Continuous sedimentation theory. Effects of density gradients and velocity profiles on sedimentation efficiency

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CONTINUOUS SEDIMENTATION THEORY

EFFECTS OF DENSITY GRADIENTS AND VELOCITY PROFILES ON SEDIMENTATION EFFICIENCY

T W M WOUDA, K RIETEMA and S P P OTTENGRAF
Laboratory for Physical Technology, Eindhoven University of Technology, P O Box 513 Eindhoven, Netherlands

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Abstract—A theory is presented on continuous sedimentation In case the solids concentration is small and uniformly distributed over the inlet height, the theory predicts independent sedimentation efficiencies on velocity distributions in a longitudinal vertical plane A velocity profile in a horizontal plane on the other hand will have a negative effect on the efficiency Experiments, carried out on a laboratory-scale model, have shown that even small density differences in the basin can have a significant effect on the velocity distribution The measured efficiencies are in good agreement with the theory

1 INTRODUCTION

Gravity sedimentation is an important unit operation in separating particles from a dispersion In waste water treatment it is one of the most commonly used techniques in the mechanical purification field

Hazen [1] was one of the first to investigate this subject and as early as 1904 developed a theory of continuous sedimentation The most important result of his study was the conclusion that the sedimentation efficiency in an ideal basin depends only on the settling velocity of the particles and the overflow rate of the basin

In practice, however, ideal sedimentation generally does not occur because of flocculation and hindered settling of the particles Non-uniform velocity profiles are also generally assumed to influence the sedimentation process (see Fig 1) These profiles can be induced by several causes, such as the shape of the basin, wind or density gradients

Density gradients are always present, thanks to differences in the solids concentration, at the inlet and the outlet of the basin Temperature differences may also cause differences in density Because of the higher density of the inlet, the inlet stream will move downwards in the basin and flow along the bottom in a layer decreasing in thickness

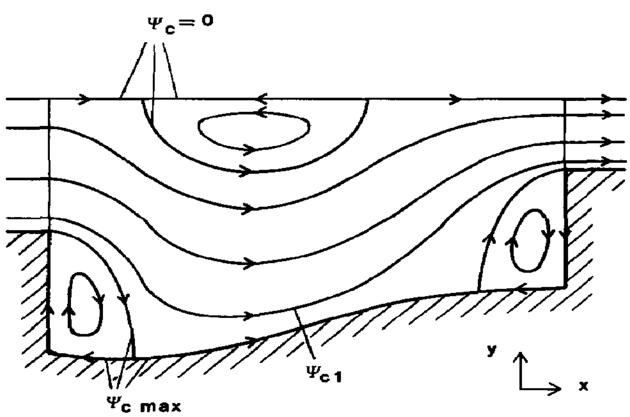

Fig 1 Streamlines of the continuous phase in a two-dimensional sedimentation basin In the circulations the streamlines are closed (in the x-direction the indicated dimensions are presented on a reduced scale)

The result of this effect will be a non-uniform velocity profile, but circulating flow may even occur in parts of the basin The expected influence of a non-uniform velocity distribution on the sedimentation efficiency has been the subject of many investigations reported in literature

Some of them [2, 3] state that the influence on the efficiency can be predicted with the aid of the residence time distribution of the basin

Takamatsu *et al* [4] tried to describe the efficiency as a function of the recirculation ratio (i e the ratio between the backward flow and the forward flow) A restriction of their model is that it only predicts efficiencies in the case of real backflow, while a basin with varying forward velocities is not considered

Clements *et al* and Price *et al* [5–7] predict that velocity distributions in a longitudinal vertical plane will have little effect on sedimentation, while a velocity profile in a horizontal plane will give lower efficiencies They introduce a parameter to give a qualitative prediction of the effect of velocity profiles, this parameter being known as the time ratio

2 THEORY

2 1 *Basic assumptions*

A general theory on continuous sedimentation will be developed based on the following assumptions

1 The sedimentation basin may be considered as two-dimensional In the case of a rectangular basin, the only one considered here, this means that the problem can be described with two coordinates the horizontal coordinate x and vertical coordinate y In the direction of the third coordinate z there are neither velocities nor velocity gradients

In the case of a circular basin the problem can similarly be described with the coordinates r and y (see Appendix)

2 The planes of inlet and outlet of the sedimentation basin are vertical and perpendicular to the horizontal component of the flow vector

3 The sedimentating system is in the steady state

4 The flow through the basin is laminar, turbulent mixing, at least, can be neglected, so that streamlines of both the liquid and the dispersed phase (particle trajectories) can be indicated

5 There is no proceeding flocculation and the settling particles or flocs are considered to be uniform in size and density so that their settling velocity is a function only of the liquid volume fraction ϵ

6 Once a settling particle has reached the bottom it is considered to be separated Slip of separated particles along the bottom (because of shear or an inclined bottom plane) is assumed not to affect the separation

7 The densities of solids and liquid are constant

No specific assumptions are necessary about the height of inlet and outlet or about the shape of the bottom It is also not necessary for inlet and outlet to extend to the bottom Because of density gradients in the suspension which will occur in the sedimentating system an overall circulation might arise in the basin while, because of wind and kinetic energy of the inlet flow, secondary circulations may be induced as indicated in Fig 1

2 2 *Conclusions from the continuity equations*

We will conceive the sedimentating suspension as a dispersed system consisting of a continuous phase, the liquid, and a dispersed phase, the settling solids To such a system we may apply the continuity equations, one for the continuous phase and one for the dispersed phase

With ϵ as the volume fraction of the continuous phase and $\mathbf{v}_c$ the linear velocity vector of this phase, its continuity eqn runs as follows

$$\frac{\partial \epsilon}{\partial t} = -\operatorname{div}(\epsilon \mathbf{v}_c) \tag{1}$$

With $\mathbf{v}_d$ the linear velocity of the dispersed phase its continuity eqn becomes

$$\frac{\partial \epsilon}{\partial t} = \operatorname{div}\{(1-\epsilon)\mathbf{v}_d\} \tag{2}$$

We will further define the slip velocity $\mathbf{v}_s$ of the settling solids as

$$\mathbf{v}_s = \mathbf{v}_d - \mathbf{v}_c \tag{3}$$

For this slip velocity it applies that in gravitational sedimentation there is only a vertical component v_{sy} which is related to the settling velocity $v_{s\infty}$ at infinite dilution rate by

$$v_{sy} = v_{s\infty}\epsilon^{n-1} \tag{4}$$

where n is the exponent in the well-known Richardson–Zaki eqn We will consider only the stationary state determined by $\partial\epsilon/\partial t = 0$ It then follows from (1), (2) and (3) respectively that

$$\epsilon \operatorname{div} \mathbf{v}_c + \mathbf{v}_c \cdot \operatorname{grad} \epsilon = 0 \tag{5}$$

and

$$(1-\epsilon)\operatorname{div}\mathbf{v}_c + (1-\epsilon)\operatorname{div}\mathbf{v}_s - (\mathbf{v}_c + \mathbf{v}_s)\cdot\operatorname{grad}\epsilon = 0 \tag{6}$$

By addition we find

$$\operatorname{div}\mathbf{v}_c + (1-\epsilon)\operatorname{div}\mathbf{v}_s - \mathbf{v}_s \cdot \operatorname{grad}\epsilon = 0 \tag{7}$$

Inserting this result again in eqn (6) we get

$$\mathbf{v}_d \cdot \operatorname{grad}\epsilon = (1-\epsilon)\{\epsilon \operatorname{div}\mathbf{v}_s + \mathbf{v}_s \cdot \operatorname{grad}\epsilon\} \tag{8}$$

With the help of eqn (4) and realizing that $\mathbf{v}_s$ has only a vertical component, eqn (8) can finally be worked out to give

$$\mathbf{v}_d \cdot \operatorname{grad}\epsilon = n(1-\epsilon)v_{sy}\left(\frac{\partial\epsilon}{\partial y}\right) \tag{9}$$

Case 1 At low solids concentration $\epsilon \approx 1$ while grad ϵ and $\partial\epsilon/\partial y$ are of the same order of magnitude

Since generally also $v_{sy} \ll |v_d|$ we may conclude that in this case

$$\mathbf{v}_d \cdot \operatorname{grad}\epsilon = 0$$

which means that along a streamline of solids $\partial\epsilon/\partial l$ disappears or that the solids concentration along such a streamline is constant If in this case the solids concentration at the inlet of the basin is constant over the height then this concentration is the same anywhere below the solids streamline $\psi_d = 0$

If there is a concentration distribution at the inlet it may be concluded that inside a stream channel of the solids (with boundaries ψ_{d1} and $\psi_{d1} + \Delta\psi_d$) the average concentration is constant

Case 2 The solids concentration cannot be neglected or ϵ differs sufficiently from unity

$$\mathbf{v}_d \cdot \operatorname{grad}\epsilon = v_{dx}\frac{\partial\epsilon}{\partial x} + v_{dy}\frac{\partial\epsilon}{\partial y} \tag{10}$$

because $v_{sx} = 0$ it follows from eqn (3) that $v_{dx} = v_{cx}$ Combining now eqn (9) with eqn (10) gives

$$\frac{\partial\epsilon}{\partial x}\bigg/\frac{\partial\epsilon}{\partial y} + \frac{v_{dy}}{v_{dx}} = \frac{n(1-\epsilon)v_{sy}}{v_{cx}}$$

When α is the angle which the local solids velocity vector makes with the horizontal and β the angle which grad ϵ makes with the horizontal, it follows that

$$\tan(90-\beta) + \tan\alpha = \frac{n(1-\epsilon)v_{sy}}{v_{cx}} \tag{11}$$

From this eqn, if α is known the direction of grad ϵ can be calculated

If the right-hand side is $\to 0$ it follows that grad ϵ is perpendicular to $\mathbf{v}_d$. Since the right-hand side in most cases is $\ll 1$ the deviation of this perpendicular relation is only small

2.3 *Conclusions from the stream functions*

By analogy with the definition of a stream function in stationary two-dimensional one-phase flow we will here define stream functions for the continuous phase and the dispersed phase

Let ψ_c be the stream function of the continuous phase where $\psi_c = 0$ indicates the upper streamline and $\psi_c = \psi_{c\,\max}$ the lower streamline of this phase, while ψ_{c1} also indicates the amount of liquid per unit breadth which flows through the stream channel between the streamlines $\psi_c = 0$ and $\psi_c = \psi_{c1}$

Then

$$\psi_{ci} - \psi_{c\,\max} = -\int_{y \text{ on } \psi_{c\,\max}}^{y_1 \text{ on } \psi_{ci}} \epsilon v_{cx}\, dy = \int_{x \text{ on } \psi_{c\,\max}}^{x_1 \text{ on } \psi_{ci}} \epsilon v_{cy}\, dx \tag{12}$$

It follows that

$$v_{cx} = -\frac{1}{\epsilon}\frac{\partial \psi_c}{\partial y}$$

and

$$v_{cy} = \frac{1}{\epsilon}\frac{\partial \psi_c}{\partial x}$$

In the same way the stream function ψ_d of the dispersed phase is defined as

$$\psi_{di} - \psi_{d\,\max} = -\int_{y \text{ on } \psi_{d\,\max}}^{y_1 \text{ on } \psi_{di}} (1-\epsilon) v_{dx}\, dy = \int_{x \text{ on } \psi_{d\,\max}}^{x_1 \text{ on } \psi_{di}} (1-\epsilon) v_{dy}\, dx \tag{13}$$

from which

$$v_{dx} = -\frac{1}{1-\epsilon}\frac{\partial \psi_d}{\partial y}$$

and

$$v_{dy} = \frac{1}{1-\epsilon}\frac{\partial \psi_d}{\partial x}$$

Of course ψ_d is only defined on places where $\epsilon < 1$. From relation (3) it now follows that

$$\frac{1}{1-\epsilon}\frac{\partial \psi_d}{\partial y} = \frac{1}{\epsilon}\frac{\partial \psi_c}{\partial y}$$

and

$$\frac{1}{1-\epsilon}\frac{\partial \psi_d}{\partial x} = \frac{1}{\epsilon}\frac{\partial \psi_c}{\partial x} + v_{sy}$$

Or more generally that

$$\frac{\epsilon}{1-\epsilon}\frac{\partial \psi_d}{\partial l} = \frac{\partial \psi_c}{\partial l} + \epsilon v_{sy}\frac{\partial x}{\partial l} \tag{14}$$

We now integrate eqn (14) along a specific streamline of the dispersed phase (for which $\psi_d = \psi_{di}$). Along such a streamline $\partial\psi_d/\partial l = 0$

With the boundary condition that at $x = 0$ the liquid streamline which starts at the same point as the streamline ψ_{di} has a value ψ_{ci} it is found that

$$\psi_{ci} = \psi_c^* + \int_0^{x_1} \epsilon v_{sy}\, dx \tag{15}$$
(along streamline ψ_{di})

In this eqn ψ_c^* is the value of the liquid streamline which intersects the dispersed-phase streamline ψ_{di} at $x = x_1$

As stated in Section 2 on the basis of eqn (9), at not too high solids concentration this concentration and therefore also ϵ is constant along a solids streamline. As v_{sy} is only a function of ϵ it follows that ϵv_{sy} is constant along a solids streamline

In this case eqn (15) may be written as

$$\psi_{ci} = \psi_c^* + \epsilon v_{sy} x_1 \tag{16}$$

This eqn means that the distance x_1 at which the liquid streamline ψ_c^* intersects the solids streamline ψ_{di} (to which the value ψ_{ci} belongs) can be calculated from eqn (16) and that to calculate x_1 no knowledge is needed about the shape of the sedimentation basin nor about circulating flows which occur in this basin

The streamline $\psi_{c\,\max}$ which always runs along the bottom of the basin (also in the case of circulating flows) can be chosen for the liquid streamline ψ_c^* as well

A particle, therefore, can be considered to be separated if its streamline intersects the liquid streamline $\psi_{c\,\max}$. This will happen at a point x_1 which again can be calculated from eqn (16)

The efficiency of the sedimentation basin can be calculated by realizing that particles which are just not separated are following a solids streamline which intersects the liquid streamline $\psi_{c\,\max}$ at $x_1 = L$ where L is the horizontal coordinate of the outlet (see Fig 2)

Let this specific streamline be $\psi_{d\eta}$ while the value of the liquid streamline which starts at the same point of the inlet is $\psi_{c\eta}$, then

$$\psi_{c\eta} = \psi_{c\,\max} + \epsilon v_{sy} L \tag{17}$$

If the solids concentration (and therefore also ϵ) is constant over the height of the inlet, ϵ is constant anywhere below $\psi_d = 0$ and it can easily be shown that at the inlet ($x = 0$) and at the same height

$$\frac{1}{1-\epsilon}\psi_d = \frac{1}{\epsilon}\psi_c$$

In that case

$$\psi_{c\eta} = \frac{\epsilon}{1-\epsilon}\psi_{d\eta}$$

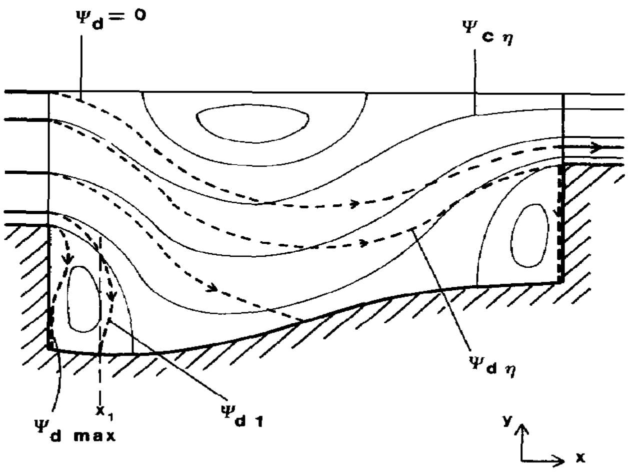

Fig 2 Streamlines of the dispersed phase (dashed lines) in a two-dimensional sedimentation basin The solid lines indicate the corresponding streamlines of the continuous phase, which start at the same inlet height

while

$$\psi_{c\,max} = \frac{\epsilon}{1-\epsilon}\psi_{d\,max} = \epsilon\Gamma$$

in which $\Gamma = \psi_{c\,max} + \psi_{d\,max}$ = the volumetric load of the suspension per unit breadth

It now follows from eqn (17) that

$$\psi_{d\eta} = \psi_{d\,max} + (1-\epsilon)v_{sy}L$$

The efficiency is defined by

$$\eta = \frac{\psi_{d\,max} - \psi_{d\eta}}{\psi_{d\,max}} \tag{18}$$

$$\eta = \frac{-(1-\epsilon)v_{sy}L}{\psi_{d\,max}} = \frac{|v_{sy}|L}{\Gamma} \tag{19}$$

which in fact is the well known *Hazen equation*

If the solids concentration is not constant over the height of the inlet the efficiency is also determined by this solids concentration distribution

If this distribution is, however, such that $(1-\epsilon) \ll 1$ anywhere, $\psi_{d\eta}$ is still determined by $\psi_{c\eta}$, which follows from eqn (17) From the concentration distribution at the inlet, the part of the solids which is separated and with it the efficiency can be derived

2 4 *Effect of circulations*

In terms of stream functions, a circulation means an area in which the streamlines of the liquid phase are closed (see Fig 1) It is interesting to follow a solids streamline which traverses a circulation (see Fig 2)

Let ψ_{di} intersect the boundary liquid streamline of the inlet circulation at $x = x_1$, while it reaches the bottom at $x = x_2$ Integrating eqn (15) along ψ_{di} gives

$$\psi_{c2} = \psi_{c1} - \int_{x_1}^{x_2} \epsilon v_{sy}\, dx \ (x \text{ along } \psi_{d1})$$

in which ψ_{c1} and ψ_{c2} are the liquid streamlines that intersect streamline ψ_{d1} at $x = x_1$ and x_2 respectively However, $\psi_{c1} = \psi_{c2} = \psi_{c\,max}$ and thus

$$\int_{x_1}^{x_2} \epsilon v_{sy}\, dx = 0$$

From this it follows that $x_1 = x_2$ and so the streamline ψ_{d1} leaves the circulation at the same horizontal distance as it enters the circulation The particles entering the basin along the solids streamline $\psi_{d\,max}$ will therefore be separated at $x = 0$

This means that if a circulation occurs below the inlet there must be a particle-free zone in this area which has the solids streamline $\psi_{d\,max}$ as a boundary

2 5 *Influence of a velocity profile in the z-direction*

In the foregoing theory it is assumed that neither velocities nor velocity gradients are present in the z-direction A velocity profile in the z-direction complicates the description of the system while it can no longer be considered as two-dimensional

However, as long as the velocity in the z-direction is zero anywhere, so that all streamlines move parallel to the side walls of the basin, the stream function theory can still be applied In a small vertical slice of the basin between z and $z + \Delta z$ the load Γ is then constant and the efficiency of this part of the basin is given by eqn (19)

$$\eta(z) = \frac{v_{sy}L}{\Gamma(z)} \quad \text{if } \Gamma(z) > v_{sy}L$$

and

$$\eta(z) = 1 \quad \text{if } \Gamma(z) \le v_{sy}L$$

η and Γ are now functions of z

The overall efficiency $\bar{\eta}$ can now be calculated from

$$\bar{\eta} = \frac{\int_0^B \eta(z)\Gamma(z)\, dz}{Q} \tag{20}$$

In which $Q = \int_0^B \Gamma(z)\, dz$ is the total suspension load of the basin It will be shown that this overall efficiency $\bar{\eta}$ is less than or equal to

$$\eta_0 = \frac{v_{sy}LB}{Q} \tag{19a}$$

which is the efficiency of the basin with a flat velocity profile in the z-direction

Assume there is a region $z_1 \le z \le z_2$ where the load is so low that all particles will be separated, so $\eta(z) = 1$

The overall efficiency is then given by the general eqn

$$\bar{\eta} = \frac{\int_{0\ \text{except}\ z_1 \le z \le z_2}^{B} v_{sy}L\, dz + \int_{z_1}^{z_2} \Gamma(z)\, dz}{Q} \tag{21}$$

Because $\Gamma(z) \leq v_{sy}L$ in the region $z_1 \leq z \leq z_2$ it follows that $\bar{\eta} < \eta_0$ Such a region, for which $\eta(z) = 1$, can generally be expected near the side walls of the basin

It may be that the streamlines are not moving parallel to the side walls of the basin This can for instance be caused by cross-wind If this is the case the stream function theory can no longer be applied

2 6 *Influence of settling velocity distribution*

If the particles are not uniform in size, shape or density there will be a settling-velocity distribution $G(v_{sy})$, (see Fig 3) with the property

$$\int_0^{\infty} G(v_{sy})\,\mathrm{d}v_{sy} = 1$$

Hence $G(v_{sy})\Delta v_{sy}$ represents the mass fraction of particles, having a settling velocity between v_{sy} and $v_{sy} + \Delta v_{sy}$, if Δv_{sy} is chosen sufficiently small

The theoretical consideration put forward in Section 2 1 still holds good, as does eqn (19), in which η is now a function of v_{sy} For the total efficiency of all particles having settling velocities between v_{sy1} and v_{sy2} it follows in this case that

$$\eta = \int_{v_{sy1}}^{v_{sy2}} \eta(v_{sy})G(v_{sy})\,\mathrm{d}v_{sy} \tag{22}$$

If there is a critical velocity v^*_{sy} for which $\eta(v^*_{sy}) = 1$, η is given by

$$\eta = \int_{v_{sy1}}^{v^*_{sy}} \eta(v_{sy})G(v_{sy})\,\mathrm{d}v_{sy} + \int_{v^*_{sy}}^{v_{sy2}} G(v_{sy})\,\mathrm{d}v_{sy} \tag{23}$$

In case the velocity profile in the x–z plane is not flat (Section 2 2) $\eta(v_{sy})$ is given by eqn (21) ($\epsilon \approx 1$) (z_1 and z_2 are now functions of v_{sy}) The total efficiency now is given again by eqn (22) If v_{sy} reaches a critical value for which $z_1 = z_2$, $\eta(v^*_{sy})$ will be $= 1$ Now here the efficiency for all particles follows from eqn (23)

3 EXPERIMENTAL

Experiments were carried out in a so-called two-dimensional laboratory basin with a free interface The dimensions of the basin are $91 \times 3 \times 13$ cm ($L \times B \times H$) The experimental set-up is shown in Fig 4

The suspension is made up of resin pellets in water The pellets have a density of 1040 kg/m^3 The settling-velocity distribution G of the suspended resin pellets in tap water has been determined with a sedimentation balance at a concentration of about 13 kg/m^3 The result is shown in Fig 3 The suspension is kept in a storage tank[1] (70 l) From this tank it enters the distribution tank[2] from where the suspension is fed uniformly over the total height into the sedimentation basin[3]

The outlet stream, again uniformly over the total height, is mixed up and passes a Sigrist photometer[4] This photometer is used to determine when the conditions are stationary The flow is measured with a flow meter[5]

The suspension is then received in a tank[7] This is done because pumping it back in the storage tank during an experiment would cause a time-depending inlet concentration of the suspension

To vary the velocity profile in an x–y plane over the height different solids concentrations are used It was experimentally verified that this velocity profile was nearly flat when the basin was fed only with water (of constant temperature)

4 RESULTS

Experiments have been carried out using four different solids concentrations 5, 10, 15 and 20 kg/m^3, with corresponding values of $1 - \epsilon_i = 4\,8, 9\,6, 14$ and 19×10^{-3}

These values of $1 - \epsilon_i$ are so low, that they have a negligible effect on the settling velocity v_{sy} (see eqn 4) On the other hand the change in solids concentration will cause a marked effect on the velocity profile in the x–y plane Figures 5 and 6 clearly show that, as a consequence of the different values of ϵ_i different density-gradients did appear, resulting in velocity profiles with velocities in the lower part of the basin much greater than the superficial

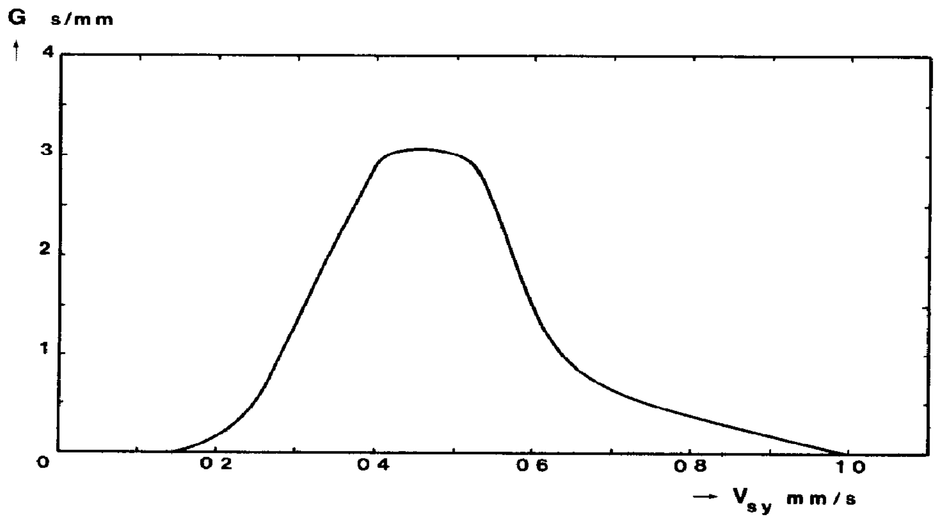

Fig 3 Settling-velocity distribution $G(v_{sy})$ as a function of the settling velocity v_{sy} of resin pellets in tap water

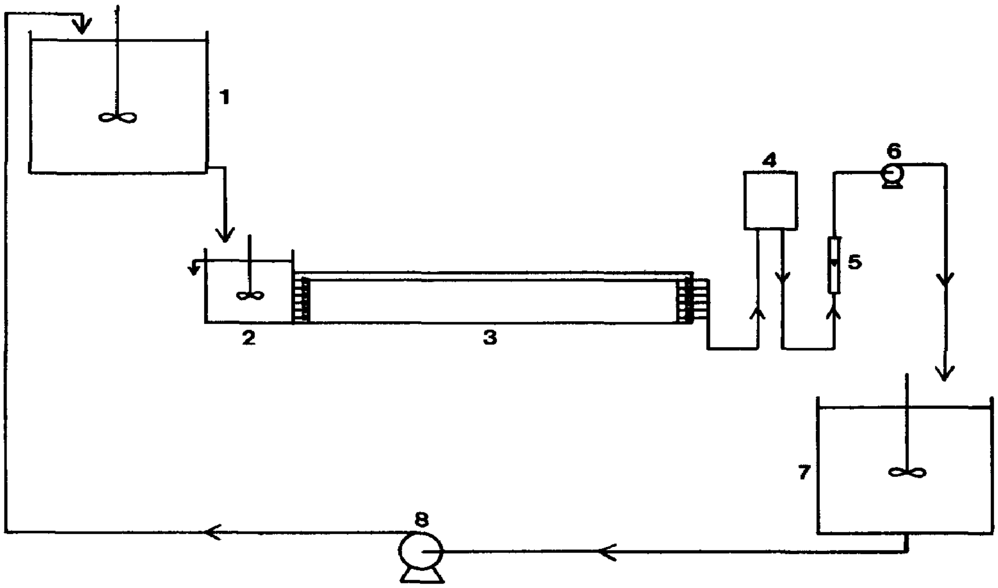

Fig 4 Experimental set-up (1), storage tank, (2), distribution tank, (3), sedimentation basin, (4), Sigrist photometer, (5), flow meter, (6), pump, (7), storage tank, (8), pump

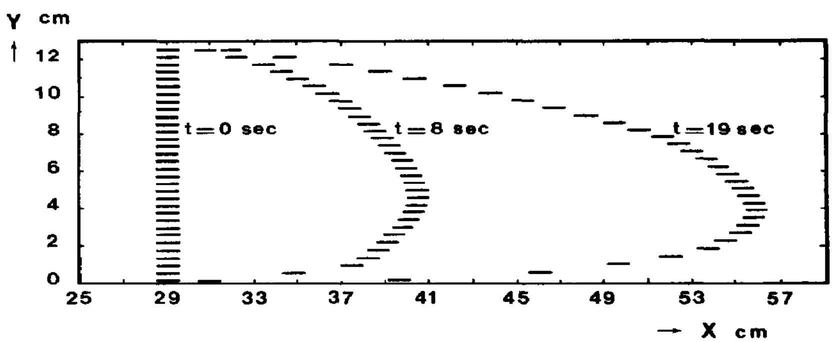

Fig. 5 Velocity profile in the x–y plane when the basin is fed with a suspension, the solids concentration of which is 5 kg/m^3 The superficial velocity v_{co} = 0 7 cm/s The dye is injected over the whole height of the basin The figure shows the position of the tracer at t = 0, 8 and 19 sec respectively

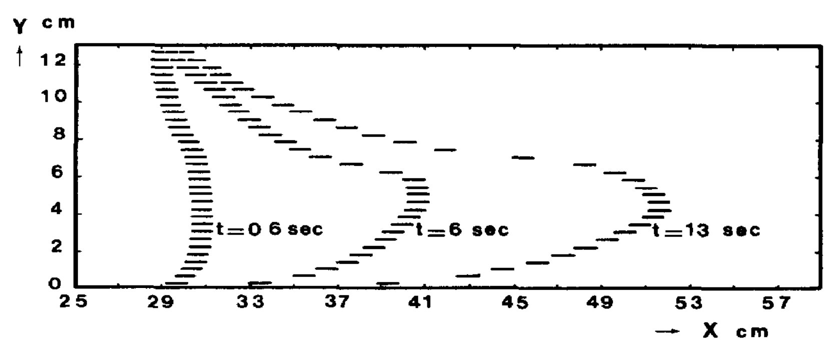

Fig 6 Velocity profile in the x–y plane when the basin is fed with a suspension, the solids concentration of which is 20 kg/m^3 The superficial velocity v_{co} = 0 7 cm/s The figure shows the position of the tracer at t = 0 6, 6 and 13 sec respectively

velocity The figures show examples with a superficial velocity v_{co} = 0 7 cm/s while $1-\epsilon_i$ = 4 8 and 19×10^{-3} respectively The densities of the inlet suspension are $\bar{\rho}$ = 1000 8 and 1000 2 kg/m^3 respectively (ρ_c = 1000 kg/m^3)

Using a value for v_{co} = 0 49 cm/s, real backflow is noted in the upper part of the basin for $1-\epsilon_i = 19\times10^{-3}$

The measured sedimentation efficiencies are given in Fig 7 The surface load (=Q/BL) is varied between 0 5 and 2 0 mm/s, resulting in Re numbers sufficiently low to

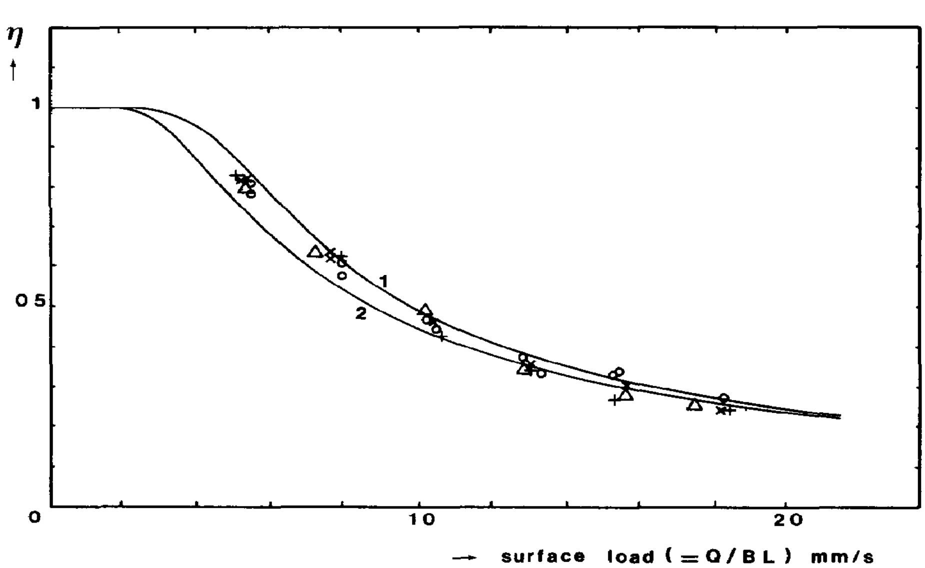

Fig 7 Measured efficiencies vs surface loading at several velocity profiles in the basin The different profiles were created by varying the solids concentration 5(○), 10(×), 15(△), and 20(+) kg/m³ Line 1 represents the theoretical efficiency for a flat profile in the x–z plane and follows from Fig 3, eqn (19a) and (23) Line 2 corresponds to a parabolic profile in the x–z plane and follows from Fig 3, eqn (21) and (23)

prevent turbulent mixing ($Re_{max} = \rho_c Q d_h / B\, H\, \mu = 810$ in which $d_h = 4\, B\, H / (B + 2H)$)

The lines represent theoretical curves and have been calculated from the settling velocity distribution, using eqns (19a), (22) and (23) for line 1 and eqns (21), (22) and (23) for line 2 successively Line 1 corresponds to a flat velocity profile, while line 2 corresponds to a parabolic profile in the x–z plane having the same mean velocity

$$v_c = \frac{3}{2} v_{c0} \left[1 - \frac{(2z - B)^2}{B^2} \right]$$

With this parabolic velocity profile the efficiency for particles having a settling velocity v_{sy} follows from eqn (20)

$$\eta(v_{sy}) = 1 - \left[1 - \frac{2}{3} \eta_0 \right]^{3/2} \qquad (\eta_0 < 1)$$

and the overall efficiency from eqns (22) and (23)

Parabolic profiles in the two-dimensional basin were measured using pure tap water As soon as the basin is fed with a suspension the velocity profile flattens due to small pressure gradients which arise in the z-direction These gradients are caused by the density differences which arise because particles near the side walls of the basin are settling closer to the inlet than particles in the middle The result is that real efficiencies are found between line 1 and 2

Equation (16) has an interesting implication, which is discussed below

$$\psi_{c1} = \psi_c^* + \epsilon v_{sy} x_1$$

If $\psi_{c\,max}$ is chosen for ψ_c^* (as discussed in Section 2 3), x_1 indicates the place where the solids streamline ψ_{d1} reaches the bottom of the basin

If ϵ is constant over the inlet height, then

$$\psi_{c1} = \frac{\epsilon}{1-\epsilon} \psi_{d1} \quad \text{and} \quad \psi_{c\,max} = \frac{\epsilon}{1-\epsilon} \psi_{d\,max}$$

It follows that

$$\psi_{d1} = \psi_{d\,max} + (1 - \epsilon) v_{sy} x_1$$

The second term on the right hand represents the amount of particles separated between $x = 0$ and $x = x_1$ This means that the sediment layer will have the same height throughout the whole length of the basin, independent of the shape of the bottom

This theoretical result has been confirmed experimentally

5 CONCLUSIONS

The values of $1-\epsilon$ used in the experiments are low enough to compare the results with the theory worked out in case 1 of Section 2 2, where the solids concentration $1-\epsilon$ is assumed to be small and uniformly distributed over the inlet height The theory then predicts a sedimentation efficiency which is not influenced by a velocity profile in an x–y plane

It is derived also that a velocity profile in an x–z plane will influence the performance of the basin negatively Although the influence of the different solids concentrations on velocity profile in an x–y plane is clearly demonstrated, the measured efficiencies are not influenced by this phenomenon The values of η are in good agreement with the theoretical ones Since the velocity profile in an x–z plane lies between a flat profile and a parabolic one, the values of η are expected to lie between line 1 and 2 in Fig 7 The results confirm this

In case the solids concentration cannot be neglected (case 2, Section 2 2) additional information about the velocity profile of the dispersion will be needed to determine the course of ϵ along a solids streamline and from it the efficiency

Acknowledgement—The authors would like to express their thanks to J J J den Ridder for his contribution to this investigation during the last year of his course in engineering

NOTATION

- B breadth of the basin, m
- d_h hydraulic diameter of the basin, m
- G settling velocity distribution of the particles, s/m
- H height of the basin, m
- l local direction of solids movement, m
- L length of the basin, m
- n exponent in the Richardson-Zaki relation
- Q total suspension load of the basin, m^3/s
- R outer radius of the circular basin, m
- $\mathbf{v}_c$ velocity vector of the continuous phase, m/s
- $\mathbf{v}_d$ velocity vector of the dispersed phase, m/s
- $\mathbf{v}_s$ slip velocity vector of the settling particles, m/s
- x, y, z coordinates in cartesian system, m
- r, y, θ coordinates in cylindrical system, m, rad

Greek symbols

- α angle between the local solids velocity vector and the horizontal plane, rad
- β angle between grad ϵ and the horizontal plane, rad
- Γ volumetric suspension load per unit breadth, m^2/s
- ϵ volume fraction of the continuous phase
- η sedimentation efficiency of the basin
- ρ_c density of the continuous phase, kg/m^3
- ρ_d density of the dispersed phase, kg/m^3
- $\bar{\rho}_i$ density of the inlet suspension, kg/m^3
- ψ_c stream function of the continuous phase, m^2/s
- ψ_d stream function of the dispersed phase, m^2/s

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APPENDIX

If in the case of a circular basin, the system has neither velocities nor velocity gradients in the θ direction, the problem can again be considered as two-dimensional and described with the horizontal coordinate r and the vertical coordinate y It will be shown that the theoretical results obtained in Section 2 apply

The definition of the liquid stream function now follows from

$$\psi_{c1} - \psi_{c\,\max} = -\int_{y \text{ on } \psi_{c\,\max}}^{y_1 \text{ on } \psi_{c1}} \epsilon r v_{cr}\, dy = \int_{r \text{ on } \psi_{c\,\max}}^{r_1 \text{ on } \psi_{c1}} \epsilon r v_{cy}\, dr \tag{12a}$$

Where ψ_c now represents the amount of liquid per *radial* Thus

$$v_{cr} = -\frac{1}{\epsilon r}\frac{\partial \psi_c}{\partial y}$$

$$v_{cy} = \frac{1}{\epsilon r}\frac{\partial \psi_c}{\partial r}$$

By analogy

$$v_{dr} = -\frac{1}{(1-\epsilon) r}\frac{\partial \psi_d}{\partial y}$$

$$v_{dy} = \frac{1}{(1-\epsilon) r}\frac{\partial \psi_d}{\partial r}$$

Equation (14) now is modified in

$$\frac{\epsilon}{1-\epsilon}\frac{\partial \psi_d}{\partial l} = \frac{\partial \psi_c}{\partial l} + r\epsilon v_{sy}\frac{\partial r}{\partial l} \tag{14a}$$

Following the same procedure and making the same assumptions as in Section 2 3, eqn (17) becomes

$$\psi_{c\eta} = \psi_{c\,\max} + \epsilon v_{sy}\frac{1}{2}R^2 \tag{17a}$$

where R is the outer radius of the basin Again it can be shown that

$$\psi_{c\eta} = \frac{\epsilon}{1-\epsilon}\psi_{d\eta} \quad \text{and} \quad \psi_{c\,\max} = \frac{\epsilon}{1-\epsilon}\psi_{d\,\max} = \epsilon\Gamma$$

where Γ is now the volumetric load of the suspension per *radial* It follows for

$$\eta = \frac{|v_{sy}|\frac{1}{2}R^2}{\Gamma} \tag{19b}$$

The influence of a velocity profile in the θ direction is given by eqn (20a)

$$\bar{\eta} = \frac{\int_0^{2\pi} \eta(\theta)\Gamma(\theta)\, d\theta}{Q} \tag{20a}$$

where

$$Q = \int_0^{2\pi} \Gamma(\theta)\, d\theta$$

and

$$\eta_0 = \frac{v_{sy}\pi R^2}{Q} \tag{19c}$$