Cylinder characteristics: (a) configuration of the cylinder, (b) temperature difference between cylinder-rear and -front end as function of $Ri_D$, + marks scaled according to the left axis, * according to the right axis)

...
power spectra of the accelerations in \( x - , y - \) and \( z - \)direction were calculated (fig. 4.3b). It turns out that the most significant frequency for the \( y - \) and \( z - \)directions is 8 Hz while for the \( x - \)direction also at 6 Hz a significant peak in the power spectrum can be observed. The corresponding amplitudes for all these frequencies are approximately \( O(10^{-3}D) \). Therefore it is justified to assume that the translation induced vibrations are of minor influence on the vortex shedding process. To reduce the induced vibrations even further the two guiding rails were cleaned from dust and lubricated with silicon oil before each experiment. To prevent a discontinuous translation speed of the carriage, the variations of the separation between the two guiding rails were brought down to \( \pm 0.05 \) mm.

A major drawback of any large water tank facility, is the sensitivity for temperature differences between the water in the tank and the surrounding temperature in the laboratory room. This difference can cause significant thermally-driven fluid motions. These can be minimised, however, by positioning the water tank in a temperature controlled room. In addition, the water tank was thermally insulated from the laboratory room. The resulting background velocity appears to be about 0.2 mm/s and has a spatial variation of about the same size as the cross-sectional dimensions of the set-up (see also section 4.4). The temperature differences responsible for the background motion turn out to be approximately 0.1 K and are mainly found close to the test section walls.

The previously discussed error sources arising from the set-up can be related to error sources in the relevant quantities as inflow velocity \( U_0 \) and temperature \( T_0 \), cylinder wall temperature \( T_1 \) and measured position vector \( \vec{x} \) (from which velocities can be derived). The deduced error sources are summarised in table 4.1.
Table 4.1: Test-rig-induced errors for different inflow conditions and quantities

<table>
<thead>
<tr>
<th>Quantity</th>
<th>$\bar{U}_0$</th>
<th>$T_0$</th>
<th>$\Delta T$</th>
<th>$\bar{x}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error</td>
<td>$\pm 0.02 \bar{U}_0$</td>
<td>$\pm 0.05$ K</td>
<td>$\pm 0.02 \Delta T$</td>
<td>$\pm 0.01$ mm</td>
</tr>
</tbody>
</table>

### 4.2 Flow visualisation

To obtain qualitative information about the flow field fluid flow visualisation experiments can be performed. In this way insight into the various flow phenomena can be obtained, useful for example for planning more detailed quantitative experiments. Within this research project two different visualisation methods were used: bubble wire and fluorescent dye visualisations. In this section both methods will be briefly introduced and the specific configurations will be described.

In the bubble wire experiments, small gas bubbles are generated to visualize the flow. The bubbles are created by the process of electrolysis. When a potential difference (typically between 20 and 40 Volts for the present configuration) is applied, hydrogen bubbles are formed at the cathode (negative) side according to the following chemical reaction

$$4H_2O^+ + 4e^- \rightarrow 2H_2(g) + 4H_2O.$$  \hspace{1cm} (4.1)

At the anode (positive) side oxygen bubbles are formed according to

$$4OH^- + 4e^+ \rightarrow 2H_2O + O_2(g).$$  \hspace{1cm} (4.2)

In general, the hydrogen bubbles are used for visualisation experiments. From the chemical reactions it can be seen that the production rate of hydrogen bubbles is twice that of oxygen bubbles. Also, the size of the hydrogen bubbles is smaller than that of the oxygen bubbles. This results in a lower bubble rise velocity for the hydrogen bubbles and a shorter response time on fluid flow changes. The anode is placed far downstream and the oxygen bubbles produced will not affect the measurements.

The hydrogen bubbles are created along a thin platinum wire. For a 40$\mu$m wire diameter, the bubble diameter becomes about 40 $\mu$m (Matsui et al. (1977)) with a corresponding rise velocity of about 1 mm/s. For the experiments this means that the bubble rise velocity is about 10% of the horizontal main flow velocity. The rising of the bubbles forms a major disadvantage of this measurement method, although this effect becomes smaller for decreasing bubble diameter or increasing main flow velocity. The wire is positioned in the flow with a fork contraption (fig. 4.4a). The fork is placed about 10$D$ upstream of the cylinder allowing the formed bubbles to catch up with the flow. By taking the fork dimensions such that the corresponding $Re_D < 30$, the disturbance of the main flow remains limited.

The generated bubbles will show a streak-line pattern which can be recorded with a camera. For the experiments the camera is placed such that the optical axis of the camera coincides with the cylinder axis. This allows to view the behaviour of the shed 2D vortex structures.

More detailed qualitative information is obtained from the fluorescent dye visualisations. In contrast to bubble wire visualisations, buoyancy effects are then almost absent. The visualisation
4.3 PARTICLE VELOCIMETRY

Using tracer particles for fluid velocity measurements is a well-established technique. For this, tracer particles are seeded in a fluid and illuminated within a defined area. The images of the moving particles in this area can be recorded and processed. Currently, several techniques based on particle visualisation have been developed to measure the velocity. When the defined area is a thin light sheet, 2D techniques as 2D Particle Tracking Velocimetry (2D PTV) and Particle Image Velocimetry (2D PIV) can be applied. In 2D PTV techniques, individual particles in subsequent images are tracked, whereas in 2D PIV the average displacement is determined within a segment of an image. For 3D investigations extensions of the previous techniques can be applied on an illuminated volume.

Within this research project mostly particle tracking methods were used. The accuracy of the particle tracking techniques is determined by the response time of particles on flow velocity vari-
Figure 4.5: Flow visualisation results: (a) bubble wire results, (b) dye visualisations viewed from above, (c) dye visualisation viewed along cylinder axis

ations and the accuracy with which the position of the particles can be determined from the obtained images. The accuracy of these positions is highly determined by the quality of the obtained image. The image quality is directly related to the light source and camera used and by the particle location algorithm (Blob Detection, fig. 4.6). It turns out that the particle localisation can be performed with sub-pixel accuracy. For a $1024^2$ pixels camera, and a measurement area of about $10 \times 10 \text{ cm}^2$, the particle location error is approximately 0.005 mm. This error then causes a defect in the measured velocity equal to $\approx 0.1 \text{ mm/s}$, which is about 1% of the main stream velocity.

In case the density of the particles is slightly different from their surrounding fluid, inertia forces acting on the seeding particles will cause that the particles do not respond directly to velocity fluctuations. The corresponding error is directly related to the size of the particles, the fluctuations in the flow field and the density difference between the particle and the ambient fluid. During all experiments the seeding particles had a maximum diameter of 50 $\mu$m (for 3D PTV). The nominal density of the hollow glass particles was 1000 kg/m$^3$ with a density variation of about 0.5%. For these particles the typical response time is $10^{-4} \text{s}$ which is considerably smaller than the typical time scale of the flow ($\approx 1 \text{s}$). Therefore, the particles used follow the flow almost perfectly.

### 4.3.1 2D High Resolution Particle Velocimetry

For the 2D experiments a hybrid measurement technique consisting of PTV and PIV is used. This results in a so-called Super Particle Tracking (Hart (1999)) or High Resolution Particle Velocimetry method (HiRes-PV). The algorithm functions according to the following steps (fig. 4.6): First the captured images (Image Reader) are dynamically thresholded to remove background intensity variations (Dynamic Thresholding). Then the images are processed to obtain the pixel coordinates of the particles present in the images (Blob Detection). Next, pixel coordinates are re-mapped to a physical coordinate set (Mapping). Finally, every particle in frame $f - 1$ is matched with a candidate particle in frame $f$ where the candidate particles are defined as all particles within a given
maximum matching distance $\Delta S_{\text{max}}$ from the particle in frame $f - 1$. From all candidate particles, the one situated closest to the position of the particle in frame $f$ is matched (Matching). The matching algorithm is improved by using an estimation for the particle position in frame $f$. This estimated position is provided by a prediction algorithm (Prediction).

In case the previous matching result gives insufficient information for a prediction, external information from file can be retrieved for each frame on demand (Background Velocity Reader). For HiRes-PV the external information used is the result obtained by PIV. A more detailed description of the relevant algorithm parts can be found in van der Plas and Bastiaans (1999). This paper also presents the performance tests of the algorithm, based on synthesised data. Tests on synthesised data are quite common and show the ideal properties of the method. Performance tests are rarely applied on real fluid flow data, although only then the real performance of the method can be checked. Therefore in this section, the method is tested on measurement data obtained from a towing tank experiment in order to show the ‘real’ performance with respect to the discussed physical problem.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{algorithm_diagram.png}
\caption{Schematic structure of the particle tracking algorithm.}
\end{figure}

**Performance**

The performance of the tracking algorithm is influenced by the quality of the image sequence and the setting of the tracking parameters. The quality of the measured image sequence determines whether or not tracking results representative for the measured flow field can be obtained. The quality is determined by the image quality ($q_i$) and sampling quality ($q_s$). A good image quality
means low noise and well detectable particle blobs in an image. The sampling quality indicates how well the particles can be tracked. This will be elucidated below.

For the sampling quality of an image sequence, the displacement of the particles with respect to mean inter-particle distance $d_n$ is of importance. A good measure of this particle distance $d_n$ is the mean minimum particle distance defined as $d_n = 1/2 \sqrt{\frac{A}{n_p}}$ with $n_p$ the total number of particles and $A$ the considered flow area. The sampling quality can then be expressed as the ratio between $d_n$ and the estimated maximum particle displacement $\Delta S_{\text{max}}$, occurring between two images,

$$q_s = \frac{d_n}{\Delta S_{\text{max}}}.$$ \hspace{1cm} (4.3)

The sampling quality gives an indication of the trackability of an image sequence. This can be understood by considering the effect of an increasing $\Delta S_{\text{max}}$ for a constant $d_n$. Assume a $\Delta S_{\text{max}} \gg d_n$, meaning that between two captured images, $f$ and $f + 1$, the displacement of the particles is much larger than the average distance between the particles. For this case it is very difficult to detect which particle in frame $f$ corresponds to a particle in frame $f + 1$. In this situation an erroneous candidate is more likely to be closer to the original particle position in frame $f$ than the correct particle. An erroneous matching is then unavoidable, resulting in a decreasing accuracy of the measured velocity field.

The accuracy and performance of the HiRes-PV algorithm is tested by comparing it to a standard PTV-algorithm. In order to make a fair comparison, thresholding, blob detection and mapping are done in a similar manner (fig. 4.6). For the PTV code, no background reader was used. Prediction was done on the information of the previous matching. For HiRes-PV a background velocity field was present and used for the prediction of the new particle location. The background velocity field was obtained by running a PIV code on the same images as used for the tracking.

The tests on the code performance were carried out on a data set with a high image quality. The images are obtained from an experiment where the evolution of the shed vortex structures was studied for $Ri_D = 1.3$ and $Re_D = 75$. In the vector field a shed upper and lower vortex can be observed (fig. 4.7). For the tracking process, the blob detection and dynamical threshold parameters were optimised so that a maximum number of particles could be detected. These parameters remained fixed during all tests. With this optimal set of parameters about $12000 \pm 500$ blobs were detected resulting in a non-varying mean minimum particle distance $d_n$ equal to $4.6 \pm 0.2$ camera pixels.

To get a good impression of the experimental performance of the algorithm it is sufficient to vary the sampling quality. This sampling quality can either be changed by varying $d_n$ or $\Delta S_{\text{max}}$ (eq. 4.3). In the tests, applied on the same sequence of acquired images, $q_s$ is varied by varying $\Delta S_{\text{max}}$. This variation can be obtained by skipping frames in the sequence of analysed images. For a frame skip equal to 0, the subsequently grabbed camera frames are analysed while for a frame skip of $k$ only the $k$th grabbed images are analysed. By doing so $\Delta S_{\text{max}}$ increases and therefore $q_s$ decreases (Table 4.2).

A decrease in $q_s$ affects the obtained vector field evidently for both PTV (fig. 4.7a,c,e,g) and HiRes-PV (fig. 4.7b,d,f,h). For $q_s = 6$, both vector fields (fig. 4.7a,b) show no significant difference. The observed stray vectors are mostly caused by errors in the particle location, whose
importance increases for increasing $q_s$ (small displacements with respect to the inter-particle distance). As $q_s$ decreases ($q_s = 3$, fig. 4.7c,d), first the quality of the vector field improves. A further decrease of $q_s$ (down to $q_s = 1.2$, fig. 4.7e,f), results for PTV in a vector field with more stray vectors, while for HiRes-PV the quality remains constant or even improves. For the smallest investigated $q_s$ ($q_s = 0.33$, fig. 4.7g,h), the PTV vector field appears as a random field of vectors and no coherent flow structures can be detected anymore. This means that no correlation can be found between particles in subsequent frames. The HiRes-PV results on the other hand, still represent the field satisfactorily. Only in the areas with large velocity gradients, regions appear where less vectors are found. These regions were mainly caused by the absence of valid background velocity vectors. These background vectors are generated by running a PIV algorithm on the same image sequence as used for the tracking run. Especially in the regions with large velocity gradients, the correlation between the images was lost for these large frame skips (small $q_s$).

The absence of valid background velocity vectors in these regions results in a local decrease of the number of matchings.

Another approach to analyze the quality of the tracking algorithm is by means of a performance measure. This performance measure can be defined as a combination of the total number of matched particles and the quality of the resulting vector field. For the number of matched particles the fractional velocity yield $\gamma$ is used, where $\gamma$ describes the ratio between the matched particles and detected blobs. The quality of the vector field is expressed by the mean standard deviation $\sigma_{u,v}$ for the $u$ and $v$ component of the velocity. This standard deviation is calculated by dividing the flow field in $N_i$ interrogation areas. In each area the local standard deviation of the flow field is calculated with respect to the average velocity within this area. The mean standard deviation $\sigma_{u,v}$ is then calculated by averaging the local standard deviation over all interrogation areas. For all tests it turned out that $\sigma_u \approx \sigma_v$. Therefore all further discussions are based on $\sigma_u$.

One should note that when $N_i$ is taken too small (in other words: for large interrogation areas), physical velocity gradients will influence this mean average standard deviation strongly. On the other hand, by taking $N_i$ too large, the number of vectors evaluated for the calculation of the standard deviation in the interrogation area becomes very small. In the presented results $N_i$ is chosen to be 256 ($16 \times 16$). The performance of the tracking algorithm $\eta$ is then defined as

$$\eta = \frac{\gamma}{\sigma_u / U_0}.$$  \hspace{1cm} (4.4)

For the tests as presented here, the HiRes-PV results show a better performance than the PTV-results (fig. 4.8). For all chosen $q_s$, $\eta$ is higher for the HiRes-PV results. Only for the largest $q_s$, the difference is small. For decreasing $q_s$, the PTV performance reaches its maximum already at $q_s \approx 3$ while the HiRes-PV performance peaks at $q_s \approx 1$. Furthermore, the PTV performance decreases rapidly for $q_s < q_{s,\text{max}}$ while the HiRes-PV performance remains relatively high after reaching its maximum.
Figure 4.7: Calculated vector fields for different sampling qualities at $t=5$ [s]: (a) PTV results, (b) HiRes-PV
The performance $\eta$ is a combination of the separate quantities $\gamma$ and $\sigma_u$. In the figures 4.9a and 4.9b the separate quantities are presented. It shows that for the PTV results $\gamma$ increases as $q_s$ decreases (fig. 4.9a, o-symbols) meaning that for increasing frame skip an increasing number of detected particles can be matched. For the HiRes-PV results, $\gamma$ decreases continuously for decreasing $q_s$ but most rapidly for $q_s < 2$ (fig. 4.9a, *-symbols). This implies that the highest spatial resolution is achieved for the highest $q_s$.

From the standard deviation (fig. 4.9b) it can be seen that the quality of the PTV vectors decreases rapidly for $q_s < 2$ (increasing $\sigma_u$). For the smallest $q_s$, this standard deviation becomes almost of the same order as the velocity itself. The accuracy of the HiRes-PV vector field increases continuously for decreasing $q_s$ (Fig. 4.9b). This behaviour can partly be understood by considering the particle location error, which becomes of minor importance for large particle displacements (small $q_s$).

**Concluding remarks**

The results presented here show that the performance of the HiRes-PV algorithm turns out to be better than that of the PTV algorithm, especially for image sequences where the particle displacements are larger than the inter-particle distances ($q_s < 1$). This compares fairly well with the results of tests with synthesised image sequences (van der Plas and Bastiaans (1999)). In the tests with the synthesised data, where the particle positions are known very accurately, errors in the measured velocity vectors $\sigma_u$ are solely caused by erroneous matchings. In the now presented results the errors in the measured velocity vectors are a combination of the erroneous matchings and the errors in the particle positions. Especially for large values of $q_s$ (very small displacements), the latter errors are the most dominant ones for both PTV and HiRes-PV (relative small displacements with respect to the position errors).

Although the number of matched particles decreases for the HiRes-PV tests, the accuracy of the obtained velocity vectors remains high (small $\sigma_u$). This means that the validity of the found
vectors is preserved despite the decrease in sampling quality. This is in contrast to the PTV algorithm where $\sigma_u$ reaches a minimum at $q_s = 3$, and then increases fast for further decreasing $q_s$ (accuracy is lost for small $q_s$).

From optical inspection of the vector fields it can be concluded that the HiRes-PV algorithm performs optimally at $q_s \approx 1$. This deduced optimum coincides very well with the optimum as found from the performance measure $\eta$. It is also found that the quality of the background velocity is of crucial importance to the performance of the HiRes-PV algorithm. Stray vectors in this background velocity cause a decrease of the velocity yield and therefore a decrease in performance. Thus in practice, the possibility to generate adequate background velocity vectors determines the lowest limit of $q_s$ for HiRes-PV. This lowest limit can be derived from the behaviour of $\gamma$ and is defined as the position of $q_s$ for which $\gamma$ starts to decay significantly (around $q_s = 0.5$).

### 4.3.2 3D Particle Tracking Velocimetry

In the 2D techniques the flow is analysed in a thin light sheet and only the vector components within this sheet can be evaluated. Hence, a fully 3D approach is demanded for investigation of flows which are or become 3D. Although a few methods exist for analysing 3D velocities in a point (3D Laser Doppler Anemometry) or plane (stereo PIV), only a fully three-dimensional technique applied on a volume will give the information needed to understand these flows. In general, two different methods can be applied. The first method is an extension of the 3D stereo-PIV technique. In this method velocity information in a volume is obtained by acquiring several slices of the flow field using a scanning technique (Rockwell et al. (1993), Brucker (1995)). The second method is a 3D extension of the particle tracking method. Instead of tracking particles in a thin light sheet,
the particles are now tracked in an illuminated volume. The obtained 3D particle trajectories can be used to calculate the 3D vector field. Because the actual path of the particles is analysed, particle tracking techniques in general are more accurate than PIV based techniques (Adrian (1991) and Cowen and Monismith (1997)). Besides, the trajectories of the particles are analysed in a volume. In contrast to a method using several thin slices of the volume in spanwise direction, now a 3D continuously spatially sampled flow field is obtained. This technique of 3D particle tracking was already introduced by Chang and Taterson (1983) and further developed by among others Racca and Dewey (1988) and Maas et al. (1993). In most of these investigations a 3D particle localisation algorithm is used based on a so-called epipolar lines method. In this method several transformations between the physical 3D domain and the camera images are needed, resulting in long computational times (for details see Maas (1996)). For the purpose of this research project, the developed 2D particle tracking algorithm is extended into a 3D tracking method by applying a new 3D localisation method.

In 3D PTV methods at least 2 synchronised cameras need to be used. Only then stereo images can be obtained required to determine the 3D position of the particles (comparable to the human eye). By using only 2 cameras, the possibility exists, that especially for high seeding densities, particles are hiding behind each other. To decrease this effect of particle ‘hide and seek’ a third synchronised camera is applied which looks at the same volume as the other two cameras (fig. 4.10a). A third camera also reduces the ambiguity occurring during the 3D localisation.

From the obtained images, the 3D position of the particles can be determined using a 3D localisation algorithm (fig. 4.10b). The algorithm functions comparable to the 2D PTV algorithm (fig. 4.6). First the captured images are dynamically thresholded (Dynamic Thresholding). Within each image the 2D representation of a particle is detected (Blob Detection). From the particles located in the three cameras, a 3D position can be deduced (Mapping to Lines of Possible Position and 3D Localisation). At that point, the 3D processing algorithm differs from the 2D algorithm. As soon as the 3D position of the particles are known, Matching and Path Storage are almost equivalent to the 2D algorithm.

For the 3D particle location, the domain viewed by the three cameras needs to be calibrated very accurately. Only then the particles viewed in the three separate camera images can be coupled, resulting in the 3D position. These calibration and 3D particle localisation methods will be discussed separately in the next section.

3D localisation

The main idea behind the 3D localisation of particles is that, under normal circumstances, a particle projected on a camera image will have a position in the world coordinate system somewhere along the particle projection line (fig. 4.11a). The ‘normal circumstances’ criterion means basically that no abrupt change in refractive index of the medium is allowed (no ‘mirror’ effects) and that possible changes are known and stable or predictable in time. This is not a serious restriction since the applicability of seeding techniques for flow visualisation in general satisfies this criterion. Note that this also holds for different refractive indices (e.g. a glass - water interface), or for deformable media (e.g. lens-like deformations in the glass wall) without affecting the above statement. In the special (but often present) case of a medium with a uniform refractive
index, the lines of possible positions will be straight. This property is used for the construction of a transformation between pixel coordinates of each camera to a line in world coordinates. When this transformation is known for all cameras, the detected blobs in the images can be transformed to a set of lines of possible particle positions (fig. 4.11b). The points in space where the lines from the different cameras cross, are the position of the particles. This 3D localisation method, based on crossing projection lines, only requires the construction of these lines. In the so-called epi-polar line method, as used by various researchers, the virtual images of the particle projection lines belonging to cameras 1 and 2, need to be constructed in camera 3. This demands for an additional back-transformation from world coordinates to camera coordinates causing an increase of the computational effort.

The 'crossing-line method' allows to use only two cameras. However, in high-seeding-density flows accidental crossing of two lines is quite common, resulting in multiple crossings and consequently in ambiguities determining the 3D particle position. Besides, particles can also hide behind one another resulting in no crossings at all. The use of three cameras is therefore almost obligatory. The chances of accidental crossings of three lines through one point are relatively

![diagram](image-url)
4.3. PARTICLE VELOCIMETRY

A blob, detected in each camera image, is now represented by three spatial lines crossing somewhere in the measuring volume. Due to optical disturbances, random camera noise and errors in the calibration data, an exact crossing of the three lines will be unlikely. Therefore the line crossing is defined as a minimum approaching or neighboring distance $\Delta c$ between the three lines. Depending on the quality of the calibration data and camera/lens characteristics, $\Delta c$ should be set to an optimal value. In general, a too large value results in an increase of crossing possibilities especially for experiments where a large seeding density is used. A very small value, on its turn, would result in no crossings at all.

For the transformation of the detected 2D blobs to lines in world coordinates, a mathematical model can be developed based on the measured geometry of the experimental set-up and camera characteristics. The derivation of the transformation parameters would be a time-consuming task and some kind of check of the transformation has to be performed with in-situ objects. Since the experimental technique by itself requires a good optical access, the in-situ calibration is the preferred technique.

**Calibration**

The position of the tracer particles in 3D can only be determined if the camera parameters for each camera, such as the position and orientation with respect to the measuring volume, are known. These parameters can be determined by traversing a well-defined object through the measuring volume. The calibration process should be performed very accurately. Not only the accuracy of the 3D position is fully dependent on the calibration data, the data also determines whether 3D particles can be found or not.
To obtain the calibration data a calibration system is developed, allowing to position very accurately a precisely manufactured 2D grid on well-known positions inside the measuring volume (fig. 4.12). This grid consists out of a blackened square copper-bronze foil \((200 \text{ mm} \times 200 \text{ mm} \times 0.1 \text{ mm})\) glued on a flattened opaque glass plate. In this foil, pin-holes with a diameter of 0.1 mm are etched with a mutual spacing of 5 mm. The distance between the pin-holes as well as their diameter have an accuracy of 0.1 \(\mu\text{m}\). In the middle of the pin-hole pattern, three larger pin-holes, forming a capital \(L\), are etched (fig. 4.12). These pin-holes are used to determine the orientation of the cameras and to establish the origin, \((x/D, y/D) = (0, 0)\), which is located in the lower left corner of the \(L\) image. By illuminating the grid from behind, the pin-holes in the foil are visualised and can be captured by the cameras. Translation of the calibration grid is applied by using a traversing device as used on precision lathes. This allows to position the calibration grid with an accuracy of \(\pm 5 \mu\text{m}\) on well-known positions inside the measuring volume. The calibration data is then obtained by acquiring images of the calibration grid at several stepwise distances \(\Delta z\) within the measuring volume, where the first grid position is marked as \(z = 0\).

The calibration data is used in the following way. For each grid position, the \((x, y, z)\) positions of the detected pin-holes are exactly known. This information is used to determine which \((x, y, z)\) position is projected on each camera pixel. Use is made of \(n_z\) different \(z\)-positions, meaning that at each camera pixel \(n_z\) \((x, y, z)\) positions are projected. By applying a least square fit through these points, the coefficients of the straight-unique line are established for every camera pixel. Because blob detection is performed on sub-pixel accuracy, a continuous function, describing the line coefficients within a camera image, is needed. Therefore a 2D \(n_f\)-th order polynomial (with \(n_{f,c}\)-th cross-order terms) is fitted through the coefficients belonging to each pixel. For this particular experiment, it turns out that for \(n_f \geq 3\) and \(n_{f,c} \geq 3\) the error in \(z\)-position becomes independent of the chosen orders (fig. 4.13).
Figure 4.13: Dependency of the accuracy in position on the order of the fit in the calibration procedure

**Performances**

**3D localisation** The performance of the method is first tested on its capability to localize well-known positioned markers somewhere in the calibrated measuring domain. To that end, the calibration grid is placed at several known \( z \)-positions in this volume. The grid could be positioned with an accuracy \((±5 \mu m)\). By illuminating the grid from behind, the regular pattern of pin-holes is visualized and captured by all three cameras. The positions of the pin-holes are now reconstructed and compared with the known positions. A measure of the average error \( \epsilon_i \) in \( i \)-th component of the located pin-hole-position vector is the standard deviation in the located position with respect to the known position. This can be defined according to

\[
\epsilon_i = \sqrt{\frac{\sum_{k=1}^{n_d} (x_{i,k}(\text{known}) - x_{i,k}(\text{located}))^2}{n_d - 1}} \tag{4.5}
\]

with \( x_{i,k}(\text{known}) \) the \( i \)-th vector component of the known position vector of the \( k \)-th pin-hole, \( x_{i,k}(\text{located}) \) the \( i \)-th component of position vector as calculated by the 3D localisation algorithm and \( n_d \) the total amount of located pin-holes.

It turns out that the error in the located positions \( \epsilon_i \) is dependent on the camera configuration (fig. 4.14a). The camera configuration is determined by the camera-viewing direction and the optical path \( l_w \) through water. The camera viewing directions \( \alpha_{1,2,3} \) are defined as the angles between the optical line of the cameras and \( z \)-direction (central line in fig. 4.14a). Here \( \alpha \) is measured as the real angle between the optical axis and the viewing direction after refraction by the air/test-section interface.

The dependency of \( \epsilon_i \) on these variables is investigated by three sets of experiments where
in the first set the camera viewing angles are equal to \( \alpha_1 = \alpha_2 = \alpha_3 = \alpha \). In the second set \( \alpha_1 \approx \alpha_2 \approx 5^\circ \) and \( \alpha_3 \) is varied while in the third set \( l_w \) is varied. During these experiments the accuracy of the installed angle was estimated to be \( 1^\circ \) and the accuracy of the measured optical length was 0.5mm.

For all three sets it was found that \( \epsilon_x \) and \( \epsilon_y \) are more or less independent of the varied variable and equal both to about \( 10 \mu m \). The error in \( z \) position \( \epsilon_z \), on its turn, strongly varied as function of the chosen parameters (fig. 4.14b). For a changing \( \alpha \) (depicted with the black circles and scaled according to the bottom axis) a strongest change in \( \epsilon_z \) can be observed. For increasing \( \alpha \), \( \epsilon_z \) decreases rapidly for \( 5^\circ < \alpha < 20^\circ \) and then becomes almost constant for \( \alpha > 20^\circ \).

The decrease in \( \epsilon_z \) is to be expected if one considers the localisation method. By viewing the measuring volume from larger angles, the crossing position of the projection lines can be determined more accurately. In other words, by viewing at a larger angle, more 3D information is gained. One should therefore expect that \( \epsilon_z \) would decrease continuously when \( \alpha \) is enlarged. The reason that this dependency of \( \epsilon_z \) only holds for \( \alpha < 20^\circ \) is probably caused by the increasing relative influence of other error sources as camera noise, calibration errors or nonlinear deformation of the images at the air/water interface when viewing with an angle.

![Camera configuration](image1)

![Dependency of accuracy in z-position on camera geometry](image2)

(a) Camera configuration  (b) Dependency of the accuracy in \( z \)-position on the camera geometry. Three quantities have been varied.

**Figure 4.14:** Performance variables and their influence on the \( z \)-position

For the second set (denoted with a triangle and scaled with the upper axis, fig. 4.14b), in which only \( \alpha_3 \) was varied, an increasing \( \alpha_3 \) also results in a decrease of \( \epsilon_z \), but less strong than for the
first set. Besides, at $\alpha_3 = 25^\circ$ the accuracy of the $z$-position is still improving, while for a varying $\alpha$ this improvement seems to stop for $\alpha > 20^\circ$.

The influence of the optical path length is determined for $\alpha \approx 13^\circ$ by varying the position of the measuring volume within the test section (denoted with a square and scaled with the middle axis, fig. 4.14b). By doing so, it turns out that this variation has no detectable effect on the accuracy in $z$-position. Only a slight improvement can be observed if the measuring volume is positioned closer to the air/water interface, but this improvement is relatively small.

In conclusion one can state that the most optimal camera configuration would be a symmetric one, in which the viewing angle of all three cameras is about $20^\circ$. By a further increase of this angle no significant improvements in the accuracy can be gained. The remaining error in $z$-position $\epsilon_z$ is approximately 0.003 cm. Comparing this accuracy with the accuracy in $x$- and $y$-position, it is found out that $\epsilon_z$ is about 3 to 4 times larger than $\epsilon_{x,y}$.

![Distribution of measured speeds](image)

**Figure 4.15: Performance of the 3D-matching algorithm**

**3D matching** For the most optimal camera configuration ($\alpha \approx 20^\circ$, $l_w \approx 150$ mm) the 3D tracking of particles is tested. In this test, the calibration grid is translated through the measurement domain with a constant step size $\Delta z = 0.1$ cm. As this test is applied on a plane with
points, no ambiguities during the localisation (occurrence of two crossing possibilities) or the phenomenon of 'hiding' particles occur. Furthermore, only about 200 white dots are viewed by the cameras corresponding to a low seeding density. At every position three independent camera images are captured from which the pin-hole positions are determined. Doing this for a sequence of 10 steps, a tracking run is simulated from which a virtual velocity can be detected between two subsequent frames. The accuracy of the matching in \( z \)-direction can then be determined by plotting the distribution of all found \( z \)-velocities during the entire tracking run (fig. 4.15). The accuracy of the determined \( z \)-velocity was found to be \( 2\sigma = 0.002 \text{ cm/s} \).

For the real-fluid flow test, images are captured of the flow behind the heated cylinder. For this flow no exact solution exists and therefore no accuracy measure can be derived. The tracking results still show how the performance of the 3D PTV code behaves. In the sequence of the ac-

![Figure 4.16: Characteristic result showing the 3D PTV performance on a real fluid flow for \( Re_D = 75, Ri_D = 1.2 \)](image)

quired three camera images, about 1800 ± 100 particle images (fig. 4.16a) or blobs are detected. From the blobs found, about 50% can be reconstructed into a 3D particle position (fig. 4.16b). In order to determine a velocity vector, a particle needs to be located at least twice. To decrease the effect of erroneous matched particles a velocity vector is calculated from 3 subsequently located particle positions. By doing so, about 60% of the located particles can be used for the calculation of the vector field (fig. 4.16c) resulting in about 500 velocity vectors.

The particles discarded in the processing as discussed above are due to several causes. Firstly, a high seeding density increases the chance of hiding particles dramatically. For a seeding density as described above it is estimated that this effect results in about 20% loss of particles. The total loss of velocity vectors is even higher, because calculation of the velocity demands that the particle is found in 3 subsequent sets of captured images. Secondly, the ambiguity of multiple crossing possibilities is also responsibly for a severe particle loss. When a high seeding density is used, the chance of having lines with multiple crossing possibilities increases. The number of ambiguities of this kind increases for increasing seeding density. Also recording error sources as camera noise and interlace effects or calibration errors have an increasing negative influence on the performance for increasing seeding density.
4.4 Concluding remarks

The 3D PTV method as described in the previous section uses a more efficient 3D localisation method compared to commonly used methods. The performance of the used method is at least equivalent to the methods reported in literature. From the localisation accuracy tests it is deduced that the particles can be located with an accuracy of 0.003 cm, which corresponds to a pixel accuracy of 0.5 pixels. This localisation error is about 5 times less than found by for example Hagiwara et al. (1999), which gives a fair indication that the used localisation performs quite well.

For higher seeding densities it turns out that the number of resulting vectors with respect to the found blobs drops significantly. In comparison to other reported results (Maas (1996)), this decrease is quite common and is mainly caused by the strong increase of ambiguities for increasing seeding density. By extension of the algorithm it is expected that this observed drop-out can be decreased. In chapter 7 we will return to this topic.

4.4 Measurement procedure

Preparations

Before the actual experiment is started, several preparations have to be made. For the particle tracking experiments a seeding is suspended in the water tank and uniformly distributed by stirring (care is taken that the fluid motions have completely damped out before the start of experiment). For the 2D HiRes-PV experiments, particles are used with a nominal diameter of 10 \( \mu \text{m} \) and 50 \( \mu \text{m} \) during the 3D PTV experiments. The gravitational settling of the 10 \( \mu \text{m} \) particles takes more than 24 hours while it takes about 5 hours for the 50 \( \mu \text{m} \) particles. These times determine the maximum waiting period as mentioned.

Experimentally it is found that already after a few hours the fluid motions created by the stirring are reduced to about 0.8 mm/s. A further decrease in background velocity could not be obtained unless special attention is given to the thermally driven fluid motions. These motions are induced by the temperature difference between the water in the tank and the conditioned laboratory environment. By insulating the entire water tank after stirring, and waiting for about 5 hours or more, the remaining fluid velocity could be further reduced to 0.2 mm/s (fig. 4.17b).

The experiment is started by first heating the cylinder without translating it (fig. 4.17a). As soon as a constant temperature is reached, the translation is started (at \( t = 120 \text{ s} \)) causing a drop in cylinder wall temperature. The cylinder wall reaches finally a steady temperature (for \( t > 150 \text{ s} \)) after which the acquisition of data can start.

Experiments

In principle two types of experiments are performed. The first type involves the investigation of the vortex wake behaviour and is carried out by mounting the camera on the translation system. By focusing the camera on an illuminated area situated just behind the cylinder the vortex wake behaviour within this area can be investigated. The typical duration of such an experiment is about 2 to 3 minutes. The equipment to create the light sheet is also translated by the cylinder-camera
CHAPTER 4. EXPERIMENTAL METHODS

contraption. The light source can be a slide projector (mainly used for the visualisation experiments), or a mirror/lens combination illuminated by an externally placed laser (mainly used for the particle tracking experiments). The lasers used are a pulsed 200 mJ, 532.8 nm Nd-YAG laser for the 2D HiRes-PV experiments and a continuous Argon-ion laser for the 3D PTV experiments. The high output power of the Nd-YAG laser allows the use of very small particles. Even particles with a diameter of 10 µm can be detected easily. For the Argon-Ion laser particles with typical diameter of 50 µm need to be used.

A second type of experiments involves the investigation of the evolution of the shed vortex structures. These experiments are carried out by placing the camera at a fixed position with respect to the water tank. The camera is focussed on a certain area within the water tank through which the cylinder is translated. The structures, shed from the cylinder, have only a small horizontal velocity and remain inside the illuminated area for about one minute. Therefore the evolution of these structures can be investigated. The illuminated measuring area is now created by a light source positioned on a fixed ‘world’ position. Again this light source can be a slide projector or laser.

For all measurements a data-acquisition system is used consisting of a pentium-I, 166MHz-PC with a 40-Gb RAID0 volume. This volume is made up from 2 SCSI-controllers with five 4 Gb harddisk each. With this system a data rate of 30 Mb/s is reached. The images from the camera are captured by a frame-grabber. For the 2D PTV experiments a digital frame-grabber is used. The camera is a high resolution $1024^2$ pixels fully digital non-interlaced camera (Kodak ES-1.0) with a capturing rate of 30 images per second. A non-interlaced camera is needed when a pulsed light source is used. Only then emission of the light pulse and the reading of the odd and even lines take place at exactly the same moment. The synchronisation of the camera with the laser
is obtained by using the camera as 'master' controlling the generation of the laser pulses. For the 3D PTV experiment a three-colour-input frame-grabber is used. Rather than having a single colour camera, now three monochrome (interlaced) cameras are used. Each camera is connected to one of the colour inputs, resulting in a combined image. In the post-processing phase, the three-colour-image is again converted to three monochrome images.

**Post processing**

The results of the particle tracking experiments are the particle positions as a function of time. From these particle trajectories the velocity \((U_x, U_y, U_z)\) can be calculated in the following way

\[
U_x = \frac{x^{i+1} - x^{i-1}}{2\Delta t}, \quad U_y = \frac{y^{i+1} - y^{i-1}}{2\Delta t}, \quad U_z = \frac{z^{i+1} - z^{i-1}}{2\Delta t}
\]  

(4.6)

where the superscript denotes the frame number, \((x^i, y^i, z^i)\) the particle location in frame number \(i\) and \(\Delta t\) the time step between 2 subsequent frames. A velocity vector which differs significantly from its surrounding average is removed. As criterion a difference of more than two times the standard deviation found in the surrounding vectors is used.

In principle the measured velocity field is an unstructured field. To calculate derived quantities as vorticity or divergence, the unstructured field is transformed onto a structured grid. For this transformation a Gaussian weighting method is used (Green and Gerrard (1993))

\[
U_{x,y,z}(\tilde{x}, \tilde{y}, \tilde{z}) = \frac{\sum_{j=1}^{N_v} \alpha_j(x_j, y_j, z_j)U_{x,y,z}(x_j, y_j, z_j)}{\sum_{j=1}^{N_v} \alpha_j(x_j, y_j, z_j)}
\]

\[
\alpha_j(x_j, y_j, z_j) = \exp\left(-\frac{(x_j - \tilde{x})^2 + (y_j - \tilde{y})^2 + (z_j - \tilde{z})^2}{H^2}\right)
\]

(4.7)

where \(N_v\) denotes the number of unstructured velocity vectors, \(\alpha_j\) the weighting parameter which describes the weighted contribution of a velocity vector at position \((x_j, y_j, z_j)\) on the interpolated velocity in a point of the structured grid at position \((\tilde{x}, \tilde{y}, \tilde{z})\). \(H\) determines the width of the weighting function and has a strong influence on the obtained interpolated field. It is shown (Agui and Jimenez (1987)) that the error made by the weighting procedure has a minimum value if \(H\) is chosen according to

\[
H = 1.25d_v, \quad d_v = \frac{1}{\sqrt{\pi}}\sqrt{\frac{A}{N_v}}
\]

(4.8)

where \(d_v\) describes the average minimum distance between the two closest neighbouring vectors. It is calculated by using the total domain covered by the test section \(A\) and the total number of velocity vectors \(N_v\). The interpolation error is strongly dependent on the ratio between the spatial velocity fluctuations and \(\delta\). In this experiment the ratio is about 50 resulting in a negligible interpolation error (Agui and Jimenez (1987)).

For the considered flow the vector density and therefore the local value of \(\delta\) differs strongly in the flow field, especially in the cylinder near wake. Calculation of this local value from the PTV
data yields that the local value of $\delta$ in the cylinder near wake was about 10-15% larger than the value of $\delta$ considering the entire domain. Therefore the value of $\delta$ is taken from this local area.

The vorticity $\omega$ is calculated using the interpolated data on the structured grid by using a first-order central difference scheme. In order to minimise the effect of random errors the vorticity fields are averaged over 6 samples. For the investigated 2-D problem, where the typical time scale turns out to be 6 seconds, it is presumed that this averaging does not influence the observed results.
Chapter 5

Two-dimensional wake dynamics

In order to obtain a global view of the influence of heat on the vortex wake, bubble-wire visualisation experiments are carried out for $Re_D = 75$ and varying $Ri_D$. The visualisation results clearly show for $Ri_D = 0$ the appearance of a von Kármán vortex street (fig. 5.1a). As expected, the subsequently shed vortices form an alternating vortex configuration. For increasing $Ri_D$ this configuration becomes disturbed. The trajectories followed by the shed structures then deflect in negative $y$-direction (fig. 5.1b and c), which is remarkable considering the buoyancy induced force.

![Figure 5.1](image)

Figure 5.1: Influence of heat on the vortex street for $Re_D = 75$ and (a) $Ri_D = 0$, (b) $Ri_D = 0.5$ and (c) $Ri_D = 1$ as visualised by hydrogen bubbles

A second effect of the heat input is that between two neighbouring vortices a relative motion can be observed. The lower vortex rotates around the subsequently shed upper vortex, forming a double vortex configuration (fig. 5.2). A possible explanation for this rotation is a strength difference between the upper and lower vortices. To evaluate this statement, quantitative experiments and numerical simulations are performed. The visualisation experiments show that, at least for $Ri_D < 1$ and $x/D < 25$, the flow is 2D. Therefore, in the following sections, the wake behaviour is analysed with 2D-flow measurement and simulation techniques.
5.1 Vortex trajectories

The flow behind the cylinder is examined experimentally using the HiRes-PV technique. In these experiments the camera is mounted on the translation construction. Thus, the behaviour of the shed vortex structures is analysed for several shedding periods within a fixed domain with respect to the cylinder. In the present study, the positions of the structures \( (X, Y) \) are calculated from the 2D vorticity field (see e.g. fig. 5.4a, c, e) using equation (2.17);

\[
X = \frac{1}{\Gamma} \int_A \omega_z(x, y) x dA, \\
Y = \frac{1}{\Gamma} \int_A \omega_z(x, y) y dA,
\]

with \( \Gamma = \int_A \omega_z(x, y) dA \) and \( A \) the area enclosed by a closed iso-vorticity contour \( \omega_a \). The calculated vortex position or centroid \( (X, Y) \) depends on the value of \( \omega_a \). For a too small value of \( \omega_a \) experimental noise influences the calculated centroid position. This becomes clear considering for example the vorticity contour \( |\omega_z| = 0.1 \) in figure 5.3a. This contour has an irregular shape compared to the iso-contours for higher values of \( \omega_z \). As a result, this irregular shape will considerably affect the calculated vortex position. On the other hand, for higher values of \( \omega_a \) the calculated centroid approaches the position of the vorticity extreme (fig. 5.3a, b). This becomes even more clear if for example the \( y \)-position of the calculated lower vortex centroid is compared with the \( y \)-position of its vorticity peak (fig. 5.3c). For increasing \( \omega_a \) both the numerically and the experimentally calculated centroid positions approach this peak position. For \( \omega_a < 0.2 \) the experimentally determined vortex \( y \)-positions show, compared to the numerical results, a different behaviour for increasing \( \omega_a \). The measuring-noise and the lower spatial resolution of the measurements probably cause this difference. It was therefore decided to determine the vortex positions by taking the closed iso-vorticity contour \( |\omega_a| = 0.2 \).
Figure 5.3: Effect of increasing values of $\omega_a$ on the calculated vortex centroid for $Ri_D = 1$ and $Re_D = 75$: (a) experimental result, (b) numerical result, (c) $y$-position of the lower vortex position.

For a duration of 120 s, the vortex positions are calculated from the measured vorticity fields at intervals of 1 s and the results are plotted in figures 5.4b,d,f. The cylinder is positioned in the origin $(x/D, y/D) = (0, 0)$. The figures show the trajectories of about 40 shed vortex structures. For both the upper and lower rows, a second-order function, which represents the average trajectory, is fitted through the vortex positions. The trajectories, determined for $Ri_D = 0$, $Ri_D = 0.5$ and $Ri_D = 1$, are corrected for the weak background motion still present in the water-tank (see section 4.4). To do so, the advection due to this average background velocity is subtracted from the measured vortex structure advection.

For $Ri_D = 0$ the average trajectories for the upper and lower vortices (fig. 5.4b) show an almost symmetric profile with respect to the wake axis. For increasing downstream position, both trajectories are deflected equally from the wake axis $y = 0$. This deflection appears as a widening of the vortex street. The widening is assumed to be caused by viscous spreading of the vortex cores (Green and Gerrard (1991)). For $Ri_D > 0$, an additional deflection of the vortex trajectories in negative $y$-direction is found. Both the trajectories of the upper and lower rows are below the trajectories for $Ri_D = 0$. This deflection shows a maximum for $Ri_D = 0.5$.

Around the average trajectories a variation of about $\pm 0.3D$ is found. This variation has two sources; the first is the inaccuracy in the determined vortex position. A characteristic vortex path shows a small variation in the determined position around its average path. This rather random variation, caused by measuring-errors, turns out to be $\pm 0.05D$ (fig. 5.5a). The second source is a temporal oscillation of the vortex rows. This phenomenon is analysed by considering the time dependent $y$-position of both vortex rows at $x/D = 20$ (fig. 5.5b). These row positions are constructed from the vortex position found in the examined vorticity field. Through the found positions of the upper and lower vortex structures, curves are fitted representing the temporal position of the vortex row. The vortex row positions at $x/D = 20$ (fig. 5.5b) show a slowly varying
Figure 5.4: Experimental HiRes-PV results for \( Re_D = 75 \), showing the vorticity contours at equivalent time moments for (a) \( Ri_D = 0 \), (c) \( Ri_D = 0.5 \) and (e) \( Ri_D = 1 \) and structure trajectories for (b) \( Ri_D = 0 \), (d) \( Ri_D = 0.5 \) and (f) \( Ri_D = 1 \).
5.1. VORTEX TRAJECTORIES

Figure 5.5: Analyses of the experimentally found vortex trajectories for $Re_D = 75$ and $Ri_D = 0$: (a) error in the vortex position, (b) temporal variation of the vortex row position at $x/D = 20$

behaviour. Up to $t \approx 100$ s both the upper and lower vortex rows move away from the $x$-axis resulting in a widening of the vortex street. Slightly later, both rows suddenly approach the $x$-axis again, resulting in a narrowing of the vortex street. This slow variation has a maximum amplitude of about $0.5D$ and can be considered as the major cause of the observed variation in the vortex trajectories. This behaviour was observed in several independent experiments (Appendix C). The origin of this phenomenon is not clear.

Also in both vortex rows an oscillation of about $0.16$ Hz with an amplitude of $\pm 0.15D$ can be observed. This oscillation is assumed to be caused by the earlier explained variation in the vortex position as induced by the measuring-errors. This error, which was found to be about $0.05D$, can be amplified due to interpolation or extrapolation of the vortex positions to the position $x/D = 20$.

The vortex trajectories can also be analysed using the numerical results for $Re_D = 75$, $Pr = 7$ and $Ri_D$ between 0 and 1. Use is made of the Spectral Element Method with a 9-th order approximation polynomials. The simulations are performed on a mesh as shown in Appendix B. For the implicit time step $\Delta t = 0.02$ is chosen and within such an implicit time step four explicit time steps are carried out (for details see Chapter 3).

For about four typical shedding periods, the vortex positions are calculated and plotted as presented in figure 5.6. With no heat input ($Ri_D = 0$) the vortex trajectories show a widening of the vortex street. This widening is smaller than obtained from the experimental results. This discrepancy may be caused by the long term variation in the experimental vortex trajectories as discussed before. For $Ri_D > 0$ both the upper vortex and the lower vortex move downwards with respect to the vortex trajectory as found for $Ri_D = 0$. The deflection of the lower vortex row is larger than that of the upper vortex row. Furthermore, it is found that for $Ri_D > 0$ the trajectories start at a
higher $y$-position than for $Ri_D = 0$. This shift is assumed to be caused by heat effects in the near wake and can, although less pronounced, also be observed in the experimental results (compare fig. 5.6a and 5.4b,d). More downstream, another process causes the downwards deflection of the vortex rows for $Ri_D > 0$.

At the end of the domain the strongest deflection is found for $Ri_D = 0.5$. For higher $Ri_D$-values, a less severe deflection is observed. Here one should take into account that for $Ri_D > 0$ the trajectories start at a higher $y$-position. To quantify the net deflection, the difference between the vortex position just after formation ($x/D = 8$) and its position at $x/D = 22$ (fig. 5.6b) has been determined. For $Ri_D = 0$, $\Delta Y$ is positive for the upper vortex row which is therefore deflected in positive $y$-direction. On the other hand, for the lower vortex row $\Delta Y$ is negative representing a deflection in negative $y$-direction. The value of the deflection is (in absolute sense) the same for both rows. This observed deflection represents the widening of the vortex street. By increasing $Ri_D$, the lower vortex row is deflected more severe in negative $y$-direction represented by a decreasing value of $\Delta Y$. For $Ri_D = 0.5$ the strongest deflection is observed. Further increase of $Ri_D$ results in a less severe net deflection. For the upper vortex row, an increasing $Ri_D$ also results in a downward deflection compared to the unheated situation. The largest difference with respect to the unheated situation can now be observed for $Ri_D = 0.4$ (fig. 5.6b) where the value of $\Delta Y$ is even negative. For higher heat input $\Delta Y$ increases again.

![Figure 5.6: Calculated vortex deflection for $Re_D = 75$ and varying $Ri_D$. (a) vortex trajectories and (b) net deflection $\Delta Y = (Y(x/D = 22) - Y(x/D = 8))/D$](image)

Comparison of the numerical and experimental results reveals a good qualitative agreement. The strongest deflection occurs for $Ri_D = 0.5$, higher values for $Ri_D$ result in a less severe deflection. Quantitatively some differences between the experimental and numerical results are observed. For example, the vortex street widening for $Ri_D = 0$ appears to be stronger in the experimental results as one can conclude from the net deflection $\Delta Y$ (fig. 5.6b). Although the experi-
Roman numerals are used to denote the sections,

5.2. RELATIVE MOTION OF THE VORTEX STRUCTURES

Figure 5.7: Reference frame attached to an upper vortex (Vortex 2) to analyze the relative movement of an earlier shed vortex (Vortex 3 or 4)

mental trajectories are corrected for the advection by the background motion, it appears that the observed differences between the numerical and experimental results are possibly caused by the small temporal variations in the experimental inflow conditions due to this background motion.

5.2 Relative motion of the vortex structures

From the visualisation results a relative movement between the subsequently shed vortex structures was observed. This movement is analysed by attaching a reference frame to a selected upper vortex. By doing so, the relative movement of the earlier shed lower vortex with respect to the upper one can be measured during the downstream convection of both structures (see fig. 5.7).

For $Ri_D = 0$ (fig. 5.8a, *-marks) the experimental results show that the lower vortex, starting at $((x - X_u)/D, (y - Y_u)/D) = (3, -1.4)$, moves in vertical direction away from the upper vortex and moves to $((x - X_u)/D, (y - Y_u)/D) = (3, -2.2)$ when it arrives at the end of the domain ($x/D = 23$). In the vortex trajectory (fig. 5.4b), this behaviour manifests itself as a widening of the vortex street. For the numerical results (fig. 5.8b) a similar but less severe widening can be observed. Also the vortex spacing ratio $a/b$ can be determined. For the experimental results this ratio varies as function of the downstream position between 0.23 and 0.33. This result is close to the ratio $a/b$ as found by other researchers where $a/b$ varies between 0.18 and 0.33 (Zdravkovich (1997)). The spacing ratio determined from the numerical simulations varies between 0.20 and 0.27. The difference between the experimental and numerical results can be explained by the stronger street widening in the experimental results (larger $b$-value) which is probably caused by the earlier explained vortex row oscillation. Both the experimental and numerical results agree quite well with the point-vortex prediction by von Kármán ($a/b = 0.281$).

For $Ri_D > 0$ the relative positions between the upper and lower structures differ from the ones as found for $Ri_D = 0$. This difference already exists the moment the structures are shed (at $x/D \approx 8$) and therefore the vortex street configuration becomes disturbed. During the downstream convection, the relative vertical movement increases for $Ri_D > 0$ resulting in a larger
vortex street widening. For the different values of $Ri_D > 0$, this widening hardly varies, which means that for $Ri_D > 0$ the street widening is constant. Besides a vertical movement, a strong horizontal movement in negative $x$-direction can be observed as $Ri_D$ increases. In combination with the vertical movement this results in a rotation of the lower vortex around the upper vortex. This rotation increases for increasing $Ri_D$.

Considering the relative position of an upper vortex (Vortex 4) with respect to a subsequently shed other upper vortex (Vortex 2), no significant influence of $Ri_D$ on their initial relative position can be observed (fig. 5.9). The variation in the experimental results is mainly caused by the scatter in the vortex trajectories. Also during the downstream convection, the two vortices remain at constant distance. From this result, in combination with the effect of heat on the relative position of a lower vortex, it can be concluded that the vortex street configuration changes due to heat addition. For $Ri_D > 0$, two subsequently shed structures seem to form some kind of linked or combined structure. Between the different linked structures a spacing can be observed which is already present after the vortices are shed. The latter can be concluded from the shift in relative distance between vortex 2 and vortex 3 at $x/D = 8$ for increasing $Ri_D$. Because the distance between vortex 2 and 4 remains almost constant, this shift shows that the distance between the linked structures increases for increasing heat input. The spacing between the linked structures

**Table 5.1: Strouhal number at $x/D = 10$ as function of $Ri_D$**

<table>
<thead>
<tr>
<th>$Ri_D$</th>
<th>0</th>
<th>0.25</th>
<th>0.5</th>
<th>0.75</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$St$</td>
<td>0.1550</td>
<td>0.1542</td>
<td>0.1534</td>
<td>0.1527</td>
<td>0.1527</td>
</tr>
</tbody>
</table>

Figure 5.8: Relative movement of a lower vortex (vortex 3) with respect to an upper vortex (vortex 2) for $Re_D = 75$: (a) experimental results, $Ri_D = 0, 0.5$ and 1, (b) numerical results $Ri_D = 0, 0.25, 0.5, 0.75$ and 1.
is probably caused by a considerably faster formation of an upper vortex compared to the formation of a lower vortex while the total shedding time of two vortices (an upper and a lower) is almost independent of the Richardson number. The latter can be concluded from the Strouhal number (Table 5.1) which only shows a slight decrease for increasing $Ri_D$. During the downstream convection the distance between two successively linked structures remains the same. Only the spacing between two of these structures increases due to the rotation of the vortices within such a linked structure which is illustrated in fig. 5.10 showing the calculated vorticity contour plot for $Ri_D = 0.75$.

The relative movement of two subsequently shed structures can be analysed for a longer period by performing experiments with a fixed camera position. By doing so, the position and characteristics (fig. 5.11) of two subsequently shed vortex structures are analysed for about 40 s. This corresponds to a maximum downstream position of $x/D = 40$.

The vorticity contours, taken at about 30 s after passing of the cylinder, show that for increasing heat input, the shape of the vortex structures slightly deforms. Also the effect of the relative rotation can be observed. For $Ri_D > 0$ the lower structure moves underneath the upper vortex. This becomes even more clear by considering the vortex trajectories (fig. 5.12). This figure shows the trajectories of the shed vortices with respect to a world co-ordinate system. Therefore, the horizontal axis is more or less arbitrary. Reference to the cylinder can be made by considering that at $t = 0$, the cylinder has just left the domain at the left side on a vertical position $y/D = 0$. The trajectories show that for $Ri_D = 0$, the vortices move into the direction of the cylinder, as is to be expected considering the interaction between the vortices. Also the widening of the vortex street can be observed.
CHAPTER 5. TWO-DIMENSIONAL WAKE DYNAMICS

Figure 5.10: Calculated vorticity contours for $Re_D = 75$ and $Ri_D = 0.75$

Figure 5.11: Vorticity contours at $t = 25$ s for $Re_D = 75$ and (a) $Ri_D = 0$, (b) $Ri_D = 0.5$ and (c) $Ri_D = 1$. The cylinder is positioned at $x_c/D = -25$. 
Figure 5.12: Structure evolution experiments showing the movement of the lower and upper vortices in a fixed reference frame for $Re_D = 75$ and (a) $Ri_D = 0$, (b) $Ri_D = 0.5$ and (c) $Ri_D = 1$.

For $Ri_D = 0.5$, both the upper and lower vortices move continuously downwards. This is directly related to the earlier observed vortex street deflection. Again for $Ri_D = 0.5$ the strongest deflection can be observed. For $Ri_D = 1$, the upper vortex first moves down and later slowly moves upwards again. This behaviour was already observed in the numerical results (see fig. 5.6a) where at the end of the domain, the upper vortex row moves up again. In the vortex evolution experiments the analysed length of the vortex trajectories is about twice as long as for the numerical results. Therefore, the upward movement of the upper vortex can be observed more clearly. For $Ri_D > 0$, the relative movement between the two subsequently shed structures is clearly visible in the trajectory plots and increases for increasing $Ri_D$. For $Ri_D = 1$ the lower vortex has moved even underneath the upper vortex and finds as upstream neighbour another lower vortex with same circulation sign.

A remarkable result is the direction in which the upper vortex moves for $Ri_D = 1$. In contrast to $Ri_D = 0$ and $Ri_D = 0.5$, the upper vortex moves now in positive $x$-direction. This behaviour is possibly initiated by the strong interaction between the lower vortex and a subsequently shed upper vortex.

### 5.3 Structure strength

At the calculated vortex position also the vortex strength $\Gamma$ is calculated, which is defined as

$$\Gamma = \int_A \omega_z dA,$$

with $A$ the area enclosed by the vorticity contour $\omega_a = 0.2$. During the downstream convection of a structure the calculated circulation will be influenced by viscous effects. The initial amount of vorticity confined by the contour of $\omega_a$ will be spread and partly ends up outside the area enclosed
by $\omega_a$. This causes $\Gamma$ to decrease, while the vortex may be of a constant strength. As viscous diffusion is hardly affected by buoyant effects, the viscous spreading is supposed to be similar for all $Ri_D$. Hence, $\Gamma$ can be used to represent the vortex strength. During the downstream convection of the structures, $\Gamma$ is calculated at each vortex position. This results in a set of data points representing the structure strength and position (fig. 5.13). For the experimental results a function is fitted through these data points. The variation around the fitted function is observed to be $\pm 0.1$ (see Appendix D).

Figure 5.13: Vortex strength for $Re_D = 75$ and $Ri_D = 0, 0.5$ and 1: (a) experimental results, (b) numerical results

An increase of the $Ri_D$-value results in an increasing (negative, clockwise directed) circulation of the upper vortices, while a decrease in (positive, anti-clockwise directed) circulation can be observed for the lower structures (fig. 5.13). Considering $\Gamma$ as the strength of a structure, heat causes the upper vortices (negative $\Gamma$) to become stronger while the lower vortices are becoming weaker. This behaviour can be found in both the experimental (fig. 5.13a) and the numerical results (fig. 5.13b). The strength difference between the upper and lower vortices $\Delta |\Gamma| = |\Gamma_u| - |\Gamma_l|$ (fig. 5.14) shows that for $Ri_D = 0$ no strength difference can be found between the upper and lower vortex rows. By increasing $Ri_D$ an increasing negative $\Delta |\Gamma|$ can be observed. Comparison of the experimental and numerical results shows that the tendency of the strength difference as function of $Ri_D$ is quite the same. This strength difference is already present at $x/D = 8$ and then increases during the downstream convection. This suggests that a considerable part of the strength difference is initiated during the vortex formation process. This can be observed more clearly in the numerical results, where the strength difference at position $x/D = 8$ is much more pronounced than in the experimental results.

Further downstream, the discrepancy between the experimental and numerical $\Delta |\Gamma|$ becomes smaller. The observed disagreement for $x/D < 12$ possibly finds its roots in the too small spatial measuring-resolution in this region. This becomes clear when comparing the vorticity contours
5.3. STRUCTURE STRENGTH

Figure 5.14: \( \Delta |\Gamma| \) between upper and lower vortices for \( Re_D = 75 \) and varying \( Ri_D \): (a) experimental results, (b) numerical results

as shown in figure 5.3 for the experiments and numerical simulations. The latter results show strong gradients within the structure which are not captured by the experimental results. Downstream viscous effects cause these gradients to decay, allowing the measuring-method to capture the structure characteristics more accurately.

Another measure of the strength of the structures are the vorticity extremes of the structures. The extremes for the upper and lower vortices are presented as function of their downstream position (fig. 5.15). For increasing \( Ri_D \), the extremes of the negative upper vortices increase which is already observed just after the formation of the structures. On the other hand, the lower vortex extremes are hardly influenced by \( Ri_D \). The increasing peak values of the upper vortices cause a difference \( \Delta |\omega_{\text{max}}| = |\omega_{\text{max}}(l)| - |\omega_{\text{min}}(u)| \) between the upper and lower vortex extremes (fig. 5.16). This difference is already present at \( x/D = 8 \). More downstream, the peak-value difference decreases. The difference in extreme values becomes most clear in the numerical results. The experimentally determined differences are about half of the numerical predicted ones. Considering the peak values one can see that for \( Ri_D = 0 \), the experimental and numerical peak values are almost equal. Only for increasing \( Ri_D \), the values of the experimental determined upper vortex extremes (negative vorticity) are observed to be smaller than for the numerical results. Again it seems justified to conclude that the experimental method is not fully capable of capturing the high velocity gradients.

Considering the results of the peak vorticity and the integrated vorticity, it can be concluded that due to heat addition the upper vortices become stronger and the lower vortices slightly weaker. Therefore a strength difference is observed between the structures in the two vortex rows and the upper vortices are stronger than the lower vortices. The results also show that this strength difference is already present just after the formation of the structures.
Figure 5.15: Absolute vorticity extremes for $Re_D = 75$ and varying $Ri_D$: (a) experimental results, (b) numerical results

Figure 5.16: Difference in absolute vorticity extremes for $Re_D = 75$ and varying $Ri_D$: (a) experimental results, (b) numerical results
5.4 Point-vortex simulations

The observed strength difference between the upper and lower vortices is supposed to be responsible for the observed phenomena as deflection and relative rotation. In order to verify this statement, point-vortex simulations were performed in which only the effect of a strength difference on the vortex street behaviour is analysed. To that end, no changes are made in the street configurations on forehand. For the point-vortex strengths the numerically calculated circulations at \( x/D = 12 \) (see Table 5.2) are taken. At this position it is assumed that the vortices have reached a developed stage and that heat has a minor effect on their downstream characteristics.

As described in Chapter 2, the point vortices enter the considered domain at \( x_{pv}/D = 0 \) with \( x_{pv} \) the downstream co-ordinate of the point vortices. The spacing between the vortices at entrance is set to the spacing as found by the von Kármán stability criterion. The behaviour of the point vortices downstream of this position is then fully determined by the strength of the vortices. The entrance position \( x_{pv}/D = 0 \) can be seen as a position just behind the vortex-formation region.

The point-vortex simulations show for an increasing strength difference an increasing deflection of both vortex rows in negative \( y \)-direction (fig. 5.17a). In contrast to the numerical and experimental results, the deflection increases continuously. This becomes even more pronounced considering the vertical difference in vortex row positions \( \Delta Y_{pv} \) between \( x_{pv}/D = 8 \) and \( x_{pv}/D = 22 \) (fig. 5.17b). For increasing \( \Delta |\Gamma| \) the deflection increases for both the upper and lower vortex rows. The deflection of the lower vortex row turns out to be larger, a behaviour also observed in the experimental and numerical results (see fig 5.6b).

From the vortex trajectories, the relative movement between two subsequent structures can be deduced (fig. 5.18, see also fig. 5.7). For \( \Delta |\Gamma| = 0 \) (or, equivalent, \( Rii_D = 0 \)) almost no relative movement between these two structures occurs. An increasing strength difference results again in a rotation of the lower vortex around the upper vortex. The vertical displacement, which can be seen as a widening of the vortex street, also appears in the point-vortex simulations. Therefore, it can be concluded that the additional widening for \( Rii_D > 0 \) is almost fully caused by \( \Delta |\Gamma| \) and not by the viscous spreading of the vortex cores.

It is interesting to observe that for all strength differences, the relative movement of the lower vortices is along the same path (fig. 5.18a). However, for an increasing strength difference the length of this path also increases (fig. 5.18b). This can not be observed in the presented numerical and experimental results (fig. 5.8). In these results the direction of the paths changes for varying \( Rii_D \). The change in path direction is probably caused by a horizontal shift in position between two subsequent structures at the moment the structures are shed (at \( x/D = 8 \)). Therefore, the interaction between the structures is different for varying \( Rii_D \). For the point-vortex simulations the relative distance between two subsequent structures at the entrance of the domain is equal for

<table>
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<th>Table 5.2: Point-vortex strength</th>
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Figure 5.17: (a) Point-vortex trajectories for different $\Delta \Gamma$ and (b) net deflection $\Delta Y_{pv} = (Y_{pv}(x/D = 22) - Y_{pv}(x/D = 8))/D$

all $\Delta |\Gamma|$. 

Figure 5.18: Relative movement between the lower point vortex and upper point vortex $(X_u, Y_u)$ for increasing strength difference as calculated by the point-vortex model: (a) paths for different $\Delta |\Gamma|/\Gamma_u$, (b) paths plotted separately
5.5 Discussion

The presented results show that for moderate Ri\textsubscript{D} numbers (0 < Ri\textsubscript{D} < 1) an increasing heat input results in a deflection of the vortex street in negative y-direction and in a relative movement of two subsequently shed vortices. These two vortices, one negative and one positive, form a combined linked structure in which a strong interaction takes place. This results in a relative rotation of the lower vortex around the upper vortex. The mutual rotation between the vortices causes a larger widening of the vortex street. Both the deflection and rotation are caused by a strength difference ΔΓ (which has a negative sign) between the upper and lower vortices where the upper vortices are slightly stronger than the lower ones.

Most of the observed phenomena are captured more or less by the point-vortex model. It can therefore be concluded that the deflection and rotation are a result of the heat induced strength difference ΔΓ. This behaviour can be understood considering a small part of a point-vortex street (fig. 5.19). In the street, which is situated in a potential flow with main velocity U\textsubscript{0}, the vortices are distributed as they are in a von Kármán vortex street (fig. 5.19a). The strength difference ΔΓ between the upper and lower vortices causes the lower vortices to move slower downstream than the upper vortices. The upper vortices move with a velocity equal to \( U = U_0 - \delta u_1 \) and the lower vortices with \( U = U_0 - \delta u_2 \), where \( \delta u_2 > \delta u_1 \). This results in a distorted configuration where the distance between two subsequent vortices (first a lower followed by an upper vortex) becomes smaller (fig. 5.19b). As a consequence the velocity component \( v \) of the vortices as induced by the other vortices does not cancel out anymore (as it did in the initial situation). Therefore a negative \( v \) equal to \( \delta v_1 \) remains which causes the vortex street to deflect in negative y-direction (fig. 5.19c). In the experiments and numerical simulations the deflection of the vortex street is most significant for Ri\textsubscript{D} = 0.5. A further increase of Ri\textsubscript{D} results in a growing influence of the buoyant force and consequently a weaker deflection. The buoyancy force is not present in the point-vortex model.

Figure 5.19: Bend off of the vortex street due to strength differences: (a) initial situation, (b) distortion of the position in x-direction resulting in a velocity component \( v \), (c) deflection of the vortex street.
Chapter 6

Vortex formation and shedding process

The analysis of the wake flow for $0 < Ri_D < 1$ revealed that at $x/D = 8$ (just after the formation of the structures) the upper vortices are already stronger than the lower vortices. Therefore, processes in the near wake are probably responsible for this observed difference in strength. To investigate these processes, the vortex shedding process is considered. A description of the vortex shedding mechanism for $Ri_D = 0$ is given by Green and Gerrard (1993) who present for $Re_D = 73$ the experimentally determined evolution of vorticity and shear stress distribution in the near wake. In the present investigation, the shedding and formation process for $Re_D = 75$ and $Ri_D = 0.5$ is chosen as basis for the analysis. In the previous Chapter it is found that for this situation the strongest deflection of the vortex street occurs. The shedding and formation processes are first discussed by considering the evolution of the vorticity distribution and baroclinic vorticity production. From this discussion several monitor quantities are derived like the cylinder wall vorticity, the vortex formation length and the baroclinic vorticity production. These monitor quantities are presented for $0 < Ri_D < 1$.

6.1 Vorticity distribution

The heat influenced vortex shedding process is analysed by considering a sequence of the numerically calculated contours of $\omega_z$ (fig. 6.1 left) and $Q = 1/2(S_1^2 + S_2^2 - \omega_2^2)$ (fig. 6.1 right for definition see Chapter 2). These contours are presented at 6 different stages during the shedding period $T$ for $Ri_D = 0.5$ and $Re_D = 75$.

The sequence starts with the formation of an upper vortex indicated by the stretching of the vorticity strand located at the upper cylinder shoulder (fig 6.1a). The downstream end or tip of the vorticity strand coincides with an area where $Q < 0$ (fig. 6.1b at $x/D \approx 1.5, y/D \approx 0.5$). Inside such an area, vorticity advected from the boundary layers is relatively unaffected by shear stress and can therefore organise into a coherent structure. This can already be seen one step later, where at the tip of the vorticity strand a local vorticity extreme is forming (fig. 6.1c at $x/D \approx 2.0, y/D \approx 0.5$). This extreme is still positioned within an area of strongly negative $Q$. In the following steps, the local vorticity extreme develops further and is slowly moving in downstream direction. Simultaneously, the vorticity strand (which connects the local vorticity extreme with its
source, the boundary layer), starts to constrict between $x/D \approx 2.0$ and $x/D \approx 3.0$ at $y/D \approx 0.5$ (fig. 6.1 i, k). The area where this strand constricts, coincides with an area of $Q > 0$ (fig. 6.1h, j, l) indicating that strain rate dominates over vorticity. The area of $Q > 0$ is rapidly increasing and reaches a maximum in figure 6.1j. As soon as the vorticity structure reaches its developed stage and is disconnected from the vorticity strand, advection causes the structure to accelerate and to move in the downstream direction (fig. 6.1k, a).

The formation of a lower vortex is supposed to start half a period later (fig. 6.1g). Again the vorticity strand is stretched while its tip coincides with an area of $Q < 0$ (fig. 6.1g and h at $x/D \approx 1.5$, $y/D \approx -0.5$). Comparing this area with the one as found during the formation of an upper vortex (fig. 6.1b, d, f with 6.1h, j, l) the simulations show that the extreme value of $Q$ is more negative for the upper vortex. However, the size of the area occupied by $Q < 0$ is slightly larger for the lower vortex. Therefore, organization of vorticity during the formation of a lower vortex takes place in a larger area and is more affected by the strain rate. This causes the local vorticity extreme to appear more downstream or equivalently on a later stage during the formation process (compare fig. 6.1c at $x/D \approx 3$, $y/D \approx -0.5$ with fig. 6.1c at $x/D \approx 2$, $y/D \approx 0.5$). Also, during the constriction process the area of $Q > 0$ seems to be more intensive for the lower vortex (compare fig. 6.1f at $x/D \approx 2.8$, $y/D \approx -1.0$ with fig. 6.1j at $x/D \approx 2.1$, $y/D \approx 0.5$).

The analysis above shows that, owing to heat effects, the formation process of a lower vortex differs from the formation process of an upper vortex. During the organization of the boundary-layer produced vorticity into a coherent structure the relative influence of strain rate is less for the upper vortex (lower $Q$ level). Therefore, the formation process of the upper vortex is more effective than for the lower one. Besides, the formation of an upper vortex takes place closer to the cylinder than it does for a lower vortex. Therefore the distance between the vorticity source and the formation area decreases. This prevents strain rate, which dominates in the strand, to have a big influence on the advected vorticity. The shift in vortex formation position becomes even more pronounced considering the formation length $L_f$ for various $Ri_D$ numbers (fig. 6.2a), where $L_f$ is based on the definitions of a closing contour $0.1 \omega_{max}$ (see for definition Chapter 2). The results show for increasing $Ri_D$ the lower vortex to be formed further downstream. The upper vortex, on the other hand, is formed closer to the cylinder.

Furthermore, for increasing $Ri_D$ the position for which at the tip of the vorticity strand a local vorticity extreme appears, moves into the direction of the cylinder. This can be concluded from the starting positions of the lines representing the local vorticity extremes as function of their downstream position (fig. 6.2b). For the upper vortex (negative $\omega_z$) this position is shifted upstream for increasing $Ri_D$, while the position for the lower vortex is shifted downstream. From the behaviour of the negative local vorticity extremes, it can be seen that the decrease of these extremes for $Ri_D > 0$ differs from the decrease for $Ri_D = 0$. The value of the extremes are more or less equal just after their formation. Slightly downstream, the extreme values for $Ri_D > 0$ show almost no decrease (for $Ri_D = 0.5$), or even increases (for $Ri_D = 1$). This difference in behaviour, which takes place between $1.5 < x/D < 4$, causes the observed strength difference in the value of the extremes as already indicated in figure 5.15.
6.1. VORTICITY DISTRIBUTION

Figure 6.1: Near-wake flow behaviour calculated numerically for $Re_D = 0.5$ and $Re_D = 75$: (left) vorticity $\omega_z$ with 15 contour levels between -3 and 3, (right) $Q$ with 15 contour levels between -2 and 2.
6.2 Sources of vorticity

Close around the cylinder \((-1 < x/D < 1)\)

The observed differences in strength of the upper and lower vortices are possibly related to the effect of heat on the vorticity sources. The first source to be considered is the cylinder wall vorticity, here expressed as the period-averaged wall vorticity \(\bar{\omega}_w\) defined as \(1/T \int_0^T \omega_z(R, \phi, t)dt\) (fig. 6.3). An increasing \(Ri_D\) causes the level of the negative vorticity on the upper side of the cylinder \(0^\circ < \phi < 120^\circ\) to become lower. Between the two separation points \(120^\circ < \phi < 240^\circ\) the negative wall vorticity decreases while the positive wall vorticity increases. Upstream of the lower separation point \(240^\circ < \phi < 360^\circ\) the wall vorticity also increases. Thus one can state that heat causes the negative wall vorticity to decrease and the positive one to increase. As a consequence the vorticity level in the strands (which originate from the wall vorticity upstream of the separation point) decreases the strength of the upper vortex and increases the strength of the lower vortex. Also the wall vorticity downstream of the separation point becomes more positive. All effects on the wall vorticity are acting oppositely with respect to the observation that the upper vortices become stronger than the lower ones when adding heat.

From the period-averaged wall vorticity the position of the separation and the stagnation points can be derived. Within this work the separation points are characterised by the position at which the period-averaged wall vorticity \(\bar{\omega}_w(wall)\) is zero. The influence of heat on these stagnation points shows that the shoulder separation points \(S_{p1}\) and \(S_{p2}\), definition see Chapter 2) move slightly in the direction of positive \(\phi\) (fig. 6.4a,b). The front side stagnation point shows a stronger shift in positive \(\phi\) direction (fig. 6.4c). Here a clockwise shift of about 5° can be observed. For the second stagnation point \(St_2\), found at the cylinder rear side, an even stronger dependency on \(Ri_D\)
Figure 6.3: Effect of heat on the period-averaged wall vorticity $\bar{\omega}_{\text{wall}}$ for $Re_D = 75$

is observed (fig. 6.4d). Again the stagnation point moves in clockwise direction for increasing heat input, which results in a displacement of about $45^\circ$ for $Ri_D = 1$. The latter shift is induced by the increasing influence of the heat induced vertical upward flow. This flow stimulates the clockwise rotating recirculation area and weakens the anti-clockwise rotating one.

The second source of vorticity is the baroclinic production term $Ri_D \partial \Theta / \partial x$. Its effect on the structure strength during the formation process is analysed from the sequence of figures showing the iso-production contours between -0.5 and 0.5 for $Ri_D = 0.5$ and $Re_D = 75$ (fig. 6.6). It can be seen that close at the cylinder almost no temporal variation in this production term occurs. At the front side of the cylinder ($x/D < 0$), the production term is positive and at the rear side ($0 < x/D < 1$) the term is negative. The net effect of the production term is therefore not obvious. An estimation of the cumulative effect of the very-near-wake vorticity production and changing wall vorticity level on the finally formed vortex structures strength is derived from the local period-averaged flux of vorticity $\Phi_{\omega_z}(y) = \int_T w_\phi z dt / T$ through a vertical cross section at $x/D = 1$ (6.5a). This local vorticity flux represents all upstream produced vorticity which possibly ends up in the vortex structures. As one can conclude (fig. 6.5a), an increasing $Ri_D$ causes the flux of negative vorticity to decrease, while the flux of positive vorticity increases. The period-averaged total vorticity $\Omega_{\omega_z}$ transported over the boundary $x/D = 1$ can be calculated by integrating the local vorticity flux $\Phi_{\omega_z}(y)$ over the cross-section length $2L$

$$\Omega_{\omega_z} = \int_{-L}^{L} \Phi_{\omega_z}(y) dy.$$  \hspace{1cm} (6.1)

This net vorticity flux (fig. 6.5b) shows that due to heat more positive than negative vorticity enters the region $x/D > 1$. Therefore, from the very-near-wake produced vorticity the earlier observed difference in strength between the upper and lower vortex structures can not be explained.
Figure 6.4: Influence of heat addition on the separation points $S_{p1}$ (a) and $S_{p2}$ (b) and on the front side stagnation point $St_1$ (c) and the rear side stagnation point $St_2$ (d)
6.2. SOURCES OF VORTICITY

Figure 6.5: The effect of heat addition on (a) the period-averaged vorticity flux $\overline{F_{\omega_z}}$, (b) the integrated averaged vorticity flux $\Omega_{\omega_z}$ both evaluated at the cross-section $x/D = 1$

**Downstream of the cylinder** ($1 < x/D < 6$)

The structure formation is further analysed in the downstream region between $1 < x/D < 6$ where the situation changes completely. At the moment an upper vortex is initiated, an area of negative baroclinic vorticity production is located at the tip of the (negative) vorticity strand (fig. 6.6b at $x/D \approx 1.7, y/D \approx 0.5$). The production term contributes therefore to the vorticity strand to be formed. Slightly upstream at $x/D \approx 1, y/D \approx 0.5$, a small area of positive production can be found. This area coincides with the area where somewhat later the constriction of the vorticity strand takes place. While the area of positive production almost disappears (fig. 6.6d), the negative one is growing in size and strength (figs. 6.6d, f, h). Therefore, in the first half of the formation period the production term adds to the newly formed negative upper structure.

In the second half of the formation period (figs. 6.6h, j, l), the results show that the area of negative production hardly grows. On the other hand, a new area of positive production is found in figure 6.6f at $x/D \approx 2.1, y/D \approx 0.3$. This area grows and at the end of the formation process (fig. 6.6b at $x/D \approx 5.1, y/D \approx 0.5$) it becomes almost as large as the negative production area. This area of positive production now falls within the vortex structure and contributes to the vortex strength. The contour plots suggest that between $1 < x/D < 3$, which is in the first half of the formation process, a net amount of negative vorticity is produced within the upper vortex. This production causes the upper vortex to become stronger with respect to the situation for $Ri_D = 0$. The latter was already observed in behaviour of the vortex structure extreme in this region (fig. 6.2b). More downstream also positive vorticity is produced within the vortex structure, which counteracts the production of negative vorticity. The difference in strength with respect to the situation for $Ri_D$ then hardly changes (see fig. 6.2b).

During the formation of a lower vortex, also an area of negative vorticity production arises at
Figure 6.6: Near-wake vorticity (left) and vorticity production (right) for $R\tilde{D} = 0.5$ and $Re_D = 75$. 

- (a) $t = 0$
- (b) $t/T = 1/6$
- (c) $t/T = 2/6$
- (d) $t/T = 3/6$
- (e) $t/T = 4/6$
- (f) $t/T = 5/6$

The diagrams illustrate the evolution of vortical structures in the wake of a cylinder at different normalized times $t/T$. The vorticity is shown on the left, and the vorticity production is on the right.
6.2. SOURCES OF VORTICITY

the tip of the vorticity strand (fig. 6.6h at \( x/D \approx 1.5 \) and \( y/D \approx -0.5 \)). Only now, the vorticity sign in the strand is positive, which implies a counteracting effect of the production term. The production term is then, among others, possibly responsible for the delayed appearance of the local vorticity extreme at a more downstream position (fig. 6.2b). Here one should note that the production term in extreme value and size is smaller than for the upper vortex. Also a small area of production of positive vorticity can be found upstream of the negative vorticity production area, as observed in figures 6.6h, j at \( x/D \approx 1 \) and \( y/D \approx -0.3 \). This area seems to be rather small and only has a small effect on the constriction process. In the second part of the formation process, a new island of positive vorticity production appears (fig. 6.6i at \( x/D \approx 2, y/D \approx -0.4 \)) which grows again in size and strength and falls within the area covered by the formed lower vortex structure. From that moment on the positive production contributes to the strength of the structure and therefore counteracts the negative production term. The production of vorticity within a lower vortex is thus first primarily negative, causing the positive vortex structure to become weaker. More downstream, positive production of vorticity is increasing, and competes with the negative production.

![Figure 6.7: Integrated vorticity production for \( Re_D = 75 \) and varying \( Ri_D \): (a) in the upper vortex and (b) in the lower vortex](image)

The quantitative contributions \( \Gamma_{prod} \) of the baroclinic vorticity production to the circulation of the structure is analysed by calculating the integrated contribution of the confined areas of negative and positive vorticity production separately according to

\[
\Gamma_{prod} = \int_A Ri_D \frac{\partial \Theta}{\partial x} dA, \tag{6.2}
\]

where the area \( A \) is defined as the area bounded by the threshold production level \( |Ri_D(\partial \Theta)/(\partial x)| = 0.1 \). The integrated productions of the positive areas as well as the negative
areas are plotted for the upper vortex in figure 6.7a during their downstream movement. It can be seen that for $x/D < 4$, the negative vorticity production is significantly larger than the positive vorticity production. Especially in the region $1 < x/D < 3$ the positive production is relatively small and almost disappears, resulting in only a negative vorticity contribution to the structures. This disappearance of the positive vorticity production was also observed in the sequence of vorticity production contour plots (fig. 6.6). From these contour plots it was concluded that the positive vorticity production island which disappears, does not fall within the vortex structure. Therefore it does not contribute to the structure strength. In the region $x/D > 3$ the positive vorticity production increases and becomes as strong as the negative vorticity production at $x/D \approx 5$. At this downstream position the positive vorticity production area falls within the vortex structure, as shown in the vorticity production contour plots, and contributes to the vortex strength.

For the lower structure, the integrated vorticity production term shows a more or less similar behaviour (fig. 6.7b). Again, the negative vorticity production is stronger than the positive vorticity production. Only for the lower vortex the positive vorticity production remains negligible up to $x/D \approx 2$. Beyond this point, the positive vorticity production increases but remains smaller than the negative vorticity production within the scope of the figure (fig. 6.7b).

In conclusion, the negative vorticity production for both the upper and lower vortex is stronger than the positive vorticity production. Therefore the upper vortices become stronger while the lower vortices become weaker.

6.3 Temperature distribution

Besides advection of vorticity, also heat is convected into the cylinder wake. The transport and redistribution of heat during the vortex formation is analysed by the sequence of iso-thermal plots (fig. 6.8a,c,..., k). This sequence of plots reveals that heat is convected from the cylinder wall into the near wake in a way similar to the advection of vorticity. A strand of warm fluid originates at the shoulders of the cylinder and stretches into downstream direction (fig. 6.8a). At the tip of this strand a local temperature extreme can be found at $x/D \approx 1.9$, $y/D \approx 0.5$ for the upper blob (fig. 6.8c) and at $x/D \approx 2.1$, $y/D \approx -0.5$ for the lower one (fig. 6.8k). These extremes develop further into a confined area of heated fluid (fig 6.8e,g and 6.8a,c). The strand, connecting the confined area of warm fluid with the cylinder, constricts causing the area to become an isolated hot blob (fig. 6.8i, k, a and 6.8c, e, g).

Comparing the positions of the hot blobs with the positions of the vortex structures (fig. 6.9a) it is seen that the vortex structures coincide with areas enclosing a local temperature extreme. The similarity between advection of vorticity and convection of heat implies that the vortex structures are hot isolated areas with a strong circulation. Furthermore, the hot blob is more compact than the vortex structure. Due to the fact that heat is captured in the kernel of the vortex structure, only diffusion processes can cause the hot blob to spread. Therefore, the hot blobs are more or less preserved as long as the vortex structure remains stable.

Although the formation process of the upper and lower hot blobs is almost comparable, the local extreme values of temperature differ (fig. 6.9b). The upper blobs turn out to be warmer than the lower ones and the difference increases for larger $Re_D$ values. During the formation of an up-
Figure 6.8: Temperature distribution showing 15 iso-thermals between $0 < \Theta < 0.6$ (left) and streamlines (right) for $Ri_D = 0.5$ and $Re_D = 75$.
per vortex, this temperature difference causes the negative temperature gradients upstream of the vorticity strand to become larger than for the lower vortex. Considering the vorticity production term \( \left( \frac{\partial \omega}{\partial x} \right) \), this inherently means a larger production of negative vorticity in the upper vortex.

In order to understand the formation of the warmer upper vortex, the convection of heat (but also vorticity) is analysed from the instantaneous streamlines (fig. 6.8b, d,..., l). Transport of any quantity will be along the streamlines as long as diffusive transport can be neglected with respect to the advection process. For a qualitative discussion, this assumption is justified in the specific problem. From the streamline patterns at the initial stage of the upper structure formation (fig. 6.8b and also previous stages j, l) a clockwise rotating recirculation area can be found at \( \frac{x}{D} = 0.7 \) and \( \frac{x}{D} = 1.5 \). This circulation area causes warm fluid, located in the near wake of the cylinder, to be transported to the growing upper structure. Even when the recirculation area disappears the strong advection, denoted by the vertical streamlines at \( \frac{x}{D} = 1 \), is still present (fig. 6.8d). In a later stage of the shedding process this transport mechanism disappears.

During the formation of a lower structure, a counter-clockwise rotating recirculation area can be found in the near wake at \( \frac{x}{D} \approx 1.5 \) and \( \frac{y}{D} \approx -0.3 \) (fig. 6.8h). Compared to the clockwise rotating recirculation area, this circulation area exists for a shorter time (fig. 6.8h-j). Considering the velocity field close to the cylinder, this means that the counter-rotating circulation area becomes weaker and remains for a shorter time and the clockwise rotating one becomes stronger and remains for a longer time. This was already observed from the wall vorticity between \( 120^\circ < \phi < 240^\circ \) which becomes more positive for \( Ri_D < 0 \). Therefore, the process responsible for the convection of warm fluid from the region close to the cylinder into the region...
where a lower vortex is formed is less effective compared to the upper vortex. This causes the upper vortices to become warmer than the lower vortices and hence stronger.

The averaged streamline pattern shows this behaviour even more clearly (fig. 6.10). For the upper circulation area, the averaged streamlines are located close to the cylinder. Heat induced by the cylinder is therefore effectively convected into the upper wake region. For the lower circulation area, almost no downward directed streamlines are located close to the cylinder. The transport of heat into the lower wake region is therefore considerably smaller. The streamlines also show that a net amount of fluid is transported from the lower half of the wake into the upper half. This net transport is depicted by the dashed streamlines. Fluid situated between the cylinder and these streamlines first moves underneath the cylinder but finally ends up in the upper half of the wake.

6.4 Discussion

The presented results show that various heat-induced effects play a role during the vortex formation process. All together these effects are responsible for the observed difference in structure characteristics as temperature and strength of the upper and lower vortices. To address all effects, one needs to separate the effects of heat in the region close at the cylinder (\(1 < x/D < 1\)) from the effects in the formation region (\(1 < x/D < 6 - 8\)).

In the area close at the cylinder the presented results show that due to heat the net produced vorticity is positive. The main sources responsible for this phenomenon are depicted in figure 6.11. The first process is the effect of heat on the boundary-layer thickness and consequently on the wall vorticity (depicted with a triangle and a sign indicating the sign of vorticity which the source is adding). In the upstream half of the cylinder (\(0^\circ < \phi < 90^\circ\)) heat induces an additional acceleration of the fluid in the boundary layer (heat contributes to the main flow direction). Therefore the gradients in the boundary layer are expected to increase and thus the wall vorticity should increase. At the lower cylinder side, between \(270^\circ < \phi < 360^\circ\), heat causes the velocity
in the boundary layer to decrease (heat creates an upward motion). Therefore, the velocity gradients in the boundary layer are expected to become weaker and the wall vorticity is expected to decrease. In the results showing the wall vorticity, this behaviour can not be observed. More precisely, the opposite takes place. Considering the downstream side of the cylinder, heat contributes to the clockwise-rotating circulation area. Therefore the wall vorticity which is then positive, increases. Half a period later, the counter-clockwise rotating recirculation area becomes weaker due to the induced heat. The, then negative, wall vorticity decreases. The averaged wall vorticity over the entire shedding period becomes therefore more positive. This can be observed in the results showing the wall in the considered area.

![Diagram showing wall vorticity](image)

Figure 6.11: Schematic representation of the effect of heat on the vorticity sources close to the cylinder. Only the advection of vorticity into the region $x/D > 1$ is considered.

The rather unexpected change in wall vorticity for $x/D < 0$ can be explained by the second process which is a change in flow profile around the cylinder (its schematic effect is depicted with a square, fig. 6.11). For increasing $Ri_D$ all separation points move in clockwise direction. Therefore, it appears that the angle between the inflow direction and the gravity vector decreases (fig. 6.12a), causing more fluid to be advected underneath the cylinder than above. The velocity below the cylinder increases, causing the gradients in the boundary layer and therefore the wall vorticity to increase. In the region above the cylinder the flow velocity decreases for $Ri_D > 0$. Hence, the wall vorticity (in absolute sense) decreases. The latter is supported by the results showing the period averaged velocity component $u/U_0$ at a cross section $x/D = 0$ (fig. 6.12b). For $y/D < 0$ the velocity increases as $Ri_D$ is enlarged and decreases for $y/D > 0$. From the numerically calculated streamlines pattern for $Ri_D = 1$ and experimental dye visualisations for $Ri_D = 1.1$ the change in flow profile becomes even more clear. The streamlines and dye streaklines (which can be compared because the flow behaves more or less stationary for $x/D < 0$) show a deflection downwards, resulting in an increased flow velocity below the cylinder. The latter can also be concluded from the spacing between the streamlines which decreases in the area below the cylinder.

This change in flow pattern or deflection of streamlines possibly originates from the net flux of...
6.4. DISCUSSION

Figure 6.12: Effect of heat on upstream inflow direction: (a) schematic representation, (b) period-averaged $u$-velocity component at $x/D = 0$

Figure 6.13: Effect of heat on upstream stream line pattern for $Re_D = 75$: (a) numerical results for $Ri_D = 1$, (b) dye visualisations for $Ri_D = 1.1$
fluid from the lower half of the wake ($y < 0$) to the upper half. As the period-averaged streamline pattern has shown, this transport takes place in the area close behind the cylinder (fig. 6.10) and is driven by the temperature differences in this area. Due to this net flux of mass in $y$-direction, conservation of mass demands an increasing flux in $x$-direction of fluid underneath the cylinder. This explains the increasing velocity component $u$ underneath the cylinder for increasing $Ri_D$ and the decreasing velocity above. Due to the changing velocity profile the force exerted by the flow on the cylinder (and vice versa) changes. The period-averaged lift-force shows that a negative lift is acting on the cylinder. From the calculations it turned out that this negative lift is mainly caused by the pressure difference between the upper and lower wall (fig. 6.14a). Considering only the wall-shear stresses a positive upward lift would arise. However, this contribution is an order of magnitude smaller than the pressure induced effect. The shear-stresses cause a net torque on the cylinder (fig. 6.14b)

![Figure 6.14: Period-averaged forces on the cylinder induced by the flow: (a) lift force, (b) torque](image)

The third source in the narrow region around the cylinder is the baroclinic vorticity production (depicted with a circle in fig. 6.11). The results show that at the upstream cylinder side, the baroclinic vorticity production is positive while on the downstream side the production is negative. Although no irrefutable proof is yet present, the net effect hardly seems to influence the vorticity level in the near wake.

In order to understand the formation of stronger upper vortices, it is more convenient to consider the flux of vorticity into the region where the vortex structures are generated. Therefore within the present study the narrow region around the cylinder $-1 < x/D < 1$ can be considered as a vorticity source from which vorticity is advected into the downstream region $x/D > 1$ (fig. 6.11). The results have shown that for $Ri_D > 0$ slightly more positive than negative vorticity is produced and transported into the region $x/D > 1$ (fig. 6.5). This is in contrast with the results
found for the fully developed structures found for \( x/D > 8 \), where the negative vortex structures are stronger than the lower ones (with positive vorticity).

The in Chapter 5 observed strength difference between the shed vortex structures must therefore originate in the region where the formation and shedding of the structures take place. In general there are two processes responsible for the observed phenomena. The first one is the difference in the shedding and the formation processes of the upper and lower vortices. The analysis of the shedding process by the sequence contours of \( \omega_z \) and \( Q \) has shown that the upper vortex is formed at a more upstream position. Furthermore, the level of \( Q \) in the upper formation region is considerably more negative than in the lower formation region. In combination with the fact that the absolute vorticity level is slightly lower for the upper vortex (less negative vorticity is advected into the downstream region), it can be concluded that the strain rate level is lower in the upper vortex formation region. Therefore, the vortex formation process of the upper vortex is less affected by strain (overall lower \( Q \) level) than the lower vortex. This lower shear stress level is caused by the changing flow profile around the cylinder. As explained before, due to heat addition less fluid flows over the top of the cylinder than underneath. As a result, the upper shear layer decreases in strength while the lower shear layer increases (fig. 6.15a). The average velocity component \( u \) in a cross-section at \( x/D = 1 \) supports this statement (fig. 6.15b). From this result it can be seen that for increasing \( R_{iD} \) the velocity in the shear layer \( \bar{u}_{sl} \) increases in the lower shear layer and decreases in the upper shear layer. Therefore, the velocity gradient in the lower shear layer increases and consequently the strain rate increases. The opposite holds for the upper shear layer.

The decreased strain rate level in the upper shear layer causes a less severe loss of vorticity (Green and Gerrard (1993), Weiss (1991)) compared to the situation in the lower layer. This implies that, although less negative vorticity is produced, the negative upper vortex can become stronger than the lower vortex. Furthermore, the higher velocity in the lower region causes the lower structure to be formed more downstream, as can be concluded from the behaviour of the formation length for varying \( R_{iD} \). Therefore strain rate has a stronger influence on the advected boundary-layer vorticity than during the formation of an upper vortex.

The second process and possibly the most important one, is the baroclinic vorticity production in the region \( 1 < x/D < 3 \). In this region, the vorticity production is mostly negative. Also the areas where vorticity is produced coincide with the region where a vortex structure is positioned. This means that vorticity production in the considered region adds to the negative vorticity structures and causes the positive vorticity structures to become weaker. The former can be observed in the peak vorticity of the upper structure extremes which increase between \( 1 < x/D < 3 \) for \( R_{iD} > 0 \) (fig. 6.2).

In conclusion, it may be stated that the strength difference between the upper and lower vortices, in case heat is added, is caused by complicated processes which may induce counteracting effects but at the end result in the observed strength difference. The strength difference on its turn is responsible for the remarkable downward deflection of the entire vortex row and the relative movement between the wake vortex structures. Furthermore, it is shown that heat is captured within the vortex structures and advected downstream as hot isolated blobs. Therefore no direct mixing with the surroundings takes place after the vortex structures are formed, at least not for \( R_{iD} < 1 \).
Figure 6.15: Effect of heat addition on the shear layer strength (a) schematic model, (b) the period-averaged velocity component $u$ over the shear layer at $x/D = 1$.
Chapter 7

Wake transition

For $Ri_D < 1$ and within the region $x/D < 25$, the flow in the wake was observed to be two-dimensional and periodic. The effect of increasing $Ri_D$ was first investigated by performing bubble wire visualisations (fig. 7.1). Although the rising of the bubbles has an influence on the observed phenomena, in the visualisation results an early 3D transition of the vortex street can be observed. A description of this transition process is the subject of this chapter.

7.1 Global process

The stability of the vortex wake for $Ri_D > 1$ is globally analysed by dye visualisations (fig. 7.2). These visualisations are performed using a 7 needle-injection probe which creates a pattern of dye streaks. The dye streaks are recorded using 2 cameras. The first camera views the wake in span-wise direction (fig. 7.2a, c,..., k) resulting in a side view of the wake flow. The cylinder is positioned at $(x/D, y/D) = (0, 0)$ and appears in the images as a slanted rectangle with a bright and a dark half.

The images are captured every 2.5 s, which is almost equivalent to $2/5T$, with $T$ the typical shedding period ($T = 6.5$ s). Clearly visible in the sequence is the entrainment of injected dye from outside the upper shear layer into the cylinder near wake. Therefore, first the upper vortex becomes visible. More downstream dye is also advected into the lower vortex which becomes clearly observable for $x/D > 10$. More downstream the side view images show an additional dye blob on top of an upper vortex. This blob appears approximately at $x/D \approx 15$ (fig. 7.2a, denoted with the white arrow) and develops as it is convected downstream. At about $x/D \approx 20$ the dye blob escapes out of the primary upper vortex and develops into a mushroom-like shape. The mushroom shape is even better observable for a more downstream dye blob (called secondary structure and denoted with the white dashed arrow in figure 7.2a, c,..., k). The upstream secondary structure (solid arrow) develops in a similar way and reaches the described stage at approximately the same downstream position (fig. 7.2k).

The second camera views the flow from an oblique angle above (fig. 7.2b,d,..., l), thus providing information about span-wise variations. The images show that up to $x/D \approx 12$ the dye streaklines remain almost parallel implying that no span-wise variations occur. In this region the flow is
assumed to be 2D. More downstream, the development of the secondary structure takes place as indicated by the white dashed and solid arrows in figure 7.2b,c, ..., l. The secondary structure becomes apparent as the upward deflection of certain dye streaks. Therefore a span-wise variation in the streak line behaviour can be observed, which indicates that the flow transforms into a 3D flow. However, the third velocity component seems to be small. This can be concluded from the dye streak-lines which show almost no deflections in span-wise direction. Analysis of the recorded images has shown that at least at two span-wise positions a secondary structure develops. The span-wise distance between the two structures is estimated to be about $4D$ to $5D$.

During the downstream convection, the escaping secondary structure develops into a vortex ring structure. This can be clearly seen from the escaping thermal structure as depicted by the dashed arrow in figure 7.2f. In the planar view such a vortex ring is represented by a mushroom shape (fig. 7.2e).

### 7.2 2D analysis of the transition process

The development and escape of a secondary structure as observed in the dye visualisations is analysed by performing 2D HiRes-PV experiments in the $x - y$ plane. The experiments are carried out with a fixed camera position. Therefore the evolution of the structures can be analysed in detail over a relatively long period (about 6 to 8 typical shedding periods). Reference to the cylinder can be made using the cylinder position $x_c/D$, as indicated in the figures.

For $Re_D = 75$ and $Ri_D = 1.3$ the process of a growing secondary structure is analysed by considering the vorticity contours at 5 different time stages (fig. 7.3a, c, ..., i, iso-vorticity contours between -1.6 and 1.6 with step size 0.1). At the first stage, about 10 seconds after the cylinder has left the domain at $x/D = 0, y/D = 0$ (fig. 7.3a, $t_0 = 10s = 1.5T$), the shed upper vortex structure can be observed at $x/D = 2.5, y/D = 0.5$ and the lower structure at $x/D = 4, y/D = -1$. These structures appear as the regular vortex structures as observed in the 2D wake analysis. Only some slight deformation of the structures can be observed. This deformation causes the
7.2. 2D ANALYSIS OF THE TRANSITION PROCESS

Figure 7.2: Dye visualisations for $Re_D = 75$ and $Ri_D = 1.3$ (left) side view, (right) oblique view. The dashed and solid arrows point in both sequences to the same structure.
upper vortex to appear as a horseshoe-shaped structure, whereas the lower vortex appears to be stretched in positive $y$-direction.

About half a period later (fig. 7.3c), inside the negative upper vortex an area of positive vorticity is growing. This area grows further and moves in positive $y$-direction (fig. 7.3e). Simultaneously, the surrounding negative structure stretches and an upward moving vorticity strand is formed. At the tip of this strand, a local extreme can be observed (fig. 7.3e at $x/D = 3.0$, $y/D = 3.0$) which becomes separated from the primary structure. This separated blob of negative vorticity forms a combined vortex structure with the further developed area of positive vorticity. This combined vortex structure which appears in the 2D slice as a dipole structure accelerates and leaves the interrogation area (fig. 7.3g,i). An exact copy of this process can also be observed in the upper vortex positioned somewhat more downstream (around $x/D \approx 8.0$).

Considering the evolution of the primary vortex structures, the vorticity contours show that after the dipole structure has escaped, the upper vortex seems to restore again but becomes significantly weaker. Also the lower vortex, which moves underneath the upper vortex, becomes weaker. As soon as this lower vortex has moved entirely underneath the upper vortex, it seems to disappear. Therefore in the last analysed stages only the upper vortices and the tail of the escaped vortex structure can be observed. In the vector plots the escaping secondary dipole structure appears as an outburst of fluid from the primary upper vortex structure (fig. 7.3d at $x/D = 2$, $y/D = 2$). This upward motion, originating in the primary vortex, increases (fig. 7.3f) and develops as an upward moving plume. At the top of the plume the development of a dipole-like structure can be observed (see fig. 7.3h). As soon as the dipole has escaped, the upper vortex seems to restore again. This can be clearly seen in figure 7.3j in which the velocity vectors around $(x/D, y/D) = (2, 0.5)$ show a circulation area again. At the position of the lower vortex almost no circulation can be observed in the last two vector plots (fig. 7.3h, j).

In order to analyse the development from a 2D to a 3D flow, the experimental results (showing the $u$, $v$ component of the 3D velocity field) can be compared with the results of 2D simulations. The simulations are performed for $\text{Re}_D = 75$. The development of the flow is analysed by a sequence of vorticity contours between $1 < t/T < 3$ (iso vorticity contours between -1.6 and 1.6 with step size 0.1). The position of the cylinder $x_c/D$ within the reference frame of the figures is also presented next to the presented figures.

Considering the vortex pair of which the upper vortex is positioned at $(x/D = 2.5$, $y/D = 0.5$ and the lower vortex at $x/D = 4$, $y/D = -1$, both the numerical and experimental results show a similar structure shape at $t/T \approx 1$ (fig. 7.4a, b). Within the upper vortex a small disturbance can be observed which appears as an additional vorticity island causing the earlier observed structure deformation. Half a period later (fig. 7.4c, d) this disturbance has developed further into a local area of positive vorticity inside a negative upper vortex.

For later stages, a difference appears between the experimental and the numerical results. In the numerical results, the secondary structure continues to grow at a relatively fixed position within the surrounding primary upper vortex. This upper vortex shows the appearance of an upward stretched strand of negative vorticity (fig. 7.4f). In the experimental results the secondary structure starts moving in positive $y$-direction rather than growing inside the primary upper vortex (fig. 7.4e-g). During this upward movement the secondary structure grows in size and strength. The deformation of the primary upper vortex, as observed in the numerical results, can also be seen
7.2. 2D ANALYSIS OF THE TRANSITION PROCESS

Figure 7.3: 2D experimental analysis showing the development of the wake structures for $Re_D = 75$ and $Ri_D = 1.3$: (left) $\omega_z$, (right) vector field. The position of the cylinder is denoted with $x_c$. 

- (a) $t/T = 1$
  $x_c/D = -6.5$

- (b) $t/T = 1.5$
  $x_c/D = -9.8$

- (c) $t/T = 2.0$
  $x_c/D = -13$

- (d) $t/T = 3.0$
  $x_c/D = -19.5$

- (e) $t/T = 5.0$
  $x_c/D = -33$
Figure 7.4: Development of the vorticity distribution for $Re_D = 75$ and $Ri_D = 1.3$: (a) experimental results, (b) numerical results. The position of the cylinder is denoted with $x_c$. 

$t/T = 1$
$x_c/D = -6.5$

$t/T = 1.5$
$x_c/D = -9.8$

$t/T = 2.0$
$x_c/D = -13$

$t/T = 2.5$
$x_c/D = -16.3$

$t/T = 3.0$
$x_c/D = -19.5$
in the experimental results. Only in these results, the upward stretching of a negative vorticity strand is observed to be stronger. The stretching is also extended up to a higher $y$ position and at the tip of the strand a local negative vorticity extreme appears (fig. 7.4e). The formation of the dipole structure (fig. 7.4g) and the escape of a secondary structure are processes which can only be observed in the experimental results. This allows to conclude that initially the development of the secondary structure is a 2D process, but that the escaping process (formation of a dipole structure and acceleration) is a 3D phenomenon. The statement is supported by the dye visualisation results in which this 3D transition was observed. Here the transition was observed as the appearance of additional structures at certain span-wise locations.

Considering the lower vortex structure it is remarkable to see that during the entire analysed time-span the experimentally and numerically determined structure behaviour is very similar. Therefore, it can be assumed that the lower vortex maintains, at least within the considered time span, its two-dimensional character.

The flow development towards a 3D stage can be analysed considering the divergence of the measured 2D velocity field ($\partial u/\partial x + \partial v/\partial y$) which becomes non-zero when the flow develops into a 3D stage (i.e. $\partial w/\partial z$ becomes non-zero). However, analysis of the experimental results shows that the 2D divergence has a rather small value which is, for detection of the 3D transition, too strongly influenced by the measuring noise. Consequently, it can not be used as a criterion for the 3D transition.

From the comparison between the experimental and numerical results it was observed that the movement of the secondary structure in positive $y$-direction could only be detected in the exper-
imental results. The position where this upward movement starts will be denoted as the critical point \( x_{cr} \) in the transition towards a 3D flow field. The localisation of this point is based on a qualitative analysis of the measured vorticity distribution. Therefore the accuracy of the determined location is rather poor and turns out to have a scatter of about \( \pm 1.5D \). Still, from the results a clear dependency of \( x_{cr} \) on \( Ri_D \) can be observed (fig. 7.5). For decreasing \( Ri_D \) this critical point shows a tendency to move to a more downstream location. For \( Ri_D < 1 \) no escape of a secondary structure could be observed, at least not within the window of observation \( 0 < x/D < 35 \). For increasing \( Ri_D \) this critical position seems to reach a limiting value, which is about \( x/D = 10 \) to 12.

From the numerical results also the temperature field and the baroclinic production of \( \omega_z \) can be derived during the development of the secondary structure. The isothermals are plotted within the vorticity contours (dashed lines) for \( 0 < \Theta < 0.5 \) using 21 levels (fig. 7.6a, c,..., i). The iso-production contours are also plotted within the vorticity contours for \( -0.5 < Ri_D \partial \Theta / \partial x < 0.5 \) using 21 levels (fig. 7.6b, d,..., j). The development of the secondary structure is analysed between \( t/T = 1 \) and \( t/T = 3 \).

The plots of the temperature distribution show that heat is distributed within the flow field as isolated warm blobs (fig. 7.6). These warm blobs are captured within the vortex structures and advected downstream without being influenced too much by mixing processes with their surroundings. Only heat diffusion processes can cause the extreme values to decrease. During the downstream convection the behaviour and deformation of the relatively warm blobs show a behaviour similar to that of the vortex structures. Also a strand, now containing warm fluid, is stretched in positive \( y \)-direction (figs. 7.6e, g, h). This strand coincides with the stretched strand of negative vorticity. Relating this observation to the experimental results, it is expected that the experimentally observed vorticity strand, which is stronger and stretched more severely, contains a considerable amount of the heat originally contained by the primary structure. The escape of the secondary structure can possibly be seen as an effective way of releasing heat out of the primary structure.

The lower vortex structure seems to be hardly influenced by the induced heat. Only a slight increase of the structure temperature can be observed. The temperature of the lower structure appears to be about one order of magnitude smaller than the temperature of the upper vortex. As a result, the upper vortex is more affected by buoyancy than the lower vortex. This asymmetry in temperature distribution was already observed in the 2D wake analysis for \( 0 < Ri_D < 1 \).

The presence of the temperature gradients in horizontal direction give rise to the baroclinic vorticity production (7.6b,d,..., j). Inside an upper vortex structure a considerable area of positive and negative vorticity production can be found. A closer analysis reveals that the area of positive vorticity production coincides with the developing area of positive vorticity within the negative structure. This allows to conclude that the vorticity production is responsible for the development of the secondary structure. The area of negative production is found inside the primary upper vortex and stretched vorticity strand. This means that baroclinic vorticity production contributes to the strength of the vortex structure and adds to the stretching process of the negative strand.

Considering the lower structure, almost no additional vorticity is produced within the structure. Only just after the formation, an area of negative and positive vorticity production can be observed which seems to be responsible for the deformation of the lower vortex structure (fig. 7.6 b,c).
Figure 7.6: Development of the (a) isothermals (solid line) and (b) baroclinic vorticity production (solid lines) within the vorticity distribution (dashed lines) for $Re_D = 75$ and $Ri_D = 1.3$. The position of the cylinder is denoted with $x_c$. 
7.3 3D analysis

The 3D transition of the vortex wake flow and wake structures is investigated further by applying 3D-PTV experiments for $Ri_D > 1$. The experiments are performed with a fixed camera position, allowing to analyse in detail the evolution of the shed vortex structures towards a 3D stage. The flow field is analysed in the domain $-5 < x/D < 5, -3 < y/D < 4, 0 < z/D < 5$ and considering the previously presented visualisation and HiRes-PV experiments it is to be expected that within this domain a secondary escaping structure can be observed.

The 3D velocity vectors were calculated from particles which could be tracked in 5 subsequent frames. Through these paths 3rd order functions are fitted from which the instantaneous velocity vector can be derived. By doing so, in every frame about 400-500 velocity vectors are obtained. The noise in the velocity vectors is reduced by averaging the vector fields found over 5 subsequent samples. The averaging procedure is only allowed if the averaging-period is considerably smaller than the typical time scale of the flow. This is indeed the case for the considered flow field where the typical flow period $T$ is approximately 6 s and the averaging-period is 0.2 s. From the averaged velocity field $\omega_z$ is calculated. The vorticity calculation is performed on a structured grid of $30 \times 30 \times 15$ elements. The unstructured velocity field is interpolated from the measured points (unstructured) to the grid (see also section 4.4).

The 3D wake transition process is analysed considering the span-wise component $\omega_z$ of the vorticity vector $\vec{\omega}$. As the 2D analysis has shown, this component shows the development of the secondary structure. The development is elucidated considering a negative and a positive iso-vorticity surface within the 3D domain, where the negative vorticity contour is presented by a shaded surface and the positive vorticity by a meshed surface. In figure 7.7 the surfaces for $\omega_z = -0.25$ and $\omega_z = 0.15$ are presented. These surfaces represent clearly the primary upper and lower vortex structures and appear as 2D vortex tubes. Measuring noise possibly causes the observable span-wise variations. In a cross-section at $z/D = 1$ the primary upper and lower vortices can be clearly observed. The distribution is similar as the one observed in the 2D HiRes-PV results, although the latter results show much more details of the flow.

At $t/T = 2.3$, the iso-surfaces of $\omega_z = -0.1$ and $\omega_z = 0.25$ show that around $(x/D, y/D, z/D) = (1, 2, 1)$ a patch of fluid is growing which contains positive vorticity (fig. 7.7c). From the vorticity distribution at $z/D = 1$ it can be seen that this volume of positive vorticity is already linking with the negative vorticity of the primary structure resulting in the dipole like secondary structure (fig. 7.7d). This dipole structure then escapes from the primary upper vortex. This can be seen more clearly in the results at $t/T = 3$ (fig. 7.7e,f). Here a larger area of positive vorticity can be observed which forms a 3D linked structure with the strand of negative vorticity. The formed 3D-vortex ring structure escapes from the measuring domain. In the cross-section at $z/D = 1$ the vortex ring is observed as a dipole-like structure (fig. 7.7e). The $\omega_z$ distribution in the cross-section shows a strong similarity with figure 7.4f.

The 3D character of the flow becomes even more pronounced considering the cross-sections $z/D = 1, z/D = 2$ and $z/D = 4$ (fig. 7.8). The secondary structure is hardly visible at the cross-section at $z/D = 4$. Only a very small area of positive vorticity can be found at $(x/D, y/D) = (1, 2)$ and an upward stretched part of the upper vortex can be seen. In the cross-section at $z/D = 2$ the secondary structure can be observed already more clearly. Here a distinguished area of posi-
Figure 7.7: Development of the vorticity distribution: (a, c, e) 3D iso-vorticity contour surfaces, (b, d, f) iso-vorticity contours between $-1 < \omega_z < 1$, $\Delta \omega_z = 0.1$ in 2D slice at $z/D = 1$ for $Re_D = 75$ and $Re_D = 1.3$.
Figure 7.8: Three cross-sections at (a) $z/D = 4$, (b) $z/D = 2$ and $z/D = 1$ considered at $t/T \approx 2.7$ for $Ri_D = 1.3$ and $Re_D = 75$ ($x_c/D = -17.5$)

tive vorticity can be seen at $(x/D, y/D) = (1, 3)$ as well as a separate area of negative vorticity at $(x/D, y/D) = (2, 1)$. For $z/D = 1$ the secondary structure appears as a fully developed dipole structure.

A similar process was observed in all of the performed experiments for $Ri_D > 1$. Only the position at which the subsequent stages could be observed shows a dependency on $Ri_D$. For example for $Ri_D = 1.15$ (fig. 7.9) the escape of the secondary structure was observed to take place about $6D$ more downstream than for $Ri_D = 1.3$ (fig. 7.7e).

\section*{7.4 Discussion}

The results presented have shown that for $Ri_D > 1$ a 3D transition of the 2D vortex street is observed within the measurement area ($0 < x/D < 35$). The transition involves the following subsequent steps

- Deformation of the upper vortex structure.
- Appearance of a local area of positive vorticity, generated by baroclinic vorticity production. This area can be found within the negative upper vortex.
- Formation of a dipole-like secondary structure consisting of the area of positive vorticity and a part of the negative upper vortex
- Escape of the secondary vortex structure from the primary parental upper vortex.

In the first two steps the flow is observed to remain 2D. Therefore these processes can also be observed in the 2D simulations. The formation of the dipole (which turns out to be a vortex-ring-like structure in 3D) and its escape (step 3 and 4) is only observed in the experimental results. The 3D-PTV experiments as well as the 3D-dye visualisations have shown that these phenomena are 3D processes. Therefore, between step 2 and step 3 a transition to a 3D flow occurs. Within the present investigation as critical point the downstream position is taken at which the secondary
7.4. DISCUSSION

structure starts to accelerate in positive y-direction. This critical point shows a rather strong dependency on $Ri_D$. Considering the lower vortices, the results have shown that these vortices remain 2D during the entire transition process. Again the earlier observed rotation of the lower vortices around the upper ones can be observed as well as the strength difference. Furthermore, the numerical simulations have shown that the upper vortices become isolated warm patches of fluid which are, due to their strong structure circulation, isolated from the surrounding environment. The lower vortices on the other hand seem to be hardly affected by heat. Therefore buoyancy will have a more significant influence on the upper vortices than on the lower vortices.

The 3D visualisations show that the escape of a secondary structure takes place at distinguished span-wise positions. Although precise measurement results are not yet available, the present visualisation results show that at about every 4 to 5 $D$ span-wise location such a secondary structure can be observed. This span-wise distribution coincides with the critical span-wise wave number of the low Reynolds number instability for a forced convection ($Ri_D = 0$) as observed by Williamson (1996) and Barkley and Henderson (1996). Their investigations have shown that for $180 < Re_D < 259$ the flow is most sensitive for disturbances with a typical wavelength of $4D$. Although for the $Re_D$ considered in the present investigation the stability analysis of Barkley and Henderson (1996) predicts an unconditionally stable 2D flow field, it is remarkable to see that the span-wise distribution of the secondary structures have the same wavelength.
Chapter 8

Recommendations on experimental techniques

Further research on the 3D-transition process calls for accurate measurements of the 3D-velocity field and the 2D- and 3D-temperature distribution. The results of the 3D-PTV experiments suggest that the spatial resolution is relatively poor. To that end, in the next section some recommendations will be presented for further improvement of the 3D-PTV method. Measurement of the temperature distribution could also provide more information about the thermally induced effects responsible for the observed transition. Therefore, the development of a non-intrusive temperature measurement technique is initiated. This technique, referred to as Laser Induced Fluorescence (LIF), will be discussed in the second section.

8.1 3D PTV

The presented 3D-PTV results have shown that the number of 3D vectors found remains limited. Especially when the number of particles detected in the 2D images is considered, it appears that the vector yield is very low. Considering the cameras used (512 × 512 pixels) an estimated maximum of approximately 2500 particles are detected in each 2D image. About 80% of these particles remain within these 2D images during 5 subsequent frames (corresponding tracking length is 5). This implies that one can expect at most approximately 2000 3D-velocity vectors. From the results presented in figure 8.1 it is seen that this estimated maximum number is not reached. Only about 500 velocity vectors can be determined (based on a tracking length of 5), which is about 25% of the predicted maximum number.

Furthermore, the processing time was found to be long. The processing time scales with the square of the number of particles as found in the 2D images. Analysis of the algorithm revealed that most of the processing time is consumed in the 3D-localisation process. For a typical 3D-tracking run with about 2500 2D blobs in each image, a typical processing time on a PII-350 PC of 1000 s is needed to process one set of 2D images. In order to improve the performance (yield and processing time) some recommendations with respect to the hardware and algorithm are discussed.
8.1.1 Yield

The low 3D-velocity vector yield finds its roots basically in three sources:

- Accuracy and number of detected 2D blobs.
- Accuracy and efficiency of the 3D-particle localisation out of the 2D blobs.
- Efficiency of the tracking process.

2D blob detection

The accuracy of the detected 2D blobs can be increased by improving the quality of the camera images. Within the present investigation three analogue interlaced monochrome cameras were used with $512 \times 512$ pixels each. These cameras appear to have a noise level of about 5% causing the accuracy of the detected blob position to decrease. Furthermore, the accuracy of the 2D detected blob positions is also influenced by the relative low resolution of the cameras. For an increasing camera resolution the pixel density increases, consequently resulting in a more accurate representation of a particle image. More important for the accuracy is the interlacing of the cameras which causes a deformation of the particle image. This deformation depends on the velocity of the flow field and the sample rate of the camera. For a typical particle velocity of 5 pixels between 2 subsequent frames the deformation results in a particle position error which is estimated to be 2 camera pixels. This position error also influences the accuracy of the projection line. This can finally lead to an increasing number of erroneously localised 3D particles and consequently to a decreasing vector yield.
3D localisation

The performance tests showed that 3D-particle positions can be determined very accurately. However, in these tests all particles were positioned in a flat plane. Therefore no multiple-(accidental)-crossing possibilities of the projection lines were observed which could result in ambiguities for the 3D localisation (fig. 8.2a). In real fluid flow experiments these multiple crossings do occur. In the 3D-localisation algorithm used only the first found crossing possibility is used. Other crossing possibilities are not taken into account and this information is lost. During the particle matching it can turn out that an erroneous crossing is chosen. This will result in no matching with previous
located particles and therefore cause a decrease in the velocity yield. At this point, the algorithm can be improved by using all crossing possibilities for matching. The incorrect crossings then do not match with other particles and automatically drop out during the matching procedure.

The accuracy of the calibration data has a large influence on the vector yield. The number of ambiguities increases for decreasing calibration accuracy. A poor calibration accuracy calls for a large value of the minimum neighbouring distance $\Delta c$. This increases the chance on ambiguities and, within the present method, results in a decrease of the vector field accuracy. Also the angles between the optical axis of the cameras have a large influence on the number of ambiguities. For an increasing angle, the crossing-angles between the projection lines increase leading to more accurately determined line crossings. To that end, it is recommended to calibrate the domain with a camera set-up with $\alpha_1 = \alpha_2 = \alpha_3 \approx 90^\circ$. This recommendation asks for an adjusted calibration system where two cameras view the measuring volume from the side plane and one from above.

For a high seeding density, the possibility exists that two or more particles are located on one particle-projection line (hiding particles, fig. 8.2b). This results in only one particle image in the camera frame. Within the present algorithm, every 2D localised blob can only be used once, resulting in a loss of 3D particles. Allowing multiple usage of a 2D blob will overcome this problem and will consequently result in an increased number of 3D localised particles.

**Efficiency of the tracking algorithm**

In the present algorithm at every time step the 3D-particle locations are independently determined from the 2D detected blobs without considering information of previous time steps (fig. 8.3a). The independent determination in combination with the previously discussed sources of particle losses causes the average length of the particle paths (average value of fig. 8.1) to become relatively short. In order to improve this path length and the number of 3D localised particles use can be made of the previously determined particle positions (fig. 8.3b).

The suggested method starts with the detection of 3D particles as performed by the crossing-line method. In the second step, the 2D representations of the 3D particles are then marked in each 2D image. Because it is now known which set of 2D particles represents a 3D particle, tracking can be applied on the sequence of the 2D images. At each time step, the 3D representation can then easily be determined from the newly determined 2D-particle locations using the crossing-line method. Because matching in the 2D images is far more effective, and appears to have a tracking efficiency of about 75 %, the number of correctly 3D-localised particles will increase. Furthermore, the time consumed by blob detection and tracking of particles in the separate 2D images is an order lower than the present 3D-processing time, resulting in a smaller overall processing time. Of course, implementation of this recommendation is no sinecure and care has to be taken for particles which hide and appear again (and therefore temporally disappear in the 2D camera images).

The suggested procedure allows only tracking of the particles which are initially detected. New particles entering the measuring domain need to be detected separately. To that end, the 2D blobs which are already used as representation of a 3D particle need to be removed from each camera image. The remaining images can then be used to locate the new particles and to update the list of 3D localised particles.
8.2. TEMPERATURE MEASUREMENTS USING LIF

This proposed method is less time consuming than the presently used method. A disadvantage of this method is its sensitivity for errors in the initially determined particle positions. If 2D blobs are erroneously used for 3D particles, no mechanism is present to correct this. Therefore, after a certain number of steps (for example after every 5 time steps) the list of 3D localised particles need to be fully checked and updated.

8.1.2 Processing time

A further decrease of the processing time can be obtained by processing the images in subsequent steps. In the present method, all 2D blobs are detected at once and used for the 3D localisation. As the 3D-localisation time scales with the square of the number of particles $N_p$, one can better perform the 3D localisation in $n_s$ steps where in each step $N_p/n_s$ particles are considered. The processing time then scales with $N_p^2/n_s$ rather than with $N_p^2$. To perform this subdivision, the camera images can be divided in $n_s$ sub-parts.

8.2 Temperature measurements using LIF

At this moment, results of a non-intrusive measurement of a 2D-temperature plane are not often reported, especially not in the temperature range 20 °C-60 °C. Most of the methods used are limited in their applicability due to restrictions in the temperature range, accuracy and temperature gradients. The most common used technique is based on liquid crystals. Several researchers have reported quantitative measurements using liquid crystals (Dabiri and Gharib (1990), Dabiri and Gharib (1991) and Sillekens (1995)). The accuracy relative to the temperature range is not large.
and is found to be 10% (Sillekens (1995)).

A less common technique is based on Laser Induced Fluorescence which uses temperature sensitive fluorescent dyes excited by laser light (Sakakibara et al. (1993a), Sakakibara and Adrian (1997) and Nakajima et al. (1990)). A large advantage of LIF above liquid crystals is the higher relative accuracy, which is typically a few percent of the maximum temperature range (Sakakibara et al. (1993b)). The basic process of LIF is the natural fluorescence of molecules and atoms which is induced by absorption of photons. This absorption causes a transition from the ground state to an excited state. During the residence of the molecule in the exited state different processes cause a loss of energy. Therefore, the energy emitted during the transition back to the ground state decreases, consequently resulting in an increase of the wavelength of the emitted photon. Due to this shift in wavelength between absorbed and emitted light, the emitted photons are hardly re-absorbed. The observed fluorescence intensity $I$ can then be written as (Berlman (1971) and Bindhu et al. (1996)):

$$I = I_0 A \Phi_f C \frac{\lambda_e}{\lambda_f},$$

with $I_0$ the intensity of the excitation beam, $C$ the concentration of the fluorescent dye, $\epsilon$ the molar absorptivity, $A$ the fraction of the available light collected, $\lambda_e/\lambda_f$ the wavelength ratio accounting for the energy loss in the excited state and $\Phi_f$ the quantum yield. The quantum yield turns out to be temperature dependent, resulting in a temperature dependent fluorescence intensity. This dependence makes LIF useful for temperature measurements. For Rhodamine B, a decrease of fluorescence intensity $I$ of 2% - 3% per Kelvin is not unusual (Nakajima et al. (1990) and Sakakibara et al. (1993a)). This decrease is not wavelength dependent as can be concluded from the fluorescence spectra (fig. 8.2) which are obtained by using an Ar$^+$ laser and a spectrograph (SpectraPro 275). The spectra show that a higher temperature results in an overall decrease of the emission spectrum.

For temperature measurements using LIF most commonly a CW Ar$^+$ laser ($\lambda_e = 514.5$ nm) is used as excitation source. This laser guarantees a rather constant intensity $I_0$. Within the present investigation a Nd:YAG laser is used (Spectra-Physics GCR-150) operating at 532 nm. The laser produces a 6 ns pulse containing 200 mJ at a repetition frequency of 30 Hz. Due to this short pulse duration it is possible to acquire instantaneous images with a high signal to noise ratio (Karasso and Mungal (1997)). The high laser-power output allows to examine larger flow field areas than employing an Ar$^+$ laser. Furthermore, the absorption spectrum of Rhodamine B dissolved in water shows a maximum around 550 nm (fig. 8.2). This maximum fits better with the Nd-YAG laser ($\lambda_e = 532$ nm) than an Ar$^+$ laser (514.5 nm). As a result, for an equal excitation intensity an Nd:YAG laser will result in a higher fluorescence intensity (eq. 8.1). Furthermore, the influence of non-fluorescent tracers, which can act as quenchers (Nash et al. (1995)), is reduced by dissolving the fluorescent dye Rhodamine B ($M = 379.02$ gr/mol) in 5 $\mu$m filtered de-mineralised water. The typical concentration of the dye solution is determined by re-absorption effects which should be avoided as much as possible. The concentration is then chosen to be $10^{-5}$ mol/l. In the performance and accuracy analyses these re-absorption effects will be discussed more thoroughly.

To test whether this method is applicable for the phenomena studied in this research project, some preliminary experiments will be presented in which this technique was applied for the tem-
8.2. TEMPERATURE MEASUREMENTS USING LIF

8.2.1 Temperature sensitivity

In order to use LIF as a temperature measurement technique, the relation between the emission intensity $I$ and the temperature needs to be determined. Therefore a simple experimental set-up is used as shown in figure 8.5a. The laser sheet with a thickness of approximately 0.1 mm and a total height of 30 cm (power density $2 \times 10^5 \text{ W/m}^2$) is created using a positive lens and a cylinder lens. To filter the reflections of the laser light from the fluorescence signal, a long-wave pass filter (Schott Glass OG550) is used which is placed in front of the camera. This filter blocks light with a wavelength smaller than 550 nm almost completely and transmits the emitted fluorescence intensity (see fig. 8.2). The transmitted fluorescence signal is recorded using a CCD camera (Adimec MX12, 1024 x 1024 pixels, 8 bit, interlaced) and stored using an image grabber. Again, the camera synchronises the laser. However, due to the short pulse duration (6 ns) only the odd or even lines are illuminated as the camera is interlaced, which halves the vertical resolution.

The relation between temperature and fluorescence intensity is determined by heating the test-section to several accurately known temperatures (accuracy ±0.05 °C). The reference temperature is measured in the middle of the test section. Simultaneously, the fluorescence intensity $I$ as function of the temperature was measured in 5 different experiments, each containing 9 temperature settings. Rather than using the intensity $I$, a normalised intensity $I_n$ is used to determine the calibration function. For the normalisation of the intensity $I$ two reference intensities are taken, one at a high temperature $I_{T,\text{high}}$ ($\approx 60$ °C) and one at a low temperature $I_{T,\text{low}}$ ($\approx 20$ °C). Normali-
Figure 8.5: Measurement equipment: (a) general set-up, (b) cavity set-up

Figure 8.6: Relation between the fluorescence intensity and temperature
sation of the fluorescence intensity $I$ then reads

$$I_n = \frac{I - I_{T,\text{high}}}{I_{T,\text{low}} - I_{T,\text{high}}}.$$ (8.2)

By using the normalised intensity one compensates for non-uniformities in the laser sheet, geometric scaling parameters and inter-experimental variations.

The normalised intensity shows for all the experiments a decrease of the fluorescence intensity with temperature (fig. 8.6). A third-order polynomial is fitted to the experimental data, representing the calibration function ($\sigma = 0.4 ^\circ\text{C}$). The temperature of a recorded intensity field is now obtained by calculating the normalised local fluorescence intensity (eq. 8.2) from which the temperature can be determined using the calibration function.

### 8.2.2 2D-temperature measurement

In order to quantify the performance of method, the temperature field is measured in a thermally driven cavity flow. To that end, a test-section as shown in figure 8.5b is used, with the left wall set to a uniform temperature $T_h = \text{58.8} ^\circ\text{C}$ and the right wall to a temperature $T_c = \text{14.6} ^\circ\text{C}$. The corresponding Grashof number is then $4.5 \times 10^7$ (properties taken at the mean temperature of 36.7 $^\circ\text{C}$), implying that the flow is steady (Henkes (1990) and Le Quéré and Alziary de Roquefort (1985)). The temperature field of the natural convection flow is measured 20 hours after the walls are brought to the pre-set temperatures. This is long enough for the considered flow problem to evolve to a steady flow.

For the typical cavity dimensions, the flow in the interior of the cavity can be assumed to be 2D (Mallinson and de Vahl Davis (1977)). Therefore the measurements made in the symmetry plane can be compared with 2D flow simulations obtained with a Finite Element Method. For this simulation the domain is divided in $40 \times 20$ elements. As boundary conditions for the velocity the no-slip conditions are applied at the walls. For the temperature boundary conditions at the top and bottom wall homogeneous Neumann boundary conditions are used (normal derivatives are zero). At the vertical walls the dimensionless temperature $\Theta$ is set to $\Theta = 1$ at the hot wall and $\Theta = 0$ at the cold wall. Furthermore, the thermal expansion coefficient $\beta$ and the viscosity are assumed to be temperature dependent.

From comparison of the numerical and the experimental results in a cross-section at $x/B = 0.5$ it can be seen that the stratification, which can be observed in the interior of the domain, is measured quite well (fig. 8.7a). The results show that the temperature in the interior section can be measured with an accuracy of $\pm 0.8 ^\circ\text{C}$. However, close to the vertical boundaries a larger discrepancy between the numerically and experimentally determined temperature field is observed (fig. 8.7b). At the hot side (left) the measurement strongly overestimates the temperature, while at the cold side the temperature is underestimated. These differences are primarily caused by shadowgraph effects which are introduced when large temperature gradients occur. The latter will be explained more thoroughly at the end of the next section.
8.2.3 Accuracy analysis

The overall error of the measured temperature originates from several sources. These sources can be divided in three major groups:

- Disturbing effects on the measurements of fluorescence intensity $I$. Here one can distinguish the influence of physical phenomena as photo-bleaching or re-absorption effects and the sensitivity and accuracy of the recording equipment.

- Disturbing effects on the illumination intensity $I_0$ which is influenced by the variation in the laser output and by shadowgraph effects in the laser sheet.

- The accuracy of the calibration function.

In this section these three error sources will be investigated and discussed more thoroughly.

Effects on $I$

Considering the error sources in $I$, one can distinguish hardware induced errors and errors induced by physical sources. The main hardware induced error is the limited sensitivity of the camera in combination with its noise level. From the analysed images a pixel-to-pixel variation of about 0.5% can be observed caused by the camera noise. The influence of the noise is reduced by averaging in one frame over an area of $6 \times 6$ pixels. However, this method reduces the spatial resolution of the method. For a more sensitive camera the relative influence of the camera noise decreases. The present camera used is a 8 bits camera, having 256 discrete grey scales to record the intensity. Considering the dye used, which has a 3% intensity dependency per degree Celsius, a maximum of 8 grey scales intensity decrease can be recorded for 1 °C temperature increase (3% of 256). When it is assumed that noise affects only the last bit, the intensity shows a random variation of 2
grey scales. This is equivalent to a temperature variation of 0.25 °C. For a 16 bit camera (4096 grey scales) about 120 grey scales (3% of 4096) represent one degree temperature variation. Therefore, in that way, the maximum reachable accuracy can be improved with one order.

More difficult to overcome are the physical processes influencing the accuracy of $I$. The major processes are degradation of the dye and re-absorption of the photons after emission. The degradation of the dye, introduced by the high laser power output causes the fluorescence intensity to decrease during excitation (fig. 8.8). This decrease occurs due to chemical/photochemical decomposition and is also called photobleaching (Saylor (1995)). To minimise photobleaching effects, the excitation time should be minimised during an experiment. As a rule of thumb one can state that this effect remains limited (less than 1%) for excitation times smaller than one minute. Also a correction can be made for the photobleach effects using the measured relation between the fluorescence decay and exposure time (fig. 8.8).

Re-absorption of a photon is caused by an overlap between the absorption and emission spectra (fig. 8.2). This overlap results in a partial absorption of the fluorescence signal, also called ’self-absorption’. This latter effect depends on the local dye concentration (Lemoine et al. (1996)). Part of this absorbed fluorescence signal is re-emitted. This re-emission is dependent on the temperature and the observed intensity $I$ will be a mixture of more processes. For the used concentration ($10^{-5}$ mol/l) and a small optical path of the emitted light through the test-section, this effect can be neglected. Furthermore, the re-absorption occurring in the iso-thermal situation is compensated for by normalisation of the intensity $I$.

Effects on $I_0$

The accuracy in $I_0$ is a result of the laser-output variation and shadowgraph effects. Considering the laser output variations the Nd-YAG laser appears to have a pulse-to-pulse power variation and a long-term drift of the laser output. Consequently, the measured fluorescence intensity will
respond to these variations linearly (eq. 8.1). The pulse-to-pulse variation is found to be about 3% of the set intensity $I_0$ (fig. 8.9a). To compensate for this pulse-to-pulse power variations the measured intensity $I$ is averaged over 30 images. The influence of the pulse-to-pulse variation in $I_0$ is thereby reduced to 0.55% ($\sqrt{30}$). This averaging procedure causes of course a reduction of the temporal resolution. A further reduction of this error and an increase of the temporal resolution can be obtained by performing very accurate pulse-to-pulse measurements of the laser output. However, this demands an expensive measuring-device. A second and less expensive correction method is based on the assumption that within a small time interval (about 1 s) the average temperature in the interrogation area does not vary. Furthermore, it is assumed that within this time interval the long-term variation of the laser output can be neglected. The variation of the frame-averaged fluorescence intensity can then only be caused by the pulse-to-pulse power variation. Information of the frame-to-frame variation in the frame-averaged intensity can then be used to correct for the pulse-to-pulse variation. The long term variations of $I_0$ (fig. 8.9a) are more easy to correct for. For this purpose, the laser output is measured (integrated over 1 s) using a beam-splitter.

The second perturbing phenomenon on $I_0$ is the so-called shadowgraph effect. Due to this effect, darker (shadows) and brighter streaks can be observed within the illumination plane (fig. 8.10). These shadowgraph effects are created by the local temperature variations, which result in a spatially varying refractive index $n(x, y, z)$. The spatial variation of the refractive index then results in a so-called thermal lens effect. This effect refracts the rays with excitation intensity $I_0$ resulting in the bright and dark streaks. Therefore, locally the normalised intensity can be far more larger than 1 or smaller than 0. For these intensity values the fitted 3rd-order calibration function is not valid, resulting in a large error of the measured temperature.
8.2. TEMPERATURE MEASUREMENTS USING LIF

In order to correct for this shadowgraph effect, the paths \(x_r, y_r, z_r\) of the initially parallel light rays through the medium with the varying refractive index \(n(x, y, z)\) can be considered which can be written as (Merzkirch (1974));

\[
\frac{\partial^2 z_r}{\partial x_r^2} = [1 + \left(\frac{dz_r}{dx_r}\right)^2 + \left(\frac{dy_r}{dx_r}\right)^2] \left\{ \frac{1}{n(x_r, y_r, z_r)} \frac{\partial n(x_r, y_r, z_r)}{\partial z_r} \right\} - \frac{dz_r}{dx_r} \frac{1}{n(x_r, y_r, z_r)} \frac{\partial n(x_r, y_r, z_r)}{\partial x_r},
\]
\[
\frac{\partial^2 y_r}{\partial x_r^2} = [1 + \left(\frac{dz_r}{dx_r}\right)^2 + \left(\frac{dy_r}{dx_r}\right)^2] \left\{ \frac{1}{n(x, y, z)} \frac{\partial n(x, y, z)}{\partial y_r} \right\} - \frac{dy_r}{dx_r} \frac{1}{n(x_r, y_r, z_r)} \frac{\partial n(x_r, y_r, z_r)}{\partial x_r}. \tag{8.3}
\]

For the boundary conditions one can take \(dz_r/dx_r = dy_r/dx_r = 0\) and known path positions \((x_r, y_r, z_r)\) at the entrance. The intensity \(I^*\) in a point \((x, y, z)\) then results from all light rays mapped on this point according to equation 8.3. To do so, the refractive index needs to be written explicitly as function of \(x, y, z\). The calculation has to be carried out numerically. Therefore, the thermal sensitivity of the refractive index \(n(x, y, z)\) in water (with \(T\) the temperature in Celsius) is used which can be written as (Washburn (1930))

\[
n(T) = n(\lambda_e, 20^\circ) - 10^{-5}(0.12(T - 20) + 0.2(T^2 - 400) - 5 \cdot 10^{-5}(T^4 - 1.6 \cdot 10^4)),
\]
\[
n(\lambda_e) = \sqrt{1.76 - 1.31 \cdot 10^{-2}(\lambda_m)^2 + \frac{6.5 \cdot 10^{-2}}{\lambda_m^2 - 0.115^2}},
\]
\[
\lambda_m = \lambda_e 10^6, \tag{8.4}
\]

with \(T = T(x, y, z)\).

A small but important artefact of the above discussed correction method is that it is based on an a-priori-known temperature field \(T(x, y, z)\). Therefore the correction method is not directly applicable in a real experiment. However, correction for shadowgraph effects can be obtained by applying an iterative procedure. The first correction is made on the temperature field which follows directly from the measured intensity field. On basis of the first correction as described above, a ‘new’ temperature field can be deduced. Again a correction of the temperature field can
be calculated using the newly derived temperature field. The sequence can be proceeded until the temperature field converges. A more elegant approach would be an in-situ correction. This can be obtained by using two fluorescent dyes simultaneously with one dye that is not sensitive for temperature variations. This method is already frequently used by other researchers (see for example Sakakibara and Adrian (1997) and Sakakibara et al. (1993a)).

Shadowgraph effects also influence the emitted fluorescence signal. The temperature field between the illumination plane and the camera is then responsible for this effect. In practice the introduced error will be of minor influence and at least a few orders smaller than the earlier discussed shadowgraph effects.

**Accuracy in calibration function**

The intensity variation as function of temperature turns out to be only 3 percent per degree. Therefore errors in the calibration function will result in a large error of the finally calculated temperature. In the results presented here, the calibration function introduces an uncertainty of $\pm 0.5 \, ^\circ C$. Applying more measurements to determine the calibration curve can decrease this uncertainty. Furthermore, by recording the intensity with a more sensitive camera, for example a 12 bits digital camera, the accuracy of the calibration function can also be improved.

### 8.2.4 Conclusion

From this preliminary investigation it can be concluded that using the LIF method indeed reliable temperature measurements in water can be performed. However, shadowgraph effects restrict the applicability of the measurement method and they are responsible for the largest error source. Using a single Nd-YAG laser with only one dye, the 2D-temperature distribution can be measured as long as the temperature gradients are not too large. For increasing gradients, a correction needs to be applied for shadowgraph effects. This correction can be done in an iterative procedure. At this moment the convergence of this iteration method is not known. A more straight-forward approach would be an in-situ correction as it is carried out by using two fluorescent dyes. This demands for two camera systems, preferably 12 bits. By using then two lasers with a distinguishable difference in wavelength $\lambda_c$, a 2D or even 3D-temperature measurement method can be developed with a typical accuracy of $\pm 0.1 \, ^\circ C$. 
Chapter 9

Concluding remarks

In the present study the behavior of the wake behind a heated horizontal cylinder exposed to a horizontal cross flow is investigated. For this investigation, both experiments and numerical simulations were carried out for $Re_D = 75$ and for varying heat input ($0 < Ri_D < 1.8$). It was found that for a relative low heat input ($0 < Ri_D < 1$) the cylinder wake remains 2D and exists of two rows of oppositely rotating vortices. During the downstream convection, the vortices in the wake deflect in negative $y$-direction relatively to the forced convection situation ($Ri_D = 0$). Furthermore, between the vortex structures a relative movement is observed which manifests itself as a rotation of a lower vortex around an upper one. This relative movement increases for increasing $Ri_D$. A similar deflection and relative movement are also found in the point-vortex simulations when a small difference in strength between the upper and lower vortices is prescribed. Analyses of the experimentally and numerically determined vortex strengths show indeed a difference in strength between the upper and lower vortices. The deflection of the vortex rows reaches a maximum for $Ri_D = 0.5$. Although the strength difference between the upper and lower vortices increases, a larger $Ri_D$ value results in a smaller deflection. This decreasing deflection is assumed to be caused by the increasing influence of the upward directed buoyant force.

From the structure strength as function of the downstream position (in terms of vorticity extreme and circulation of the structure) it is found that the strength difference originates during the shedding and formation of the vortex structures. Detailed analyses of the structure shedding and formation process show that close to the cylinder ($-1 < x/D < 1$) the net produced vorticity is positive, which is in contradiction to the found structure strength difference for $x/D > 8$. This difference originates in the region between $1 < x/D < 4$. Here, the baroclinic vorticity production, coinciding with the vortex structures, is mainly negative, resulting in stronger upper vortices and weaker lower vortices. Furthermore, due to the heat addition an asymmetry arises in the vortex formation process leading to an increasing influence of strain rate on the formation of a lower vortex. Consequently the formation of a lower structure is less effective resulting in a weaker vortex structure. The asymmetry originates from the changing flow profile around the cylinder for increasing $Ri_D$. The numerical results also show that advection of vorticity and convection of heat are comparable. Therefore, the shed vortex structures become areas with an increased temperature compared to their environment. Heat is convected downstream as hot blobs, captured within the vortex structures. The latter effect turns out to be stronger for the upper vortices, resulting in
the upper vortices being warmer than the lower vortices. This difference in temperature increases for increasing $Ri_D$.

For $Ri_D > 1$, an early collapse of the vortex street is observed to take place within the area of interrogation ($x/D < 35$). Three-dimensional visualisation results reveal that from an upper vortex a secondary structure arises. These structures can only be observed at certain span-wise locations and appear with a mutual spacing of about $4 - 5D$. The structures develop further into mushroom-shaped vortex structures which escape out of the primary upper vortex. The shape of the escaping structure indicates that heat-induced buoyancy effects are responsible for the formation. Detailed measurements of the velocity components $u$ and $v$ of the velocity field have shown that the development of the structure takes place in four separate stages. In the first stage the upper vortex deforms, resulting in an upward stretched strand of negative vorticity. In the second stage, an area of positive vorticity appears within the negative primary structure. In the third stage, this area develops further and forms, together with a part of the upward stretched parental-upper vortex a vortex-ring like structure which appears in the 2D HiRes-PV results as a dipole-type structure. In the last stage, the vortex-ring structure accelerates and escapes from the primary upper vortex. From the comparison of the measurements with 2D simulations it appears that the first two steps in the process are 2D and that baroclinic vorticity production is responsible for the occurrence of the positive vorticity area. The formation and consequently the escape of the dipole structure are not observed in the simulations and are therefore assumed to be 3D processes. The position at which this secondary structure escapes, depends on $Ri_D$. For increasing $Ri_D$ the 3D transition takes place closer to the cylinder. Considering the lower vortex, the results have shown that it remains stable and 2D during the last two stages. Also, from the numerical results it is observed that the temperature of the lower vortex hardly differs from the ambient temperature, while the upper vortex temperature is considerably higher. Therefore the upper vortices are affected more strongly by buoyant effects than the lower vortices.

Within the present investigation three new measuring techniques were developed. From the performance tests and the presented results it is concluded that the 2D HiRes-PV technique allows the measurement of the flow field with a high spatial resolution. In each experiment it was possible to deduce a vector field consisting of 7000 to 10000 vectors. The error in the measured velocity is found to be less than $0.1 \text{ mm/s}$. The remaining error is mainly caused by the particle localisation procedure. In conclusion, it can be stated that the HiRes-PV technique is an excellent technique to investigate transitional flow phenomena.

Considering the 3D-PTV results, it can be concluded that with the present method it is possible to measure 3D-flow features and that the results are promising. Especially the accuracy of the particle location turns out to be high. Only the spatial resolution remains relatively poor. In most of the experiments performed, about 500 velocity vectors could be deduced per 3D-interrogation volume. This shortcoming of the method needs to be resolved before it can give really valuable data for transitional processes. Several suggestions to improve the method are presented in chapter 7.

For the measurement of the 2D-temperature distribution a non-intrusive technique was introduced. The technique, referred to as Laser Induced Fluorescence (LIF), uses the temperature sen-
sitivity of Rhodamine B dissolved in water between 20 °C and 60 °C. From the presented results it can be concluded that a fluorescence intensity decrease of 2% per °C can be measured. With the currently used camera (Kodak EOS 1.0, 1024 × 1024 pixels, 8 bits) a maximum accuracy of ±0.5 °C can be achieved. By using a more sensitive camera (12 or 16 bits) or image amplifier this accuracy can be improved up to ±0.1 °C. This accuracy can only be achieved if temperature gradients perpendicular to the illumination light-beam direction remain small. For large gradients shadowgraph effects will introduce a significant measuring-error. These shadowgraph effects can be reduced by applying an iterative correction procedure or by using an in-situ correction. The latter could be performed by using two fluorescent dyes, only one of which responds to the temperature variations.

Although the results as presented here show the basic features of the development into a 3D wake, quite some information is missing for a complete understanding of the transition process. Considering for example the critical point $x_{cr}$ at which transition takes place, additional experiments need to be carried out in order to establish the dependency of $x_{cr}$ on $Ri_D$ accurately. For this purpose, one has to define an easily measurable characteristic property of this transition. Furthermore, it is observed that the transition to a 3D state takes place at a certain downstream location. The spanwise distribution was roughly estimated to be about $4 - 5D$. This estimate needs to be validated more extensively in order to define the most sensitive span-wise wave number for transition. These results can be obtained by performing more 3D-PTV experiments using the improved 3D-PTV algorithm as well as detailed visualisation experiments.

The sensitivity of the flow to span-wise disturbances can also be investigated by performing 3D Direct Numerical Simulations. These investigations can be carried out by means of a 3D Floquet stability analysis as done for the forced convection wake transition by Barkley and Henderson (1996). Although the 3D solver has already been developed, a further improvement is advised with respect to memory usage and processing time.

It appears that the transition is a strong interaction between vorticity induced effects and buoyancy effects. The influence of the vorticity induced effects is hardly investigated. It appears that vortex capturing effects delay the escape of the thermal energy. Therefore it is expected that the $Pr$ number has a strong influence on the observed phenomena. Furthermore, most of the structure characteristics originate during the vortex formation and shedding process. This process depends on the Reynolds number. Therefore, in order to understand the phenomena in more detail, the investigations should be continued with a focus on a wider range of Reynolds numbers. This also allows one to investigate the influence of buoyancy on the transition to a 3D wake as is observed for the forced convection situation (Williamson (1996)).


Appendix A

Explicit 3rd order Taylor-Galerkin scheme

Integration of the convection equations is performed with an explicit 3rd-order integration scheme, which for the momentum equation reads:

\[
\tilde{u}^{m+\frac{1}{3}} = \tilde{u}^m + \frac{\Delta s}{3} C_{\tilde{u}}^m \cdot \tilde{u}^m \\
\tilde{u}^{m+\frac{1}{3}} = \tilde{u}^m + \frac{\Delta s}{2} C_{\tilde{u}}^{m+\frac{1}{3}} \cdot \tilde{u}^{m+\frac{1}{3}} \\
\tilde{u}^{m+1} = \tilde{u}^m + \Delta s C_{\tilde{u}}^{m+\frac{1}{3}} \cdot \tilde{u}^{m+\frac{1}{3}},
\]

and for the energy equation reads:

\[
\tilde{\Theta}^{m+\frac{1}{3}} = \tilde{\Theta}^m + \frac{\Delta s}{3} C_{\tilde{\Theta}}^m \Theta^m \\
\tilde{\Theta}^{m+\frac{1}{3}} = \tilde{\Theta}^m + \frac{\Delta s}{2} C_{\tilde{\Theta}}^{m+\frac{1}{3}} \Theta^{m+\frac{1}{3}} \\
\tilde{\Theta}^{m+1} = \tilde{\Theta}^m + \Delta s C_{\tilde{\Theta}}^{m+\frac{1}{3}} \Theta^{m+\frac{1}{3}},
\]

\[
\tilde{\Theta}^0 = \Theta^n \rightarrow \tilde{\Theta}^{m_{tot}} = \tilde{\Theta}^n \quad (A.2)
\]

with \(\Delta s\) the explicit time step, \(C_{\tilde{u}}\) and \(C_{\tilde{\Theta}}\) the convection matrices for the velocity and temperature, \(m\) the counter for the intermediate explicit time levels and \(m_{tot}\) the total number of explicit time steps within an implicit time step \(\Delta t\).
Appendix B

Grid refinement

The influence of the grid spacing is investigated for three different meshes. Grid 3 is the coarsest grid on which local grid refinement is applied in the cylinder wake and close around the cylinder. The finest grid used is grid 1. Within each individual grid cell, a 9th-order approximation polynomial is defined.

Figure B.1: Grid 1, 41238 nodal points, 504 elements and a 9th order approximation polynomial
Figure B.2: Grid 2, 26910 nodal points, 328 elements and a 9th order approximation polynomial

Figure B.3: Grid 3, 14031 nodal points, 170 elements and a 9th order approximation polynomial
Appendix C

Vortex trajectory variation

The experimental results show a variation of about $\pm 0.3D$ in the vortex trajectories. To understand this variation, the position of the vortex row is analyzed. For the experiments with the strongest variations ($Re_D = 0$ and $Re_D = 0.5$) the vortex row positions show a long-term variation with an amplitude equivalent to the observed trajectory variation (figs. C.1 and C.2). For $Re_D = 1$, neither this long term variation nor the variation in the trajectories were observed (fig. C.3). The explanation for this observation is still unclear.

![Figure C.1: Temporal variation of the vortex row positions at $x/D = 20$ for $Re_D = 75$ and $Re_D = 0$](image-url)

Figure C.1: Temporal variation of the vortex row positions at $x/D = 20$ for $Re_D = 75$ and $Re_D = 0$
Figure C.2: Temporal variation of the vortex row positions at \( x/D = 20 \) for \( Re_D = 75 \) and \( Ri_D = 0.5 \)

Figure C.3: Temporal variation of the vortex row positions at \( x/D = 20 \) for \( Re_D = 75 \) and \( Ri_D = 1 \)
Appendix D

Vortex strength

The measured vortex strength is presented in the main text as the averaged fitted strength as function of the downstream position $x$. Here, the raw data are presented showing a certain variation around this averaged strength, which is approximately $\pm 0.1$.

Figure D.1: Scatter plot of the circulation $\Gamma$ as function of the downstream position $x$ for $Re_D = 75$: (a) $Re_D = 0$ and (b) $Re_D = 1$

The vorticity extremes are directly determined from the vorticity field. Therefore, the measuring noise has a stronger influence on the vortex extreme than on the calculated circulation (which is an integrated quantity). The presented scatter plots of the vortex extremes show indeed that the variation is about twice as strong as compared to the calculated circulation (fig D.2).
Figure D.2: Scatter plot of the vorticity extremes as function of the downstream position $x$ for $Re_D = 75$: (a) $Ri_D = 0$ and (b) $Ri_D = 1$
Nomenclature

\( A \) \quad \text{area [m}^2]\)
\( \mathcal{A} \) \quad \text{Aspect ratio of the cylinder (}l/D\text{)}
\( a \) \quad \text{stream wise distance between the vortices in a row [m]}
\( B_1 \) \quad \text{numerical inflow length [m]}
\( B_2 \) \quad \text{numerical wake length [m]}
\( b \) \quad \text{width of the vortex street [m]}
\( C \) \quad \text{convection matrix}
\( C \) \quad \text{concentration [mol/l]}
\( \mathcal{C} \) \quad \text{convection operator}
\( D \) \quad \text{cylinder diameter [m]}
\( D_u \) \quad \text{diffusion matrix}
\( D_{\bar{u},\Theta} \) \quad \text{diffusion operator for} \( \bar{u} \) and \( \Theta \)
\( d_n \) \quad \text{mean minimum particle distance [m]}
\( d_v \) \quad \text{mean minimum vector distance [m]}
\( F \) \quad \text{preconditioning matrix}
\( f \) \quad \text{vortex shedding frequency [1/s]}
\( f^* \) \quad \text{force vector [kgm/s}^2\text{]} \text{ or dimensionless force vector []}$
\( g \) \quad \text{gravity acceleration [m/s}^2\text{]}
\( \bar{g} \) \quad \text{gravity vector [m/s}^2\text{]} \text{ or dimensionless gravity vector []}$
\( Gr_D \) \quad \text{Grashof number} \quad \left(= \frac{\beta g (T_H - T_C) D^3}{\nu^2}\right)
\( H \) \quad \text{weighting window [m]}
\( h \) \quad \text{cylinder depth [m]}
\( I \) \quad \text{fluorescence intensity [W/m}^3\text{]}
\( I_0 \) \quad \text{emission intensity [W/m}^3\text{]}
\( I_n \) \quad \text{normalised intensity []}$
\( I_{max} \) \quad \text{maximum intensity[W/m}^3\text{]}
\( I_{T,low} \) \quad \text{fluorescence intensity at low temperature [W/m}^3\text{]}
\( I_{T,high} \) \quad \text{fluorescence intensity at high temperature [W/m}^3\text{]}
\( I^* \) \quad \text{disturbed intensity [W/m}^3\text{]}
\( J \) \quad \text{transformation Jacobian}
\( K \) \quad \text{Laplacian}
\( \mathcal{K} \) \quad \text{wave number of the solution []}$
\( L \) \quad \text{domain height [m]}
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l$</td>
<td>Cylinder length [m]</td>
</tr>
<tr>
<td>$L_d$</td>
<td>divergence matrix</td>
</tr>
<tr>
<td>$L_f$</td>
<td>formation length [m]</td>
</tr>
<tr>
<td>$L_r$</td>
<td>recirculation length [m]</td>
</tr>
<tr>
<td>$L_{no}$</td>
<td>order of the Legendre polynomial</td>
</tr>
<tr>
<td>$M$</td>
<td>mass matrix</td>
</tr>
<tr>
<td>$N$</td>
<td>number of integration points</td>
</tr>
<tr>
<td>$n_v$</td>
<td>number of point vortices</td>
</tr>
<tr>
<td>$n(x,y,z)$</td>
<td>position dependent refractive index</td>
</tr>
<tr>
<td>$n_{el}$</td>
<td>number of elements</td>
</tr>
<tr>
<td>$n_o$</td>
<td>order of the approximation polynomial</td>
</tr>
<tr>
<td>$N_{el}$</td>
<td>number of nodal points in an element</td>
</tr>
<tr>
<td>$n$</td>
<td>normal direction</td>
</tr>
<tr>
<td>$p$</td>
<td>pressure [kg/ms^2] or dimensionless pressure []</td>
</tr>
<tr>
<td>$p_h = \rho g h$</td>
<td>hydrostatic pressure [kg/ms^2]</td>
</tr>
<tr>
<td>$Pr$</td>
<td>Prandtl number [= $\nu/\kappa$]</td>
</tr>
<tr>
<td>$q$</td>
<td>pressure correction term</td>
</tr>
<tr>
<td>$q_i$</td>
<td>image quality []</td>
</tr>
<tr>
<td>$q_s$</td>
<td>sampling quality []</td>
</tr>
<tr>
<td>$Q(t^*, t)$</td>
<td>integration operator</td>
</tr>
<tr>
<td>$Q$</td>
<td>strain rate squared minus vorticity squared [1/s^2]</td>
</tr>
<tr>
<td>$Re_D$</td>
<td>Reynolds number [= $U_0 D/\nu$]</td>
</tr>
<tr>
<td>$Ri_D$</td>
<td>Richardson number [= $Gr_D/Re_D^2 = \beta g D \Delta T/U_0^2$]</td>
</tr>
<tr>
<td>$r$</td>
<td>distance from a point vortex [m]</td>
</tr>
<tr>
<td>$S_{p1}$</td>
<td>upper separation point [m]</td>
</tr>
<tr>
<td>$S_{p2}$</td>
<td>lower separation point [m]</td>
</tr>
<tr>
<td>$S_t$</td>
<td>Strouhal number [= $fD/U_0$]</td>
</tr>
<tr>
<td>$S_{t1}$</td>
<td>upstream stagnation point [m]</td>
</tr>
<tr>
<td>$S_{t2}$</td>
<td>downstream stagnation point [m]</td>
</tr>
<tr>
<td>$S_1$</td>
<td>normal strain rate [1/s]</td>
</tr>
<tr>
<td>$S_2$</td>
<td>shear strain rate [1/s]</td>
</tr>
<tr>
<td>$t$</td>
<td>time [s]</td>
</tr>
<tr>
<td>$T$</td>
<td>shedding period [s]</td>
</tr>
<tr>
<td>$T$</td>
<td>temperature [K] or dimensionless temperature []</td>
</tr>
<tr>
<td>$T_H$</td>
<td>cylinder wall temperature [°C]</td>
</tr>
<tr>
<td>$T_C$</td>
<td>free stream temperature [°C]</td>
</tr>
<tr>
<td>$U_0$</td>
<td>free stream velocity [m/s]</td>
</tr>
<tr>
<td>$U_x$</td>
<td>interpolated x-velocity [m/s]</td>
</tr>
<tr>
<td>$U_y$</td>
<td>interpolated y-velocity [m/s]</td>
</tr>
<tr>
<td>$\vec{u} = [u_1, u_2, u_3]$</td>
<td>velocity vector [m/s] or dimensionless velocity vector []</td>
</tr>
<tr>
<td>$\vec{\tilde{u}} = [\tilde{u}_1, \tilde{u}_2, \tilde{u}_3]$</td>
<td>dimensionless convected velocity []</td>
</tr>
<tr>
<td>$\vec{u}_g = [u_g, v_g]$</td>
<td>point-vortex group velocity vector [m/s]</td>
</tr>
</tbody>
</table>
\( \bar{u}_{sl} = [u_{sl}, v_{sl}] \)  velocity in the shear layer [m/s]

\( \bar{u}_w = [u_w, v_w] \)  velocity in the cylinder wake [m/s]

\( \vec{x} = [x, y, z] \)  position vector [m] or dimensionless position vector []

\( x_c \)  x-position cylinder [m]

\( x_{cr} \)  critical x-position [m]

\( X \)  x-position vortex [m]

\( Y \)  y-position vortex [m]

\( X_u \)  x-position upper vortex [m]

\( Y_u \)  y-position upper vortex [m]

\( w \)  weighting function

**Greek symbols**

\( \alpha_i \)  viewing angle camera i

\( \alpha_j \)  weighting parameter []

\( \beta \)  thermal expansion coefficient [1/°C]

\( \beta_i \)  numerical coefficients of the BD scheme

\( \delta u_i \)  vortex induced x-velocity on vortex \( i \) [m/s]

\( \delta v_i \)  vortex induced y-velocity on vortex \( i \) [m/s]

\( \Delta \Gamma \)  heat induced disturbances on vortex strength [m/s]

\( \Delta s_{max} \)  maximum matching distance [m]

\( \Delta t \)  time step [s]

\( \Delta x \)  x-grid spacing [m]

\( \Delta y \)  y-grid spacing [m]

\( \Delta Y \)  y-deflection of the vortex street [m]

\( \Delta |\Gamma| \)  absolute difference in circulation, lower minus upper vortex [m²/s]

\( \Delta |\omega_{max}| \)  absolute extreme value difference, lower minus upper vortex [1/s]

\( \eta \)  dynamic viscosity [kg/ms ]

\( \varepsilon \)  numerical error

\( \epsilon \)  molar absorptivity [m²/mol]

\( \gamma \)  fractional velocity yield []

\( \gamma_n \)  relaxation parameter for preconditioning

\( \Gamma \)  circulation or vortex strength [m²/s]

\( \Gamma_{prod} \)  instantaneous circulation production rate [m²/s²]

\( \kappa \)  thermal diffusity [W/m°C]

\( \lambda_e \)  excitation wave length [m]

\( \lambda_f \)  fluorescence wave length [m]

\( \phi \)  angle measured from upstream cylinder side

\( \Phi_I \)  quantum yield

\( \Phi_{\omega_s} \)  period-averaged flux of span-wise vorticity [m/s]

\( \nu \)  kinematic viscosity [m²/s²]

\( \rho \)  density [kg/m³]

\( \sigma \)  stress tensor or component [kg/ms²]

\( \sigma_{u,v} \)  standard variation in the measured velocity components u and v [m/s]
ψ  basis function of the interpolant
Ψ  approximation function
\( \tilde{\omega} = [\omega_x, \omega_y, \omega_z] \) vorticity vector [1/s]
\( \omega_{\text{ext}} \) local vorticity extreme value [1/s]
\( \omega_a \) iso-vorticity value defining a vortex [1/s]
\( \omega_{\text{max}} \) vorticity peak [1/s]
Ω  considered domain
\( \Omega_{\omega_s} \) integrated vorticity flux [m²/s²]
Θ  dimensionless temperature difference \([= (T - T_C)/(T_H - T_C)]\)

**Subscript**

\( c \) correction term
\( e \) element level
\( gl \) Gauss-Lobatto
\( h \) approximated solution
\( i \) counter
\( j \) counter
\( l \) lower vortex
\( k \) counter
\( 0 \) reference state
\( p \) prediction terms or counter
\( q \) counter
\( r \) light ray or counter
\( u \) upper vortex
\( - \) column notation

**Superscript**

\( * \) dimensionless quantity
\( k \) pointer in iterative procedure
\( n \) time position
Summary

For many engineering and environmental processes, heat transfer is strongly influenced by the presence of coherent fluid structures. The stability and behaviour of these structures for example, highly influence the thermal mixing properties of a flow field. The behaviour and stability of the structures may be, on their turn, influenced by addition of heat. In this investigation the effect of heat addition on coherent vortex structures is analysed by considering the flow behind a heated horizontal cylinder. By exposing the cylinder to a forced uniform horizontal cross flow, structures are formed behind the cylinder which, after they are shed, determine the wake characteristics. The effect of the induced heat on these wake structures is investigated for a relatively low Reynolds number ($Re_D \approx 75$). For the unheated situation a laminar 2D periodic flow field occurs with two stable rows of vortices which form a vortex street. The effect of heat on this stable configuration is investigated for a Richardson number between $Ri_D = 0$ and 2 where $Ri_D$ denotes the relative importance of the induced heat with respect to the cooling flow.

For this investigation a water tank facility has been designed in which it is possible to apply various measurement techniques. The main advantages of this facility are the absence of free stream turbulence and boundary layers. Early visualisation results revealed that for a relatively small heat input the trajectories of the shed vortex structures were directed downwards. This behaviour is also found in detailed Particle Tracking experiments using a High Resolution Particle Tracking routine (HiRes-PV). From these particle tracking results it turns out that for increasing heat input a strength difference occurs between the upper vortex row and the lower one. From a point vortex model it is deduced that the vortex strength difference is responsible for the negative deflection of the trajectories.

The deflection of the vortex street and the source of the strength difference is further investigated by performing numerical simulations. A Spectral Element Method is applied, a numerical method that has a spectral convergence rate and which is capable to deal with complex geometries. From the numerical results it turns out that temperature driven vorticity production is responsible for the occurrence of a strength difference between the upper and lower vortex rows. Besides, due to the heat addition, a difference between the formation of an upper and a lower vortex is observed.

By increasing the Richardson number, visualisation experiments show that the vortex street becomes unstable and collapses. This early transition of the vortex street is initiated by the development of a small additional vortex structure inside the upper vortex structures. The detailed 2D experiments and simulations reveal that this additional structure is formed by the temperature driven vorticity production. As soon as such a structure is formed, it accelerates perpendicularly
to the main flow direction and develops into a 3D thermal-mushroom type structure. The 3D character of this structure is verified by performing 3D-visualisation as well as 3D-PTV experiments. Results from these experiments show that on several span-wise locations such additional structures arise and are responsible for the early collapse of the vortex street.
Samenvatting

In veel industriële en geofysische processen wordt de overdracht van warmte beïnvloed door coherente structuren. Het gedrag, maar vooral ook de stabilité van deze structuren, bepaalt in sterke mate het thermische menggedrag van de stroming. De overgedragen warmte beïnvloedt op haar beurt weer het gedrag van de wervelstructuren. In het gepresenteerde onderzoek is deze wisselwerking onderzocht voor de stroming rond en achter een verwarmde cilinder. Hierbij is de warme cilinder horizontaal geplaatst in een eveneens horizontale koude aanstroming. Vanwege de aanstroming zullen achter de cilinder wervelstructuren worden geformd en afgeschud welke de karakteristieken van het zog bepalen. Voor de isotherme situatie en gekozen aanstroomsnelheid (gekarakteriseerd door het Reynolds getal $Re_D$, $Re_D = 75$) zal een 2D zog, bestaande uit periodiek afgeschudde wervelstructuren, ontstaan. Deze wervelstructuren vormen samen een stabiele straat van gelinkte wervels, de zogenaamde von Kármán wervelstraat. Het effect van warmte op deze structuren en het resulterende zoggedrag is vervolgens onderzocht voor $0 < Ri_D < 2$. Hierin is $Ri_D$ het Richardson getal welke de verhouding van warmttetoever ten opzichte van de geforceerde koeling uitdrukt.

Voor dit onderzoek is een sleeptank ontworpen. De voordelen van deze sleeptank zijn een lage vrije turbulentiegraad en nagenoeg geen grenslagen aan de testsectiewanden. Gedurende het onderzoek zijn diverse experimentele technieken toegepast. Zo is met behulp van bellenvisualisatie geconstateerd dat voor een geringe toevoer van warmte de gehele wervelstraat afbuigt in negatieve $y$-richting. Gedetailleerde metingen van het 2D snelheidsveld met behulp van Hoge Resolutie PTV (HiRes-PV) laten dit gedrag eveneens zien. Tevens volgt uit deze PTV experimenten dat, voor toenemende warmte-invoer, de wervels in de bovenste rij geleidelijk sterker worden dan de wervels in de onderste rij. Met behulp van een puntwervel model is verklaard dat dit verschil in sterkte verantwoordelijk is voor de geobserveerde afbuiging.

De afbuiging en de bron van dit sterkteverschil zijn tevens onderzocht met behulp van nummerieke simulaties. Hiervoor is een Spectrale Elementen Methode gebruikt, een numerieke methode welke een spectrale convergentie vertoont en waar tevens de stroming om moeilijke geometriënen gemodelleerd kan worden. De resultaten van de simulaties voor $0 < Ri_D < 1$ laten zien dat het verschil in sterkte tussen de wervels in de bovenste en onderste rij ontstaat tijdens het afschuddingsproces. Uit een gedetailleerde studie van het wervelafschuddingsproces volgt dat baroclinische vorticiteitsproductie de belangrijkste oorzaak van dit sterkte verschil is. Tevens blijkt door de warmte-invoer een verschil te ontstaan tussen het afschuddingsmechanisme van een 'bovenwervel' ten opzichte van een 'onderwervel'.

Voor grotere warmte-invoer ($Ri_D > 1$) laten visualisatie-experimenten zien dat de wervelstraat
onstabil wordt en uit elkaar valt. Dit transitieproces start met het ontstaan van een secundaire structuur binnen een bovenwervel. Gedetailleerde 2D HiRes-PV metingen en 2D simulaties laten zien dat deze structuur ontstaat door baroclinische vorticiteitsproductie. Gedurende de ontwikkeling van deze structuur treedt een transitie van het 2D zog naar een 3D stroming op. De structuur ontsnapt vervolgens uit de bovenwervel en ontwikkelt zichzelf verder naar een 3D thermische pluim met een paddestoelachtige vorm. Dit 3D karakter van het gehele proces is eveneens geanalyseerd met 3D visualisatie en 3D PTV experimenten.
Nawoord

Als ik terugkijk, lijkt alles zo eenvoudig. Dit werk, gepresenteerd in ruim 150 pagina’s, had ik daar nu echt vier jaar voor nodig? Dan besef je dat dit proefschrift eigenlijk niet het resultaat is, maar de weg naar dit proefschrift. Wellicht is dit een aardig moment om nog eens over deze weg te lopen. Kijkende naar het werk van het eerste halfjaar lijkt bijna niets terug te vinden in het huidige resultaat. In het begin gedroeg de richting van het onderzoek zich als een windwijzer in een wervel. Alle richtingen afgespeurd, alles even aangeraakt maar niets diep uitgespit. De vrijheid, die Anton en Camilo mij daarin gegeven hebben, heeft zeker geleid tot een ruimer inzicht in de stromingsleer en aanverwante onderwerpen. Daarnaast heerste in deze periode ook een gevoel van ‘ach nog 4 jaar waar maak ik mezelf druk om’. Dus genoeg tijd om eens uitgebreid te voetballen in de W-laaggang of een spelletje dart bij Maurice. Zodra de windrichting wat stabijer is geworden, verandert er iets. Het doel is concreet en het blijkt dat er nog veel moet gebeuren. Zoals het bouwen van een opstelling. Door de vele jaren ervaring van Peter en Lambert kristalliseren de wilde ideeën uit in een solide opstelling. De uiteindelijke realisatie hiervan was echter niet zonder slag of stoot en zeker de ondersteuning van Frits is van groot belang geweest. Ook in het vervolg heeft Frits veel bijgedragen aan mijn onderzoek. Van construeren tot meten, ergens onderweg kom je Gert tegen. Altijd een bron van wilde ideeën, gepresenteerd in random volgorde maar daardoor niet minder waardevol. Daarnaast hebben ook verschillende afstudeerders en stagiaires hun bijdrage geleverd aan het huidige resultaat. Zo hebben Harm en Marco een substantiële bijdrage geleverd aan de temperatuur-meetmethode, Sven en Jeroen aan de meetopstelling en PTV-metingen en Jean aan de numerieke programmatuur. De weg is in de afgelopen 4 jaar breder geweest dan het onderzoek. Zo zijn de discussies met mijn kamergenoten Misheck en Marco altijd interessante, vaak ook komische onderdelen van een dag ploeteren in W-hoog 3.145. Maar ook de discussies met mijn andere collega’s, zoals Rob en Rob, hebben op verder manieren bijgedragen aan datgene wat voor u ligt. En zo zijn zomaar vier jaar verstreken en rest nog het schrijven van slechts 160 pagina’s. Het ploeteren met keyboard en pen heeft mijn bewondering voor schrijvers als Harry Mulish en Douglas Adams zeker vergroot. Het bleek geen sinecure te zijn om een lopend verhaal op papier te zetten. Hieraan heeft zeker de input van Gert-Jan en de andere commissieleden bijgedragen. In deze laatste fase van mijn promotie ben ik toch wel enige dank verschuldigd aan Tamara, die geduldig nieuwe paragrafen en bevindingen heeft moeten aannemen. Daarnaast heeft zij voor mij de ruimte gecreëerd zodat het oneindig lijkenende schrijven toch nog eindig is geworden. Hoewel het niet mijn intentie is om op deze pagina een opsomming te geven, wil ik toch in dit slotakkoord mijn ouders bedanken en iedereen die heeft bijgedragen aan hetgeen in de afgelopen vier jaar tot stand is gekomen.