

Concatenation of graphs

Citation for published version (APA):

Engelfriet, J., & Vereijken, J. J. (1994). *Concatenation of graphs*. (Computing science reports; Vol. 9442). Technische Universiteit Eindhoven.

Document status and date:

Published: 01/01/1994

Document Version:

Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

- A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
- The final author version and the galley proof are versions of the publication after peer review.
- The final published version features the final layout of the paper including the volume, issue and page numbers.

[Link to publication](#)

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal.

If the publication is distributed under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license above, please follow below link for the End User Agreement:

www.tue.nl/taverne

Take down policy

If you believe that this document breaches copyright please contact us at:

openaccess@tue.nl

providing details and we will investigate your claim.

Eindhoven University of Technology
Department of Mathematics and Computing Science

Concatenation of Graphs

by

J. Engelfriet and J.J. Vereijken

94/42

ISSN 0926-4515

All rights reserved

editors: prof.dr. J.C.M. Baeten
prof.dr. M. Rem

Computing Science Report 94/42
Eindhoven, September 1994

Concatenation of Graphs

Joost Engelfriet

Department of Computer Science, Leiden University,
P.O. Box 9512, NL-2300 RA Leiden, The Netherlands
email: engel@friet@rulwinw.leidenuniv.nl

Jan Joris Vereijken

Department of Computing Science, Eindhoven University of Technology,
P.O. Box 513, NL-5600 MB Eindhoven, The Netherlands
email: janjoris@acm.org

September 27, 1994

Abstract

An operation of concatenation is introduced for graphs. Then strings are viewed as expressions denoting graphs, and string languages are interpreted as graph languages. For a class K of string languages, $\text{Int}(K)$ is the class of all graph languages that are interpretations of languages from K . For the class REG of regular languages, $\text{Int}(\text{REG})$ might be called the class of regular graph languages; it equals the class of graph languages generated by linear Hyperedge Replacement Systems. Two characterizations are given of the largest class K' such that $\text{Int}(K') = \text{Int}(K)$.

Context-free graph languages are generated by context-free graph grammars, which are graph replacement systems. One of the most popular types of context-free graph grammar is the Hyperedge Replacement System, or HR grammar (see, e.g., [9]). A completely different way of generating graphs (introduced in [1]) is to select a number of graph operations, to generate a set of expressions (built from these operations), and to interpret the expressions as graphs. The set of expressions is generated by a classical context-free grammar generating strings (or a regular tree grammar). It is shown in [1] that, for a particular collection of graph operations, this new way of generating graphs is equivalent with the HR grammar. Other work on the generation of graphs through graph expressions is in, e.g., [2, 3, 4, 5, 11].

We introduce a new, natural operation on graphs (which is a simple variation of the graph operations in [1]). Due to its similarity to concatenation of strings, it is called *concatenation of graphs*. Together with the sum operation of graphs (as defined in [1]) and all constant graphs, a collection of graph operations is obtained that is simpler than the one in [1], but also has the power of the HR grammar (which is our first result).

Let us be a bit more precise. We consider the multi-pointed graphs (or multi-pointed hypergraphs) of [9]. For simplicity we will restrict ourselves in this paper to graphs, but all arguments also hold for hypergraphs. A multi-pointed graph is a directed, edge-labeled graph g with a designated sequence $\text{begin}(g)$ of "begin nodes" and a designated

sequence $\text{end}(g)$ of “end nodes”. If $\text{begin}(g)$ has length m and $\text{end}(g)$ has length n (with $m, n \in \mathbb{N} = \{0, 1, 2, \dots\}$), then g is said to be of *type* (m, n) and we write $\text{type}(g) = (m, n)$. From now on we will drop the adjective “multi-pointed”. As usual we consider both concrete and abstract graphs, where an abstract graph is an equivalence class of isomorphic concrete graphs. The isomorphisms between graphs g and h are the usual ones, which, additionally, should map $\text{begin}(g)$ to $\text{begin}(h)$, and $\text{end}(g)$ to $\text{end}(h)$. In particular, isomorphic graphs have the same type. Our operations are defined on abstract graphs. The set of abstract graphs will be denoted GR. A (typed) *graph language* is a subset L of GR such that all graphs in L have the same type (m, n) , also called the type of L , and denoted by $\text{type}(L) = (m, n)$.

If g and h are graphs with $\text{type}(g) = (k, m)$ and $\text{type}(h) = (m, n)$, then their *concatenation* $g \circ h$ is the graph obtained by first taking the disjoint union of g and h , and then identifying the i th end node of g with the i th begin node of h , for every $i \in \{1, \dots, m\}$. Moreover, $\text{begin}(g \circ h) = \text{begin}(g)$ and $\text{end}(g \circ h) = \text{end}(h)$, and so $\text{type}(g \circ h) = (k, n)$. Note that the concatenation of g and h is defined only when $\text{end}(g)$ and $\text{begin}(h)$ have the same length. The *sum* $g + h$ of arbitrary graphs g and h (as defined in [1]) is their disjoint union, with $\text{begin}(g + h) = \text{begin}(g) \cdot \text{begin}(h)$ and $\text{end}(g + h) = \text{end}(g) \cdot \text{end}(h)$, where \cdot denotes the usual concatenation of sequences. Intuitively, concatenation is sequential composition of graphs, and sum is parallel composition of graphs.

We investigate some simple properties of these graph operations: they lead to a strict monoidal category. The objects of this category are the natural numbers, and each (abstract) graph of type (m, n) is a morphism from m to n in this category. Concatenation is the composition of morphisms. For each n , the identity morphism from n to itself is the (abstract graph corresponding to the) discrete graph id_n with nodes $1, \dots, n$ and $\text{begin}(\text{id}_n) = \text{end}(\text{id}_n) = (1, \dots, n)$. The fact that the category is strict monoidal means that $\text{id}_{m+n} = \text{id}_m + \text{id}_n$ and $(g + h) \circ (g' + h') = (g \circ g') + (h \circ h')$ (assuming that $g \circ g'$ and $h \circ h'$ are defined).

Let Δ be the set of operators $\{\circ, +\} \cup \{c_g \mid g \in \text{GR}\}$, where \circ and $+$ are concatenation and sum of graphs, as discussed above, and c_g is a constant standing for the graph g . A *regular tree grammar over Δ* is an ordinary context-free grammar G such that the right-hand side of each production of G is a (well-typed) expression over the operators from Δ and the nonterminals of the grammar (which should be treated as constant operators, with a given type). Obviously, the language $L(G)$ generated by G is a set of expressions over Δ (and it is called a regular tree language). But G can also be viewed as a (context-free) graph grammar, generating the graph language $\text{val}(L(G)) = \{\text{val}(e) \mid e \in L(G)\}$, where the graph $\text{val}(e)$ is the value of the expression e . Let $\text{Val}(\text{REGT}) = \{\text{val}(L(G)) \mid G \text{ is a regular tree grammar over } \Delta\}$. Intuitively, $\text{Val}(\text{REGT})$ is the class of “values of regular tree languages” over Δ (where values of expressions are graphs).

As an example, consider the context-free grammar G_b that has one nonterminal A , with $\text{type}(A) = (1, 0)$, and two productions $A \rightarrow c_g \circ (A + A)$ and $A \rightarrow c_{g'}$, where g is the triangle of type $(1, 2)$ with set of nodes $\{x, y, z\}$, set of edges $\{(x, y), (x, z), (y, z)\}$, $\text{begin}(g) = \langle x \rangle$ and $\text{end}(g) = \langle y, z \rangle$, and g' is the graph of type $(1, 0)$ with one node u , no edges, $\text{begin}(g') = \langle u \rangle$ and $\text{end}(g')$ is the empty sequence. Then $\text{val}(L(G_b))$ is the set of all graphs of type $(1, 0)$ that are obtained from binary trees by connecting each pair of brothers by an additional edge; the sequence of begin nodes consists of the root of the binary tree. This graph language is therefore in $\text{Val}(\text{REGT})$.

Our first result is that generating graph languages in the above way is equivalent to

generating them with HR grammars. As observed above, this is a simple variant of the result of [1]. Let HR denote the class of all graph languages generated by HR grammars.

Theorem 1 $\text{Val}(\text{REGT}) = \text{HR}$.

Since concatenation of graphs is associative, strings can be viewed as expressions that denote graphs. Thus, as an even simpler variation of the above approach, we can use ordinary string grammars to generate graph languages. More generally, every class K of string languages defines a class $\text{Int}(K)$ of graph languages (where Int stands for “interpretation”, which is similar to Val above). Formally, a mapping $h : \Sigma \rightarrow \text{GR}$ that associates a graph with each symbol from an alphabet Σ , is extended to a (partial) function from Σ^* to GR by:

$$h(a_1 a_2 \cdots a_n) = h(a_1) \circ h(a_2) \circ \cdots \circ h(a_n)$$

where $a_i \in \Sigma$ for all i , and $n \geq 1$. Note that the extended h is partial because the types of the $h(a_i)$ may not fit (and h is also undefined for the empty string). Thus, the only “technical trouble” is that the concatenation of graphs is typed whereas the concatenation of strings is always possible. For a string language $L \subseteq \Sigma^*$, we define, as usual, the set of graphs $h(L) = \{g \in \text{GR} \mid g = h(w) \text{ for some } w \in L\}$; note that $h(L)$ need not be a graph language (with our particular meaning of the term) because not all graphs need have the same type. Now we define $\text{Int}(K) = \{h(L) \mid L \in K, h : \Sigma \rightarrow \text{GR} \text{ with } L \subseteq \Sigma^*, h(L) \text{ is a graph language}\}$. In other words, $\text{Int}(K)$ consists of all graph languages $h(L)$, where L is any language in K and h is any mapping from the symbols of L to graphs. Intuitively, h determines the interpretation of the symbols, and the concatenation of symbols is interpreted as concatenation of graphs.

To avoid trivialities we will, in what follows, and without always mentioning it, only consider classes K that are closed under (nondeterministic) sequential machine mappings (where a sequential machine is an ordinary finite automaton that, moreover, at each step outputs one symbol). Thus, in particular, K is closed under intersection with regular languages and under finite substitutions. Note that every semi-AFL is closed under sequential machine mappings.

The first class K of interest is the class REG of regular languages. An example of a graph language in $\text{Int}(\text{REG})$, of type $(1, 0)$, is $h(ab^*)$ with $h(a) = g$ and $h(b) = g'$, where g and g' are as in the example grammar G_b , except that now g is of type $(1, 1)$ and has $\text{end}(g) = \langle z \rangle$. This graph language is the subset of $\text{val}(L(G_b))$ consisting of all sequences of concatenated triangles, where two consecutive triangles have one node in common (and each node belongs to at most two triangles).

Our second result says that the graph languages that are interpretations of a regular language are precisely those that can be generated by *linear* HR grammars, where “linear” means that there is at most one nonterminal in each right-hand side of a production of the HR grammar. This can be proved by comparing right-linear string grammars (which have productions of the form $A \rightarrow aB$ and $A \rightarrow a$) with linear HR grammars (of which, roughly speaking, the productions can be written as $A \rightarrow h(a) \circ B$ and $A \rightarrow h(a)$ for an appropriate h). Let LIN-HR denote the class of graph languages generated by linear HR grammars.

Theorem 2 $\text{Int}(\text{REG}) = \text{LIN-HR}$.

Before continuing with other classes K , we first consider a third characterization of the class $\text{Int}(\text{REG})$, corresponding to the characterization of REG by regular expressions. The operation of graph concatenation is extended to graph languages L and L' in the usual way: if $\text{type}(L) = (k, m)$ and $\text{type}(L') = (m, n)$, then their concatenation is defined by $L \circ L' = \{g \circ g' \mid g \in L, g' \in L'\}$. Then, in the obvious way, the (Kleene) star of a graph language is defined (by iterated concatenation): for a graph language L with $\text{type}(L) = (k, k)$ for some $k \in \mathbb{N}$, $L^* = \bigcup_{n \in \mathbb{N}} L^n$ where $L^n = L \circ \dots \circ L$ (n times) for $n \geq 1$, and $L^0 = \{\text{id}_k\}$. Also, $L^\oplus = L^* - \{\text{id}_k\}$ is the (Kleene) plus of L . Finally, the union $L \cup L'$ of two graph languages L and L' is defined only when $\text{type}(L) = \text{type}(L')$ (otherwise it would not be a graph language). Thus, the operations of union, concatenation, and star are also typed operations on graph languages (as opposed to the case of string languages for which they are always defined). Let $\text{REX}(\cup, \circ, *, \text{SING})$ denote the smallest class of graph languages containing the empty graph language and all singleton graph languages, and closed under the operations union, concatenation, and star. Thus, it is the class of all graph languages that can be denoted by (the usual) regular expressions, where the symbols of the alphabet denote singleton graph languages. From this it should be clear that it equals the class $\text{Int}(\text{REG})$ of interpretations of regular languages. In the proof one has to cope with the “technical trouble” of typing, in particular with the empty string. Note that, for a graph language L with $\text{type}(L) = (k, k)$, $L^* = L^\oplus \cup \{\text{id}_k\}$; from this it should be clear that $\text{REX}(\cup, \circ, *, \text{SING}) = \text{REX}(\cup, \circ, \oplus, \text{SING})$, which solves the problem with the empty string.

Theorem 3 $\text{Int}(\text{REG}) = \text{REX}(\cup, \circ, *, \text{SING})$.

By Theorems 2 and 3, $\text{LIN-HR} = \text{REX}(\cup, \circ, *, \text{SING})$. This suggests that the class LIN-HR of linear HR graph languages might be called the class of “regular” graph languages, because they can be denoted by regular expressions. The above characterization still holds after adding the sum operation (extended to graph languages in the usual way). This is because of the following simple reason.

Lemma 4 *If $\text{Int}(K)$ is closed under concatenation, then it is closed under sum.*

Proof. We first show that if M is in $\text{Int}(K)$ then so is $M + \{\text{id}_k\}$ for every k . Let $M = h(L)$ for some $L \in K$. Define $h'(a) = h(a) + \text{id}_k$ for every symbol a . Then $h'(a_1 \dots a_n) =$

$$(h(a_1) + \text{id}_k) \circ \dots \circ (h(a_n) + \text{id}_k) = (h(a_1) \circ \dots \circ h(a_n)) + (\text{id}_k \circ \dots \circ \text{id}_k)$$

because of strict monoidality, and the last expression equals $h(a_1 \dots a_n) + \text{id}_k$. This implies that $h'(L) = h(L) + \{\text{id}_k\} = M + \{\text{id}_k\}$. Similarly it can be shown that $\{\text{id}_k\} + M$ is in $\text{Int}(K)$.

Now, for arbitrary graph languages M and M' with $\text{type}(M) = (m, n)$ and $\text{type}(M') = (m', n')$, $M + M' = (M \circ \{\text{id}_n\}) + (\{\text{id}_{m'}\} \circ M') = (M + \{\text{id}_{m'}\}) \circ (\{\text{id}_n\} + M')$ by strict monoidality. Now, by the above, and the fact that $\text{Int}(K)$ is closed under \circ , $M + M'$ is in $\text{Int}(K)$. \square

It turns out that, if we allow $+$ in our regular expressions, then we do not need all singleton graph languages to start with, but only a “small” number of them, with very simple graphs only. In fact, graphs can be decomposed into very simple graphs, using concatenation and sum. Assume that the edge labels of our graphs are taken from a given set A .

For $a \in A$ let $\text{gr}(a)$ be the graph of type $(1, 1)$ with two nodes x and y , an a -labeled edge from x to y , $\text{begin}(\text{gr}(a)) = \langle x \rangle$, and $\text{end}(\text{gr}(a)) = \langle y \rangle$. For $m, n \in \mathbb{N}$, let $E_{m,n}$ be the graph of type (m, n) with one node x , no edges, $\text{begin}(E_{m,n}) = \langle x, \dots, x \rangle$ (m times), and $\text{end}(E_{m,n}) = \langle x, \dots, x \rangle$ (n times). Finally, let X be the graph of type $(2, 2)$ with two nodes x and y , no edges, $\text{begin}(X) = \langle x, y \rangle$, and $\text{end}(X) = \langle y, x \rangle$. Note also that id_0 is the empty graph. Now define the set of graphs $C_0 = \{\text{gr}(a) \mid a \in A\} \cup \{\text{id}_0, X, E_{0,1}, E_{1,0}, E_{1,2}, E_{2,1}\}$, and let $\text{REX}(\cup, \circ, *, +, \text{SING}(C_0))$ denote the smallest class of graph languages containing the empty graph language and all singleton graph languages with a graph from C_0 as element, and closed under the operations union, concatenation, star, and sum.

Theorem 5 $\text{REX}(\cup, \circ, *, \text{SING}) = \text{REX}(\cup, \circ, *, +, \text{SING}(C_0))$.

From Theorem 2 we know that $\text{Int}(\text{REG}) = \text{LIN-HR}$. It is not difficult to prove that also $\text{Int}(\text{LIN}) = \text{LIN-HR}$, where LIN is the (usual) class of languages generated by linear context-free grammars. This suggests that for graph languages the notions “regular” and “linear” coincide, as opposed to the string case. Even $\text{Int}(\text{DB}) = \text{LIN-HR}$, where DB is the class of derivation bounded context-free languages. One now wonders how much larger the class K can be made without getting a larger class $\text{Int}(K)$.

It is easy to see that for every given class K there is always a largest class K' such that $\text{Int}(K') = \text{Int}(K)$. We will call this the *extension of K* , denoted $\text{Ext}(K)$. In fact, $\text{Ext}(K) = \{L \mid \text{for every } h : \Sigma \rightarrow \text{GR with } L \subseteq \Sigma^*, \text{ if } h(L) \text{ is a graph language, then } h(L) \in \text{Int}(K)\}$. In the next theorem we give a characterization of $\text{Ext}(K)$. For a class G of graph languages, let $\text{Str}(G)$ denote the class of string languages L such that $\text{gr}(L)$ is in G . Here, $\text{gr}(L) = \{\text{gr}(w) \mid w \in L\}$, and, for a string $w = a_1 \cdots a_n$, $\text{gr}(w)$ is the graph of type $(1, 1)$ with nodes $1, \dots, n+1$, an a_i -labeled edge from i to $i+1$ for every $1 \leq i \leq n$, begin node 1, and end node $n+1$. Thus, $\text{gr}(w)$ encodes w in the obvious way: it is a path with the symbols of w as edge labels. Recall that we assume K to be closed under sequential machine mappings.

Theorem 6 $\text{Ext}(K) = \text{Str}(\text{Int}(K))$.

In the proof of this theorem it has to be shown that $\text{Int}(\text{Str}(\text{Int}(K))) = \text{Int}(K)$, and that if $\text{Int}(K') = \text{Int}(K)$ then $K' \subseteq \text{Str}(\text{Int}(K))$. The first statement is the most involved one to show. The second statement is easy to see. In fact, $\text{gr}(K') \subseteq \text{Int}(K')$: just take $h(a) = \text{gr}(a)$. Then we get $K' \subseteq \text{Str}(\text{gr}(K')) \subseteq \text{Str}(\text{Int}(K')) = \text{Str}(\text{Int}(K))$.

As a corollary of Theorem 6 we obtain that for arbitrary K and K' , both closed under sequential machine mappings, $\text{Int}(K) = \text{Int}(K')$ if and only if $\text{Str}(\text{Int}(K)) = \text{Str}(\text{Int}(K'))$. This means that the graph generating power of K is completely determined by its string generating power (with strings coded as graphs by the mapping gr).

Next we will aim at another characterization of $\text{Ext}(K)$. First we consider *controlled* linear HR grammars, in the obvious sense. Let G be a linear HR grammar with (finite) set of productions P , and let C be a string language over P (where P is viewed as an alphabet). The graph language generated by G under control C is the set of all graphs g for which there is a derivation $g_0 \Rightarrow_{p_1} g_1 \Rightarrow_{p_2} g_2 \cdots \Rightarrow_{p_n} g_n$ with $g_n = g$, g_0 is the axiom of G , and such that the string $p_1 p_2 \cdots p_n$ is in C . Of course, \Rightarrow_p denotes a derivation step of G that uses production $p \in P$. Thus, the control language C specifies the sequences of productions that the grammar G is allowed to use in its derivations.

For a class K of string languages, we denote by $\text{LIN-HR}(K)$ the class of graph languages generated by linear HR grammars under a control language from K . Generalizing Theorem 2 and its proof we obtain the next result.

Theorem 7 $\text{Int}(K) = \text{LIN-HR}(K)$.

By $2\text{DGSM}(K)$ we denote the class of images of languages from K under 2dgsms mappings, i.e., $2\text{DGSM}(K) = \{f(L) \mid f \text{ is a 2dgsms mapping and } L \in K\}$. A 2dgsms (i.e., a two-way deterministic generalized sequential machine) is a deterministic finite automaton that can move in two directions on its input tape (with endmarkers), and outputs a (possibly empty) string at each step. Generalizing the proof in [6] that $\text{Str}(\text{LIN-HR})$ equals the class of output languages of 2dgsms mappings, we prove the next result.

Theorem 8 $\text{Str}(\text{LIN-HR}(K)) = 2\text{DGSM}(K)$.

Taking these (quite obvious) generalizations together, we obtain from Theorems 6, 7, and 8 our second characterization of the class $\text{Ext}(K)$.

Theorem 9 $\text{Ext}(K) = 2\text{DGSM}(K)$.

This characterization allows us to use known formal language theoretic results regarding $2\text{DGSM}(K)$ to investigate $\text{Int}(K)$. In fact, quite a lot is known about the class $2\text{DGSM}(K)$, see, e.g., [7]. As an example, it equals the class of languages generated by K -controlled ETOL systems of finite index.

The other way around, we note that from $\text{Int}(\text{Str}(\text{Int}(K))) = \text{Int}(K)$ follows that $\text{Str}(\text{Int}(\text{Str}(\text{Int}(K)))) = \text{Str}(\text{Int}(K))$, i.e., the known result that $2\text{DGSM}(2\text{DGSM}(K)) = 2\text{DGSM}(K)$. This shows that $\text{Ext}(K)$ is closed under 2dgsms mappings.

Thus, for $K = \text{REG}$, $\text{Ext}(K)$ is the class $2\text{DGSM}(\text{REG})$ of output languages of 2dgsms mappings. Since it is well known that the class DB of derivation-bounded context-free languages is contained in $2\text{DGSM}(\text{REG})$ (see, e.g., [10]), this implies the previously mentioned result that $\text{Int}(\text{LIN}) = \text{Int}(\text{DB}) = \text{Int}(\text{REG})$. Also, since there is a context-free language not in $2\text{DGSM}(\text{REG})$, see [8, 7], $\text{Int}(\text{REG})$ is properly included in $\text{Int}(\text{CF})$, where CF is the class of context-free languages. We finally note that $\text{Int}(\text{CF})$ is properly included in HR, the class of graph languages generated by HR grammars. The inclusion follows from the fact that context-free grammars generating graph expressions built from concatenation and all constant graphs, can be simulated by HR grammars, by Theorem 1. Properness of the inclusion follows from the more general fact that for any class K , $\text{Int}(K)$ contains graph languages of bounded path-width only. Thus, the set of all binary trees (which is in HR) does not belong to any $\text{Int}(K)$.

References

- [1] M. Bauderon, B. Courcelle; Graph expressions and graph rewritings, *Math. Syst. Theory* 20 (1987), 83-127.
- [2] B. Courcelle; Graph rewriting: an algebraic and logic approach, in *Handbook of Theoretical Computer Science, Vol. B* (J. van Leeuwen, ed.), Elsevier, 1990, pp. 193-242.
- [3] B. Courcelle, J. Engelfriet, G. Rozenberg; Handle-rewriting hypergraph languages, *JCSS* 46 (1993), 218-270.

- [4] F. Drewes; Transducibility - symbolic computation by tree-transductions, University of Bremen, Bericht Nr. 2/93, 1993.
- [5] J. Engelfriet; Graph grammars and tree transducers, Proc. CAAP'94 (S. Tison, ed.), Lecture Notes in Computer Science 787, Springer-Verlag, Berlin, 1994, pp. 15-36.
- [6] J. Engelfriet, L.M. Heyker; The string generating power of context-free hypergraph grammars, JCSS 43 (1991), 328-360.
- [7] J. Engelfriet, G. Rozenberg, G. Slutzki; Tree transducers, L systems, and two-way machines, JCSS 20 (1980), 150-202.
- [8] S. Greibach; One-way finite visit automata, Theor. Comput. Sci. 6 (1978), 175-221.
- [9] A. Habel; *Hyperedge replacement: grammars and languages*, Lecture Notes in Computer Science 643, Springer-Verlag, Berlin, 1992.
- [10] V. Rajlich; Absolutely parallel grammars and two-way finite state transducers, JCSS 6 (1972), 324-342.
- [11] J.J. Vereijken; *Graph Grammars and Operations on Graphs*, Master's Thesis, Leiden University, 1993.

In this series appeared:

- | | | |
|-------|---|--|
| 91/01 | D. Alstein | Dynamic Reconfiguration in Distributed Hard Real-Time Systems, p. 14. |
| 91/02 | R.P. Nederpelt
H.C.M. de Swart | Implication. A survey of the different logical analyses "if...,then...", p. 26. |
| 91/03 | J.P. Katoen
L.A.M. Schoenmakers | Parallel Programs for the Recognition of P -invariant Segments, p. 16. |
| 91/04 | E. v.d. Sluis
A.F. v.d. Stappen | Performance Analysis of VLSI Programs, p. 31. |
| 91/05 | D. de Reus | An Implementation Model for GOOD, p. 18. |
| 91/06 | K.M. van Hee | SPECIFICATIEMETHODEN, een overzicht, p. 20. |
| 91/07 | E.Poll | CPO-models for second order lambda calculus with recursive types and subtyping, p. 49. |
| 91/08 | H. Schepers | Terminology and Paradigms for Fault Tolerance, p. 25. |
| 91/09 | W.M.P.v.d.Aalst | Interval Timed Petri Nets and their analysis, p.53. |
| 91/10 | R.C.Backhouse
P.J. de Bruin
P. Hoogendijk
G. Malcolm
E. Voermans
J. v.d. Woude | POLYNOMIAL RELATORS, p. 52. |
| 91/11 | R.C. Backhouse
P.J. de Bruin
G.Malcolm
E.Voermans
J. van der Woude | Relational Catamorphism, p. 31. |
| 91/12 | E. van der Sluis | A parallel local search algorithm for the travelling salesman problem, p. 12. |
| 91/13 | F. Rietman | A note on Extensionality, p. 21. |
| 91/14 | P. Lemmens | The PDB Hypermedia Package. Why and how it was built, p. 63. |
| 91/15 | A.T.M. Aerts
K.M. van Hee | Eldorado: Architecture of a Functional Database Management System, p. 19. |
| 91/16 | A.J.J.M. Marcelis | An example of proving attribute grammars correct: the representation of arithmetical expressions by DAGs, p. 25. |

- 91/17 A.T.M. Aerts
P.M.E. de Bra
K.M. van Hee
Transforming Functional Database Schemes to Relational Representations, p. 21.
- 91/18 Rik van Geldrop
Transformational Query Solving, p. 35.
- 91/19 Erik Poll
Some categorical properties for a model for second order lambda calculus with subtyping, p. 21.
- 91/20 A.E. Eiben
R.V. Schuwer
Knowledge Base Systems, a Formal Model, p. 21.
- 91/21 J. Coenen
W.-P. de Roever
J.Zwiers
Assertional Data Reification Proofs: Survey and Perspective, p. 18.
- 91/22 G. Wolf
Schedule Management: an Object Oriented Approach, p. 26.
- 91/23 K.M. van Hee
L.J. Somers
M. Voorhoeve
Z and high level Petri nets, p. 16.
- 91/24 A.T.M. Aerts
D. de Reus
Formal semantics for BRM with examples, p. 25.
- 91/25 P. Zhou
J. Hooman
R. Kuiper
A compositional proof system for real-time systems based on explicit clock temporal logic: soundness and completeness, p. 52.
- 91/26 P. de Bra
G.J. Houben
J. Paredaens
The GOOD based hypertext reference model, p. 12.
- 91/27 F. de Boer
C. Palamidessi
Embedding as a tool for language comparison: On the CSP hierarchy, p. 17.
- 91/28 F. de Boer
A compositional proof system for dynamic process creation, p. 24.
- 91/29 H. Ten Eikelder
R. van Geldrop
Correctness of Acceptor Schemes for Regular Languages, p. 31.
- 91/30 J.C.M. Baeten
F.W. Vaandrager
An Algebra for Process Creation, p. 29.
- 91/31 H. ten Eikelder
Some algorithms to decide the equivalence of recursive types, p. 26.
- 91/32 P. Struik
Techniques for designing efficient parallel programs, p. 14.
- 91/33 W. v.d. Aalst
The modelling and analysis of queueing systems with QNM-ExSpect, p. 23.
- 91/34 J. Coenen
Specifying fault tolerant programs in deontic logic, p. 15.

91/35	F.S. de Boer J.W. Klop C. Palamidessi	Asynchronous communication in process algebra, p. 20.
92/01	J. Coenen J. Zwiers W.-P. de Roever	A note on compositional refinement, p. 27.
92/02	J. Coenen J. Hooman	A compositional semantics for fault tolerant real-time systems, p. 18.
92/03	J.C.M. Baeten J.A. Bergstra	Real space process algebra, p. 42.
92/04	J.P.H.W.v.d.Eijnde	Program derivation in acyclic graphs and related problems, p. 90.
92/05	J.P.H.W.v.d.Eijnde	Conservative fixpoint functions on a graph, p. 25.
92/06	J.C.M. Baeten J.A. Bergstra	Discrete time process algebra, p.45.
92/07	R.P. Nederpelt	The fine-structure of lambda calculus, p. 110.
92/08	R.P. Nederpelt F. Kamareddine	On stepwise explicit substitution, p. 30.
92/09	R.C. Backhouse	Calculating the Warshall/Floyd path algorithm, p. 14.
92/10	P.M.P. Rambags	Composition and decomposition in a CPN model, p. 55.
92/11	R.C. Backhouse J.S.C.P.v.d.Woude	Demonic operators and monotype factors, p. 29.
92/12	F. Kamareddine	Set theory and nominalisation, Part I, p.26.
92/13	F. Kamareddine	Set theory and nominalisation, Part II, p.22.
92/14	J.C.M. Baeten	The total order assumption, p. 10.
92/15	F. Kamareddine	A system at the cross-roads of functional and logic programming, p.36.
92/16	R.R. Seljée	Integrity checking in deductive databases; an exposition, p.32.
92/17	W.M.P. van der Aalst	Interval timed coloured Petri nets and their analysis, p. 20.
92/18	R.Nederpelt F. Kamareddine	A unified approach to Type Theory through a refined lambda-calculus, p. 30.
92/19	J.C.M.Baeten J.A.Bergstra S.A.Smolka	Axiomatizing Probabilistic Processes: ACP with Generative Probabilities, p. 36.
92/20	F.Kamareddine	Are Types for Natural Language? P. 32.

92/21	F.Kamareddine	Non well-foundedness and type freeness can unify the interpretation of functional application, p. 16.
92/22	R. Nederpelt F.Kamareddine	A useful lambda notation, p. 17.
92/23	F.Kamareddine E.Klein	Nominalization, Predication and Type Containment, p. 40.
92/24	M.Codish D.Dams Eyal Yardeni	Bottom-up Abstract Interpretation of Logic Programs, p. 33.
92/25	E.Poll	A Programming Logic for F_{ω} , p. 15.
92/26	T.H.W.Beelen W.J.J.Stut P.A.C.Verkoulen	A modelling method using MOVIE and SimCon/ExSpect, p. 15.
92/27	B. Watson G. Zwaan	A taxonomy of keyword pattern matching algorithms, p. 50.
93/01	R. van Geldrop	Deriving the Aho-Corasick algorithms: a case study into the synergy of programming methods, p. 36.
93/02	T. Verhoeff	A continuous version of the Prisoner's Dilemma, p. 17
93/03	T. Verhoeff	Quicksort for linked lists, p. 8.
93/04	E.H.L. Aarts J.H.M. Korst P.J. Zwietering	Deterministic and randomized local search, p. 78.
93/05	J.C.M. Baeten C. Verhoef	A congruence theorem for structured operational semantics with predicates, p. 18.
93/06	J.P. Veltkamp	On the unavoidability of metastable behaviour, p. 29
93/07	P.D. Moerland	Exercises in Multiprogramming, p. 97
93/08	J. Verhoosel	A Formal Deterministic Scheduling Model for Hard Real-Time Executions in DEDOS, p. 32.
93/09	K.M. van Hee	Systems Engineering: a Formal Approach Part I: System Concepts, p. 72.
93/10	K.M. van Hee	Systems Engineering: a Formal Approach Part II: Frameworks, p. 44.
93/11	K.M. van Hee	Systems Engineering: a Formal Approach Part III: Modeling Methods, p. 101.
93/12	K.M. van Hee	Systems Engineering: a Formal Approach Part IV: Analysis Methods, p. 63.
93/13	K.M. van Hee	Systems Engineering: a Formal Approach

- 93/14 J.C.M. Baeten
J.A. Bergstra
Part V: Specification Language, p. 89.
On Sequential Composition, Action Prefixes and
Process Prefix, p. 21.
- 93/15 J.C.M. Baeten
J.A. Bergstra
R.N. Bol
A Real-Time Process Logic, p. 31.
- 93/16 H. Schepers
J. Hooman
A Trace-Based Compositional Proof Theory for
Fault Tolerant Distributed Systems, p. 27
- 93/17 D. Alstein
P. van der Stok
Hard Real-Time Reliable Multicast in the DEDOS system,
p. 19.
- 93/18 C. Verhoef
A congruence theorem for structured operational
semantics with predicates and negative premises, p. 22.
- 93/19 G-J. Houben
The Design of an Online Help Facility for ExSpect, p.21.
- 93/20 F.S. de Boer
A Process Algebra of Concurrent Constraint Program-
ming, p. 15.
- 93/21 M. Codish
D. Dams
G. Filé
M. Bruynooghe
Freeness Analysis for Logic Programs - And Correct-
ness?, p. 24.
- 93/22 E. Poll
A Typechecker for Bijective Pure Type Systems, p. 28.
- 93/23 E. de Kogel
Relational Algebra and Equational Proofs, p. 23.
- 93/24 E. Poll and Paula Severi
Pure Type Systems with Definitions, p. 38.
- 93/25 H. Schepers and R. Gerth
A Compositional Proof Theory for Fault Tolerant Real-
Time Distributed Systems, p. 31.
- 93/26 W.M.P. van der Aalst
Multi-dimensional Petri nets, p. 25.
- 93/27 T. Kloks and D. Kratsch
Finding all minimal separators of a graph, p. 11.
- 93/28 F. Kamareddine and
R. Nederpelt
A Semantics for a fine λ -calculus with de Bruijn indices,
p. 49.
- 93/29 R. Post and P. De Bra
GOLD, a Graph Oriented Language for Databases, p. 42.
- 93/30 J. Deogun
T. Kloks
D. Kratsch
H. Müller
On Vertex Ranking for Permutation and Other Graphs,
p. 11.
- 93/31 W. Körver
Derivation of delay insensitive and speed independent
CMOS circuits, using directed commands and
production rule sets, p. 40.
- 93/32 H. ten Eikelder and
H. van Geldrop
On the Correctness of some Algorithms to generate Finite
Automata for Regular Expressions, p. 17.

- 93/33 L. Loyens and J. Moonen ILIAS, a sequential language for parallel matrix computations, p. 20.
- 93/34 J.C.M. Baeten and J.A. Bergstra Real Time Process Algebra with Infinitesimals, p.39.
- 93/35 W. Ferrer and P. Severi Abstract Reduction and Topology, p. 28.
- 93/36 J.C.M. Baeten and J.A. Bergstra Non Interleaving Process Algebra, p. 17.
- 93/37 J. Brunekreef
J-P. Katoen
R. Koymans
S. Mauw Design and Analysis of Dynamic Leader Election Protocols in Broadcast Networks, p. 73.
- 93/38 C. Verhoef A general conservative extension theorem in process algebra, p. 17.
- 93/39 W.P.M. Nuijten
E.H.L. Aarts
D.A.A. van Erp
Taalman Kip
K.M. van Hee Job Shop Scheduling by Constraint Satisfaction, p. 22.
- 93/40 P.D.V. van der Stok
M.M.M.P.J. Claessen
D. Alstein A Hierarchical Membership Protocol for Synchronous Distributed Systems, p. 43.
- 93/41 A. Bijlsma Temporal operators viewed as predicate transformers, p. 11.
- 93/42 P.M.P. Rambags Automatic Verification of Regular Protocols in P/T Nets, p. 23.
- 93/43 B.W. Watson A taxonomy of finite automata construction algorithms, p. 87.
- 93/44 B.W. Watson A taxonomy of finite automata minimization algorithms, p. 23.
- 93/45 E.J. Luit
J.M.M. Martin A precise clock synchronization protocol,p.
- 93/46 T. Kloks
D. Kratsch
J. Spinrad Treewidth and Patwidth of Cocomparability graphs of Bounded Dimension, p. 14.
- 93/47 W. v.d. Aalst
P. De Bra
G.J. Houben
Y. Komatzky Browsing Semantics in the "Tower" Model, p. 19.
- 93/48 R. Gerth Verifying Sequentially Consistent Memory using Interface Refinement, p. 20.

- 94/01 P. America
M. van der Kammen
R.P. Nederpelt
O.S. van Roosmalen
H.C.M. de Swart The object-oriented paradigm, p. 28.
- 94/02 F. Kamareddine
R.P. Nederpelt Canonical typing and Π -conversion, p. 51.
- 94/03 L.B. Hartman
K.M. van Hee Application of Marcov Decision Prozesse to Search Problems, p. 21.
- 94/04 J.C.M. Baeten
J.A. Bergstra Graph Isomorphism Models for Non Interleaving Process Algebra, p. 18.
- 94/05 P. Zhou
J. Hooman Formal Specification and Compositional Verification of an Atomic Broadcast Protocol, p. 22.
- 94/06 T. Basten
T. Kunz
J. Black
M. Coffin
D. Taylor Time and the Order of Abstract Events in Distributed Computations, p. 29.
- 94/07 K.R. Apt
R. Bol Logic Programming and Negation: A Survey, p. 62.
- 94/08 O.S. van Roosmalen A Hierarchical Diagrammatic Representation of Class Structure, p. 22.
- 94/09 J.C.M. Baeten
J.A. Bergstra Process Algebra with Partial Choice, p. 16.
- 94/10 T. verhoeff The testing Paradigm Applied to Network Structure. p. 31.
- 94/11 J. Peleska
C. Huizing
C. Petersohn A Comparison of Ward & Mellor's Transformation Schema with State- & Activitycharts, p. 30.
- 94/12 T. Kloks
D. Kratsch
H. Müller Dominoes, p. 14.
- 94/13 R. Seljée A New Method for Integrity Constraint checking in Deductive Databases, p. 34.
- 94/14 W. Peremans Ups and Downs of Type Theory, p. 9.
- 94/15 R.J.M. Vaessens
E.H.L. Aarts
J.K. Lenstra Job Shop Scheduling by Local Search, p. 21.
- 94/16 R.C. Backhouse
H. Doornbos Mathematical Induction Made Computational, p. 36.
- 94/17 S. Mauw
M.A. Reniers An Algebraic Semantics of Basic Message Sequence Charts, p. 9.

- 94/18 F. Kamareddine
R. Nederpelt Refining Reduction in the Lambda Calculus, p. 15.
- 94/19 B.W. Watson The performance of single-keyword and multiple-keyword pattern matching algorithms, p. 46.
- 94/20 R. Bloo
F. Kamareddine
R. Nederpelt Beyond β -Reduction in Church's $\lambda \rightarrow$, p. 22.
- 94/21 B.W. Watson An introduction to the Fire engine: A C++ toolkit for Finite automata and Regular Expressions.
- 94/22 B.W. Watson The design and implementation of the FIRE engine: A C++ toolkit for Finite automata and regular Expressions.
- 94/23 S. Mauw and M.A. Reniers An algebraic semantics of Message Sequence Charts, p. 43.
- 94/24 D. Dams
O. Grumberg
R. Gerth Abstract Interpretation of Reactive Systems: Abstractions Preserving \forall CTL*, \exists CTL* and CTL*, p. 28.
- 94/25 T. Kloks $K_{1,3}$ -free and W_4 -free graphs, p. 10.
- 94/26 R.R. Hoogerwoord On the foundations of functional programming: a programmer's point of view, p. 54.
- 94/27 S. Mauw and H. Mulder Regularity of BPA-Systems is Decidable, p. 14.
- 94/28 C.W.A.M. van Overveld
M. Verhoeven Stars or Stripes: a comparative study of finite and transfinite techniques for surface modelling, p. 20.
- 94/29 J. Hooman Correctness of Real Time Systems by Construction, p. 22.
- 94/30 J.C.M. Baeten
J.A. Bergstra
Gh. Ştefanescu Process Algebra with Feedback, p. 22.
- 94/31 B.W. Watson
R.E. Watson A Boyer-Moore type algorithm for regular expression pattern matching, p. 22.
- 94/32 J.J. Vereijken Fischer's Protocol in Timed Process Algebra, p. 38.
- 94/33 T. Laan A formalization of the Ramified Type Theory, p.40.
- 94/34 R. Bloo
F. Kamareddine
R. Nederpelt The Barendregt Cube with Definitions and Generalised Reduction, p. 37.
- 94/35 J.C.M. Baeten
S. Mauw Delayed choice: an operator for joining Message Sequence Charts, p. 15.
- 94/36 F. Kamareddine
R. Nederpelt Canonical typing and Π -conversion in the Barendregt Cube, p. 19.

- 94/37 T. Basten
R. Bol
M. Voorhoeve
Simulating and Analyzing Railway Interlockings in
ExSpect, p. 30.
- 94/38 A. Bijlsma
C.S. Scholten
Point-free substitution, p. 10.
- 94/39 A. Blokhuis
T. Kloks
On the equivalence covering number of splitgraphs, p. 4.
- 94/40 D. Alstein
Distributed Consensus and Hard Real-Time Systems,
p. 34.
- 94/41 T. Kloks
D. Kratsch
Computing a perfect edge without vertex elimination
ordering of a chordal bipartite graph, p. 6.