Peak-to-average power reduction in space division multiplexing based OFDM systems through spatial shifting

Peak-to-average power reduction in space division multiplexing based OFDM systems through spatial shifting

T.C.W. Schenk, P.F.M. Smulders and E.R. Fledderus

A new method to reduce the peak-to-average power ratio (PAPR) in space division multiplexing systems applying orthogonal frequency division multiplexing is proposed. The method applies spatial shifting to partial transmit sequences to achieve a decreased PAPR on all transmit branches.

Introduction: The combination of multiple-antenna techniques, often collectively referred to as multiple-input multiple-output (MIMO), with the multicarrier transmission scheme orthogonal frequency division multiplexing (OFDM) is a very promising solution for next generation broadband wireless systems since both approaches exploit the wireless multipath channel. One of the most popular MIMO techniques is space division multiplexing (SDM), where independent datastreams are sent on the different transmitter (TX) branches, resulting in a data-rate that scales linearly with the number of TX antennas.

It is well known that one of the main disadvantages of OFDM is that the time-domain signals exhibit a high peak-to-average power ratio (PAPR). To avoid clipping and nonlinear distortion, a large back-off has to be applied when feeding OFDM signals to the power amplifier, which limits the maximum output power. Several methods were previously presented to reduce the PAPR of OFDM transmissions, see e.g. [1, 2]. An extension of these techniques for space time block coded systems is proposed in [3].

In this Letter, a method is proposed that reduces the PAPR of TX signals in SDM OFDM systems. The proposed method is based on the rearrangement of the TX vector in such a way that subparts are transmitted on those TX branches that result in the lowest overall PAPR. This technique, in contrast to [1–3], requires no multiplications for composition of the TX vectors. It is shown that a significant gain in PAPR performance can be achieved with limited complexity and signalling overhead.

MIMO OFDM notation: For a system applying N subcarriers, the transmit vector representing the mth MIMO OFDM symbol is given by

$$\mathbf{s}_m = (\mathbf{I}_{N_t} \otimes \mathbf{F}^{-1})\mathbf{S}_m \tag{1}$$

where $\mathbf{F}$ is the $N \times N$ Fourier matrix, of which the (i, k)th element equals $\exp(-j2\pi\, ik/N)$, $\mathbf{I}_{N_t}$ represents the N_t-dimensional identity matrix and $\otimes$ denotes the Kronecker product. The vector $\mathbf{S}_m$ contains the coded, interleaved complex data-symbols. The structure of $\mathbf{s}_m$, and similarly for $\mathbf{S}_m$, is given by $\mathbf{s}_m = [\mathbf{s}_{m,1}^T, \mathbf{s}_{m,2}^T, \ldots, \mathbf{s}_{m,N_t}^T]^T$, where $\mathbf{s}_{m,n_t}$ denotes the N-dimensional time domain transmit vector for the n_tth TX branch. The PAPR of the mth symbol on the n_tth TX branch is then defined by

$$\text{PAPR}_{m,n_t} = \frac{\max(\mathbf{s}_{m,n_t} \circ \mathbf{s}_{m,n_t}^*)}{\mathbb{E}(\mathbf{s}_{m,n_t} \circ \mathbf{s}_{m,n_t}^*)} \tag{2}$$

where $\max(\mathbf{y})$ produces the maximum element of the input vector $\mathbf{y}$ as its output, $\mathbb{E}(\cdot)$ is the expected value, $*$ denotes complex conjugation and $\circ$ signifies element-wise multiplication.

Spatial shifting of PTS: The data-carriers in the mth OFDM symbol are subdivided into P disjoint groups, referred to as partial transmit sequences (PTS) [2]. The pth subcarrier group for the n_tth TX branch is contained in the N-dimensional vector $\mathbf{S}_{m,n_t}^{(p)}$, where all other elements are put to zero. The time domain transmit vector is then given by $\mathbf{s}_{m,n_t} = \sum_{p=1}^{P} \mathbf{F}^{-1}\mathbf{S}_{m,n_t}^{(p)} = \sum_{p=1}^{P} \mathbf{s}_{m,n_t}^{(t)}$. Since the grouping structure is equivalent for the different spatial streams, the corresponding PTS on the different TX branches can be mutually interchanged using spatial shifting (SS). In this way different TX vectors can be constructed that represent the same data.

The mth transmit vector after SS can be written as

$$\tilde{\mathbf{s}}_{m,c} = \sum_{p=1}^{P} C(\mathbf{s}_m^{(p)}, N_{c_p}) \tag{3}$$

where $C(\mathbf{y}, c)$ denotes applying a cyclic shift of c samples to the vector $\mathbf{y}$, $c_p \in \{0, 1, \ldots, N_t - 1\}$ and $\mathbf{s}_m^{(p)} = [\mathbf{s}_{m,1}^{(p)\,T}, \mathbf{s}_{m,2}^{(p)\,T}, \ldots, \mathbf{s}_{m,N_t}^{(p)\,T}]^T$. The P-dimensional vector $\mathbf{c}$ equals $[c_1, c_2, \ldots, c_P]$. In the case of P PTS and N_t TX branches, there are thus $N_t P$ different possibilities to form $\tilde{\mathbf{s}}_{m,\mathbf{c}}$.

To achieve a power efficient transmission, we want to select and transmit the vector $\tilde{\mathbf{s}}_{m,\mathbf{c}}$ which exhibits the lowest average peak power over the different branches. We use this as selection criterium for the shift parameters $\mathbf{c}$. The procedure can be written as

$$\tilde{\mathbf{c}}_m = \arg\min_{\mathbf{c}} \left(\sum_{n_t=1}^{N} \max(\tilde{\mathbf{s}}_{m,\mathbf{c},n_t} \circ \tilde{\mathbf{s}}_{m,\mathbf{c},n_t}^*) \right) \tag{4}$$

where $\arg\min(\cdot)$ produces the argument for which the expression is minimised, $\tilde{\mathbf{s}}_{m,\mathbf{c},n_t}$ is the n_tth N-dimensional subvector of $\tilde{\mathbf{s}}_{m,\mathbf{c}}$ and $\tilde{\mathbf{c}}_m$ is the selected shift vector for the mth MIMO OFDM symbol. We can choose one of the elements of $\mathbf{c}$ to be 0, without loss of optimality, since varying this parameter would only result in spatial swapping of another group of TX vectors between the branches, which does not increase the performance. There are, thus, effectively $N_t(P-1)$ possibilities to evaluate.

Side information or transparency: For the SS method to perform optimally, optimisation has to be applied for every OFDM symbol separately, as implied by (4). For the receiver (RX) to be able to compensate for the SS applied at the TX, side information (SI) has to be transmitted to report which SS vector $\tilde{\mathbf{c}}_m$ is used. The overhead induced by the transmission of this SI is $\lceil \log_2(N_t\{P-1\}) \rceil$ bits per MIMO OFDM symbol. Here $\lceil \cdot \rceil$ denotes the smallest integer that is greater or equal to its argument. To achieve more reliable detection of these bits, (space time) block coding can be applied to the SI-data.

For some systems it might, however, be beneficial to make a transparant solution, in which the RX needs no information about the SS applied at the TX. This can be achieved by applying the SS of (3) jointly to all symbols within a packet, thus also to the part of the transmission used for MIMO channel estimation. The selection criterium in (4) then transforms into

$$\tilde{\mathbf{c}} = \arg\min_{\mathbf{c}} \left(\sum_{m=1}^{M} \sum_{n_t=1}^{N} \max(\tilde{\mathbf{s}}_{m,\mathbf{c},n_t} \circ \tilde{\mathbf{s}}_{m,\mathbf{c},n_t}^*) \right) \tag{5}$$

where M equals the number of OFDM symbols in the packet. The influence of the SS applied at the TX is now included in the *effective* MIMO channel estimate found at the RX. When the RX compensates for this effective MIMO channel, the SS is also removed. From the viewpoint of the RX the SS method can be seen as effectively interchanging the columns of the physical MIMO channel matrix. This makes the method transparant, meaning no extra SI has to be transmitted, which enables the application of this technique without standardisation. The resulting PAPR will, however, be less optimal than in the SI case since the optimisation is done jointly for the full packet.

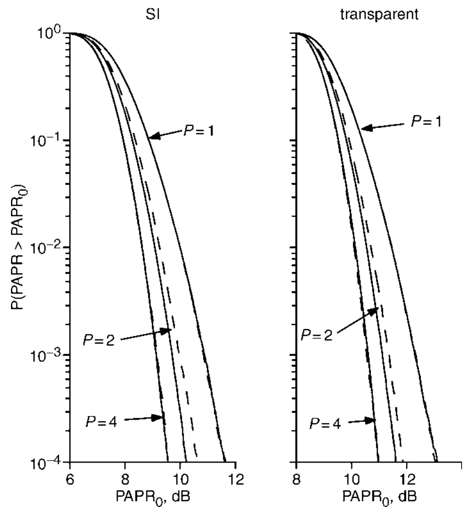

Fig. 1 *CCDF of PAPR for SS method applied per MIMO OFDM symbol (SI) and per packet (transparent) (packets of 10 OFDM symbols applied for transparant method)*

– – – – $N_t = 2$
——— $N_t = 4$

Numerical results: The performance of the proposed method is illustrated by Fig. 1, which shows results from Monte-Carlo simulations for a system applying $N=256$ carriers. The performance is given in terms of the complementary cumulative distribution function (CCDF) of the PAPR. The number of TX branches equals $N_t=2$ and 4. The number of PTS is $P=1$, 2 and 4. QPSK modulation is used on all carriers. Results are given for optimisation according to (4) and (5), the results of which are given in the left and right Figures, respectively. It is noted that the PAPR for the transparent method is calculated for the entire 10 OFDM symbol packet.

For $P=1$, i.e. the reference case without SS, the performance is independent of the number of antennas. For $P=2$ the case with 4 TX branches performs better than the one with 2 TX branches. For $P=4$, however, the difference in performance for $N_t=2$ and $N_t=4$ is very marginal. It can be concluded from Fig. 1 that significant improvement in PAPR performance can already be achieved for small values of P and N_t. Furthermore, the PAPR performance of the transparent method seems worse than that of the SI-based method, even for the reference case, which is explained by the fact that the PAPR is here calculated over the whole packet, inherently yielding higher PAPR values. The improvement in PAPR through SS, however, is for the transparent mode similar to that for the SI-based case. For both cases an improvement of about 2 dB is achieved at a clipping rate of 10^{-4}. For lower clipping rates the gain will increase.

Conclusions: The proposed spatial shifting (SS) scheme for PAPR reduction in SDM based OFDM systems is of low complexity and induces a limited signalling overhead. The scheme can be applied in a transparent and a signalling-based mode. A substantial improvement in PAPR performance is shown to be achieved for both modes with only a limited number of partial transmit sequences. Further improvements can be achieved by combining the proposed SS technique with the methods proposed in [1–3].

8 May 2005

Electronics Letters online no: 20051677

doi: 10.1049/el:20051677

T.C.W. Schenk, P.F.M. Smulders and E.R. Fledderus (*Department of Electrical Engineering, Eindhoven University of Technology, P.O. Box 513, 5600 MB Eindhoven, The Netherlands*)

E-mail: t.c.w.schenk@tue.nl

References

1 Bauml, R., Fischer, R., and Huber, J.: 'Reducing the peak-to-average power ratio of multicarrier modulation by selected mapping', *Electron. Lett.*, 1996, **32**, pp. 2056–2057

2 Muller, S., and Huber, J.: 'OFDM with reduced peak-to-average power ratio by optimum combination of partial transmit sequences', *Electron. Lett.*, 1997, **33**, pp. 368–369

3 Moon, J.-H., You, Y.-H., Jeon, W.-G., Kwon, K.-W., and Song, H.-K.: 'Peak-to-average power control for multiple-antenna HIPERLAN/2 and IEEE802.11a systems', *IEEE Trans. Consum. Electron.*, 2003, **39**, pp. 1078–1083