Problem 87-11: Monotonicity of Bessel functions

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PROBLEMS AND SOLUTIONS

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All problems and solutions should be sent, typewritten in duplicate, to Muray S. Klamkin, Department of Mathematics, University of Alberta, Edmonton, Alberta, Canada T6G 2G1. *An asterisk placed beside a problem number indicates that the problem was submitted without solution. Proposers and solvers whose solutions are published will receive* 5 *reprints of the corresponding problem section. Other solvers will receive just one reprint provided a self-addressed stamped (U.S.A. or Canada) envelope is enclosed. Proposers and solvers desiring acknowledgment of their contributions should include a self-addressed stamped postcard (no stamp necessary outside the U.S.A. and Canada). Solutions should be received by December* 31, 1987.

PROBLEMS

Monotonicity of Bessel Functions

Problem 87-11*, *by* S. W. RIENSTRA (Katholieke Universiteit, Nijmegen, the Netherlands).

The eigenvalue equation related to a problem of sound propagation in hard-walled annular ducts is given by

$$x^2\{J_n'(x)Y_n'(xh)-Y_n'(x)J_n'(xh)\}=0$$

with $n=0, 1, 2, \cdots, 0<h<1$, where J_n' and Y_n' denote derivatives of Bessel functions, while h is the ratio between the inner and outer duct radii. We are interested in the solutions $x=\alpha(h)$ as a function of h. In particular, we want to show that

$$\frac{d\alpha}{dh}=\frac{\alpha f_n(\alpha)}{h\{f_n(\alpha h)-f_n(\alpha)\}} \quad \text{where} \quad f_n(x)=\frac{J_n'(x)^2+Y_n'(x)^2}{1-n^2/x^2},$$

is always finite or $f_n(\alpha h)-f_n(\alpha)\neq 0$.

If $n=0$, the latter is true since $f_0(x)=J_1(x)^2+Y_1(x)^2$ is a decreasing function [1]. It is also true for $n\geqq 1$ if $\alpha h<n$ and $\alpha>n$ ($\alpha\leqq n$ does not occur). If $\alpha h=n$, $d\alpha/dh=0$. Finally, it will also be true for the case $\alpha h>n$ if $f_n(x)$ is decreasing for $x>n$. In view of numerical evidence that this is so for $n=0, 1, \cdots, 100$, it is conjectured to be true. Prove or disprove.

REFERENCE

[1] G. N. WATSON, *A Treatise on the Theory of Bessel Functions*, Cambridge University Press, New York, 1948, p. 446.

A Column Vector Problem From Numerical Integration

Problem 87-12*, *by* M. M. CHAWLA (Indian Institute of Technology, New Delhi, India).

Let $w(x)>0$ be a weight function defined on $[a, b]$ with associated orthonormal polynomials $p_n^*(x)$, $n\geqq 0$. Let $\{x_k\}_{k=0}^N$ denote the $N+1$ Gaussian abscissae (the zeros of $p_{N+1}^*(x)$) and let $\{\lambda_k\}_{k=0}^N$ be the corresponding (Gaussian) weights for the $(N+1)$-point (Gaussian) quadrature formula for $\int_a^b w(x)f(x)\,dx$.