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Modeling and Identification of Centrifugal Compressor Dynamics with Approximate Realizations

Jan van Helvoirt, Bram de Jager, Maarten Steinbuch, and Jan Smeulers

Abstract— **This paper deals with the parameter identification of a model for the dynamic behavior of a large industrial centrifugal compression system. Experimental results are presented to evaluate a new approach for determining the parameters of a modified version of the well-known Greitzer model. This approach is based on an approximate realization algorithm that constructs an LTI model from step response data. It is shown that the method is capable of providing estimates for a limited set of model parameters. In a subsequent sensitivity analysis it is proven that the identifiable parameter are all related to the dominant dynamics of the system. The main conclusion is that the suggested approach is capable of providing estimates for the various model parameters. However, identification is limited to those parameters that are associated with dynamics that can be excited sufficiently during forced response experiments.**

I. INTRODUCTION

Surge is an unstable operational mode of a compressor that can occur at low mass flows. The instability is characterized by a limit-cycle oscillation in compressor flow and pressure rise that reduces compressor performance and efficiency. Surge can also endanger the safe operation of a compressor due to the large mechanical and thermal loads involved.

The most widely used dynamic model to describe transients and the unstable surge behavior of a compressor is the so-called Greitzer model. Determination of the parameters in this model is not straightforward due to the complex geometry of actual centrifugal compression systems. The authors of [1–4] solved this by tuning one of the parameters to get good agreement between model predictions and experimental data. However, we point out that the tuning approach can still lead to different parameter values for one compression system, as becomes clear from [3–7].

In order to circumvent the problem of obtaining accurate model parameters, a different method to obtain them for a centrifugal compression system was proposed in [7]. In this new approach, one of the parameters is calculated from a linear time-invariant (LTI) system that approximates the relevant dynamics of the actual system. The required LTI system can be extracted from step response data, measured at stable operating conditions, by using a modified approximate realization algorithm.

In this paper we will present experimental results that illustrate the capabilities and shortcomings of the proposed approach. After introducing the experimental set-up and the dynamic compressor model, we provide the outcomes of the approximate realization algorithm for numerous step response experiments. Next we evaluate these results and conclude that the identification method is only capable of providing accurate estimates for a limited set of model parameters. In a subsequent sensitivity analysis we illustrate the existence of a clear distinction between fast and slow (dominant) dynamics in the investigated compression system. We also show how the various model parameters are related to either the fast or slow dynamics. Based on the presented findings, we conclude that successful identification of model parameters is possible with the suggested approach, but it is limited to those parameters associated with the system dynamics that are excited sufficiently during forced response experiments.

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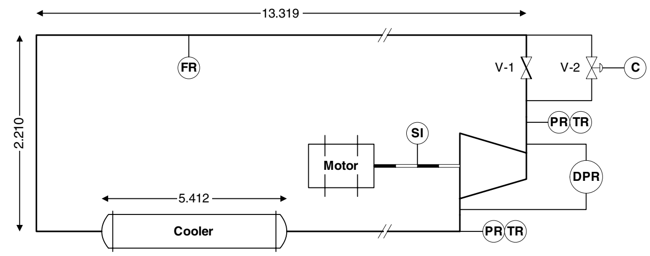

Fig. 1. Scheme of compressor test rig.

II. COMPRESSION SYSTEM

All experiments described in this paper were conducted on a large centrifugal compression system. This section briefly introduces the main components and instrumentation of the installation. Furthermore, we introduce the equations for the nonlinear lumped parameter model, similar to the original Greitzer model. We validate the model by comparing simulation results with data from actual surge measurements. These results are also used to illustrate that not all of the model parameters are uniquely determined.

A. Experimental set-up

The system under study is a single stage centrifugal compressor rig that is normally used to test industrial compressors for the oil and gas industry. The whole installation is schematized in Fig. 1. The compressor is driven by an 1.7 MW electric motor that is connected to the shaft through a gearbox. The rotational speed of the compressor can be varied between $6,000$ and $16,000$ rpm. The compressor operates in a closed circuit that makes it possible to use different pure gases or gas mixtures. Furthermore, this configuration allows for varying the average pressure in the system between 1 and 15 bar. All results presented in this paper were obtained from experiments with N_2 gas ($28.0134 \cdot 10^{-3}$ kg/mol, R = 296.8 J/kg $\cdot$ K), an average suction pressure of 10 bar and at rotational speeds between $9,000$ and $16,000$ rpm.

Throttling of the compressor is done by means of a butterfly valve (v-1 in Fig. 1). The parallel control valve (v-2 in Fig. 1) is used for more precise adjustments of the mass flow rate. The return piping contains a measurement section with a flow straightener, an orifice flow meter with a diameter ratio of 0.375, and a gas cooler. With the combination of the throttle, return piping, and cooler, the conditions of the gas at the compressor inlet were kept approximately constant.

The compression installation is equipped with numerous temperature probes and static pressure transducers to determine the steady-state performance of the compressor, see also Fig. 1. A dedicated data-acquisition and control system is used for operating the installation, converting and recording sensor outputs, and for online monitoring. The sample time for all static measurements is 6 s. Additional dynamic pressure transducers were installed in the suction and discharge pipes to measure the pressure rise fluctuations during experiments. All dynamic pressure signals were measured at a sampling rate of 1.28 kHz.

B. Compressor model

In order to describe the dynamic behavior of the compression system we used a lumped parameter model, following the geometry of Fig. 2. The model is analogous to [8] and the same assumptions were made during the derivation. However, we neglected the inertial effects in the throttle and the time lag associated with rotating stall development, in accordance with [9], [10]. Therefore, we omitted the impulse balance for the throttle and the relaxation equation of the original Greitzer model.

By applying the principles of mass and momentum conservation we obtained

$$\frac{dm_c}{dt} = \alpha\left(\Delta p_c(m_c, N) - \Delta p\right) \quad (1)$$

$$\frac{d\Delta p}{dt} = \beta\left(m_c - m_l(\Delta p, u_l) - m_s(\Delta p, u_s)\right) \quad (2)$$

where $\Delta p = p_2 - p_1$ denotes the pressure difference between the two volumes. The term Δp_c denotes the quasi-stationary compressor characteristic, see also Fig. 3. The parameters α and β are defined as

$$\alpha = \frac{A_c}{L_c} \quad (3)$$

$$\beta = \frac{a_1^2}{V_1} + \frac{a_2^2}{V_2} \quad (4)$$

where the following values are assumed: $A_c = 0.0034$ m^2, $L_c = 0.386$ m, $V_1 = 3.0693$ m^3, and $V_2 = 0.3204$ m^3.

Another difference with the original Greitzer model is that in Fig. 2 the return piping couples the throttle exit to the compressor inlet. The effect of this coupling is accounted for by the additional term a_1^2/V_1 in β.

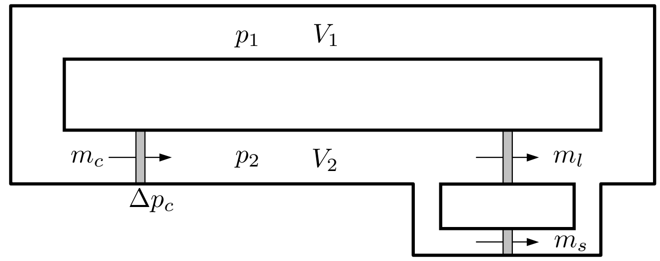

Fig. 2. Lumped parameter model.

C. Model validation

In order to evaluate the developed compressor model we compared the simulation results with data from actual surge measurements. In total 14 surge measurements were done at three different rotational speeds. During each measurement the small throttle was closed and the compressor was brought into surge by adjusting the large throttle valve opening, after which the dynamic pressure oscillations were measured for 6.4 s. Data from the static pressure and temperature measurements were used to initialize the simulation model.

One typical measurement and the result obtained with the simulation model are shown in Fig. 4. We remark that, according to the classification suggested in [11], the compressor exhibits deep surge. The plot shows that the pressure rise fluctuations and frequency of the surge oscillations were predicted remarkably well. Note that no noise was incorporated in the simulations and hence the spectrum of the simulated pressure signal is lower in between the frequency peaks. Due to the large inaccuracy of the static pressure and mass flow data during surge, we allowed for $\pm 0.5\%$ adjustments of the initial operating point and therefore of the throttle constant. We stress that no other model parameters were tuned.

In the introduction we argued that tuning the geometric model parameters of the Greitzer model can lead to ambiguous results. See also [7]. From literature we concluded that

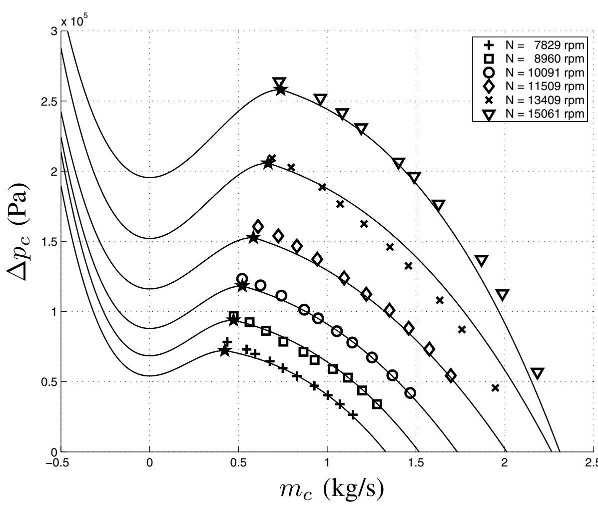

Fig. 3. Steady-state compressor map $\Delta p_c(m_c, N)$ with measured operating points and points on the approximated surge line ($\star$).

TABLE I

ADDITIONAL NOMENCLATURE

Symbol	Description	Unit
a	speed of sound	m/s
A	area	m^2
L	length	m
m	mass flow rate	kg/s
N	rotational speed	rpm
p	pressure	Pa
t	time	s
u	normalized valve position	-
V	volume	m^3
Subscript	**Description**	
1	piping, suction side	
2	plenum, discharge side	
c	compressor	
l	large throttle	
s	small throttle	
t	combined throttle	

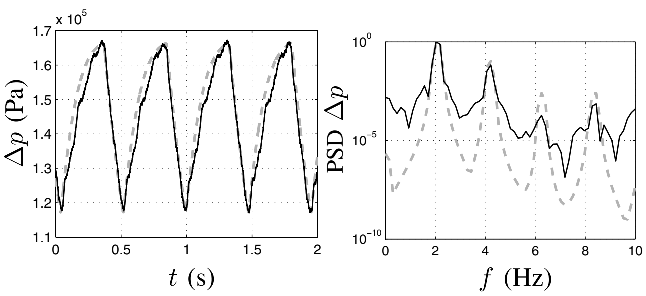

Fig. 4. Pressure difference and power spectral density of surge measurement (-) and simulation (- -) at N = 11,284 rpm.

it is common practice to select L_c as tuning parameter. In order to illustrate the difficulty when L_c is used to tune the simulation model, we performed a simulation with $L_{c,new}$ = $2L_c$ = 0.772 m. Initially, the model predictions with $L_{c,new}$ indicated a 7% lower surge frequency and a lower value for the minimum pressure during the surge oscillations with respect to the measurement.

In [7] it was suggested that it is allowed to compensate for the different minimum pressures by adjusting the valley point of the compressor curve. By trial and error we found that a 2.5% increase of $\Delta p_c(0)$ was sufficient to obtain the result shown in Fig. 5. Note that the increased valley point not only compensated for the lower minimum pressure but also for the observed underestimation of the surge frequency.

Note that decreasing L_c has an opposite effect that can be compensated by selecting a slightly lower valley point for the compressor curve. Furthermore, increasing L_c is equivalent to decreasing A_c and vice versa, as can be easily seen from equation 1. From the forgoing we conclude that the ration $\alpha = A_c/L_c$ is not a suitable tuning parameter for

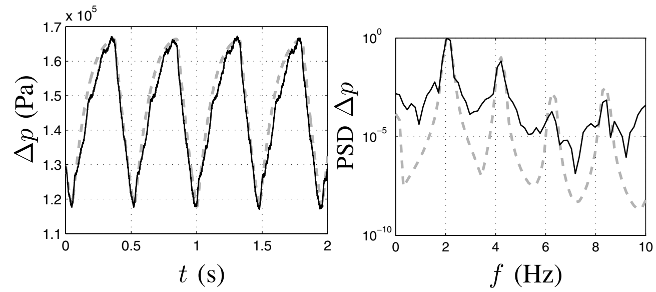

Fig. 5. Pressure difference and power spectral density of surge measurement (-) and simulation (- -) at N = 11,284 rpm. Model parameter L_c increased by 100 % and $\Delta p_c(0)$ increased by 2.5 % for this simulation.

the compression system under study, because variations of α have a small effect on surge amplitude and frequency. In the next section we discuss a new approach to define suitable and unique values for the various model parameters.

III. MODEL IDENTIFICATION

The identification method, initially proposed for this application in [7], is based on an approximate realization algorithm that constructs an LTI system from step response data. These step response sequences are relatively easy to obtain from forced response measurements on the compression system, this in contrast to for example frequency response data. Benefits of the approximate realization algorithm are its straightforward implementation and computational simplicity, while providing enough flexibility and insight in order to tune its performance.

We will now start with discussing the step response measurements and the results from these experiments. We also compare the experimental results with model simulations. Then we will treat the approximate realization algorithm and its application to the measurements in more detail. Finally we will present and discuss the results obtained with the parameter identification method.

A. Step response measurements

Step response data from the compression system can be obtained by applying a step input to the control valve and measuring the resulting pressure rise over a certain period of time. Before the step is applied, the compressor must be in a steady-state operating point to avoid that other transient dynamics of the compression system influence the measurements. Moreover, the size of the applied step must be large enough to result in a measurable response. However, the perturbation must remain small to keep the operating point of the compressor close to its initial value. Otherwise, linear approximations of the system dynamics are invalid.

In total 10 different experiments were performed at various operating points, rotational speeds and with different step sizes. The step signal was generated at the 0–10 V output channel of the data-acquisition system and electronically converted to a 4–20 mA current. Before the step input was applied, the compressor was brought to a stable operating

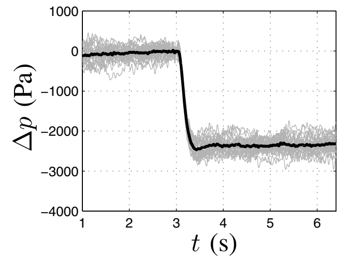

Fig. 6. Filtered step response data (gray) and mean value (black) at N = 9,432 rpm and $u_{s,0} = 0.409$, $\delta u_s = 0.287$.

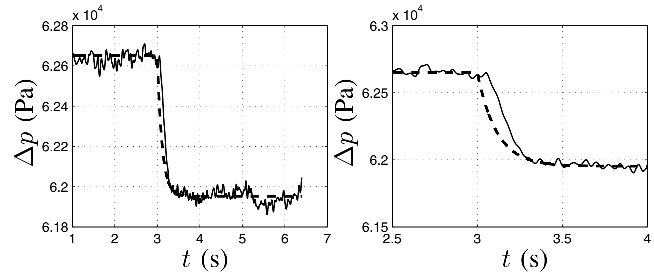

Fig. 7. Measured step response data (-) and model predictions (- -) at N = 9,432 rpm, $u_{s,o} = 0.409$, $\delta u_s = 0.287$. Whole time-span (left figure) and zoomed plot around $t = 3$ s. (right figure).

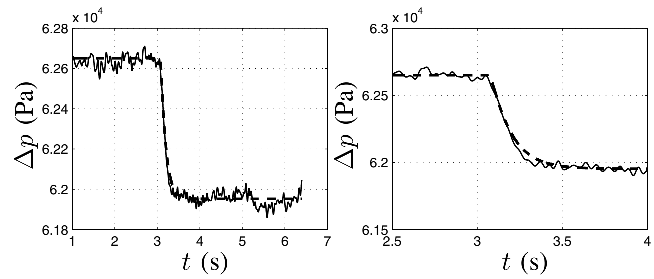

Fig. 8. Measured step response data (-) and model predictions with valve dynamics (- -) at N = 9,432 rpm, $u_{s,o} = 0.409$, $\delta u_s = 0.287$. Whole time-span (left figure) and zoomed plot around $t = 3$ s. (right figure).

point. The measurement time was 6.4 s in which 8192 samples of the signals p_1, p_2, T_1, and T_2 were collected.

For each experiment 25 measurements were done to minimize the effects of measurement noise on the results. Measurement noise was also reduced through anti-causal filtering of the data with a 5th order, low-pass Butterworth filter with a cut-off frequency of 14 Hz. Note that the filter cut-off is chosen very low to remove significant effects from electro-magnetic coupling with the surrounding equipment and from the large flow noise inside the compression system.

Further analysis of the filtered data from all experiments showed that the smallest variance was obtained with an initial valve opening $u_{s,0}$ of 40.9% and a step size δu_s of 28.7%. Therefore, from now on we will only present results obtained with these settings, unless stated otherwise.

Results of the step response measurements for one operating condition and with the selected step input from 40.9% to 69.6% are shown in Fig. 6. We remark that the measured signals were shifted to obtain an initial operating point around zero. This shifting was done by subtracting the average value of 1000 samples (≈ 0.8 s.) prior to the step input from the measured data. Note that, despite the filtering, relatively large differences exist between the 25 measurements. These differences were mainly caused by flow turbulence and varying operating points.

It is illustrative to perform step response simulations with the dynamic model of the compression system while using the same settings as during the measurements. A comparison of one particular step response simulation with the corresponding measurement is shown in Fig. 7. Similar results were obtained for other measurements. From this result we conclude that the compressor model is capable to give a good qualitative prediction of the transient dynamics of the compressor when subject to a step on the control valve input. However, when we look closer around t = 3 s. a significant difference can be observed.

The following explanations were found for the observed differences. A detailed analysis of the control valve dynamics revealed that the valve response to the step input is too slow to be neglected and the model did not account for these slow valve dynamics. Furthermore, we concluded that the valve has a considerable time delay of 37 samples at a sample frequency of 1.28 kHz. In addition, we found that an additional time delay of 16 samples was present in the anti-aliasing filters of the data-acquisition system. See [12] where we have discussed the valve dynamics in general and the time delay in particular.

In order to account for the valve dynamics and time delay, we placed the following filter $H_s(s)$ between the input u_s and the valve characteristic in the simulation model

$$H_s(s) = e^{-\frac{53}{1280}s} \frac{155208.9566}{(s+30.79)(s^2+60.32s+5042)} \qquad (5)$$

We remark that this filter is slightly different from the original approximation for the valve dynamics given in [12], since we replaced the (non-minimum phase) zeros in the transfer function by a static gain to assure a stable inverse. Through step response simulations we verified that this further simplification of the valve dynamics did not lead to noticeably different responses.

By taking the time delays and valve dynamics into account, the simulation model was able to accurately predict the measured step responses, as can be seen in Fig. 8. From this analysis of the step response measurements, we will proceed by discussing the application of the approximate realization algorithm to these data and the outcomes of the parameter identification.

B. Approximate realizations

The goal of processing the step response data with the approximate realization algorithm is to obtain an LTI system for the compressor dynamics. This idea was first suggested in [7]. An overview of the used algorithm, other related algorithms, and the underlying realization theory can be found in [13].

The most important step in the algorithm is to select the order of the realization. Usually, a trade-off between relevant dynamics and noise is made with the help of the singular value decomposition (SVD), as proposed in [14]. However, linearization of the compressor model showed that the input-output behavior $H_c(s)$ has the characteristics of a first order system in the frequency range of interest as can be seen from the Bode diagram in Fig. 9. Therefore, despite the fact that the developed nonlinear model is second order, we choose to construct first order approximate realizations from the step response data. A further justification for this choice will be given in the next section.

Next to the realization order, a choice must be made for the length of the sequence that is fed into the algorithm. Through various trials we determined that using 80 samples from a time interval of 3.125 s. gave reliable results with acceptable computer time (< 1 s.).

Moreover, some additional pre-processing of the measured step responses is required. First of all, the data should be scaled with the applied step size because the algorithm assumes a unit step input. Secondly, given the non-ideal dynamics of the control valve as discussed above, the data should be filtered with the inverse of equation (5) to compensate for differences between the applied input and the ideal step input. We used an additional 3rd order low-pass, Butterworth filter with a cut-off frequency of 108 Hz to make the inverse filter proper, without compromising the accuracy of the inverse in the region of interest (< 20 Hz).

In Fig. 10 the results of the approximate realization algorithm are shown for particular measurements during three different experiments. From these left plots we see that the measured step response are accurately reproduced by those of the approximated first order LTI systems. To illustrate the effect of poor step response data, the result for a bad experiment is shown in the lower right plot.

The corresponding pole locations[1] for all measurements during the three experiments are shown in Fig. 11. Again the results for one of the bad experiments are shown in the lower right figure. This latter plot clearly reveals the large variance

[1]Roots of the characteristic equation $\det(s\mathbf{I} - \mathbf{A}) = 0$

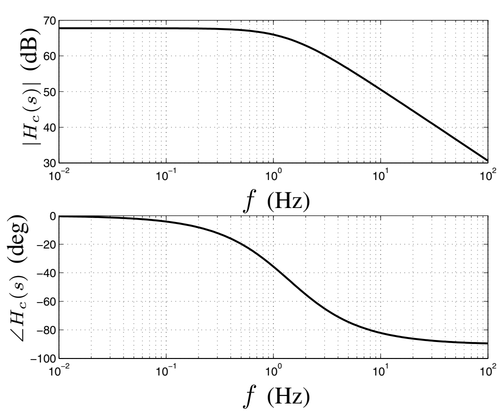

Fig. 9. Bode diagram for the linear compression system at $N = 9{,}432$ rpm and $u_{s,0} = 0.409$.

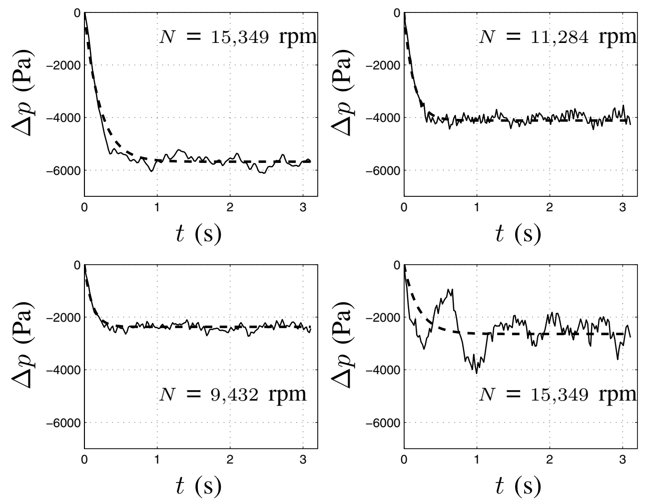

Fig. 10. Measured step responses (-) and the step responses of the corresponding approximated LTI models (- -). Results in lower right figure obtained with $u_{s,0} = 0.409$, $\delta u_s = 0.172$.

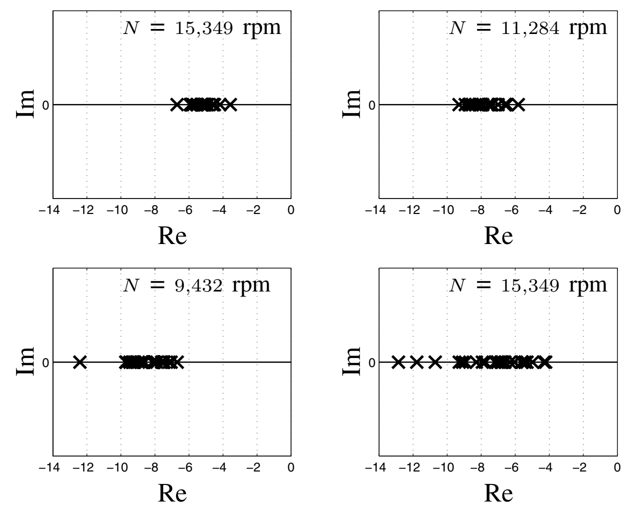

Fig. 11. Pole locations of the LTI approximations for all step response measurements at different operating conditions. Results in lower right figure obtained with $u_{s,0} = 0.409$, $\delta u_s = 0.172$.

in the LTI system pole, caused by the poor measurement quality. These bad experiments were excluded from the data set in subsequent analyses.

Note that the variance in the pole locations from the other experiments is not negligible either. This spread in the calculated poles is due to the sensitivity to measurement noise of the approximate realization algorithm. Therefore, a large number of step response measurements at one operating condition are required in order to obtain reliable results.

C. Discussion

The negligible effect of L_c and A_c on the surge amplitude and frequency (in contrast to the effect of V_1 and V_2), was the main motivation to investigate a parameter identification method as an alternative for the common practice of tuning a compressor model. See also [7].

The presented results of this investigation show that the compressor dynamics of the studied system in the stable flow regime can be approximated by a first order LTI system. This apparent first order dynamic behavior implies that only two model parameters can be determined independently from the approximate realizations. For the developed second order compressor model this means that no unique values for all parameters can be found but only for a subset or for specific combinations of them.

More precisely, we concluded that the identification method is also not capable to provide a reliable estimate for either one of the parameters L_c and A_c. However, reasonable estimates can be obtained for V_2 or to a lesser extent for V_1. We will elaborate on these findings in the next section.

Finally, we note that a step signal only excites a system at low frequencies. Hence, it seems worthwhile to investigate whether applying a richer excitation signal, in combination with a more sophisticated identification algorithm, enables us to identify all model parameters separately.

IV. Linear sensitivity analysis

In order to further investigate the effect of the model parameters on the dynamic behavior, a sensitivity analysis was performed. The results of this analysis will provide explanations for the previously discussed observations.

The fact that both the simulated and measured step responses indicate that the compressor dynamics are first order, can be illustrated by investigating the sensitivity of the eigenvalues (poles) of the linearization of the compressor model (1)-(2). For this reason, the eigenvalues $\lambda_{1,2}$ were calculated by solving $\det(\lambda\mathbf{I} - \mathbf{A}) = 0$, yielding

$$\lambda_{1,2} = \frac{\alpha d_c - \beta d_t \pm \sqrt{(\alpha d_c + \beta d_t)^2 - 4\alpha\beta}}{2} \tag{6}$$

where d_c and d_t represent the partial derivatives of the compressor and combined throttle characteristics, respectively.

Numerical values for λ_1 and λ_2 were obtained by initializing the model with experimental data from one particular experiment in a stable operating point at $N = 9,432$ rpm. This resulted in eigenvalues $\lambda_1 \approx -8$ and $\lambda_2 = -870$, that are clearly located far apart. This implies that the system pole associated with λ_1 is much slower and therefore dominant over the one associated with λ_2.

The term $-4\alpha\beta$ in (6) contributes only 2% to the term under the square root sign and therefore it can be neglected. In that case we obtain the following approximations for the two eigenvalues

$$\begin{aligned} \lambda_1 &\approx -\beta d_t && (7) \\ \lambda_2 &\approx \alpha d_c && (8) \end{aligned}$$

From these approximations we immediately see that the parameter α barely influences λ_1. This confirms our observation that L_c and A_c have a small effect on the dominant compression system dynamics. Furthermore, the approximation for λ_1 confirms that d_t and β, hence the volumes V_1 and V_2, have a large influence on the dominant dynamics.

Note that the above approximation is not valid with respect to the influence of the slope of the compressor curve d_c. The variation of d_c can be large and can even change sign, see Fig. 3. In that case linear theory, based on small perturbations around an equilibrium point, does not hold.

To complete the analysis we give the expressions for the zero z and static gain K of the linearized compressor model.

$$\begin{aligned} z &= \alpha d_c && (9) \\ K &= -\beta d_s && (10) \end{aligned}$$

where d_s represents the partial derivative of the small throttle characteristic with respect to the throttle opening. From (9) we see that positive values of d_c that occur in the unstable flow regime, the linearized system is non-minimum phase. This implies that the pressure difference can initially move away from the new equilibrium, commanded by a change in throttle valve opening, before the pressure difference goes to the new steady-state value. The static gain term (10) implies that opening or closing the small throttle valve has little effect on the pressure difference Δp when the system volumes V_1 and V_2 are large, or in other words when β is small.

Finally, we remark that the above analysis is valid for the stable flow regime of the compressor. In the unstable flow regime a limit cycle oscillation will occur, so linear theory and hence the approximations for (6) do not longer apply.

V. Conclusions

In this paper we discussed the experimental evaluation of a new parameter identification approach for a large industrial compression system. First, we introduced a dynamic model for the compression system and we validated the model by comparing simulations results with surge and step response measurements. We then illustrated that the commonly used approach of parameter tuning to match model predictions with experimental data can lead to ambiguous results.

For this reason we investigated an alternative approach to uniquely define the parameters of the compressor model, based on an approximate realization algorithm that constructs an LTI system from step response data. The obtained LTI system accurately described the local dynamic behavior although the algorithm proved to be sensitive to system and measurement noise. We then illustrated that the proposed method only provided reasonable estimates for a limited set of model parameters. Identifying all model parameters could be achieved by excitation the system in a wider frequency range, for example via loudspeakers.

These findings were confirmed by a subsequent sensitivity analysis of the linearized compressor model. The sensitivity analysis showed that, next to the slope of the compressor curve, the volumes are the dominant parameters for the dynamic behavior of the investigated compression system. To complete the picture, a more detailed analysis of the nonlinear dynamic behavior in the unstable flow regime is required. Information about the shape and period time of the limit cycle oscillations could also be used to tune the model and thereby obtaining unique values for all parameters. However, this requires reliable mass flow data that are difficult to obtain in practice.

REFERENCES

[1] O. O. Badmus, S.Chowdhury, K. M. Eveker, and C. N. Nett, "Control-oriented high-frequency turbomachinery modeling: Single-stage compression system one-dimensional model," *ASME J. Turbomach.*, vol. 117, pp. 47–61, 1995.

[2] G. L. Arnulfi, P. Giannattasio, C. Giusto, A. F. Massardo, D. Micheli, and P. Pinamonti, "Multistage centrifugal compressor surge analysis: Part II—Numerical simulation and dynamic control parameter evaluation," *ASME J. Turbomach.*, vol. 121, pp. 312–320, 1999.

[3] F. Willems, W. P. M. H. Heemels, B. de Jager, and A. A. Stoorvogel, "Positive feedback stabilization of centrifugal compressor surge," *Automatica*, vol. 38, pp. 311–318, 2002.

[4] J. T. Gravdahl, F. Willems, B. de Jager, and O. Egeland, "Modeling of surge in free-spool centrifugal compressors: Experimental validation," *J. Prop. Power*, vol. 20, pp. 849–857, 2004.

[5] C. Meuleman, F. Willems, R. de Lange, and B. de Jager, "Surge in a low-speed radial compressor," in *Proc. ASME Turbo Expo* 1998, Stockholm, Sweden, 1998.

[6] C. Meuleman, R. de Lange, and A. van Steenhoven, "Surge dynamics in a centrifugal compressor system," in *Proc. 3rd European Conf. on Turbomachinery–Fluid Dynamics and Thermodynamics (ImechE)*, London, UK, 1999, pp. 895–904.

[7] J. van Helvoirt, B. de Jager, M. Steinbuch, and J. Smeulers, "Stability parameter identification for a centrifugal compression system," in *Proc.* 43*rd IEEE Conf. on Decision and Control*, Atlantis, Paradise Island, Bahamas, 2004, pp. 3400–3405.

[8] E. M. Greitzer, "Surge and rotating stall in axial flow compressors: Part I—Theoretical compression system model," *ASME J. Eng. Power*, vol. 98, pp. 190–198, 1976.

[9] R. S. Mazzawy, "Discussion of [15]," *ASME J. Fluids Eng.*, vol. 103, p. 395, 1981.

[10] F. P. T. Willems, "Modeling and bounded feedback stabilization of centrifugal compressor surge," Ph.D. dissertation, Technische Universiteit Eindhoven, Eindhoven, The Netherlands, 2000.

[11] B. de Jager, "Rotating stall and surge control: A survey," in *Proc.* 34*th IEEE Conf. on Decision and Control*, New Orleans, LA, USA, 1995, pp. 1857–1862.

[12] J. van Helvoirt, O. Bosgra, B. de Jager, and M. Steinbuch, "Approximate realization of valve dynamics with time delay," in *Proc.* 16*th IFAC World Congress*, Prague, Czech Republic, 2005.

[13] B. De Schutter, "Minimal state-space realization in linear system theory: An overview," *J. Comp. Appl. Math.*, vol. 121, pp. 331–354, 2000.

[14] S. Kung, "A new identification and model reduction algorithm via singular value decompositions," in *Proc.* 12*th Asilomar Conf. on Circuits, Systems, and Computers*, Pacific Grove, CA, USA, 1978, pp. 705–714.

[15] K. E. Hansen, P. Jørgensen, and P. S. Larsen, "Experimental and theoretical study of surge in a small centrifugal compressor," *ASME J. Fluids Eng.*, vol. 103, pp. 391–395, 1981.