

Problem 87-11: Monotonicity of Bessel functions

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PROBLEMS AND SOLUTIONS

EDITED BY MURRAY S. KLAMKIN

COLLABORATING EDITORS: CECIL C. ROUSSEAU OTTO G. RUEHR

All problems and solutions should be sent, typewritten in duplicate, to Muray S. Klamkin, Department of Mathematics, University of Alberta, Edmonton, Alberta, Canada T6G 2G1. An asterisk placed beside a problem number indicates that the problem was submitted without solution. Proposers and solvers whose solutions are published will receive 5 reprints of the corresponding problem section. Other solvers will receive just one reprint provided a self-addressed stamped (U.S.A. or Canada) envelope is enclosed. Proposers and solvers desiring acknowledgment of their contributions should include a self-addressed stamped postcard (no stamp necessary outside the U.S.A. and Canada). Solutions should be received by December 31, 1987.

PROBLEMS

Monotonicity of Bessel Functions

*Problem 87-11**, by S. W. RIENSTRA (Katholieke Universiteit, Nijmegen, the Netherlands).

The eigenvalue equation related to a problem of sound propagation in hard-walled annular ducts is given by

$$x^2\{J'_n(x)Y'_n(xh) - Y'_n(x)J'_n(xh)\} = 0$$

with $n = 0, 1, 2, \dots$, $0 < h < 1$, where J'_n and Y'_n denote derivatives of Bessel functions, while h is the ratio between the inner and outer duct radii. We are interested in the solutions $x = \alpha(h)$ as a function of h . In particular, we want to show that

$$\frac{d\alpha}{dh} = \frac{\alpha f'_n(\alpha)}{h\{f_n(\alpha h) - f_n(\alpha)\}} \quad \text{where} \quad f_n(x) = \frac{J'_n(x)^2 + Y'_n(x)^2}{1 - n^2/x^2},$$

is always finite or $f_n(\alpha h) - f_n(\alpha) \neq 0$.

If $n = 0$, the latter is true since $f_0(x) = J_1(x)^2 + Y_1(x)^2$ is a decreasing function [1]. It is also true for $n \geq 1$ if $\alpha h < n$ and $\alpha > n$ ($\alpha \leq n$ does not occur). If $\alpha h = n$, $d\alpha/dh = 0$. Finally, it will also be true for the case $\alpha h > n$ if $f_n(x)$ is decreasing for $x > n$. In view of numerical evidence that this is so for $n = 0, 1, \dots, 100$, it is conjectured to be true. Prove or disprove.

REFERENCE

- [1] G. N. WATSON, *A Treatise on the Theory of Bessel Functions*, Cambridge University Press, New York, 1948, p. 446.

A Column Vector Problem From Numerical Integration

*Problem 87-12**, by M. M. CHAWLA (Indian Institute of Technology, New Delhi, India).

Let $w(x) > 0$ be a weight function defined on $[a, b]$ with associated orthonormal polynomials $p_n^*(x)$, $n \geq 0$. Let $\{x_k\}_{k=0}^N$ denote the $N + 1$ Gaussian abscissae (the zeros of $p_{N+1}^*(x)$) and let $\{\lambda_k\}_{k=0}^N$ be the corresponding (Gaussian) weights for the $(N + 1)$ -point (Gaussian) quadrature formula for $\int_a^b w(x)f(x) dx$.