On subset selection from Logistic populations

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Summary

Some distributional results are derived for subset selection from Logistic populations, differing only in their location parameter. The probability of correct selection is determined. Exact and numerical results concerning the expected subset size are presented.

Keywords and Phrases: subset selection, Logistic distribution, probability of correct selection, expected subset size.

1. Introduction

An important class of problems is concerned with selection of the best of $k(\geq 2)$ populations $\pi_1, \pi_2, \cdots, \pi_k$. These populations may be among other things treatments or production processes. Given are k random variables $X_1, X_2, \cdots, X_k$, which may be sample means, associated with these populations. We assume that the distributions of these random variables differ only in their location parameter. The problem considered in this paper is to select a non-empty subset, as small as possible, such that the probability of selecting the best population in the subset is at least equal to a specified value $P^*(k^{-1} < P^* < 1)$. This so called subset selection procedure has been introduced by Gupta (1965).

In selection problems populations with large (small) values of a certain parameter are usually considered good. We define the population with the largest of the unknown values of the k location parameters to be the best. If there are more than one contenders of the best, we suppose that one of these is appropriately tagged.

In Van der Laan (1989) some results are given concerning Bechhofer's Indifference Zone selection procedure for Logistic populations. In Han (1987) and Gupta and Han (1987) the relevant distribution theory has been solved using Edgeworth expansions. Lorentzen and McDonald (1981) considered the problem of selecting the best Logistic population using sample medians. In this paper we shall study the distribution theory for subset selection procedures, starting from Logistic populations. Using the Logistic distribution it is possible to solve analytically certain distribution problems. An interesting point is the striking resemblance between the Normal and Logistic distribution for a suitable choice of the parameters. An illustration of this resemblance can be given using results from Van der Laan (1989). Using the subset selection rule with the model assumption of Normality, whereas the distribution used is in fact Logistic, each with variance one, the actual lower bound of the probability of correct selection has been given in the next table for some values of k and P^*.

k \ P^*	.90	.95	.99
2	.9063	.9510	.9872
5	.9000	.9444	.9835
10	.8897	.9368	.9801

If X_i $(i = 1, 2, \cdots, k)$ are the sample means based on $n \geq 2$ independently and Logistically distributed observations, the deviations will tend to become smaller for increasing n. For $n = 10$ and a simulation with 5000 runs (cf. Van der Laan and Van Putten (1989)) this tendency can be illustrated by presenting some results in the following table.

P^* k	.90	.95	.99
2	.899	.952	.989
5	.892	.952	.989
10	.889	.963	.987

Finally, some results concerning the expected subset size and its maximum value in a specific subspace of the parameter space are provided.

2. Subset selection from Logistic populations

Let $X_1, X_2, \cdots, X_k$ be $k(\geq 2)$ independent random variables with probability densities $f(x-\theta_1), f(x-\theta_2), \cdots, f(x-\theta_k)$, respectively. These k random variables characterize populations $\pi_1, \pi_2, \cdots, \pi_k$. We are interested in choosing the population with the largest value of θ (the best population). The ranked location parameters $\theta_1, \theta_2, \cdots, \theta_k$ are denoted by $\theta_{[1]} \leq \theta_{[2]} \leq \cdots \leq \theta_{[k]}$. If there are more than one contenders of the best, we suppose that one of these is appropriately tagged. We choose a non-empty subset such that the probability is at least P^* (with $k^{-1} < P^* < 1$) that the selected subset contains the population with the largest value of θ by following the next selection rule:

$$\text{Select } \pi_i \text{ iff } x_i \geq x_{\max} - d \ ,$$

where x_i is the observed value of $X_i\,(i = 1, 2, \cdots, k)$ and $x_{\max}$ is the observed value of $X_{\max} = \max\limits_{1 \leq i \leq k} X_i$. A correct selection CS means selection of any subset which includes the best one. The probability of CS is equal to

$$P(CS) = P(X_{(k)} \geq X_{\max} - d) \ ,$$

where $X_{(k)}$ is the unknown random variable which is associated with $\theta_{[k]}$. Now we can write (cf. Gupta (1965))

$$P(CS) = \int_{-\infty}^{\infty} f(t) \prod_{i=1}^{k-1} F(t + d + \theta_{[k]} - \theta_{[i]})\, dt \ ,$$

where $F(\cdot)$ and $f(\cdot)$ are the distribution function and the density, respectively, of $X_i - \theta_i$ $(i = 1, 2, \cdots, k)$ and

$$\inf P(CS) = \int_{-\infty}^{\infty} f(t) F^{k-1}(t+d)\, dt \ ,$$

which is attained for $\theta_{[1]} = \theta_{[k]}$. The smallest value of d has to be chosen for which

$$\int_{-\infty}^{\infty} f(t) F^{k-1}(t+d)\, dt = P^* \ ,$$

to be sure that $P(CS) \geq P^*$ for all configurations of $\theta_1, \theta_2, \cdots, \theta_k$.

Assuming $X_1, X_2, \cdots, X_k$ are Logistically distributed with known scale parameter λ we get

$$f(x-\theta_i) = \lambda\, e^{-\lambda(x-\theta_i)} \, \{1 + e^{-\lambda(x-\theta_i)}\}^{-2}$$

and

$$F(x-\theta_i) = \{1 + e^{-\lambda(x-\theta_i)}\}^{-1}$$

for $-\infty < x < \infty$ and $i = 1, 2, \cdots, k$. The probability requirement can now be written as follows

$$P^* = \int_{-\infty}^{\infty} \lambda\, e^{-\lambda x} \, \{1 + e^{-\lambda x}\}^{-2} \, \{1 + e^{-\lambda(x+d)}\}^{-(k-1)}\, dx$$

$$= a^{k-1} \int_{0}^{\infty} (s+1)^{-2} \, (s+a)^{-k+1}\, ds$$

with $a = e^{\lambda d}$. From Van der Laan (1989) it can easily be seen that this equality reduces to

$$P^* = 1 - (k-1)\, a^{k-1} (a-1)^{-k} S_a(k-1) \ , \qquad (1)$$

where for $c > 0$ and integer m the following definition holds: $S_c(m) = \ln c - \sum_{i=1}^{m} \frac{1}{i} (1 - \frac{1}{c})^i$ and $\sum_{i=1}^{m} \frac{1}{i} (1 - \frac{1}{c})^i = 0$ voor $m \leq 0$.

In table 1 values of $\sqrt{3}\pi^{-1}\lambda d$ are given for which (1) holds. The value of d can be deduced in a simple way.

3. Expected size of the subset

Consistent with the probability requirement, we would like the size of the selected subset to be as small as possible. This size S is a random variable with possible outcomes $1, 2, \cdots, k$. A criterion for the efficiency of the selection procedure is the expected value $E(S)$ of S. We have

$$E(S) = \sum_{i=1}^{k} P(\text{selecting the population with } \mu_{[i]})$$

$$= \sum_{i=1}^{k} P(X_{(i)} \geq X_{\max} - d)$$

$$= \sum_{i=1}^{k} \int_{-\infty}^{\infty} f(t) \prod_{\substack{j=1 \\ j \neq i}}^{k} F(t + d + \theta_{[i]} - \theta_{[j]})\, dt \ .$$

From Gupta (1965) we have

$$\max_{\Omega} E(S) = k \int_{-\infty}^{\infty} F^{k-1}(x+d) f(x)\, dx$$

$$= kP^* \ ,$$

where Ω is the parameter space consisting of all configurations of θ's. In the subset $\Omega(\delta)$, defined by $\theta_{[i]} \leq \theta_{[k]} - \delta$ for $i = 1, 2, \cdots, k-1$ and $\delta > 0$, $E(S)$ takes on its maximum value M when $\theta_{[i]} = \theta_{[k]} - \delta$ for all i ($\leq k-1$) and hence

$$M = \max_{\Omega(S)} E(S)$$

$$= \int_{-\infty}^{\infty} F^{k-1}(x+d+\delta)\, dF(x) + (k-1) \int_{-\infty}^{\infty} F^{k-2}(x+d)\, F(x+d-\delta)\, dF(x) \ .$$

Now we can prove the following theorem.

Theorem 1. For Logistic populations we have for all $k \geq 2$

$$M = \begin{cases} \frac{1}{2}(k+1) - \frac{k-1}{a^2}\left(\frac{a^2}{a^2-1}\right)^k S_{a^2}(k-1) - \frac{1}{2}(k-1)(k-2)\left\{\frac{1}{a} - (k-1)\frac{a^{k-2}}{(a-1)^k} S_a(k-1)\right\} \\ \qquad\qquad \text{for } d=\delta \\ \frac{ab}{ab-1} - \frac{k-1}{ab}\left(\frac{ab}{ab-1}\right)^k S_{ab}(k-2) + \frac{k-1}{a-b}\left[a - (k-2)\left(\frac{a}{a-1}\right)^{k-1} S_a(k-2) - \right. \\ \qquad \left. - \frac{ab}{a-b}\left\{\left(\frac{a}{a-1}\right)^{k-2} S_a(k-3) - \left(\frac{b}{b-1}\right)^{k-2} S_b(k-3)\right\}\right] \\ \qquad\qquad \text{for } d \neq \delta \end{cases}$$

where $b = e^{\lambda\delta}$.

Proof: We write

$$M = I_1 + (k-1)\, I_2 \ ,$$

where

$$I_1 = \int_{-\infty}^{\infty} F^{k-1}(x+d+\delta)\, dF(x)$$

$$= \lambda (e^{\lambda(d+\delta)})^{k-1} \int_{-\infty}^{\infty} (e^{\lambda(d+\delta)} + e^{-\lambda x})^{-k+1} (1+e^{-\lambda x})^{-2} e^{-\lambda x}\, dx$$

$$= (a\, b)^{k-1} \int_{0}^{\infty} (t+ab)^{-k+1} (t+1)^{-2}\, dt$$

$$= \left[\frac{ab}{ab-1} \right]^k \{ \frac{ab-1}{ab} - (k-1) \frac{ln(ab)}{ab} + \frac{1}{ab} \sum_{i=1}^{k-2} \frac{k-1-i}{i} (1-\frac{1}{ab})^i \}$$

$$= \frac{ab}{ab-1} - \frac{k-1}{ab} \left[\frac{ab}{ab-1} \right]^k S_{ab}(k-2)$$

and

$$I_2 = \int_{-\infty}^{\infty} F^{k-2}(x+d)\, F(x+d-\delta)\, dF(x)$$

$$= \frac{a^{k-1}}{b} J$$

with

$$J = \int_{0}^{\infty} (t+a)^{-k+2} (t+\frac{a}{b})^{-1} (t+1)^{-2}\, dt \ .$$

First we consider the case $d = \delta$ (or $a = b$), then

$$I_1 = \int_{-\infty}^{\infty} F^{k-1}(x+2d)\, dF(x)$$

$$= \frac{a^2}{a^2-1} - \frac{k-1}{a^2} \left[\frac{a^2}{a^2-1} \right]^k S_{a^2}(k-2)$$

$$= 1 - \frac{k-1}{a^2} \left[\frac{a^2}{a^2-1} \right]^k S_{a^2}(k-1)$$

and

$$I_2 = a^{k-2} \int_0^\infty (t+1)^{-3}(t+a)^{-k+2}\, dt$$

$$= \frac{1}{2} - \frac{1}{2}(k-2)\, a^{k-2} \int_0^\infty (t+1)^{-2}(t+a)^{-k+1}\, dt$$

$$= \frac{1}{2} - \frac{1}{2}(k-2)\, \{\frac{1}{a-1} - (k-1)\, \frac{a^{k-2}}{(a-1)^k}\, S_a(k-2)\} \ .$$

After some elementary calculations the result of theorem 1 follows.

Now we consider the case $d \neq \delta$ (thus $a \neq b$). For $k \geq 4$ we get

$$J = \frac{b}{a-b} \int_0^\infty (t+a)^{-k+2}(t+1)^{-1}\{(t+1)^{-1} - (t+\frac{a}{b})^{-1}\}\, dt$$

$$= \frac{b}{a-b}\, \{a^{-k+3}(a-1)^{-1} - (k-2)(a-1)^{-k+1} S_a(k-3)\} -$$

$$- (\frac{b}{a-b})^2\, \{\int_0^\infty (t+a)^{-k+2}(t+1)^{-1}\, dt - \int_0^\infty (t+a)^{-k+2}(t+\frac{a}{b})^{-1} dt\}$$

$$= \frac{b}{a-b}\, \{a^{-k+3}(a-1)^{-1} - (k-2)(a-1)^{-k+1}\, S_a(k-3)\} -$$

$$- (\frac{b}{a-b})^2\, \{(a-1)^{-k+2} S_a(k-3) - b^{k-2} a^{-k+2} (b-1)^{-k+2} S_b(k-3)\} \ .$$

From this it follows

$$M = \frac{ab}{ab-1} - \frac{k-1}{ab}\, (\frac{ab}{ab-1})^k S_{ab}(k-2) + \frac{k-1}{a-b}\, [\frac{a^2}{a-1} - (k-2)(\frac{a}{a-1})^{k-1} S_a(k-3) -$$

$$- \frac{ab}{a-b}\, \{(\frac{a}{a-1})^{k-2} S_a(k-3) - (\frac{b}{b-1})^{k-2} S_b(k-3)\}]$$

and the result of theorem 1 follows.

For $k = 2$ we get

$$I_1 = (\frac{ab}{ab-1})^2 \{\frac{ab-1}{ab} - \frac{ln(ab)}{ab}\}$$

and

$$I_2 = \frac{a}{b} \int_0^a (t + \frac{a}{b})^{-1} (t+1)^{-2}\, dt$$

$$= \frac{a}{a-b} \int_0^\infty (t+1)^{-1} \{(t+1)^{-1} - (t+\frac{a}{b})^{-1}\}\, dt$$

$$= \frac{a}{a-b} \{1 - \frac{b}{a-b} \ln(\frac{a}{b})\} \ ,$$

thus

$$M = \frac{ab}{ab-1} - \frac{ab}{(ab-1)^2} \ln(a\, b) + \frac{a}{a-b} \{1 - \frac{b}{a-b} \ln(\frac{a}{b})\}$$

and this is the general expression for $d \neq \delta$ with $k = 2$. For $k = 3$ we get

$$I_2 = \frac{a^2}{b} \int_0^\infty (t+1)^{-2} (t+a)^{-1} (t+\frac{a}{b})^{-1} dt$$

$$= \frac{a^2}{a-b} [\int_0^\infty (t+1)^{-2}(t+a)^{-1}\, dt - \int_0^\infty (t+1)^{-1}(t+a)^{-1}(t+\frac{a}{b})^{-1}\, dt]$$

$$= \frac{a^2}{a-b} [\frac{1}{a-1} - \frac{1}{(a-1)^2} \ln a - \frac{b}{a-b} \{\int_0^\infty (t+1)^{-1}(t+a)^{-1} dt - \int_0^\infty (t+a)^{-1}(t+\frac{a}{b})^{-1} dt\}]$$

$$= \frac{a^2}{a-b} [\frac{1}{a-1} - \frac{\ln a}{(a-1)^2} - \frac{b}{a-b} \{\frac{\ln a}{a-1} - \frac{b}{a} \frac{\ln b}{b-1}\}]$$

and I_1 can be determined as for $k = 4$, thus

$$M = \frac{ab}{ab-1} - \frac{2}{ab} (\frac{ab}{ab-1})^3 S_{ab}(1) + \frac{2a^2}{a-b} [\frac{1}{a-1} - \frac{\ln a}{(a-1)^2} - \frac{b}{a-b} \{\frac{\ln a}{a-1} - \frac{b}{a} \frac{\ln b}{b-1}\}]$$

and it can easily be seen that this result is equal to the general expression of theorem 1 for $k = 3$.

For different values of d, δ and k the value of M has been computed. In table 2 the results can be found.

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Table 1 Values of $\sqrt{3}\pi^{-1}\lambda d$ fullfilling the probability requirement for some values of k and P^*.

k \ P^*	.75	.90	.95	.99
2	.9000	1.7581	2.3109	3.4578
3	1.3636	2.1906	2.7311	3.8635
4	1.6151	2.4319	2.9682	4.0955
5	1.7878	2.5995	3.1338	4.2585
6	1.9193	2.7280	3.2610	4.3842
7	2.0255	2.8322	3.3643	4.4865
8	2.1146	2.9198	3.4514	4.5728
9	2.1912	2.9954	3.5265	4.6474
10	2.2586	3.0619	3.5926	4.7131
25	2.8112	3.6104	4.1394	5.2577
50	3.2084	4.0063	4.5348	5.6524

Table 2 $M = \max_{\Omega(\delta)} E(S)$ for some values of d, δ and k.

d	δ	k = 2	3	5	10	25	50	75	100
0.1	0	1.0604	1.0922	1.1256	1.1576	1.1809	1.1896	1.1926	1.1942
	0.1	1.0602	1.0920	1.1254	1.1575	1.1809	1.1896	1.1926	1.1942
	0.2	1.0596	1.0914	1.1249	1.1572	1.1808	1.1896	1.1926	1.1942
	1	1.0439	1.0712	1.1043	1.1421	1.1744	1.1870	1.1913	1.1933
	2	1.0186	1.0325	1.0532	1.0854	1.1296	1.1580	1.1708	1.1780
	5	1.0003	1.0006	1.0011	1.0022	1.0049	1.0087	1.0119	1.0148
0.5	0	1.2942	1.4806	1.7108	1.9831	2.2337	2.3442	2.3855	2.4072
	0.1	1.2933	1.4795	1.7097	1.9825	2.2334	2.3442	2.3855	2.4072
	0.2	1.2907	1.4761	1.7065	1.9803	2.2326	2.3439	2.3854	2.4071
	1	1.2181	1.3731	1.5855	1.8719	2.1734	2.3160	2.3691	2.3964
	2	1.0959	1.1738	1.2975	1.5082	1.8309	2.0632	2.1777	2.2461
	5	1.0016	1.0032	1.0060	1.0124	1.0288	1.0514	1.0711	1.0888
1	0	1.5453	1.9546	2.5511	3.4388	4.5370	5.1668	5.4375	5.5891
	0.1	1.5440	1.9527	2.5490	3.4369	4.5361	5.1664	5.4373	5.5890
	0.2	1.5399	1.9470	2.5422	3.4310	4.5328	5.1649	5.4364	5.5884
	1	1.4255	1.7681	2.3004	3.1605	4.3291	5.0432	5.3559	5.5311
	2	1.2079	1.3902	1.7017	2.2894	3.3212	4.1695	4.6290	4.9208
	5	1.0042	1.0083	1.0161	1.0342	1.0814	1.1481	1.2063	1.2592
2	0	1.8511	2.6230	3.9973	6.7879	12.3802	17.7885	21.1225	23.4347
	0.1	1.8501	2.6214	3.9949	6.7847	12.3769	17.7859	21.1205	23.4331
	0.2	1.8471	2.6164	3.9875	6.7747	12.3662	17.7771	21.1134	23.4274
	1	1.7532	2.4511	3.7186	6.3618	11.8352	17.2687	20.6639	23.0336
	2	1.4956	1.9686	2.8578	4.8221	9.3068	14.2786	17.6475	20.1223
	5	1.0194	1.0386	1.0762	1.1676	1.4238	1.8086	2.1571	2.4791

Table 2 (continued).

		k							
		2	3	5	10	25	50	75	100
d	δ								
3	0	1.9611	2.8959	4.7010	8.9202	19.8791	34.6453	46.6745	56.8543
	0.1	1.9608	2.8953	4.6999	8.9183	19.8762	34.6417	46.6708	56.8506
	0.2	1.9597	2.8933	4.6966	8.9126	19.8670	34.6304	46.6589	56.8385
	1	1.9211	2.8215	4.5686	8.6798	19.4534	34.0790	46.0458	56.1973
	2	1.7717	2.5322	4.0222	7.5881	17.2137	30.7241	42.0496	51.8073
	5	1.0744	1.1487	1.2949	1.6590	2.7255	4.4283	6.0523	7.6088
4	0	1.9911	2.9755	4.9266	9.7162	23.4916	44.9606	64.9996	83.8868
	0.1	1.9910	2.9754	4.9263	9.7155	23.4904	44.9588	64.9973	83.8842
	0.2	1.9907	2.9748	4.9253	9.7136	23.4866	44.9530	64.9901	83.8760
	1	1.9796	2.9535	4.8857	9.6354	23.3226	44.6912	64.6580	83.4895
	2	1.9254	2.8469	4.6792	9.2007	22.3229	42.9788	62.3985	80.7882
	5	1.2273	1.4545	1.8941	3.0079	6.3243	11.7725	17.1280	22.3967
5	0	1.9981	2.9948	4.9839	9.9354	24.6340	48.7000	72.3083	95.5151
	0.1	1.9981	2.9947	4.9839	9.9352	24.6337	48.6995	72.3076	95.5143
	0.2	1.9980	2.9946	4.9836	9.9348	24.6326	48.6977	72.3052	95.5115
	1	1.9954	2.9895	4.9739	9.9147	24.5869	48.6179	72.1972	95.3788
	2	1.9805	2.9601	4.9165	9.7909	24.2862	48.0673	71.4314	94.4217
	5	1.5000	1.9999	2.9993	5.4959	12.9687	25.3681	37.7001	49.9663
6	0	1.9996	2.9990	4.9969	9.9874	24.9266	49.7326	74.4365	99.0480
	0.1	1.9996	2.9990	4.9969	9.9874	24.9266	49.7325	74.4364	99.0480
	0.2	1.9996	2.9989	4.9969	9.9873	24.9264	49.7322	74.4360	99.0474
	1	1.9990	2.9978	4.9949	9.9832	24.9167	49.7147	74.4116	99.0169
	2	1.9956	2.9909	4.9827	9.9567	24.8510	49.5914	74.2369	98.7949
	5	1.7727	2.5453	4.6335	9.1747	22.7937	45.4765	68.1404	90.7857

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