Response of wall heat transfer to flows along a cylindrical cavity and to seepage flows in the surrounding medium

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ABSTRACT

Rates of heat transfer between the wall of a cylindrical cavity (pipe/duct/tunnel/etc) and the fluid flowing along it can be influenced by changing conditions outside the cavity as well as by changing conditions within it. In the case of buried pipes or tunnels, and also in many other physical processes, the external environment may be complex, involving seepage flows through a porous medium as well as heat transfers due to both conduction and convection in the porous medium. This paper presents an analytical analysis that enables the external flow and thermal conditions to be solved simultaneously with corresponding internal conditions, enabling time-dependent wall temperatures and rates of heat transfer to be determined exactly for sufficiently simple geometrical configurations.

The analysis is used to predict time-dependent temperatures and heat fluxes at the wall of a cavity for a range of prescribed conditions, enabling the importance of timescales to be established. In some special cases, the method could be used for design in its own right, but this is unlikely to be its most important function. Rather, the analysis is envisaged as a method of providing data for the validation of numerical methods of analysis that, in turn, will be used for practical design with both simple and complex geometrical conditions. This is valuable because it can be next to impossible to obtain experimental data of sufficient quality and reliability for the validation of software used to simulate complex fluid/heat flows.

KEY WORDS

forced convection, porous medium, radial seepage, cylindrical cavity, heat transfer, composite medium, temperature profiles, analytic solutions.

NOMENCLATURE

a	cavity surface radius [m]
A, B	constants of integration
Bi	Biot number; defined by Eq.(21)
C_P	specific heat capacity at constant pressure [J kg^{-1} K^{-1}]
C_V	specific heat capacity at constant density [J kg^{-1} K^{-1}]
H	convection coefficient between the porous medium and the cavity fluid [W m^{-2} K^{-1}]
k_m	thermal conductivity of the saturated porous medium [W m^{-1} K^{-1}]
M_1, M_2	parameters defined by Eqs.(31 & 32)
q	heat flux (per unit area) [W m^{-2}]
Q_H	heat flux per unit length to the porous medium from the cavity fluid [W m^{-1}]
Q_F	radial fluid flux per unit length in the porous medium [m^2 s^{-1}]
r	radial coordinate [m]
R	non-dimensional radius; defined by Eq.(12)
R_s	non-dimensional radius of region of influence
s	Laplace transform parameter; see Eq.(24)
t	time [s]
T	absolute temperature [K]
v_r	radial filtration velocity [m s^{-1}]
w	integration variable - see Eq.(30)

Greek symbols

α	effective thermal diffusivity of the saturated medium [m^2 s^{-1}]
β	parameter defined by Eq.(24)
ε, η	parameters in Bromwich integral - see Eq.(29)
λ	Peclet number; defined by Eq.(7)
θ	non-dimensional temperature; defined by Eq.(14)
ρ	density [kg m^{-3}]
σ	ratio of heat capacities of saturated medium and water
τ	non-dimensional time; defined by Eq.(13)

τ_s	non-dimensional time to an effectively steady state
φ	additional internal energy in the medium at steady state (per unit length of cavity) [J m^{-1}]
ψ	non-dimensional heat flux; defined by Eq.(15)

Subscripts

a	cavity surface
C	cavity fluid
0	initial state in the porous medium

Superscript

$'$	Laplace transformed quantity

1. INTRODUCTION

It is frequently the case that a porous medium is penetrated by a cylindrical opening such as a borehole or tube. In some applications, there may be both heat and mass exchange between the cavity and the medium, whereas in others, there may be only heat exchange. The latter is the case, for instance, in long pipelines beneath the sea bed. It is also the case when groundwater seeping towards a railway tunnel meets an impermeable membrane around the tunnel lining. The seepage water is drained when it reaches the membrane, thus preventing the build-up of excessive pressures on the lining. This case is especially amenable to analysis because the tunnel imposes a pressure boundary condition that promotes a radial flow of seepage water in the surrounding medium.

The simultaneous existence of fluid flow and heat transfer in a porous medium is encountered in a wide range of applications. Large scale examples include geothermal energy production e.g. Gringarten *et al* (1975), Claesson & Dunand (1983), Ogino *et al* (1999); mining and tunnelling e.g. McPherson (1986), Iguchi (1985), Kimura *et al* (1992), Berner *et al* (1994), Bopp *et al* (1994), dams e.g. Yuanming (2002), ground heat exchangers e.g. Sutton et al (2003), oil reservoir engineering e.g. Kocabas (2004). Smaller scale applications include composites manufacture e.g. Simacek & Advani (2001); heat exchangers e.g. Angirasa & Peterson (1999), Raffray *et al* (2002); fuel cell applications e.g. Worth (1999). The distribution of the concentration of injected tracers is an analogous physical problem e.g. Kocabas and Islam (2000a).

Free convection in a porous medium due to a heated cavity has been considered by, for example, Bau (1984), Muralidhar (1993), Deschamps and Desrayaud (1994) and forced convection has been investigated by Thevenin (1995), Zhou and Lai (2002), Simacek and Advani (2002). Analytic solutions of the convection-diffusion equation have been reported by, for example, Chen (1986) and Philip (1994). Brown *et al* (1998) obtained transient analytic solutions for (i) constant temperature and (ii) constant heat flux conditions at the cavity surface. The resulting quadrature integrals were evaluated numerically for a range of physical conditions. Wu & Pruess (2000) also evaluated integral solutions numerically and extended the analysis by allowing for pressure-dependent permeability. Kocabas and Islam (2000b) examined the more complicated case of combined convection and dispersion, but presented analytical solutions only in the Laplace transform domain.

A solution to the diffusion problem of heat conduction in a *dry* solid bounded internally by a cylindrical opening held at constant temperature, was obtained in the form of a quadrature by Nicholson (1921) using a Green's function method. Carslaw and Jaeger (1959) used the Laplace transform method to obtain quadrature type solutions for constant temperature and for constant boundary heat flux. The Laplace transform has subsequently been the basis of many solutions of transient convection-diffusion problems e.g. Kocabas (2004). In addition, there exist useful approximate analytical solutions. For example, Kutasov (2003) reported a semi-analytic method for constant heat flux problems and Chugunov *et al* (2003) used a heat balance integral method to study a composite domain consisting of concentric shells around a well, each shell having its own physical properties.

In nearly all of the above studies, the boundary condition at the cavity surface is either prescribed-temperature or prescribed-heat flux (or equivalent conditions relating to solute concentrations). These investigations provide valuable information, but they are not good representations of the important case where a fluid is flowing axially inside the cavity. In such cases, it is more realistic to allow for convective heat transfer between the cavity surface (at unknown temperature) and the cavity fluid (at prescribed temperature). Carslaw & Jaeger (1959) presented analytic solutions for the simpler case of a cylindrical cavity in a *dry* solid, but the authors are not aware of previous work for a cavity in a porous medium in the presence of radial flow in the medium and axial flow in the cavity. Kocabas and Islam (2000b) considered a related concentration boundary value problem, but in the presence of dispersion effects.

1.1. *Objective of this paper*

The objective of this paper is to provide analytical solutions for transient heat exchange between a fluid flowing along a cylindrical cavity and a porous surrounding medium in which the effects of radial seepage are important. Analytical solutions to such problems are important for a number of reasons. First, they give an understanding of the influence of important parameters on the overall behaviour. Second, they provide benchmarking data that can be used for the validation of numerical models - e.g. finite element analyses. Neither of these purposes can be satisfied easily by experimental means, especially when the underlying flows vary strongly.

The conditions in the surrounding medium correspond to unsteady heat transfer in the presence of a steady radial seepage flow, with local thermal equilibrium being maintained between the (stationary) ground and the (moving) fluid. The conditions *at any particular axial location* inside the cavity correspond to constant velocity and temperature. The rate of heat transfer across the cavity surface (tunnel wall, say) is governed by forced convection in the cavity fluid, by conduction in the solid matrix of the surrounding medium and by advection of the seepage fluid. The implied axial variation of temperature along the cavity is governed by the usual conservation laws of mass, momentum and energy, but it is not considered explicitly herein. Instead, attention is focussed on the influence of timescales in the porous medium on the effective zone of influence of the cavity on the medium.

2. MATHEMATICAL MODEL

The assumed system consists of an infinitely long cylindrical cavity in a porous medium which extends radially to infinity. All properties of the porous medium are assumed to be uniform and independent of time and both the solid and the seepage fluid are assumed incompressible. There is a prescribed steady radial fluid flow through the porous medium and a steady axial fluid flow of an independent fluid in the cavity. Axial symmetry is assumed throughout. Figure 1 illustrates the directions of fluid and heat flow.

Heat conduction within the porous medium is assumed to be governed by Fourier’s law. In the absence of seepage, the evolution of temperature would satisfy a classical diffusion equation. In the presence of the seepage, internal energy is advected radially and, for the assumed conditions of local thermal equilibrium between the solid and fluid phases, the temperature evolution satisfies a generalized diffusion equation containing a mass transport term - see Nield and Bejan (1992).

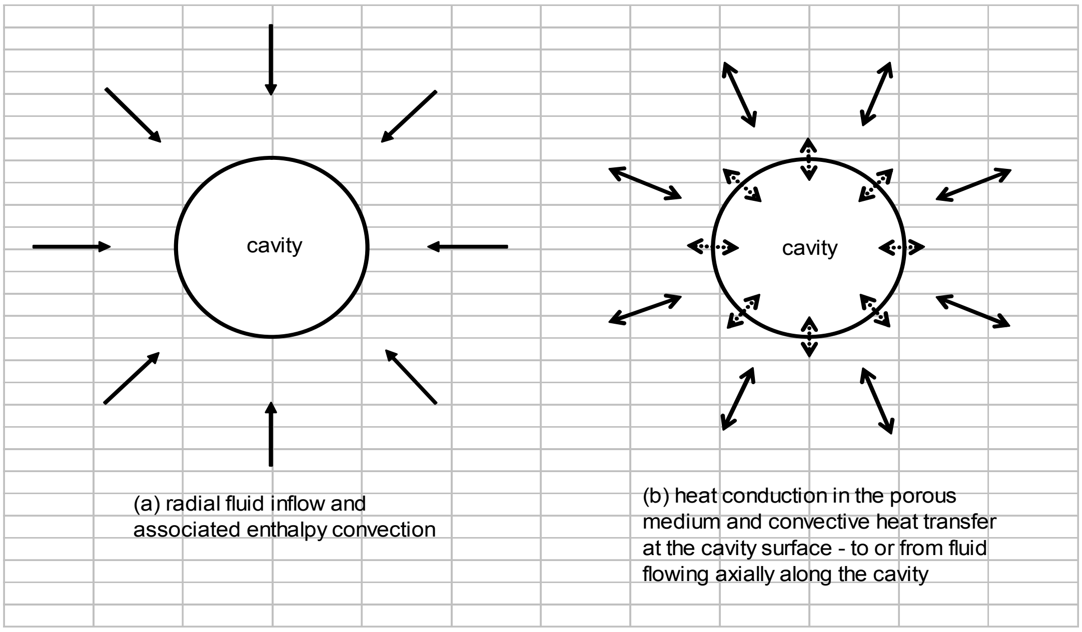

Fig. 1. Heat transfers around a cavity in the presence of inward radial fluid flow.

2.1. *Fluid flow in the porous medium*

The seepage flow is radial and steady. Conservation of mass (expressed in cylindrical co-ordinates) requires that

$$\frac{1}{r}\,\frac{\mathrm{d}(rv_r)}{\mathrm{d}r} = 0 \ , \qquad (1)$$

where v_r is the radial seepage velocity i.e. the ratio of volume flux through a cross-sectional area (of pores and solid) to that area. On integration, Eq.(1) shows that the velocity v_r is inversely proportional to radius r, i.e.

$$v_r = -\ \frac{Q_F}{2\pi r} \ , \qquad (2)$$

where Q_F, the volumetric rate of flow per unit length of cavity, is independent of both time t and distance r and is defined as positive for radially inward flow.

2.2. *Heat flux and temperature distribution*

The temperature is assumed to be axisymmetric and to vary sufficiently slowly in the axial direction that $T \equiv T(r,t)$ may be assumed. Then the heat flux per unit area q at any radius is given by Fourier's law, namely

$$q = -\ k_m \frac{\partial T}{\partial r}\ , \tag{3}$$

where k_m is the thermal conductivity of the saturated porous medium and T is the local temperature (assumed the same for the solid and fluid phases). The axi-symmetric variation of temperature in space and time satisfies – e.g. Nield and Bejan (1992) –

$$\sigma \frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} = \alpha \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right), \tag{4}$$

where σ is the ratio of the specific heat capacities of the saturated medium and the fluid and α is the thermal diffusivity of the saturated medium. It is defined as

$$\alpha \equiv \frac{k_m}{\rho C_P}\ , \tag{5}$$

where ρ and C_P are, respectively, the mass density and the specific heat capacity of the saturated porous medium. Combining Eqs.(4 & 2) leads to an augmented diffusion equation

$$\sigma \frac{\partial T}{\partial t} = \alpha \left(\frac{\partial^2 T}{\partial r^2} + \frac{1+2\lambda}{r} \frac{\partial T}{\partial r} \right), \tag{6}$$

where λ is a Peclet type number (a ratio of thermal advection to conduction), defined here as

$$\lambda \equiv \frac{Q_F}{4\pi\ \alpha}, \qquad (7)$$

The factor of 4 in this definition is introduced for notational convenience in later equations.

2.3. *Initial and boundary conditions*

The prescribed initial condition throughout the porous medium is a uniform temperature, namely

$$T(r,0) = T_0 \ . \qquad (8)$$

The temperature is assumed to remain constant at $r = \infty$ for all $t > 0$. That is,

$$T(\infty,t) = T_0 \ . \qquad (9)$$

The corresponding boundary condition at the cavity surface (r=a) is a heat flux determined by (forced) convective heat transfer between the medium at a temperature T_a≡$T(a,t)$ and the cavity fluid at a mean temperature T_C, namely

$$Q_H = 2\pi a H\ (T_a - T_C\ )\ , \qquad (10)$$

where Q_H is the rate of radial heat transfer per unit length of cavity and H is a convective heat transfer coefficient that depends on the properties and behaviour of the axial flow in the cavity. The interface temperature T_a varies with time whereas both H and T_C are assumed constant in time. Using Eq.(3), the boundary condition at the cavity surface may be expressed as

$$\left(\frac{\partial T}{\partial r}\right)_{r=a} = \frac{H}{k_m}(T_a - T_C)\ . \qquad (11)$$

Some authors replace the ratio H/k_m by h and some others use the symbol h for the parameter denoted by H in this paper. Care must be taken when interpreting these parameters.

2.4. *Non-dimensional formulation*

For generality, it is useful to express the key equations in a non-dimensional form. Accordingly, non-dimensional distance, time, temperature and heat flux are introduced, defined by

$$R \equiv r/a \ , \tag{12}$$

$$\tau \equiv \alpha \ t / a^2 \ , \tag{13}$$

$$\theta \equiv (T - T_0) \ / \ (T_C - T_0), \tag{14}$$

and

$$\psi \equiv \frac{q \ a}{k_m (T_C - T_0)} \ . \tag{15}$$

Using these parameters, Fourier's law - Eq.(3) - becomes

$$\psi = - \ \frac{\partial \theta}{\partial R} \ . \tag{16}$$

The governing Eq.(6) can be expressed as

$$\frac{\partial^2 \theta}{\partial R^2} + \frac{1 + 2\lambda}{R} \frac{\partial \theta}{\partial R} - \sigma \frac{\partial \theta}{\partial \tau} = 0 \tag{17}$$

and the initial condition throughout the porous medium – Eq.(8) – is

$$\theta(R,0) = 0 \ . \tag{18}$$

The prescribed temperature condition at infinity – Eq.(9) – is

$$\theta(\infty, \tau) = 0 \tag{19}$$

and the radial heat flux due to forced convection at the cavity surface – Eq.(10) – is

$$\psi_a = \mathbf{Bi}\ (1 - \theta_{R=1}) \ , \tag{20}$$

in which **Bi** is a Biot number defined as

$$\mathbf{Bi} \equiv aH / k_m \ . \tag{21}$$

i.e. a ratio of convection to the axial flowing fluid and conduction in the medium at the cavity surface. Using Eq.(16), the boundary condition at the cavity surface may be expressed as

$$\left(\frac{\partial \theta}{\partial R}\right)_{R=1} = \mathbf{Bi}\ (\theta_{R=1} - 1) \ . \tag{22}$$

3. GENERAL SOLUTION

The non-dimensional temperature distribution is the solution of Eq.(17) subject to the initial and boundary conditions in Eqs.(18, 19 & 22). Taking the Laplace transform of Eq.(17) along with the initial condition (18) leads to

$$\frac{\mathrm{d}^2\theta'}{\mathrm{d}R^2}+(1+2\lambda)\frac{1}{R}\frac{\mathrm{d}\theta'}{\mathrm{d}R}-\beta^2\theta'=0\ , \qquad (23)$$

where

$$\beta^2=\sigma s\ . \qquad (24)$$

Here s is the Laplace transform parameter and $\theta' \equiv \theta'(R,s)$ is the Laplace Transform of the temperature. The transforms of the boundary conditions of Eqs.(19 & 22) are, respectively,

$$\theta'(\infty,\ s)=0 \qquad (25)$$

and

$$\left(\frac{\partial\theta'}{\partial R}\right)_{R=1}=\mathbf{Bi}\ (\theta'_{R=1}-1/s)\ . \qquad (26)$$

The general solution of Eq.(23) is

$$\theta'=R^{-\lambda}\ \left[A\mathrm{I}_\lambda(\beta R)+B\mathrm{K}_\lambda(\beta R)\right]\ , \qquad (27)$$

where A and B are constants to be determined from the boundary conditions and I and K are modified Bessel functions of the first and second kind and of order λ. Using the transforms of the boundary conditions - Eqs.(25 & 26) - leads to the following expression for the transform of the temperature distribution:

$$\theta'(R,s)=\frac{\mathbf{Bi}\mathrm{K}_\lambda(\beta R)}{s\ R^\lambda\ [\ \mathbf{Bi}\ \mathrm{K}_\lambda\ (\beta)+\beta\mathrm{K}_{\lambda+1}\ (\beta)]}\ . \qquad (28)$$

The temperature distribution in the physical domain is recovered using the Bromwich integral, giving

$$\theta(R,\tau) = \frac{1}{2\pi\, i} \int_{\varepsilon - i\infty}^{\varepsilon + i\infty} e^{\eta\tau}\ \theta'(R,\eta)\ \mathrm{d}\eta\ , \qquad (29)$$

where ε is chosen such that all the singularities of the integrand lie to the left of the line $\eta = \varepsilon$.

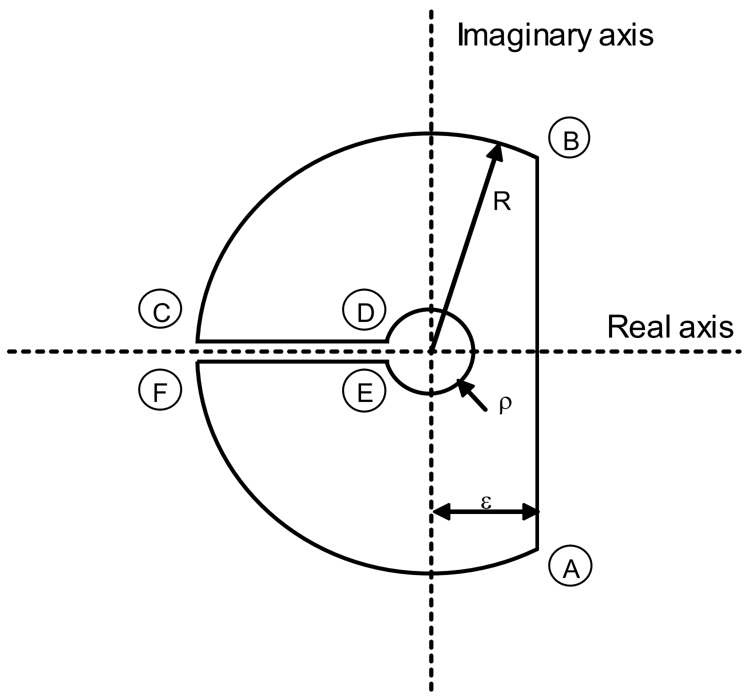

Fig. 2. Integration contour in the complex s plane.

Since θ' has a branch point at the origin, the integration is performed over the contour shown in Fig. 2, resulting in the temperature distribution

$$\theta(R,\tau) = \frac{\mathbf{Bi}}{R^{\lambda}}\left(\frac{1}{(\mathbf{Bi}+2\lambda)\ R^{\lambda}} + \frac{2}{\pi}\int_0^{\infty} \frac{e^{-w^2\tau}\ [\mathrm{J}_{\lambda}(wR\sqrt{\sigma})\, M_2\ -\ \mathrm{Y}_{\lambda}(wR\sqrt{\sigma}) M_1]\ \mathrm{d}w}{w\ [M_1^2 + M_2^2]} \right), \quad (30)$$

in which J_{λ} and Y_{λ} are Bessel functions of first and second kind and order λ,

$$M_1 = \mathbf{Bi}\,\mathrm{J}_{\lambda}\,(w\sqrt{\sigma}) + w\sqrt{\sigma}\mathrm{J}_{\lambda+1}\,(w\sqrt{\sigma}), \quad (31)$$

and

$$M_2 = \mathbf{Bi}\,\mathrm{Y}_{\lambda}(w\sqrt{\sigma}) + w\sqrt{\sigma}\ \mathrm{Y}_{\lambda+1}(w\sqrt{\sigma})\ . \quad (32)$$

The heat flux at any radius in the porous medium follows from Fourier's law – Eq.(16). The general expression, valid at any radius, is somewhat lengthy and is of limited practical interest. It is of greatest importance at the cavity surface, where it reduces to the much simpler form:

$$\psi_a = \mathbf{Bi}\left(\frac{2\lambda}{(\mathbf{Bi}+2\lambda)} + \frac{4\,\mathbf{Bi}}{\pi^2}\int_0^{\infty} \frac{\mathrm{e}^{-w^2\tau}\ \mathrm{d}w}{w\ [M_1^2 + M_2^2]} \right)\ . \quad (33)$$

The heat flux at the cavity surface may alternatively be deduced from Eq.(20) after first obtaining the surface temperature from Eq.(30).

4. STEADY STATE

As the non-dimensional time τ becomes large, the integrand in Eq.(30) tends to zero and a steady state condition is approached, with the temperature distribution satisfying

$$\theta(R,\infty) = \frac{\mathbf{Bi}}{(\mathbf{Bi}+2\lambda)\ R^{2\lambda}} \quad . \tag{34}$$

Although never reached exactly, the steady state condition closely approximates actual conditions after sufficiently long times. It can therefore provide valuable information about the influence of the material properties and the rates of flow.

Fig. 3 shows steady-state, non-dimensional temperature distributions for a range of values of λ. In all cases, the prescribed temperature of the fluid in the cavity is θ_C=1.0 and the temperature at infinity is θ_4=0.0. By inspection, the predicted temperature reduces monotonically with increasing radius and is less than 1.0 at all radii, including the cavity surface (R=1). Eq.(34) shows that the temperature at the surface is simply **Bi**/(**Bi**+2λ).

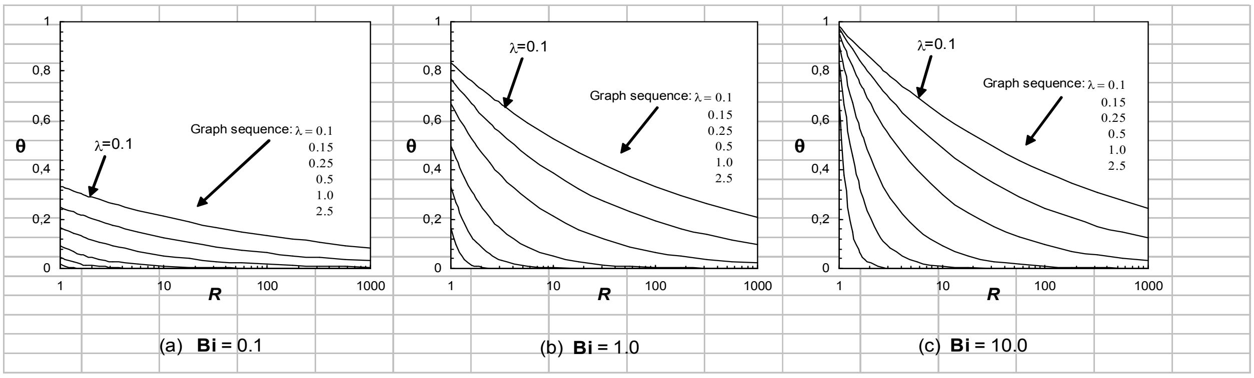

Fig. 3. Non-dimensional steady-state temperature distributions.

For any particular value of **Bi**, the temperature at any radius decreases with increasing λ; that is, the radius of (significant) influence of the cavity decreases. For any particular value of λ, however, the radius of influence increases with increasing **Bi**. Thus the steady state temperature distribution reflects the relative importance of the two rates of flow. When the cavity flow dominates (large **Bi**, small λ), the surface temperature ($\theta_{R=1}$) is large and the zone of influence is large. In the limit, this corresponds to a prescribed-temperature condition at the cavity surface. Conversely, when the seepage flow dominates (large λ, small **Bi**), the surface temperature is small and the zone of influence is small. In the limit this corresponds to an adiabatic surface condition.

The special case of λ=0 (i.e. no seepage) corresponds to pure conduction in the saturated medium. For this case, Eq.(34) shows that the temperature at all radii must be equal to the temperature in the cavity. Strictly, this is not possible at infinity - because it conflicts with the prescribed boundary condition - so the hypothesized steady state condition can never be attained. Nevertheless, the temperature at any *finite* radius will tend towards the cavity temperature, which is consistent with the trend exhibited in all graphs in Fig. 3 as λ decreases. In this limiting case, the heat flux at the cavity surface tends towards zero.

The singular nature of the steady state asymptotic condition with λ=0 arises only because the outer boundary condition is chosen at infinity. If, instead, the prescribed boundary were chosen at a finite radius, the resulting asymptotic state would be a close approximation to the true large-time condition at all radii.

4.1. *Surface heat flux*

Fig. 4 shows the steady state heat flux at the cavity surface for the same range of values of λ and **Bi** as in Fig. 3. For any particular value of **Bi**, Eq.(20) shows that the heat flux is directly proportional to the temperature difference across the cavity surface, namely ($1-\theta_{R=1}$). Hence, the trends shown in Fig. 4 could be inferred qualitatively from the surface temperatures in Fig. 3 or quantitatively from Eq.(33) with the integral set to zero.

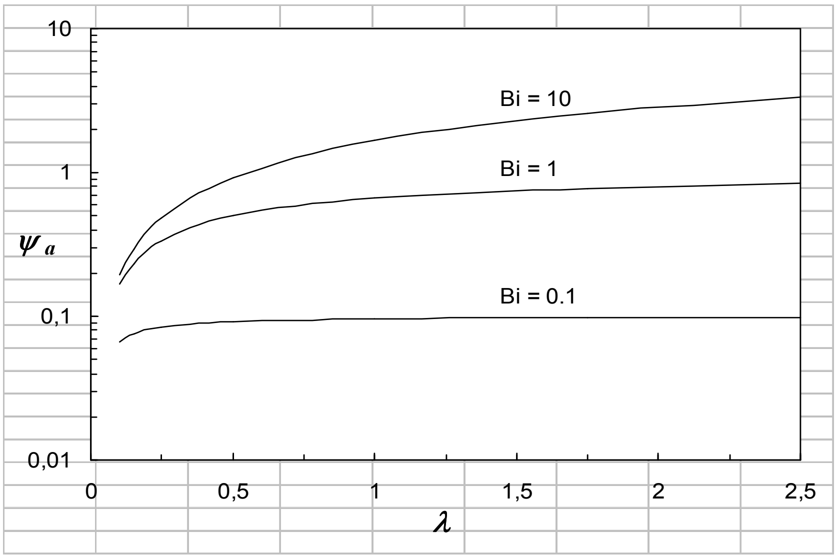

Fig. 4. Non-dimensional steady-state heat flux at the cavity surface.

For any particular value of λ, the heat flux increases with the Biot number, but not linearly. The heat flux is controlled by resistance to heat transfer in the porous medium as well as by the surface resistance. Thus, the influence of a change in the Biot number is relatively large when the surface resistance is dominant (small **Bi**, large λ), but relatively small when the resistance of the medium is dominant (small λ, large **Bi**).

4.2. *Zone of influence*

In practical applications, it can be useful to quantify the extent of the zone of influence of the cavity. Strictly, the zone is of infinite extent whatever the values of λ and **Bi.** Realistically, however, it is acceptable to define an *effective* zone of influence as the region within which the change in temperature exceeds a prescribed percentage of the overall temperature difference between the cavity and infinity. Fig. 5 shows the limits of the zones of influence defined in this way, based on differences of 95% and 99% of θ_C. The limits of the zones are strongly dependent upon the rates of radial flow, ranging from infinity in the case of no radial flow to zero at sufficiently large rates (when $\theta_{R=1}$ is smaller than the prescribed limiting difference).

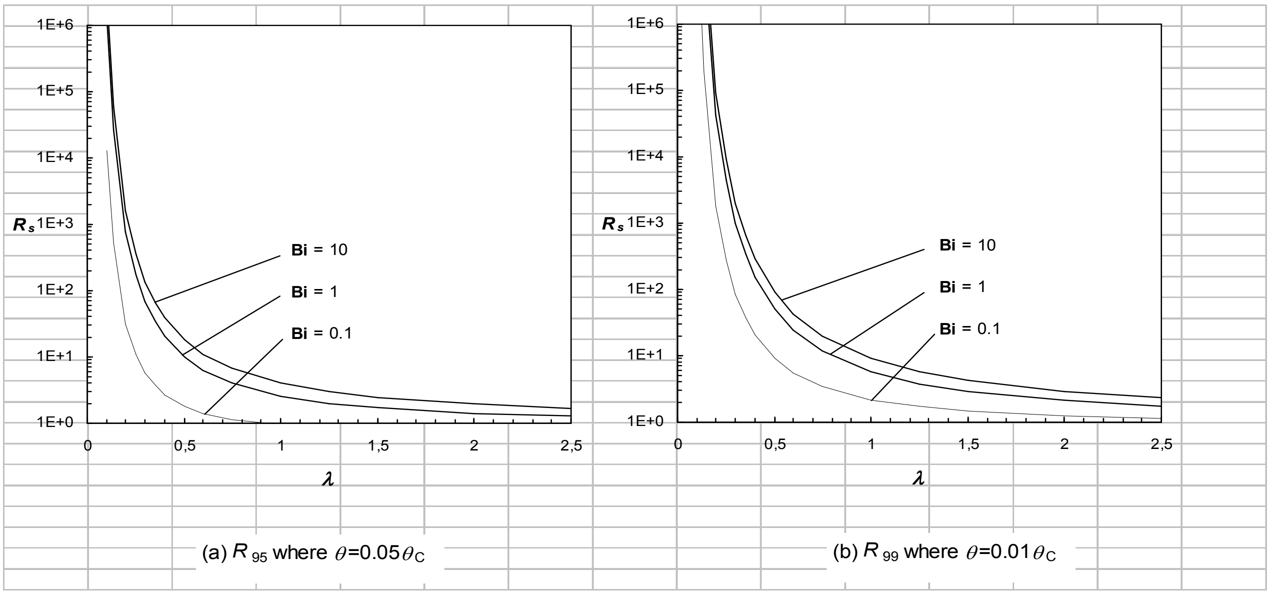

Fig. 5. Nominal limits of zone of influence of cavity - based on the temperature decay.

4.3. *Additional internal energy*

In some practical applications, the capacity of the porous medium to act as a heat source or sink is a primary consideration. For these cases, the total internal energy of the medium at steady state may be of interest. The additional internal energy in comparison with the corresponding value when θ=0 at all radii can be obtained from integration of Eq.(34) and is found to be infinite when $\lambda \leq 1$, but finite when $\lambda > 1$. In the latter case, it satisfies

$$\frac{\varphi_{\lambda>1}}{(\rho C_{\mathrm{V}})_m\, a^2\ (T_{\mathrm{C}} - T_0)} = \frac{\pi \mathbf{Bi}}{(\mathbf{Bi}+2\lambda)(\lambda-1)} \qquad (35)$$

where φ is the additional internal energy per unit length along the cavity axis and $(\rho C_{\mathrm{V}})_m$ is the heat capacity of the saturated medium.

This additional internal energy is a measure of the total amount by which the surface heat transfer can exceed the steady state value during the early stages of the process. In the steady state, the heat transfer at the surface is counterbalanced by the internal energy flux of the pore fluid reaching the surface. In the period before steady conditions prevail, additional heat transfer at the surface can be accommodated by increasing the internal energy of the medium (or by decreasing it in the case of surface heat outflow).

4.4. *Influence of material properties*

In some of the discussion in this paper, changes in the values of λ and **Bi** are tacitly regarded as indicative of changes in rates of flow. This is appropriate when all material properties and the nature of the cavity and its fluid are known *a priori*, as is often the case in practical applications.

In principle, changes in λ and **Bi** can alternatively be regarded as indicative of other causes such as changes in material properties. For example, the Peclet number λ increases with the mass density and with the specific heat capacity of the porous medium. Likewise, the Biot number **Bi** increases with the radius of the cavity and with the convective surface heat transfer coefficient H (which, in turn, depends upon the properties of the cavity fluid and the surface geometry as well as the axial rate of flow). Both λ and **Bi** are inversely proportional to the thermal conductivity of the porous medium. When required, the data presented in Figs. 3 & 4 and in subsequent figures may be interpreted as consequences of *any* of the parameters that contribute to λ and **Bi**.

5. UNSTEADY STATE

When the cavity flow is initiated suddenly at time t=0, the initial and boundary conditions are as in Eqs.(8-11). The temperature distributions shown in Fig. 6 were obtained from Eq.(30), and were evaluated to a high degree of accuracy using Maple v8. The temperature distributions evolve towards the asymptotic steady state. Qualitatively similar behaviour is observed for each of the three Biot numbers considered. That is, both the surface temperature and the radius of influence increase with time, tending eventually towards the steady state condition. Fig. 6 corresponds to λ=0.5, but similar trends exist for any value of λ. This particular value, which is near the middle of the range used in Figs. 3 and 4, is one of a small number of values for which the various Bessel functions can be evaluated relatively easily.

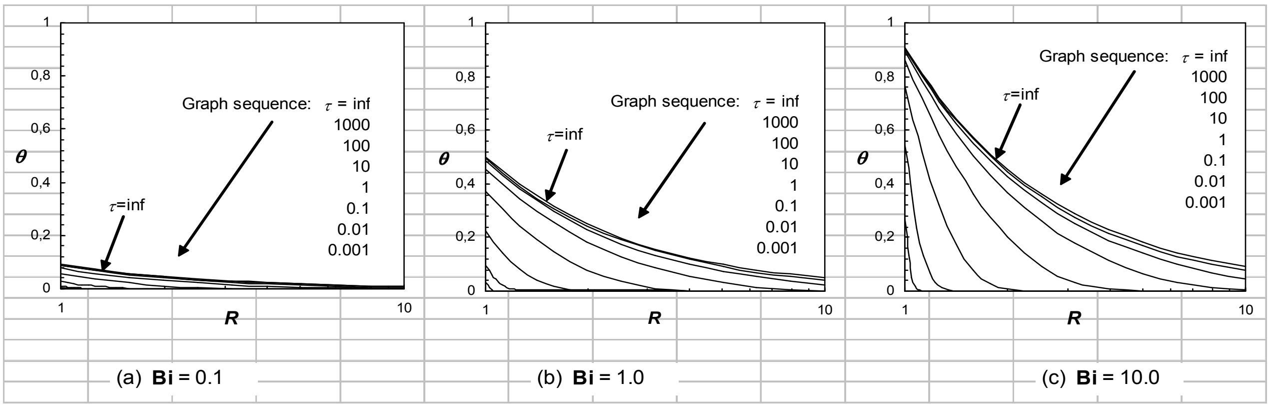

Fig. 6. Evolution of non-dimensional temperature distribution, λ=0.5.

All of the time-dependent temperature distributions are beneath the upper bounds shown by the steady-state distributions. The curves approach the asymptotes rapidly at the cavity surface [R=1] and less rapidly at increasing radius. This behaviour is captured in Fig. 7a which shows the evolution of temperature with time at the cavity surface and at R=1.26. A logarithmic axis is used for the non-dimensional time τ to enable both small and large times to be observed meaningfully. The Biot number is **Bi**=1.0 and λ is again 0.5.

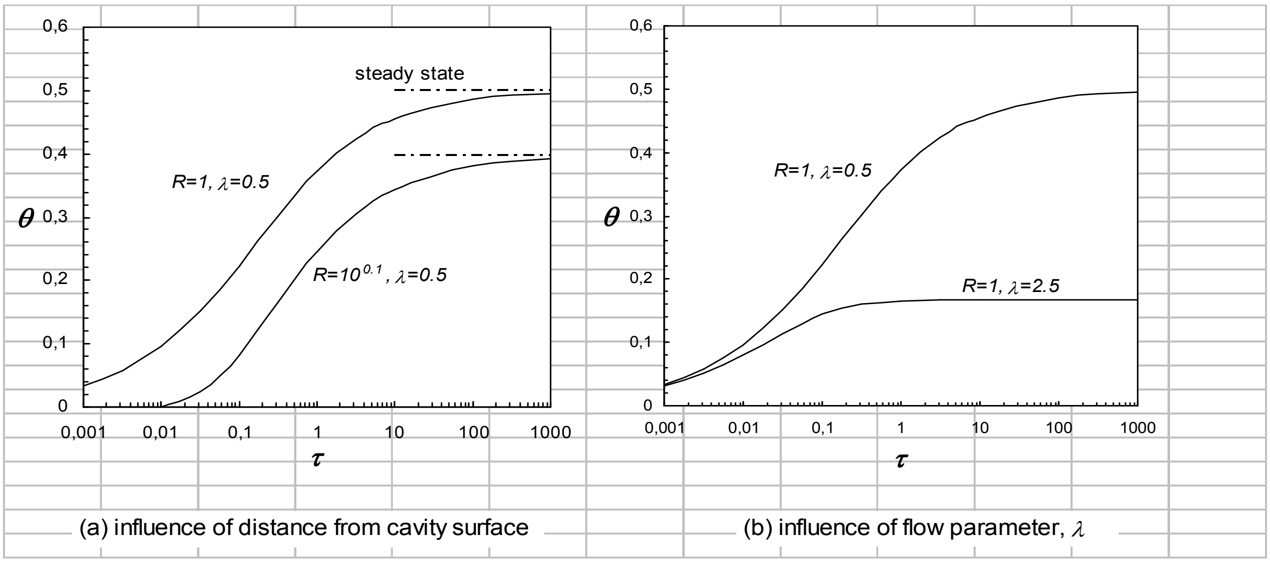

Fig. 7. Evolution of temperature at the cavity surface and within the porous medium, **Bi**=1.0.

The evolutions of temperature at the two locations are qualitatively similar. As an approximation, they could be deduced from each other by a scaling of the temperature axis and a shift along the time axis. The authors have not yet explored the physical or mathematical consequences of this observation, but they have verified it empirically by trial and error.

Fig. 7b shows the influence of λ on the temperature at the cavity surface. At sufficiently small times, the two curves are almost indistinguishable, indicating that the early-time behaviour is dominated by the cavity flow, not by the seepage flow. As time increases, the curves diverge strongly because the long-term condition depends increasingly upon the seepage flow rate (as shown in Fig. 3).

5.1. *Heat flux at the cavity surface*

Fig. 8 shows the evolution of the heat flux at the cavity surface for a range of values of λ and for three values of **Bi**. At the instant τ=0 when the imbalance is introduced, the temperature in the medium at the cavity is zero by - Eq.(18) - so that the temperature difference across the cavity surface namely ($1-\theta_{R=1}$) is unity. Consequently, using Eq.(20), the initial magnitude of the heat flux is equal to the Biot number, irrespective of the flow rate in the porous medium. This explains why the curves for different values of λ collapse onto a single curve at small times. As time increases, the influence of λ becomes increasingly strong. The larger the value of λ, the greater the heat flux at any instant and the sooner that conditions approach the steady-state asymptote.

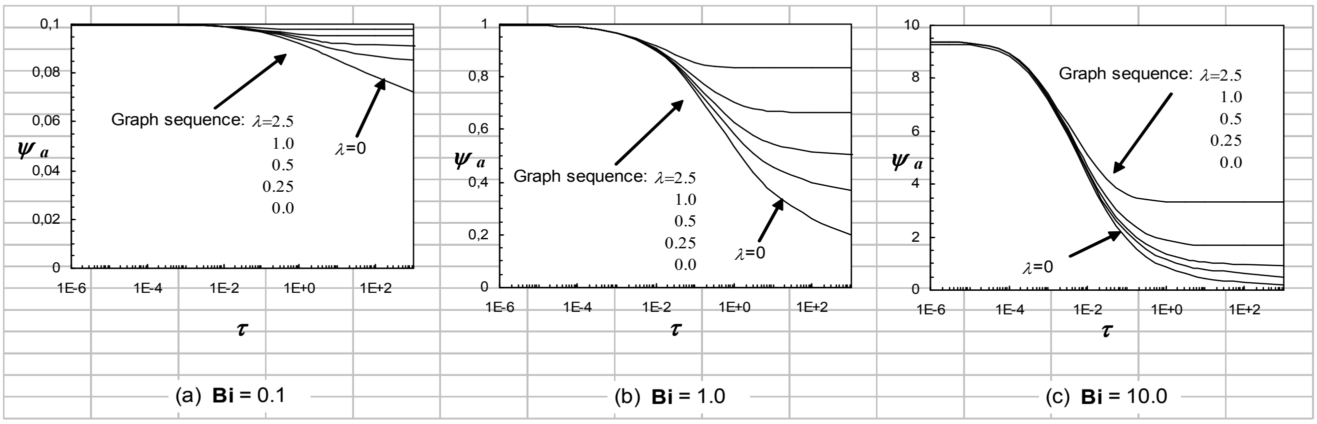

Fig. 8. Evolution of non-dimensional heat flux at the cavity surface.

5.2. *Time to approach steady-state*

In practical applications, it can be useful to quantify the time required to approach steady-state conditions. In common with the definition of the extent of the zone of influence, this is done in a structured way by evaluating the time required for the temperature change to reach a prescribed percentage of the steady-state value. Strictly, even this definition is multi-valued because the result depends upon the radial position (as noted in the discussion on Fig. 6 above). For many practical purposes, however, it is the conditions at the cavity surface that are of greatest importance. Accordingly, this is the location chosen herein.

Fig. 9 shows the times required to reach 95% of the steady state heat flux at the cavity surface. A range of values of λ is considered for each of **Bi** = 0.1, 1, $\sqrt{10}$ and 10. A striking feature of the figure is the strong dependence on λ, especially at small Biot numbers. With **Bi**=0.1, a ten-fold increase in λ causes a change by a factor of about10^5 in the time to reach 95% of the steady state surface heat transfer flux. As a practical example, consider the rate of water seepage towards a long railway tunnel deep below the ground surface. If λ=2.5, the pseudo steady-state condition might be achieved in a matter of weeks. In contrast, if λ=0.25, the equivalent period might be many centuries. In the former case, many design calculations might sensibly be based on relatively simple, steady state approximations. In the latter case, all calculations will need to take account of unsteady behaviour.

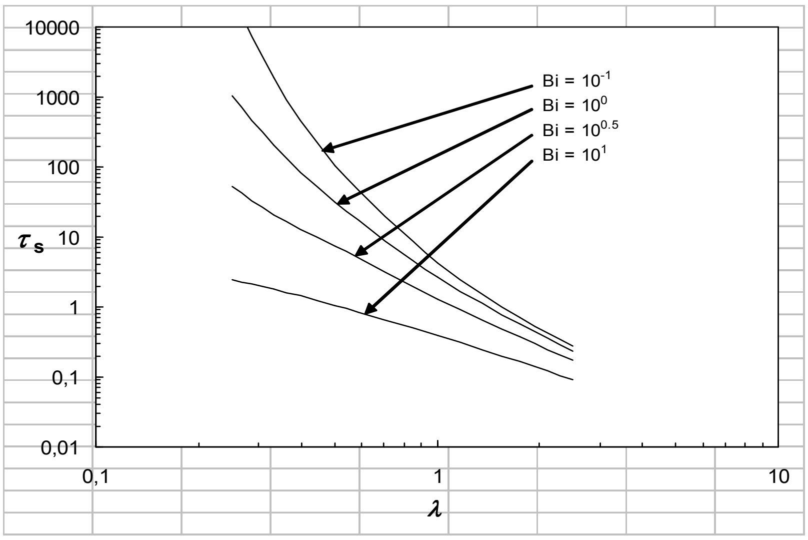

Fig. 9. Non-dimensional time for the surface heat flux to reach 95% of its steady state value.

6. CONCLUSIONS

(1) The thermal influence of seepage flow towards a cylindrical cavity in a porous medium has been studied for both steady and unsteady conditions. The most important new feature is the use of a convective heat transfer boundary condition at the cavity surface, characterising the influence of an independent fluid flow inside the cavity.

(2) The mathematical development has highlighted the importance of two non-dimensional parameters, λ and **Bi**. The first, a Peclet number, is a measure of the relative importance of radial advection and conduction in the porous medium. The second, a Biot number, characterises the relative importance of convective heat transfer to/from the cavity fluid and conduction within the porous medium.

(3) In the phenomenon under investigation, the influence of a sudden temperature change within the cavity has been studied for a range of values of λ and **Bi**. Starting with uniform conditions throughout the porous medium, successive temperature distributions lead towards a steady state asymptotic condition. This is approached most rapidly at the cavity surface and less rapidly at increasing distance from the cavity.

(4) The zone of significant influence of the cavity on temperatures in the medium increases continually, tending towards a maximum at steady state. The extent of the zone has been characterised by locating radii at which the temperature differs by a prescribed proportion of the temperature difference imposed at the cavity. The extent is greatest when the ratio **Bi**/λ is large and least when this ratio is small.

(5) The time required to approach steady state conditions is greatest when λ and **Bi** are smallest and it is least when they are largest. In some physical applications, the times may be so large that the use of simple steady state expressions will never be justified. In others, the times may be so small that engineering design may proceed on the basis of steady state assumptions alone.

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