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EINDHOVEN UNIVERSITY OF TECHNOLOGY

Department of Mathematics and Computing Science

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A mean value approach for a single server queue where part of the work can be performed during an idle period

by

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April 1983

A MEAN VALUE APPROACH FOR A SINGLE SERVER QUEUE WHERE PART OF THE WORK CAN BE PERFORMED DURING AN IDLE PERIOD

by

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Abstract.

This note considers a single server queue with Poisson arrivals. A customers service requirement consists of two parts. For the first customer in a busy period the first part of the work starts already before his arrival at the beginning of the preceding idle period. Using that Poisson arrivals see time averages, Little's formula and the expected residual life time formula, the mean response time and the mean number of customers in the system are obtained.

1. Introduction

This note deals with a single server queueing system with Poisson arrivals. The workload of a customer consists of two parts, the first of which will be called the preparatory part. For the first customer of a busy period the server starts to work on the preparatory part at the beginning of the preceding idle period, so already before the customer is present. (The system is said to be in the busy period if at least one customer is actually present in the system, so the server may be working on a preparatory part, while the system is called idle.) If the preparatory part is completed before the customer arrives the server stops and waits for the customer to come. Only then the server starts to work on the second part of the job. So during an idle period work is done for one customer only.

The situation considered here is a special case of the problem treated by Welch [1964] who considers an $M|G|1$ queue where the first customer of a busy period has a different service requirement than the other customers in that busy period.

The purpose of this note is to give a simple derivation of the mean response time and the mean number of customers in the queue by means of the following three basic results.

(i) Property PASTA (Poisson Arrivals See Time Averages).
(ii) Little's formula.
(iii) The expected residual lifetime formula.

The line of reasoning we use is very similar to the one used by Oliver [1964] to obtain the Pollaczek-Khintchin formula.

One may easily give the derivation for the more general model of Welch. However, we prefer to give it for the special case of the service requirement consisting of two parts because of the example we have in mind.

2. Example

An example of such a queueing system is the following. In a container-terminal ships have to be unloaded. A huge crane takes a container out of the hold of the ship, the preparatory part, and puts it on a trailer, the second part of the job. Think of the crane as being the server and of the trailers as being the customers. It is assumed that the trailers are not coming for a specific container, so the preparatory part can be done before the trailer arrives, and that the crane waits until a trailer arrives, if the preparatory part has been executed. If, finally, it is assumed that the trailers arrive according to a Poisson process, then we have just an example of the system described in the introduction.

3. Notations

The arrival rate of customers is λ. The work for each customer consists of two parts. A preparatory part with distribution function G_1 and a second part with distribution G_2. Further the first and second moments of G_1 and G_2 are denoted by $w_i = \int x \, d\, G_i(x)$, $w_i^{(2)} = \int x^2 d\, G_i(x)$, $I = 1,2$. The fraction of time the server is working on part 1 and part 2 of a job is denoted by $\rho_1 = \lambda w_1$ and $\rho_2 = \lambda w_2$. The fraction of time the server is not working is consequently $1 - \rho_1 - \rho_2$.

Finally, let L denote the average number of customers actually present in the system (in queue and in service) seen by a random observer and S the average response time of a customer, i.e. time in the system.

4. Derivation of mean queue length L and mean response time S

In order to obtain L and S we have to exploit property PASTA. PASTA, established under quite general conditions by Wolff [1982], implies:

(i) The average number of customers in the system seen by an arriving customer is equal to the average number of customers seen by a random observer, hence equal to L.

(ii) The probability that an arriving customer sees the server performing part i of the service of some customer is ρ_i, $i = 1,2$. The probability that the server is not working at all, so finished with a part 1 service and waiting for a customer to arrive, is $1 - \rho_1 - \rho_2$.

(iii) If an arriving customer finds the server busy with part i then the average remaining part i service time is $w_i^{(2)}/2w_i$ according to the well-known result of the mean excess lifetime of a renewal process.

These three observations enable us to write down the following expression for the expected response time of an arriving customer.

$$(1)\qquad S = (L+1)(w_1+w_2) + \rho_1\left(\frac{w_1^{(2)}}{2w_1} + w_2 - w_1 - w_2\right) + \rho_2\left(\frac{w_2^{(2)}}{2w_2} - w_1 - w_2\right) + $$
$$+ (1-\rho_1-\rho_2)(w_2 - w_1 - w_2)\ .$$

The first term on the right hand side is the mean response time if for none of the customers present nor for the arriving one any work has been done already. However, by points (ii) and (iii) we know that this need not be the case. The other three terms are the necessary corrections of the first one. For example, the second term takes into account that if the arriving customer finds the server busy with part 1 then for one of the customers (already present or just arrived) the remaining expected service time is $w_1^{(2)}/2w_1 + w_2$ instead of $w_1 + w_2$. The last term says that if upon arrival the server is idle, thus ready with part 1 and waiting for a customer to come, then the arriving customer has remaining expected service time w_2 instead of $w_1 + w_2$.

Further we have Little's formula to relate the mean number in the queue to the mean response time (cf. Little [1961]).

$$(2)\qquad L = \lambda S\ .$$

Solving (1) and (2) for L and S yields

$$S = \frac{1}{1-\rho_1-\rho_2}\left[(1-\rho_2)w_2 + \rho_1\frac{w_1^{(2)}}{2w_1} + \rho_2\frac{w_2^{(2)}}{2w_2}\right],$$

$$L = \frac{\lambda}{1-\rho_1-\rho_2}\left[(1-\rho_2)w_2 + \rho_1\frac{w_1^{(2)}}{2w_1} + \rho_2\frac{w_2^{(2)}}{2w_2}\right].$$

So the three aforementioned basic results: PASTA, Little's formula and mean excess lifetime, allow for an easy derivation of the mean response time and the mean number of customers in the queueing system.

5. References

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