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A mathematical relation between volume strain, elongational strain and stress in homogeneous deformation

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A model is presented for the volume strain of a two-phase blend which elongates homogeneously in a tensile test apparatus. In the case when only elastic deformation and crazing take place the volume strain against elongation curve can be constructed and calculated from the data of the stress–strain curve alone. When, as well as crazing and elastic deformation, shearing takes place, the data of the stress against elongation curve and the volume strain against elongation curve can be used to calculate the separate contributions of the three deformation mechanisms at any elongation. In principle, the model can be also used for any homogeneous system which deforms without necking and where one or more deformation mechanism is present.

1. Introduction

In order to study mechanical deformation of two-phase polystyrene (PS)–low-density polyethylene (PE) blends and of high impact polystyrene a dilatometer was developed to determine volume strain during tensile deformation [1–3]. The results prompted the development of a simple model that describes the volume strain as a function of stress and strain for constant strain-rate experiments. Assuming the additivity of volume strain and elongational strain caused by elasticity, crazing and shearing, as was done for creep tests by Bucknall [4–6], it is possible to write

$$\Delta V/V_0 = (\Delta V_{el} + \Delta V_{sh} + \Delta V_{cr})/V_0$$

$$= (\Delta V/V_0)_{el} + (\Delta V/V_0)_{sh} + (\Delta V/V_0)_{cr}, \tag{1}$$

where ΔV is the change in volume strain, V_0 is the zero-strain volume, and ΔV_{el}, ΔV_{sh} and ΔV_{cr} are the change in components of volume strain caused by elasticity, shearing and crazing.

$$\epsilon = \Delta l/l_0 = (\Delta l_{el} + \Delta l_{sh} + \Delta l_{cr})/l_0$$

$$= \epsilon_{el} + \epsilon_{sh} + \epsilon_{cr}, \tag{2}$$

where ϵ is the elongation strain, Δl is the change in length, l_0 is the zero-strain length, Δl_{el}, Δl_{sh}

and Δl_{cr} are the changes in length due to elasticity, shearing and crazing and ϵ_{el}, ϵ_{sh} and ϵ_{cr} are the contributions to the elongation strain caused by elasticity, shearing and crazing.

From the definition of Poisson's ratio, ν_{el},

$$(\Delta V/V_0)_{el} = (1 - 2\nu_{el})\epsilon_{el}, \tag{3}$$

while

$$\epsilon_{el} = \frac{\sigma}{E}, \tag{4}$$

where σ is the stress and E is Young's modulus. The contribution of crazing to the volume strain is given by

$$(\Delta V/V_0)_{cr} = \epsilon_{cr}, \tag{5}$$

while

$$(\Delta V/V_0)_{sh} = 0. \tag{6}$$

Thus,

$$\Delta V/V_0 = \epsilon_{el}(1 - 2\nu_{el}) + \epsilon_{cr} \tag{7}$$

or

$$\Delta V/V_0 = \epsilon_{el}(1 - 2\nu_{el}) + \epsilon - \epsilon_{sh} - \epsilon_{el}, \tag{8}$$

where ϵ_{el} may be set equal to σ/E for all values of ϵ_{sh} and ϵ_{cr} when the amount of material subjected to elastic deformation is constant.

In the case of the PS–(low-density)PE blends mentioned above, the total elongation-to-break is about 10 per cent, of which 1 to 2 per cent is

elastic. In case of crazing only, the void content is then ultimately about 8 per cent. Assuming that an approximately equal fraction of the sample material is transformed into craze-filling material, the amount of matrix available to deform elastically will always be higher than 92 per cent. This means that the elongation, ϵ_{el}, should be corrected by a factor, Δ, where $0.92 < \Delta < 1$. When the matrix also deforms by shear the same factors must be considered. As a first approximation, however, Equation 8 will be correct.

2. Crazing and elastic deformation only
In many high-impact polystyrenes shearing is negligible and Equation 8 reduces to

$$\Delta V/V_0 = (1 - 2\nu_{el})\frac{\sigma}{E} + \epsilon - \frac{\sigma}{E}. \qquad (9)$$

Again, the first term represents the elastic contribution to the volume strain and the following two terms the contribution of crazing. Rearrangement gives

$$\Delta V/V_0 = \epsilon - 2\nu_{el}\frac{\sigma}{E}. \qquad (10)$$

Equation 10 can be used to calculate the volume strain against elongational strain curve from a stress–strain curve. The stress, σ, is approximated by the engineering stress and the Young's modulus, E, is taken as the initial slope of the stress–strain curve. Poisson's ratio is either known or can be calculated from the initial slope of the volume strain against elongational strain curve. For any point, (ϵ, σ), of a given stress–strain curve (see
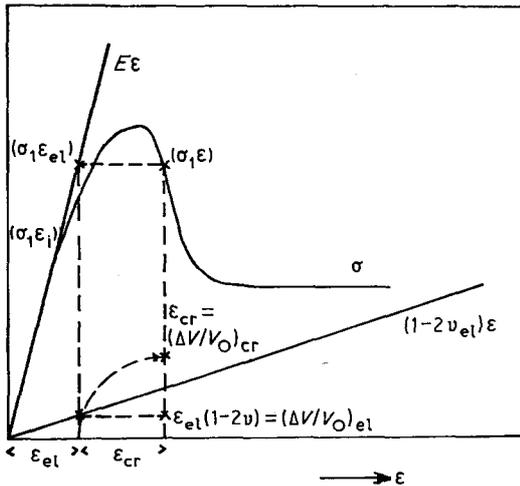

Figure 1 Construction of a volume strain curve from a known stress–strain curve.
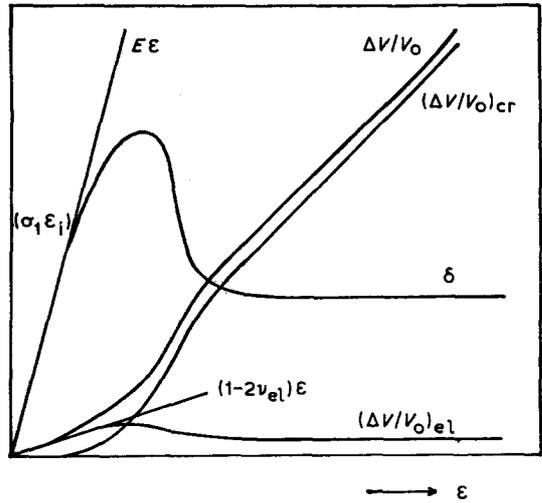

Figure 2 Construction of a volume strain curve from a known stress–strain curve.

Fig. 1), ϵ_{el} can be found by means of the proportional relation $\sigma = E\epsilon_{el}$.

Since a series model for strains is assumed, ϵ_{cr} is the difference between ϵ and ϵ_{el}. Now the contribution of the elastic deformation to the total volume strain for the point (ϵ, σ) can be found from the relation $(\Delta V/V_0) = (1 - 2\nu_{el})\epsilon_{el}$. The contribution of crazing to the total volume strain, $(\Delta V/V_0)_{cr} = \epsilon_{cr}$. In Fig. 1 (for one point) and in Fig. 2 (for all points) these contributions are calculated from a given stress–strain curve.

The slope of the volume strain against elongation curve is given by Equation 11

$$\frac{d(\Delta V/V_0)}{d\epsilon} = 1 - \frac{2\nu_{el}}{E}\frac{d\sigma}{d\epsilon}. \qquad (11)$$

Since, after yielding, $d\sigma/d\epsilon$ is negative, $d(\Delta V/V_0)/d\epsilon$ must be greater than 1. It becomes clear that a high value of gradient (> 1) results from the stress drop after the yield-point due to a rate of void formation which is high compared to the rate of elongation. At the yield-point and at high elongation values, where $d\sigma/d\epsilon = 0$, the slopes of the volume strain curve are equal to one. It is evident that void formation by crazing is initiated before the stress maximum at a point (ϵ_i, σ_i) (Fig. 2).

The general features predicted by the model are confirmed by experimental curves of PS–(low-density)PE blends. In Figs 3 and 4 the experimental results are presented for a commercial high-impact polystyrene and for a PS–PE blend
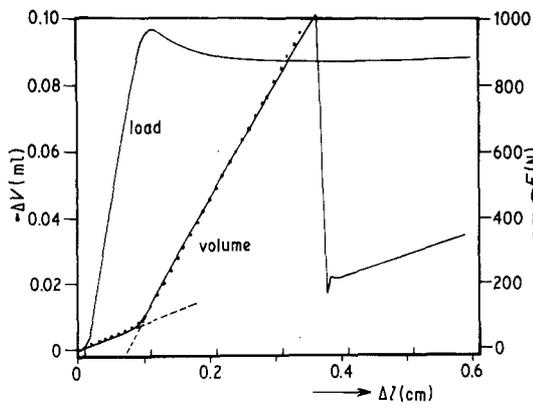
Figure 3 Experimental curves of load against elongation and of volume strain against elongational strain of high-impact polystyrene (Dow Chemical Co.). The change in sensitivity of volume strain Δl was about $\Delta l = 0.4$. Volume strain calculated using Equation 9 from the stress–strain curve is represented by dots. The experimentally determined results are represented by the full lines. Maximum slope of volume strain, $d(\Delta V/V_0)/d\epsilon = 1.06$.

[1]. The full curves are experimentally determined whereas the dots have ΔV and ϵ as co-ordinates and are calculated from Equation 9. As the true stress deviates by less than one per cent from the
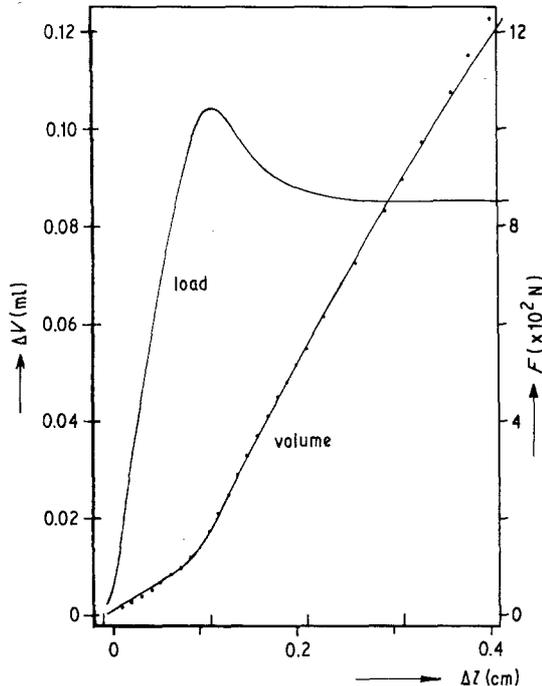

Figure 4 Experimental curves, as in Fig. 3, for a PS–(low-density)PE blend (92.5 vol% PS–7.5 vol% PE. Dots calculated using Equation 9. Maximum slope of volume strain, $d(\Delta V/V_0)/d\epsilon = 1.14$.

engineering stress, a value of the engineering stress was used instead of a true stress value. E and ν were taken from the initial slopes of the stress and volume strain curve. The agreement between the model and experimental results is very good. The maximum slopes of the volume strain curves for these cases are greater than one and are given in captions of Figs 3 and 4.

3. The deformation mechanisms present

For ABS, and for some special PS–(low-density)PE blends containing certain block co-polymers, both crazing and shearing are found [1] as deformation mechanisms. Rearrangement of Equation 8 gives

$$\epsilon_{sh} = (1 - 2\nu_{el})\frac{\sigma}{E} + \left(\epsilon - \frac{\sigma}{E}\right) - \Delta V/V_0. \tag{12}$$

The sum of the first two terms of Equation 12 represents the volume change of a hypothetical material with the same stress–strain curve in which no shearing but only elastic deformation and crazing takes place during elongation. Thus ϵ_{sh} can be calculated by combining the data from the stress–strain curve and the volume strain curve. Also, ϵ_{cr} can be found from Equation 7 by subtracting the elastic contribution to the volume strain, $(1 - 2\nu_{el})\,\sigma/E$ from the experimental volume strain giving

$$\epsilon_{cr} = \Delta V/V_0 - (1 - 2\nu_{el})\frac{\sigma}{E}. \tag{13}$$

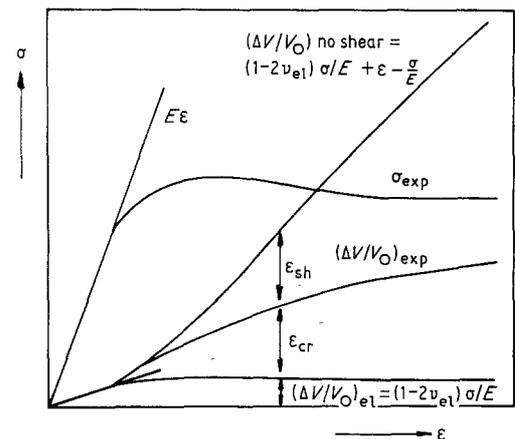These results are shown in Fig. 5 using hypothetical

Figure 5 Estimation of the contribution of shearing, crazing and elastic deformation to the total elongation from hypothetical stress and volume strain–elongation curves.

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stress—strain and volume strain—elongational strain curves. Of special interest are the parts of a stress—elongation curve where

$$d\sigma/d\epsilon = 0. \qquad (14)$$

Differentiating Equations 12 and 13 yields

$$\frac{d(\Delta V/V_0)}{d\epsilon} = 1 - \frac{d\epsilon_{sh}}{d\epsilon} = \frac{d\epsilon_{cr}}{d\epsilon} . \qquad (15)$$

This shows that for cases where $d\sigma/d\epsilon \rightarrow 0$, for instance at the yield-point or in some cases at large elongations, the slope of the volume strain curve is a direct measure of the incremental contributions $d\epsilon_{sh}/d\epsilon$ and $d\epsilon_{cr}/d\epsilon$ at the corresponding elongation. This analysis was applied in [1].

For creep experiments, where $d\sigma/d\epsilon$ is practically zero, the slope at any point on the volume strain against elongational strain curve is a measure of the strain contributions of the two mechanisms. This principle has been used by Bucknall for a number of materials that deform by crazing [7].

The model developed for heterogeneous systems should also be applicable to homogeneous materials deforming without necking, provided that mechanisms like shearing and crazing do not greatly diminish the volume of the material that is deforming elastically. However, as the volume effect on the elastic deformation is usually rather small in the region of crazing, deviations from the model will still be small, and the model will therefore still be applicable.

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