BACHELOR

Influence of polynomial fit and data frequencies on vehicle following

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Influence of polynomial fit and data frequencies on vehicle following

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1 Introduction

Autonomous driving is one of the most developing technological innovations. Nowadays more and more traffic is present on the available infrastructure, with its consequences. Autonomous vehicles could give a solution to this problem. It can increase traffic density and efficiency, while improving on the safety and comfort of the passenger. There are many companies on the market developing such vehicles with success, like Tesla who has commercialized autonomous driving. Autonomous driving also has an influence on traveling in the future. Due to a higher comfort and higher efficiency, longer travels will be more and more appealing. Nowadays, the less manually driven cars there are on the road, the more the positive effects on efficiency on the road are visible. [1]

Autonomous driving consists of many different levels of automation. Going from adaptive cruise control where the speed of the vehicle is controlled in order to follow a preceding vehicle within a desired distance, to fully controlled vehicles where no driver is needed. One approach to automated driving is vehicle following. Vehicle following is a technique that uses data from a leader vehicle in order to be followed by a follower vehicle. Data from this leader vehicle is obtained using on-board sensors and sent via vehicle-to-vehicle communication to the follower vehicle. Using this kind of measuring and communication brings certain inaccuracies with it, like sensor accuracy, communication delay, data frequency and limited computation times.

Vehicle following uses different kinds of control strategies in order for the follower vehicle to predict an accurate path. These kinds are lateral and longitudinal control. Longitudinal control uses the leader vehicle information to define a speed to follow the leader vehicle within the desired distance. Lateral control uses that information to automate the steering of the follower car, so the trajectory of the leader vehicle is followed.

In this project the focus lies on on lateral vehicle following. This lateral vehicle following is applied in a situation where a leader vehicle is driving in an x-y plane while sending the trajectory information to the follower vehicle. The data is in the form of data points with a certain frequency and noise coming from the sensors. This follower vehicle uses a lateral control strategy to follow this path as accurate as possible.

To make the data useful for the follower vehicle, such that so calculations can be done from it, polynomials are fitted through the data points. Based on those polynomials, the path of the follower vehicle can be predicted. The follower vehicle can change variables in order to process the data as accurate as possible in order to eliminate the noise on the trajectory data points as much as possible. This can be done by varying the time steps between polynomial fits for different time steps of the data.

The goal of this project is to investigate how the frequency of the data and the frequency of path prediction influences the accuracy of the predicted path. This is done by applying a lateral control strategy on a vehicle following simulation and obtaining data which indicates the accuracy of the predicted path. Therefore the following main research question can be proposed:

What is the influence of different time steps of data points and different time steps of polynomial fits on the accuracy of the predicted path of the follower vehicle?

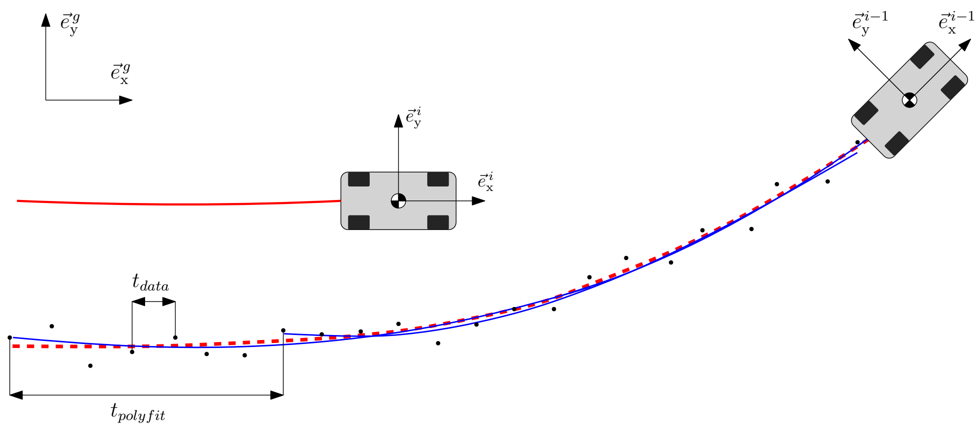

Figure 1: Situation where the problem is stated in

This report focuses on implementing a lateral controller in a vehicle following simulation. Since longitudinal control is a subject which is not treated in this report, the input is given as a constant velocity in the longitudinal direction. In chapter 2, an explanation is given on how the trajectory from the output of a vehicle model is generated, while taking into account the noise which is on data. In chapter 3, the used linear single track model is presented. In chapter 4 the method of polynomial fitting is explained followed by the error definition needed as a input for the follower vehicle model. Then the vehicle model with the longitudinal controller of the follower vehicle is explained. Eventually the path generated by the follower vehicle is presented. Chapter 5 describes the reasoning behind the tested variables and discusses the results obtained from the simulation. Chapter 6 gives the conclusions of the results and answers the research question and proposes the recommendations on the whole research.

2 Trajectory generation

Trajectory generation is a way to convert a local vehicle output over time to a trajectory in global coordinate system, which is used as the reference trajectory for vehicles which needs to follow that trajectory. In this chapter the local output from the vehicle model, the linear single track model explained in chapter 3, is used to construct a trajectory which is able to be followed by the follower vehicle.

2.1 Leader vehicle

In lateral vehicle following, state variables of the leader vehicle are measured and used to construct the vehicle's historical path. This can be applied in situations where the following vehicle needs to follow the leader vehicle as close as possible, both in longitudinal as lateral direction. Or the trajectory of the leader vehicle can be followed at another instant, where the given path always applies to a certain situation. In both situations the trajectory is generated in the same way.

The linear single track model explained in chapter 3 describes the dynamics of the leader vehicle. From this leader vehicle model, the outputs in local coordinate system needed to generate a trajectory in global coordinate system are obtained based on a given input. These output variables are indicated in blue in Figure 2, which is the component of the velocity in local x-direction v_x, the component of the velocity in local y-direction v_y and the angular velocity around the z-axis $\dot{\psi}$, also called yaw rate.

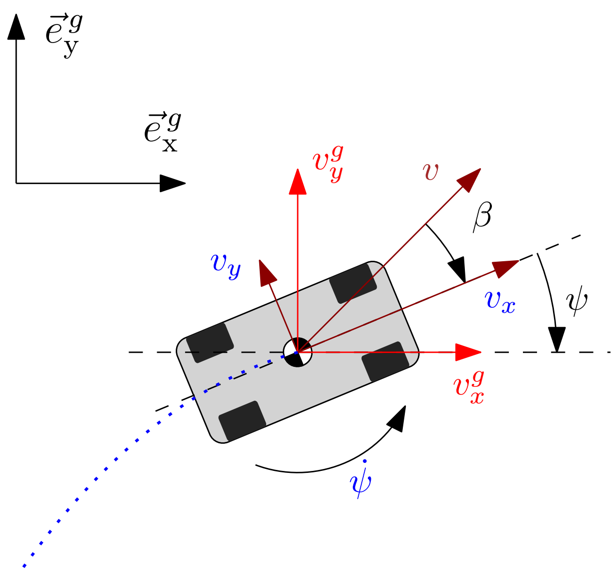

Figure 2: Schematic overview of trajectory generation

These output variables are used to construct the velocity vector indicated in Equation 1. The velocity vector is later needed to construct the x and y components on global coordinate system. The second variable needed is the side slip angle β. This variable describes the angle of the velocity vector with respect to the local x-axis of the vehicle. This is calculated using the x and y components of the velocity vector, v_x and v_y, in Equation 2. In the simulation the small angle approximation is used. This assumes that for relatively small angles, the value of the tangent is equal to the tangent itself. This does mean that $\tan\beta \approx \beta$. The velocity vector and side slip angle are described as follows:

$$\vec{v} = \begin{bmatrix} v_x \\ v_y \end{bmatrix} \tag{1}$$

$$\beta = \arctan\frac{v_y}{v_x} \approx \frac{v_y}{v_x} \tag{2}$$

Where $\vec{v}$ is the velocity vector, v_x the longitudinal velocity, v_y the lateral velocity and β the side slip angle of the vehicle.

These values are all in the local coordinate system. In order to convert the velocity vector to the global coordinate system, the yaw angle of the vehicle with respect to the x-axis of the global coordinate system $\vec{e}_{\mathrm{x}}^{g}$ needs to be used. Since the yaw rate of the vehicle over time is known, the yaw angle ψ is calculated by integrating the yaw rate over time.

$$\psi = \int \dot{\psi}\, dt \tag{3}$$

This angle is used to convert the velocity vector in the local coordinate system such that the velocity vector can be used in the global coordinate system. From the magnitude of the velocity vector, the x and y components of the velocity vector in global coordinate system are calculated. This by using the angle of the velocity vector with respect to the global x-axis. This angle is described by the sum of β and ψ. The components of the velocity vector in global coordinate system are described as follows:

$$v_x^g = ||\vec{v}|| \cos(\beta + \psi) \tag{4}$$

$$v_y^g = ||\vec{v}|| \sin(\beta + \psi) \tag{5}$$

These x and y components of this velocity vector are integrated over time to come up with the x and y coordinates of the vehicle over time, which constructs the trajectory.

$$x_g = \int v_x^g\, dt \tag{6}$$

$$y_g = \int v_y^g\, dt \tag{7}$$

2.2 Noise

Creating a trajectory with data obtained using vehicle to vehicle communication, a noise on the data needs to be taken into account. As mentioned in the introduction, this noise can have different causes. For every data point the leader vehicle sends to the following vehicle, a margin of inaccuracy is present. This inaccuracy is the noise on the data. Noise can be assumed in may different kinds: additive noise, which is a value added to the signal, multiplicative noise, which multiplies or modulates the signal or quantization error, which occurs when converting from continuous to discrete values. [2]

In this case, two kinds of noise are present. First a quantization error is assumed, this error occurs in the process of discretion of a continuous signal to an output with a finite number of elements. Since the data consists of discrete coordinates with an equal time step in between, a quantization error is present. A quantization error behaves in a similar manner to that of additive white noise, it has negligible correlation with the signal and a constant intensity. Secondly a white noise is assumed on the obtained data from the leader vehicle. This is a noise which represents a random signal having an equal intensity at different frequencies i.e. a finite variance in the lateral direction on all x positions.

This finite variance in lateral direction is a position error in lateral direction. This position error is simulated in a way that on the noiseless trajectory a random lateral error is added. This addition is described with the following formula:

$$\hat{y} = y + \mathrm{random}(-0.5, 0.5) \tag{8}$$

Where y is the y coordinate of the trajectory data points and $\hat{y}$ the y coordinate of the trajectory data points with noise.

Here a random variable is added on the reference trajectory within the range of +/- 0.5 m to get the reference trajectory with noise.

Simulating the trajectory generation using a leader vehicle and the described method of integration and noise addition, the trajectory in Figure 3 is obtained. In this figure the data points of the trajectory described by the leader vehicle are shown in blue. The noise added to these data points are indicated in orange. Those data points are used as the reference trajectory for the follower vehicle.

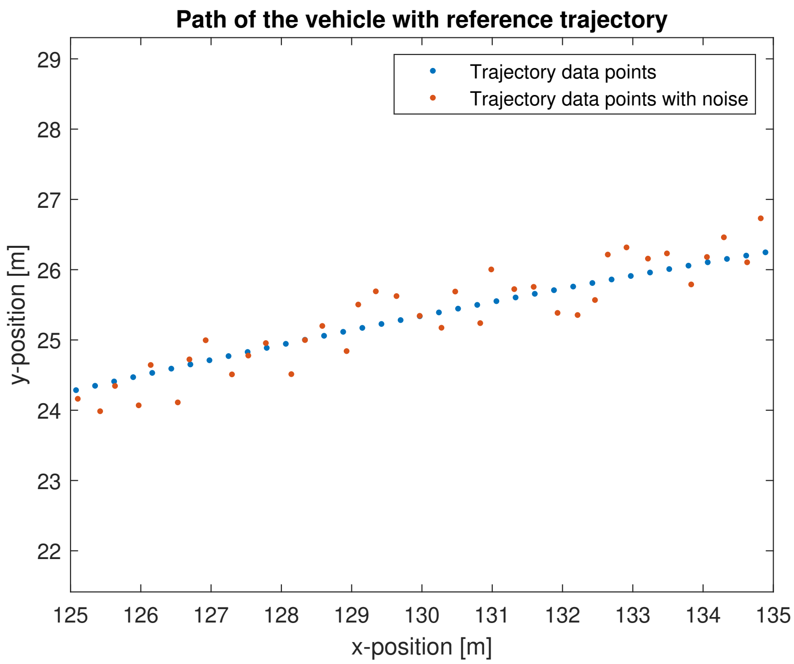

Figure 3: Trajectory data points of the leader vehicle with noise

3 Linear single track model

In this chapter the used vehicle model, the linear single track model, is elaborated on. The linear single track model is a simplified model which describes the physical driving behaviour of a vehicle. The linear single track model is, as mentioned in chapter 2, used to describe the dynamics of the leader vehicle based on a given input. Later in chapter 4, the linear single track model is used to describe the dynamics of the follower vehicle. In order to approximate the lateral dynamics of a vehicle in a linear model, a series of simplifications and assumptions are made.

3.1 Assumptions

The assumptions made on the physical vehicle in order to create a simplified linear model to make it easier to do calculations on with less variables. [3]

First, the number of degrees of freedom of the rigid body in the model is reduced to two. This is done by assuming a constant velocity of the vehicle's center of mass in the local x direction of the vehicle. Also the lifting, rolling and pitching motion will be assumed zero, so the vehicle will always be parallel to the horizontal plane and in contact with the surface. These assumptions result in a possible motion which is a rotation in the horizontal plane, also called the yaw angle ψ, occurring in the form of the yaw rate $\dot{\psi}$ in the model. The second possible motion is a translation in lateral direction, this occurs as a velocity in lateral direction v_y which is based on the side-slip angle β. Both of these variables indicate the direction of the velocity in global coordinate system, thus the direction of the movement. Both the yaw angle and the side-slip angle are based on the output coming from the vehicle model. That output is determined using the steering angle as input. Second, the small-angle assumption. This assumes that angles like the front and rear wheel side-slip angles α_f, α_r and the steering angle of the front wheel δ_f are "small". Such that the sine, cosine and tangent of the variables are approximated with a simplification using that angle. This is described as $\sin\theta \approx \theta$, $\tan\theta \approx \theta$ and $\cos\theta \approx 1 - \frac{\theta^2}{2} \approx 1$. Third, tire forces are assumed to be linear, which suggests that the tire forces do have a linear relationship with the side-slip angles. Fourth, the load distribution between the front and rear axle is assumed to be constant, so no load transfer is present between the front and rear axle.

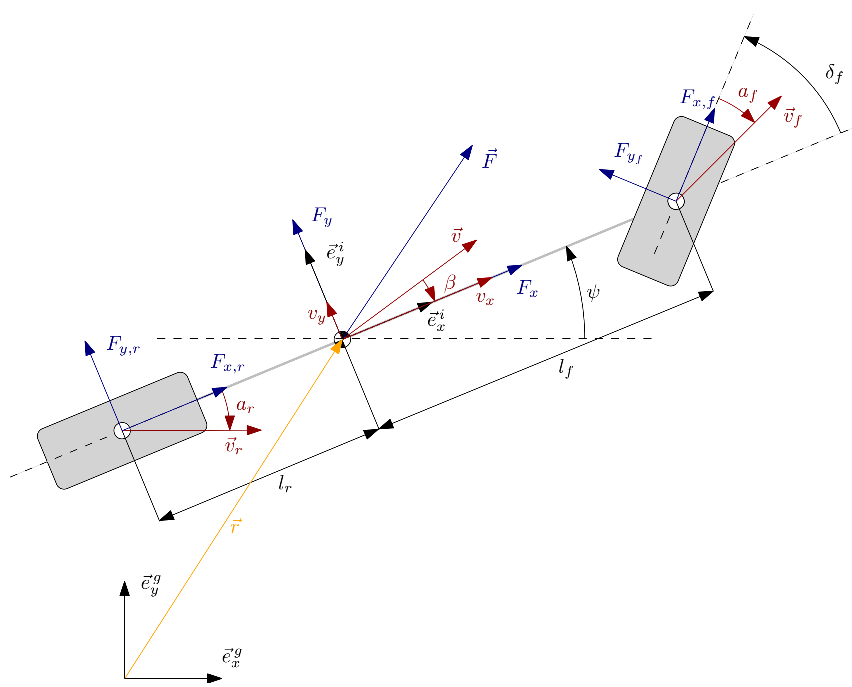

Figure 4: Geometric description of the linear single track model

3.2 Equations of motion

To come up with the equations of motion, the rigid body kinematics of the vehicle are reviewed. The kinematics are described in the local coordinate system of the vehicle: $\vec{e}^i = \begin{pmatrix} \vec{e}^i_\text{x} & \vec{e}^i_\text{y} \end{pmatrix}$. Which is positioned in a global coordinate system: $\vec{e}^g = \begin{pmatrix} \vec{e}^g_\text{x} & \vec{e}^g_\text{y} \end{pmatrix}$. The equations of motion are based on the forces acting on the vehicle, the forces are calculated using the global acceleration of the vehicle. The relation between the global frame g and the local frame of vehicle i is as follows:

$$\vec{e}^g = R(\psi)\vec{e}^i \tag{9}$$

Where,

$$R(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix} \tag{10}$$

From this, the global velocity follows as:

$$\dot{\vec{r}} = (\dot{x}\ \dot{y})R(\psi)\vec{e}^i = (v_x\ v_y)\vec{e}^i \tag{11}$$

From this, the global acceleration is described as:

$$\ddot{\vec{r}} = (\dot{v}_x\ \dot{v}_y)\vec{e}^i + (v_x\ v_y)\dot{\vec{e}}^i \tag{12}$$

$$\ddot{\vec{r}} = (\dot{v}_x - \dot{\psi}v_y\ \ \dot{v}_y + \dot{\psi}v_x)\vec{e}^i \tag{13}$$

Since $F_y = F_{y,f} + F_{y,r}$ because of the small-angle assumption, $\vec{F} = m\ddot{\vec{r}}$ and $\dot{\psi}$ is substituted by ω_z, the equations of motion can be described as:

$$F_x = m(\dot{v}_x - \omega_z v_y) \tag{14}$$

$$F_y = F_{y,f} + F_{y,r} = m(\dot{v}_y + \omega_z v_x) \tag{15}$$

$$I_{zz}\omega_z = l_f F_{y,f} - l_r F_{y,r} \tag{16}$$

With F_x as the total force in the x-direction, $F_{y,f}$ the force in y-direction on the front wheel, $F_{y,r}$ the force in y-direction on the rear wheel, v_y the component of the velocity in the y-direction, ω_z the angular velocity, $\dot{v}_y$ the derivative of the velocity in the y-direction, m the mass of the vehicle, I_{zz} the yaw moment of inertia, l_f the distance from the front axle to the center of mass and l_r the distance from the rear axle to the center of mass.

Since F_x is assumed such that v_x is constant, v_x will be a parameter instead of a state, so Equation 14 is not relevant.

The equations of motion are specified more by substituting the following equations of tire forces and slip angles:

$$F_{y,f} = -C_{\alpha,f}\alpha_\text{f} \tag{17}$$

$$F_{y,r} = -C_{\alpha,r}\alpha_\text{r} \tag{18}$$

With $C_{\alpha,f}$ and $C_{\alpha,f}$ the cornering stiffnesses of the the front and rear tire respectively which are constant since the tire forces are assumed linear and α_f and α_r which are described as follows:

$$\alpha_\text{f} = \frac{v_y + l_f\omega_z}{v_x} - \delta_\text{f} \tag{19}$$

$$\alpha_\text{r} = \frac{v_y - l_r\omega_z}{v_x} \tag{20}$$

This results in the following equations of motion:

$$\dot{v}_y = -\frac{C_{\alpha,f} + C_{\alpha,r}}{v_x m}v_y - (v_x + \frac{l_f C_{\alpha,f} - l_r C_{\alpha,r}}{v_x m})\omega_z + \frac{C_{\alpha,f}}{m}\delta_\text{f} \tag{21}$$

$$\dot{\omega}_z = -\frac{l_f C_{\alpha,f} - l_r C_{\alpha,r}}{v_x I_{zz}}v_y - \frac{l_f^2 C_{\alpha,f} + l_r^2 C_{\alpha,r}}{v_x I_{zz}}\omega_z + \frac{l_f C_{\alpha,f}}{I_{zz}}\delta_\text{f} \tag{22}$$

The constants present in the equations of motion are vehicle parameters. These vehicle parameters are given in Table 1 when the simulation is executed.

The linear single track model is described by the following state-space representation:

$$\dot{x} = Ax + Bu \tag{23}$$

$$y = Cx + Du \tag{24}$$

With vectors:

$$x = \begin{bmatrix} v_y \\ \omega_z \end{bmatrix}, \dot{x} = \begin{bmatrix} \dot{v}_y \\ \dot{\omega}_z \end{bmatrix}, y = \begin{bmatrix} v_y \\ \omega_z \\ \dot{v}_y \\ \beta \end{bmatrix} \text{ and } u = \delta_{\mathrm{f}} \tag{25}$$

The output vector is chosen such that the vehicle behaviour over time can be studied. v_y and ω_z are needed to convert the vehicle behaviour on the local coordinate system to a trajectory on the global coordinate system. $\dot{v}_y$ and β are the acceleration in lateral direction and the side-slip angle respectively. These variables are used to see whether the behaviour of the vehicle is physically possible.

The state, input, output and feedtrough matrices in the state-space representation are constructed with the equations of motion (Equations 21 and 22) and described as follows:

$$A = -\begin{bmatrix} \frac{C_{\alpha,f}+C_{\alpha,r}}{v_x m} & v_x + \frac{l_f C_{\alpha,f} - l_r C_{\alpha,r}}{v_x m} \\ \frac{l_f C_{\alpha,f} - l_r C_{\alpha,r}}{v_x I_{zz}} & \frac{l_f^2 C_{\alpha,f} + l_r^2 C_{\alpha,r}}{v_x I_{zz}} \end{bmatrix} B = \begin{bmatrix} \frac{C_{\alpha,f}}{m} \\ \frac{l_f C_{\alpha,f}}{I_{zz}} \end{bmatrix}$$

$$C = -\begin{bmatrix} -1 & 0 \\ 0 & -1 \\ \frac{C_{\alpha,f}+C_{\alpha,r}}{v_x m} & v_x + \frac{l_f C_{\alpha,f} - l_r C_{\alpha,r}}{v_x m} \\ \frac{1}{v_x} & 0 \end{bmatrix} D = \begin{bmatrix} 0 \\ 0 \\ \frac{C_{\alpha,f}}{m} \\ 0 \end{bmatrix}$$

The output of the linear single track model with an arbitrary sinusoidal input on local coordinate system is visualized for every output value over time. Together with the output, the input function δ_{f} is visualized in Figure 5. This figure the behaviour of the vehicle can be studied, together with the validation of the model used in the simulation. When the output of the system is not logically explainable, there could be something wrong when simulating the vehicle model.

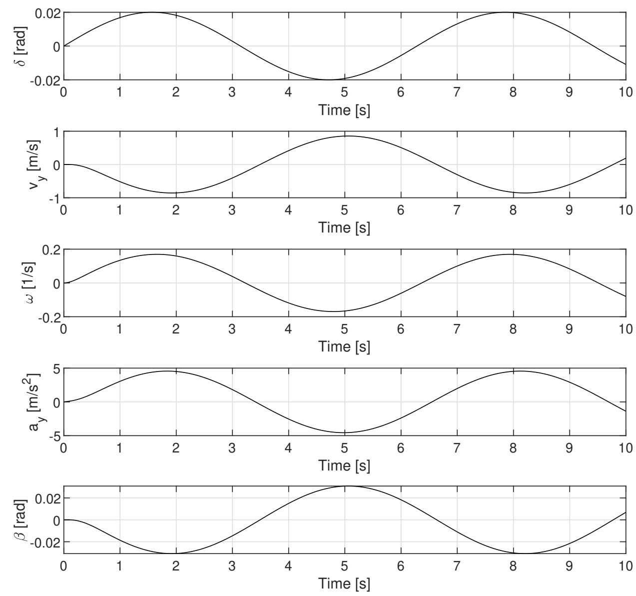

Figure 5: Input and output of the linear single track model

The lateral velocity v_y and yaw rate ω give an indication whether the velocity in lateral direction and rotational velocity is physically possible. The lateral acceleration a_y indicates whether the maximum lateral acceleration is within boundaries for a given vehicle. The side-slip angle β also gives extra information about whether the direction of the velocity vector is physically possible.

4 Follower vehicle

In this chapter the implementation of a follower vehicle with a control strategy which follows the given trajectory, is explained. This follower vehicle uses the trajectory data points which are communicated from the leader vehicle explained in chapter 2.

4.1 Polynomial regression

In order for the follower vehicle model to use the trajectory data from the leader vehicle to calculate a path, polynomial regression is used. Polynomial regression is used to fit a polynomial trough the measured trajectory data points. Polynomial regression is a form of regression analysis in which the relationship between the x and y coordinates of the trajectory data points is described by a n-th degree polynomial. The polynomial regression uses the least squares method to fit a polynomial trough the trajectory data points. The least squares method minimizes the sum of the squared residuals. Those residuals are defined as the difference between the observed and predicted value by the polynomial. [4] The polynomial fitted trough the data points is described in Equation 26. The output of a polynomial fit are the coefficients P in the power terms.

$$y = P_0 + P_1 x + P_2 x^2 + \ldots + P_n x^n + \varepsilon \tag{26}$$

With ε an unobserved random error component normally distributed with mean zero on scalar variable x. The degree of n can be chosen such that the polynomial fit is accurate.

4.1.1 Application

Polynomial fitting can be applied on vehicle following in different ways. As described in the subsection above, polynomial fitting is used to find a relationship between the x and y coordinates of the trajectory data points. Polynomial fitting can be applied in a situation where a finite length trajectory is given e.g. an intersection. One polynomial can simply be fitted trough this given data to describe it as accurate as possible. However the longer the given trajectory, the more complex and inaccurate the polynomial will be. Applying polynomial fitting in a live environment, so when a follower vehicle is following a long random trajectory described by a driving leader vehicle, it is impossible to fit a polynomial trough the whole known length. Therefore multiple polynomials are fitted over time in real time. For every polynomial fitted the path is calculated. This is schematically shown in Figure 6.

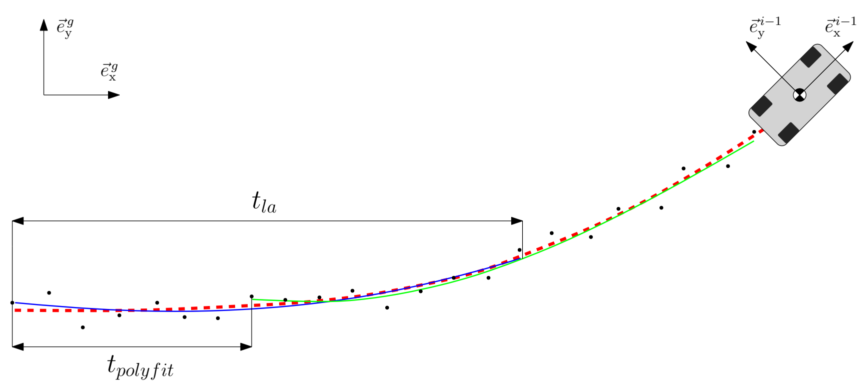

Figure 6: Schematic representation of the polynomial fit with the time steps between polynomial fits

With this way of polynomial fitting multiple variables are present. First, the order of the polynomial can be changed. The order of the polynomial is chosen as low as possible while providing an accurate fit trough the data. In this case an 3^{rd} order polynomial is assumed. This is the minimal order possible in this situation. The trajectory is differentiated twice in order to obtain the curvature of the trajectory, the is explained in subsection 4.4. Secondly, the length of the fit or the number of points the polynomial is fitted trough. In the given trajectory the data is provided with a certain frequency and the length of the polynomial is assumed a constant look ahead time. The higher the frequency of the data, the more data points the polynomial is fitted trough. Thirdly, the frequency of fitting. The frequency of fitting indicates how many times per second a new polynomial is fitted for a given length. Every time a new polynomial is fitted, new data points are used. Therefore, every polynomial will be different, using the curvature of the polynomial for that time instant, the inaccuracy of the fit polynomial which will be minimized when calculating the path of the follower vehicle. In order to get to know the effect of the frequency of the fitting on the accuracy of the path, the accuracy is calculated for different fitting frequencies.

4.2 Error definition

In order for the follower vehicle to follow the given trajectory, an error definition is made. This error definition is based on the polynomial fitted trough the trajectory data points explained in subsection 4.1. This defines the error the follower vehicle has at a certain time with respect to the trajectory. The follower vehicle is also described with the linear single track model, described in chapter 3. The reference coordinate system is the local coordinate system of the follower vehicle. Looking from this coordinate system, two types of errors can be defined. The lateral distance error $y_{e,i}$, which is the distance of the center of mass of the vehicle to the curve, perpendicular to the vehicle and the angular error $\psi_{e,i}$, which is the difference between the tangent of the curve at the point perpendicular to the vehicle and the vehicle orientation with respect to the global x axis. This is visualized in Figure 7.

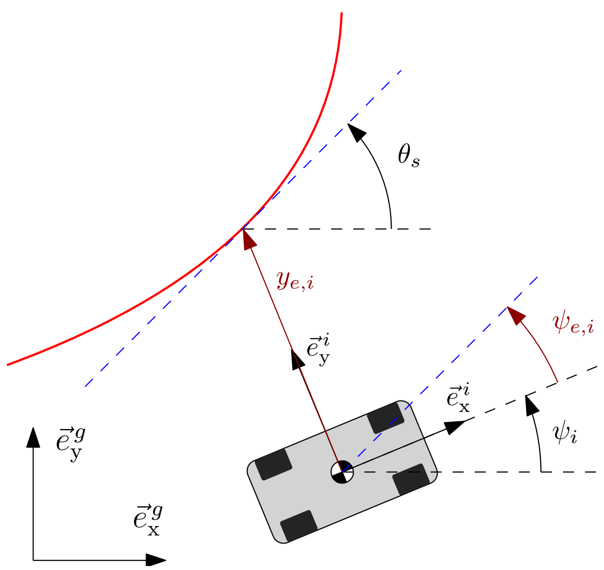

Figure 7: Error definition of the follower vehicle

Both errors can be defined as follows:

$$\psi_{e,i} = \theta_s - \psi_i \tag{27}$$

$$\dot{\psi}_{e,i} = \dot{\theta}_s - \dot{\psi}_i = \dot{\theta}_s - \omega_{z,i} \tag{28}$$

$$\dot{y}_{e,i} = -v_{y,i} + (v_{x,i} - y_{e,i}\dot{\psi}_i)\tan\psi_{e,i} \tag{29}$$

$$\approx -v_{y,i} + v_{x,i}\psi_{e,i} \tag{30}$$

With $\dot{\psi}_{e,i}$ the derivative of the angular error, $\dot{\theta}_s$ the angular rate of change of the trajectory and $\dot{y}_{e,i}$ the derivative of the lateral error.

Equation 28 and Equation 30 are the equations of motion of the error in the follower vehicle model. Those are used together with the equations of motion of the linear single track model to come up with the model for the lateral controller, which is elaborated in subsection 4.3.

4.3 Vehicle model

In this subsection the vehicle model for the follower vehicle is explained. This vehicle model is used to generate an output using the error definition described in subsection 4.2 based on the input given in the form of the polynomial fitted trough the trajectory data points described in subsection 4.1. The follower vehicle model is described as the same linear single track model as the preceding vehicle, but extended with the the equations of motion described in the error definition (Equation 28 and Equation 30). This vehicle model is described by the following state-space representation:

$$\dot{x} = Ax + B_1 u + B_2 d \tag{31}$$

$$y = Cx + Du \tag{32}$$

With vectors:

$$x = \begin{bmatrix} v_{y,i} \\ \omega_{z,i} \\ y_{e,i} \\ \psi_{e,i} \end{bmatrix}, \dot{x} = \begin{bmatrix} \dot{v}_{y,i} \\ \dot{\omega}_{z,i} \\ \dot{y}_{e,i} \\ \dot{\psi}_{e,i} \end{bmatrix}, y = \begin{bmatrix} v_{y,i} \\ \omega_{z,i} \\ y_{e,i} \\ \psi_{e,i} \end{bmatrix}, u = \delta_{f,i} \text{ and } d = \dot{\theta}_{s,i} \tag{33}$$

The output vector y is chosen such that the vehicle behaviour over time can be studied. The same as in the state-space of the linear single track model, $v_{y,i}$ and $\omega_{z,i}$ are needed to convert the vehicle behaviour on the local coordinate system to a trajectory on the global coordinate system. $y_{e,i}$ and $\psi_{e,i}$ are calculated to use as an input to the controller to calculate a steering angle input δ_f.

The state, input, output and feedtrough matrices in the state-space representation are constructed with the equations of motion (Equations 21, 22, 30 and 28) and described as follows:

$$A = -\begin{bmatrix} \frac{C_{\alpha,f}+C_{\alpha,r}}{mv_{x,i}} & v_{x,i} + \frac{l_f C_{\alpha,f} - l_r C_{\alpha,r}}{mv_{x,i}} & 0 & 0 \\ \frac{l_f C_{\alpha,f} - l_r C_{\alpha,r}}{I_{zz} v_{x,i}} & \frac{l_f^2 C_{\alpha,f} + l_r^2 C_{\alpha,r}}{I_{zz}} & 0 & 0 \\ 1 & 0 & 0 & -v_{x,i} \\ 0 & 1 & 0 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} \frac{C_{\alpha,f}}{m} \\ \frac{l_f c_{\alpha,f}}{l_{zz}} \\ 0 \\ 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{34}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{35}$$

The output y of this vehicle model on local coordinate system is visualized for every variable separate over time. This output is obtained based on a trajectory based on the same arbitrary sinusoidal input as used in Figure 5. This output is visualized in Figure 8.

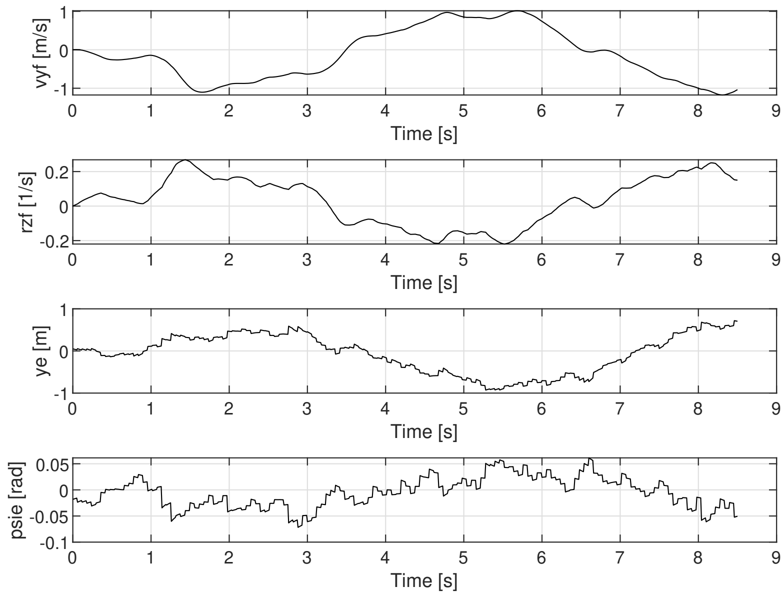

Figure 8: Output of the vehicle model

The reasoning behind this output is given above when discussing the output vector y.

4.4 Inputs

In subsection 4.3, it is explained that the vehicle model has two inputs, the controlled steering angle $\delta_{\mathrm{f,i}}$ and the angular rate of change of the trajectory $\dot{\theta}_{\mathrm{s},i}$. These inputs are discussed below.

4.4.1 Disturbance input

The first input, the angular rate of change of the trajectory is the velocity of the vehicle in x direction $v_{x,i}$ multiplied by the road curvature κ_i.

$$\dot{\theta}_{\mathrm{s},i} = v_{\mathrm{x},i}\kappa_i \tag{36}$$

The road curvature κ_i is described as the derivative of the yaw rate of the trajectory. Therefore the fitted polynomial is differentiated twice, this is described in the following equation:

$$\kappa_i = \frac{\frac{\partial^2 y}{\partial x^2}}{1 + \left(\frac{\partial y}{\partial x}\right)^2} \tag{37}$$

Where $\frac{\partial y}{\partial x}$ and $\frac{\partial^2 y}{\partial x^2}$ are the first and second derivative of the trajectory respectively. The unit of κ_i is rad/m.

4.4.2 Controlled input

The second input, the steering angle, is a controlled input. Therefore a simple feedback and feedforward controller is used described as follows:

$$\delta_{\mathrm{f},i} = \delta_{\mathrm{f},i}^{\mathrm{fb}} + \delta_{\mathrm{f},i}^{\mathrm{ff}} \tag{38}$$

Where $\delta_{\mathrm{f},i}$ is the controlled steering angle input, $\delta_{\mathrm{f},i}^{\mathrm{fb}}$ the feedback term of the controller and $\delta_{\mathrm{f},i}^{\mathrm{ff}}$ the feedforward term of the controller.

The feedforward term of the controller is described as the steer angle at steady-state cornering, so with constant velocity and constant yaw rate, using the road curvature as the variable.

$$\delta_{\mathrm{f},i}^{\mathrm{ff}} = \left(L + \eta v_{x,i}^2\right) \kappa_i \tag{39}$$

With,

$$\eta = \left(\frac{l_\mathrm{r}}{C_{\alpha,\mathrm{f}}} - \frac{l_\mathrm{f}}{C_{\alpha,\mathrm{r}}} \right) \frac{m}{L} \tag{40}$$

With m the mass of the vehicle.

The feedback term of the controller is calculated by using the lateral error $y_{e,i}$, the angular error $\psi_{e,i}$ and constants k1 and k2. The lateral error $y_{e,i}$ and the angular error $\psi_{e,i}$ are two outputs of the vehicle model shown in Figure 8

$$\delta_{\mathrm{f},i}^{\mathrm{fb}} = k_1 y_{e,i} + k_2 \psi_{e,i} \tag{41}$$

Constants k_1 and k_2 are determined using the steering angle at steady state and a formula describing a relation between the lateral error $y_{e,i}$ and the road curvature κ_i. This formula for the steering angle is described as:

$$\delta_\mathrm{f} = \frac{2\left(L + \eta v_x^2\right)}{d_a^2} \left(y_e + l_a \psi_e\right) \tag{42}$$

With, L wheel base of the vehicle, η the understeer gradient, l_a the look ahead distance and d_a the length from the rear axle to the look ahead point.

Rewriting this in the form of Equation 41, results in the following equations for k1 and k2:

$$k_1 = \frac{2\left(L + \eta v_x^2\right)}{d_a^2} \tag{43}$$

$$k_2 = l_a k_1 \tag{44}$$

4.5 Path prediction

The output of the vehicle model is used for the path generation. Two outputs are used to calculate the inputs for the controlled steering angle as elaborated in subsection 4.4. The two other outputs, velocity in lateral direction v_y and yaw rate ω_z, are used to calculate the path of the follower vehicle. These outputs are used to calculate the velocity vector on global coordinate system. The x and y components of this velocity vector are integrated over time to obtain the x and y position over time. This is done in the same way as done in subsection 2.1. Applying this on the output of the follower vehicle, the path of the follower vehicle can be visualized in the same global coordinate system as the trajectory of the leader vehicle.

4.6 Frequency of polynomial fitting

As mentioned in subsubsection 4.1.1 new polynomials are fitted while the follower vehicle is following the trajectory. This is done in a range of frequencies which is elaborated in subsection 5.4. For every time step that a new polynomial is fitted, a new piece of the trajectory is predicted. When performing this cycle for a given time, a total follower path is predicted. Every new polynomial has its unique coefficients, this does mean that the starting point of the new polynomial has other x and y coordinates and an other tangent. Therefore for every new polynomial, new initial conditions are used.

The initial conditions for the path prediction for every new polynomial are based on the starting point and angle of the new polynomial and the position, the yaw angle, the lateral velocity and the yaw rate of the follower vehicle. The initial values of the lateral velocity and the yaw rate at the new polynomial are the lateral velocity and yaw rate the follower vehicle has at that instant. The initial values of the lateral and angular error at the new polynomial are calculated with the coordinates and yaw the follower vehicle and the coordinates of the starting point of the new polynomial with the tangent at that point. The initial value for the error in local y direction $y_{e,i}$ is calculated by subtracting the y position of follower vehicle $y_{\mathrm{g,i}}$ by the y position of the starting point of the polynomial $y_{\mathrm{poly,start}}$. The initial value for the rotational error $\psi_{e,i}$ is calculated by subtracting the yaw angle of follower vehicle $\psi_{\mathrm{g,i}}$ by the angle of the starting point of the polynomial with respect to the x axis $\psi_{\mathrm{poly,start}}$. These initial values are described by the equations below.

$$y_{\mathrm{e,init}} = y_{\mathrm{poly,start}} - y_{\mathrm{g,i}} \tag{45}$$

$$\psi_{\mathrm{e,init}} = \psi_{\mathrm{poly,start}} - \psi_{\mathrm{g,i}} \tag{46}$$

As described in subsection 4.5 the output of the vehicle model is used to calculate the x and y position over time which is done by integrating velocity components over time. This integration is also done at every new polynomial, so in order to have a continuous path, the initial conditions of the yaw, x position and y position are defined. At every new polynomial the yaw, x position and y position of the follower vehicle at that instant are used as the new initial conditions.

Calculating the path for every time step and visualizing them as one path creates a path shown in Figure 9.

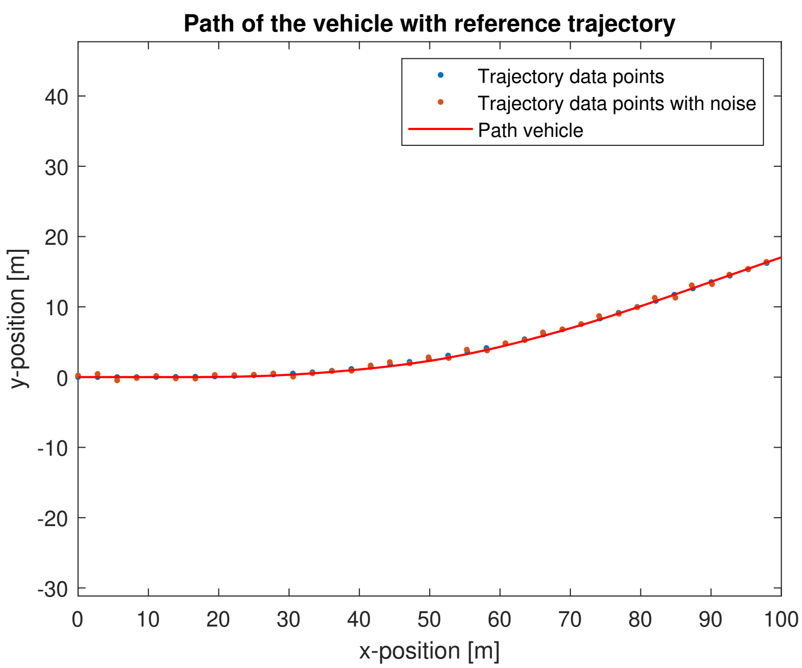

Figure 9: Path of the vehicle following the given trajectory

5 Testplan and Results

In this chapter the simulations done are explained and the results of those simulations are reviewed.

5.1 Simulation input

The simulation done is based on several input constants. Those input constants consists of vehicle model parameters, steering angle input, simulation times, noise magnitude and a look ahead time. Those constants are discussed below.

5.1.1 Vehicle model parameters

As discussed in chapter 3 and chapter 4 for both the preceding vehicle and the controlled vehicle the linear single track model is used. In this model several constants are used to approximate a vehicle as close as possible. These constants describe the dimensions and other parameters of the vehicle. These are shown in Table 1.

Table 1: Vehicle model parameters used in the simulation

Constant	**Value**
l_f	1.25 m
l_r	1.25 m
m	1500 kg
I_{zz}	1600 kg m^2
$C_{\alpha,f}$	80000 N/rad
$C_{\alpha,r}$	90000 N/rad
v_x	100 km/h

5.1.2 Inputs

In chapter 2 it is discussed that the trajectory of the leader vehicle is based from the output of the vehicle model of the leader vehicle. This output calculated based on the input to the leader vehicle. This input is the velocity in local x direction v_x and the steering angle of the front wheel δ_f. Since v_x is assumed constant, the only input is the steering angle. As the steering angle input, a sine function is used with an amplitude of 0.02 *rad*. A sine input with this amplitude is chosen such that the lateral acceleration a_y is under 5 m/s^2, which is the maximum physically possible value for the vehicle model used. [5]

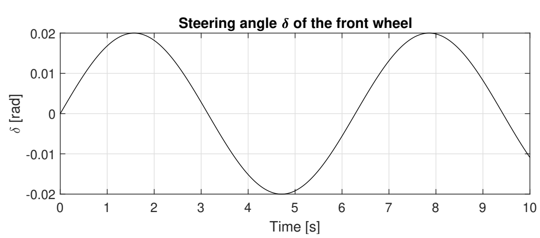

Figure 10: Input function to the preceding vehicle

The simulation time is set to 10 s. With the constant velocity of 100 km/h, a distance of roughly 275 m can be reached. This is sufficient to obtain a data set large enough to draw conclusions from.

As mentioned in subsection 2.2, noise is applied on the generated trajectory. The magnitude of this noise is chosen as +/- 0.5 m. This gives a significant distortion to the generated trajectory while being realistic [6].

The controller described in subsection 4.4 uses the curvature of the trajectory given. This curvature is calculated with the polynomial fitted trough the trajectory data points, which is also described in subsection 4.4. The length of this polynomial is determined with the look ahead time t_{la}. This look ahead time is assumed 1.5 s. [7]

5.2 Analysed variables

In order to be able to answer the research question, the two variables, the time step of the trajectory data and time step of polynomial fit trough this data, are varied within a range. For every different combination of time step in data and polynomial fit, three kinds of data sets are obtained and analysed. These are the accuracy of the polynomial fit trough the data, the lateral error $y_{e,i}$ and the angular error $\psi_{e,i}$. The accuracy of the polynomial fit is a measure for how the trajectory data points are described by a polynomial. With this data the possible cause of the lateral and angular error can be determined. The data of the lateral and angular error indicate the accuracy of the follower vehicle path on the leader vehicle trajectory.

In order to come up with these results, the ranges used for the time step of the trajectory data and time step of polynomial fit are chosen such that wide varieties of time steps are evaluated. For time steps of the trajectory data, a range between 0.1 s and 0.4 s are used. Using a time step lower than 0.1 s, the results did not have a significant difference between time steps. This can be seen in the figures in **??**. Varying the range between 0.1 s and 0.4 s will result in 15 to 4 data points respectively for the polynomial to fit trough. 4 is the minimum amount of data points needed in order for the cubic polynomial to fit trough. Time steps of polynomial fit are varied in the range between the smallest time step of trajectory data and the look ahead time t_{la}. Time steps in polynomial fit smaller than the time steps in data will result in the same polynomial, since the same data points are used to fit a polynomial trough. The maximum value of the range is the value where a polynomial is fitted behind the previous polynomial, so every data point is used only once to fit a polynomial trough. This is the case where a polynomial is fitted after passing the whole length of the previous polynomial, this length is determined by the look ahead time t_{la}.

5.3 Performance measure

In subsection 5.2 the three evaluated data sets are presented, in order to actually measure the performance of those data sets for every different combination of time steps, the root mean squared error (RMSE) is calculated. The RMSE is defined as the square root of the mean squared error. The mean squared error is defined as the mean of the sum of the difference between the observed and fitted values squared. This definition is shown in Equation 47.

$$MSE = \frac{1}{k}\sum_{i=1}^{k}(y_i - \hat{y}_i)^2 \tag{47}$$

Where y_i the observed values, $\hat{y}_i$ the estimated values and k the number of data points.

The MSE is a squared value of the data, the unit of the MSE is also a square of the unit of the data, so to come up with a value with the same unit as the data, the square root of the MSE is taken.

$$RMSE = \sqrt{MSE} \tag{48}$$

The RMSE is applied on all three the data sets. In order to measure the accuracy of the polynomial fit trough the trajectory data, the RMSE of every polynomial fitted trough the data for the given simulation length is calculated. These mean value is calculated for these RMSE values, this gives the mean RMSE value for every combination of time steps. The lower this mean RMSE value, the more accurate the data is described by the polynomials. The accuracy of the predicted path of the follower vehicle is determined by calculating the RMSE values for the lateral and angular error separately. The RMSE of the lateral and angular errors are determined by using the difference in lateral distance and angle between the trajectory and the follower vehicle. This difference is already calculated as the lateral and angular error values, which are the output values of the vehicle model described in subsection 4.3.

These RMSE values are calculated for the time steps within the ranges described in subsection 5.2 and visualized in subsection 5.4.

5.4 Results

In this subsection the results of the simulation as described in the subsections above, are shown and discussed.

5.4.1 Mean RMSE of polynomial fit

In Figure 11 the mean root mean square errors of different combinations of time step in data and polynomial fit are shown. On the x-axis the time step of the polynomial fits is shown and on the y-axis the mean RMSE values. Every individual graph represents an other time step in the trajectory data.

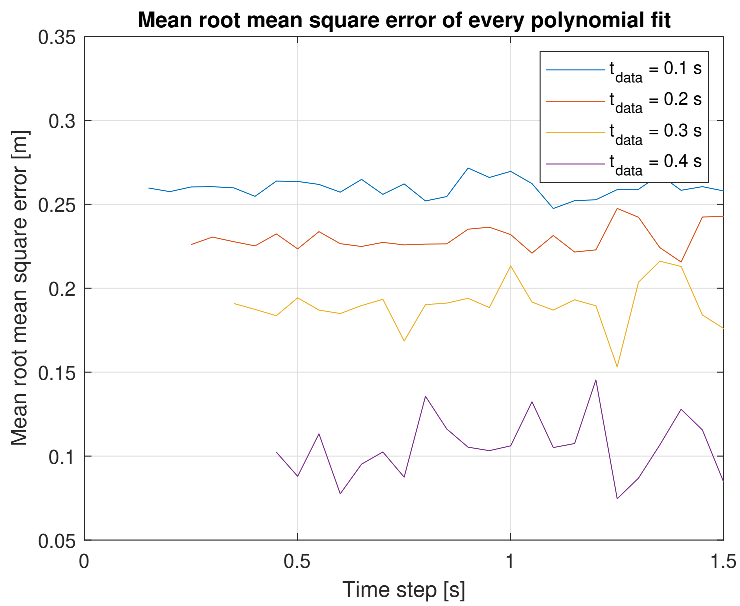

Figure 11: Mean RMSE of the polynomial fitting

Figure 11 shows that for different time steps of polynomial fit, the mean RMSE of a time step of the data points are tending to be constant. What can be seen clearly is that at lower the time steps of the data points, the higher the mean RMSE becomes. It does thus seem that the mean RMSE is only depending on the time step of the data points rather than the time step of the polynomial fit.

5.4.2 RMSE of the lateral error

In Figure 12 the root mean square errors of different combinations of time step in data and polynomial fit are shown. On the x-axis the time step of the polynomial fits is shown and on the y-axis the RMSE values of the lateral error. Every individual graph represents an other time step in the trajectory data.

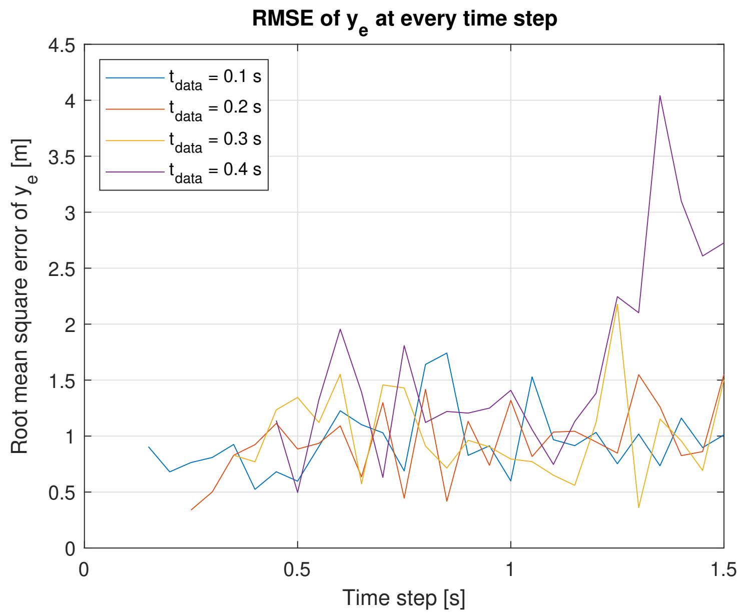

Figure 12: RMSE of the ye

In Figure 12 can be seen that there are no significant differences between different time steps in data and polynomial fit. The only remarkable thing is that for a time step in data of 0.4 s and for higher time steps in polynomial fit, the RMSE of the lateral error is significantly higher. For time steps in data between 0.1 s and 0.3 s for different time steps in polynomial fit, the RMSE of the lateral error is oscillating around 0.95 m. The results are values in the same order of magnitude as the dimensions of the vehicle. This does mean that these values do have a significant influence on the vehicle.

5.4.3 RMSE of the angular error

In Figure 13 the root mean square errors of different combinations of time step in data and polynomial fit are shown. On the x-axis the time step of the polynomial fits is shown and on the y-axis the RMSE values of the angular error. Every individual graph represents an other time step in the trajectory data.

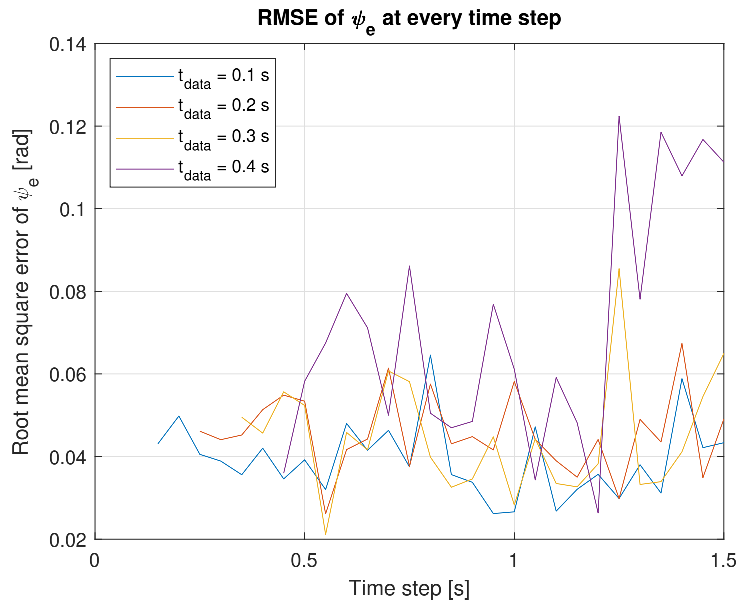

Figure 13: RMSE of the psie

Figure 13 shows the RMSE of the angular error. The range of these RMSE values is relatively small, this indicates that the different time steps of data and polynomial fit do have a relatively low impact on the accuracy of the follower path. For a time step in data of 0.4 s for all time steps of polynomial fit, the RMSE of the lateral error is significantly higher. For time steps in data between 0.1 s and 0.3 s for different time steps in polynomial fit, the RMSE of the lateral error also tend to oscillate.

6 Conclusions

In this report, the influence of different time steps in data points and different time steps in polynomial fits on the accuracy of the predicted path of the follower vehicle is investigated. This is done by simulating a leader and a follower vehicle with the linear single track model. The trajectory of the leader vehicle is communicated to the follower vehicle, which then fits multiple polynomials trough it, in order to predict a path which follows the given trajectory. This simulation is used to evaluate the accuracy of the path of the follower vehicle for different time steps in data points and polynomial fits. As mentioned in subsection 5.2, the results needed in order to answer the research question are the accuracy of the polynomial fit trough the data, the lateral error $y_{e,i}$ and the angular error $\psi_{e,i}$. Based on those results conclusions are drawn.

The RMSE values of the accuracy of the polynomial fit are not convincing. When looking at those results, it first seems that for higher time steps of the data points the overall accuracy will increase. However, there is less data to fit a polynomial with a constant length trough with higher time steps in data points. With less data to fit trough, it is more likely to find an accurate polynomial describing these data points which results in a lower mean RMSE value. Also with less data points, the noise on those data points will be more and more dominant. This does mean that the polynomial which describes the data points will not describe the trajectory of the leader vehicle as accurate as possible. It cannot be said that a higher time step of the data leads to a more accurate polynomial fit.

When comparing the lateral and angular errors for different time steps of the data points, the can be concluded that for a time step of 0.4 s, the lateral and angular errors are oscillating significantly more and with a higher absolute value than for lower time steps of the data points. This could be caused by that there are too little data points to fit the polynomials trough. As mentioned above, when there are less data points to fit trough, the noise on these data points will be more dominant. The follower path is then negatively influenced by the noise on the data, so the lateral and angular errors will increase.

Different time steps of the polynomial fits do not have a significant influence on the lateral and the angular errors. Also different time steps of the data points under 0.4 s do not have a significant influence on the lateral and angular errors. The difference in time step of the data points and polynomial fits do have more influence on the lateral error than on the angular error. This can be concluded since the order of magnitude of the lateral error is more equal to the order of magnitude of the dimensions of the vehicle than the order of magnitude of the angular error with respect to the order of the magnitude of the angles used.

Overall, different time steps of the data points and different time steps of the polynomial fits do not have a little influence on the accuracy of the predicted path of the follower vehicle. Using higher time steps of the data points, the noise on the data will be more and more dominant, such that higher lateral and angular errors will occur. Different time steps of the polynomial fits do not have a significant influence on the accuracy of the follower path for the time steps of the data points used.

7 Recommendations

There are several possible expansions and improvements that can be considered as a continuation of this project. As described in chapter 3 the linear single track model is used. This is a linear representation of a vehicle. When linearizing the dynamical behaviour, a number of assumptions were made. Every assumption made will result in inaccuracies in the results with respect to a real situation. In order to obtain more accurate results, a non-linear vehicle model can be used. This would give a more accurate representation of a real vehicle using less assumptions.

The behaviour of the follower vehicle is determined based on a sine input steering angle for defining the trajectory of the leading vehicle. A sine input steering angle would give a varying trajectory where no steady state situation will occur. Accuracies of the follower path could differ when this steering angle input is varied to a more alternating input, which would more resemble physical road situations. For example, a steady state situation with an instant gain to the steering angle.

Using other parameters could also have an influence on the accuracy of the follower path. In the simulation a longitudinal velocity of 100 km/h is assumed. Increasing or decreasing this velocity could result in a more or less accurate follower path. The controller used, uses a look ahead distance to determine errors with respect to the trajectory. This look ahead distance is assumed as 1.5 s. The influence of different look ahead distances to the accuracy of the follower path could be investigated on. The noise in this model is assumed as a white noise with an intensity of +/- 0.5 m. Differences in noise could have an influence on the accuracy of the follower path.

There has been done much research in different kinds of lateral control. In this project only one lateral controller is assumed. Other kinds of controllers could use different properties of the trajectory. The influence of time steps of the data or polynomial fit could be different, using other kinds of lateral controllers.

The main focus of this project was on lateral control, so using a constant longitudinal velocity. Expanding the lateral control strategy used with a longitudinal controller where the longitudinal velocity is controlled to follow a vehicle within a desired distance, will give a more accurate representation of a fully automated vehicle.

In subsubsection 5.4.1 it is shown that at lower the time steps of the data points, the higher the mean RMSE becomes. However the conclusion drawn from that is that it is unsure if a higher time step of the data points leads to a more accurate polynomial fit. With less data to fit trough, there is more likely to find an accurate polynomial describing that data. This does not mean that the polynomial is describing the trajectory as accurate as possible due to noise. There can be looked into what the influence of time steps of the data points including noise, on the accuracy of the polynomial fit with respect to the trajectory data points without noise.

Lastly, this project is executed completely theoretical by simulating the vehicle behaviour of a follower and leader vehicle. Real world experiments on the vehicle following can be executed. With these real world experiments the accuracies of the follower paths can be compared with the simulations vehicle models and assumptions.

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