Rule-based control of a semi-active suspension system for road holding using limited sensor information: design and experiments

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Rule-based control of a semi-active suspension system for road holding using limited sensor information: design and experiments

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ABSTRACT

This paper presents a design of a rule-based controller for a semi-active suspension system that improves the vehicle road holding behaviour. This behaviour is quantified by the evaluation of the dynamic tire deflection at each of the vehicle’s wheels. The control rule switches based on the product of the unsprung mass-and suspension deflection velocity and the excitation frequency. To determine how much the dynamic tire deformation is improved, the controller is compared to multiple passive suspension configurations as well as existing semi-active suspension controllers. The proposed controller is capable of achieving a minimal tire deformation over the entire frequency range of interest. Additionally, experiments are performed on a test vehicle which is equipped with semi-active dampers and the designed controller. In the experiments, the unsprung mass acceleration is measured since the tire deflection is impossible to measure in practice. A fixed trajectory has been driven for the controlled system and for a set of passive suspension configurations, a reduction of up to 11% of the root mean square (RMS) unsprung acceleration is seen compared to the best passive setup.

1. Introduction

An essential element of all road vehicles is the suspension system, which is composed of all the elements that connect the vehicle body to its wheels. Without a suspension system, there would be a rigid connection between the wheels and the vehicle body. As a consequence, the tire is the only element that absorbs vibrations. This leads to poor road holding behaviour as the tire contact patch and the tire forces vary hugely. Additionally, the experienced ride is bad due to the road vibrations being almost directly transmitted to the vehicle body. From the age of horse-drawn carriages, suspension systems have therefore been in use in many different variants. Most road vehicle suspensions are categorised as passive suspension systems containing a spring and damper element. Both of these elements cannot be changed after being installed, thus the spring stiffness and damping coefficient are considered constant. It is therefore straightforward to create a model of such a system, and
typically a quarter car model is chosen for this [1]. The disturbance input to any suspension system is a consequence of the road profile in combination with the forward velocity of the vehicle [2]. The desired vehicle behaviour can be achieved by selecting the suitable spring and damper elements in a suspension system. Obviously, a good estimate can be made with a suitable model.

The amount of damping in a suspension system will greatly determine its behaviour, a high damping coefficient will lead to a more rigid suspension system and in general (up to limited maximum damping) better vehicle road holding, in literature quantified by the dynamic tire deflection. A softer damper allows for more suspension movement and increases the general ride comfort, which is quantified by the sprung mass acceleration. It is important to note that it is generally assumed that the road holding and comfort criteria are conflicting and that a trade-off between these two criteria and the suspension stroke will always be present [1]. Usually, the suspension system will be designed to perform well on most types of road quality and is therefore never truly optimal [3] in all road conditions.

In the 1960s, a number of active- and semi-active suspension systems were developed, which started an innovation in terms of vehicle suspension systems. An active suspension system is characterised by the ability to add or dissipate energy, whereas a semi-active suspension system is only capable of dissipating energy [4]. Usually, in an active suspension system, the spring and damper element are replaced by an actuator which is capable of exerting suspension forces in both directions [5]. The great potential of such a system comes at the price of a power consumption [6]. In a semi-active suspension, the damping coefficient of the system is controlled. In comparison to the previously explained suspension systems, it is capable of improving the vehicle behaviour whilst achieving a negligible power consumption. This system will be considered throughout this paper. Irrespective of the actuator, the reduction of sprung mass acceleration or dynamic tire deflection of any active- or semi-active suspension system can only be achieved with a good control strategy.

The most familiar control strategy for semi-active suspension systems encountered throughout literature is the Skyhook control (SH) [7]. The control technique is often represented as a fictitious damper which is connected to the sprung mass and a fixed component in the sky and is designed to improve comfort. A basic SH controller will switch between high and low damping, which is denoted as $d_{s,\text{max}}$ and $d_{s,\text{min}}$ in this paper. The control rule is defined as

$$d_{s,\text{in}} = \begin{cases} d_{s,\text{min}}, & \text{if } \dot{z}_s(\dot{z}_s - \dot{z}_u) \leq 0, \\ d_{s,\text{max}}, & \text{if } \dot{z}_s(\dot{z}_s - \dot{z}_u) > 0, \end{cases}$$

(1)

where $\dot{z}_s$ and $\dot{z}_u$ are the sprung- and unsprung mass velocities. This control strategy is designed to minimise the sprung mass accelerations and thus improving the vehicle comfort. From the literature, it can be concluded that the SH controller is capable of minimising the sprung mass acceleration around the vehicle body eigenfrequency, which is generally excited by low-frequency road inputs. However, at higher road input frequencies which result in wheel hop movements, the performance of the SH controller is poor.

Another well-known control technique is the acceleration driven damping (ADD) [8]. The basic principle is comparable to the SH controller, however, instead of the sprung mass velocity, the sprung mass acceleration is used in the control rules. The control rule can be
written as

$$d_{s,\text{in}} = \begin{cases} d_{s,\text{min}}, & \text{if } \ddot{z}_s(\dot{z}_s - \dot{z}_u) \leq 0, \\ d_{s,\text{max}}, & \text{if } \ddot{z}_s(\dot{z}_s - \dot{z}_u) > 0. \end{cases}$$  \hspace{1cm} (2)$$

Similarly to the SH controller, it is aimed at improving the vehicle comfort by decreasing body accelerations. Switching the damper setting based on the acceleration of the sprung mass ($\ddot{z}_s$) greatly influences the controller performance in comparison to the SH controller. The ADD controller shows poor comfort behaviour at road input around the body eigen-frequency and excellent behaviour at wheel hop frequencies, which is the opposite of the SH performance.

The performance of the SH and ADD controller regarding comfort presents a possibility to combine these two control strategies. By using each of these control strategies for the frequency regions where they function good, a controller is found that reduces the body acceleration over the entire frequency range [9]. This can be done in multiple different manners [10], in general, a switch is made at a certain predefined frequency from SH to ADD control.

Besides these comfort oriented control rules there is one commonly used control rule focussed on the minimisation of the dynamic tire deflection, thereby improving vehicle road holding. The groundhook (GH) control [11,12] can visualise a fictitious damper which is connected to the wheel and either an inertial ground or the road surface. For the version where the wheel is connected to an inertial ground the control rule can be written as

$$d_{s,\text{in}} = \begin{cases} d_{s,\text{min}} & \text{if } -\dot{z}_u(\dot{z}_s - \dot{z}_u) \leq 0, \\ d_{s,\text{max}} & \text{if } -\dot{z}_u(\dot{z}_s - \dot{z}_u) > 0. \end{cases}$$  \hspace{1cm} (3)$$

An evaluation of the road holding performance of this controller shows that it is capable of reducing the dynamic tire deformation at input frequencies around the wheel hop. However, at lower frequencies, where the wheel is barely moving with respect to the road surface, the road holding performance is poor. Therefore, the objective of this paper is to improve the dynamic tire deflection over the whole frequency range.

Up to now, the majority of research with semi-active suspension has focused on optimising passenger comfort by minimising the sprung mass acceleration. With respect to road holding, the most used performance variable is the dynamic tire deflection which is reduced with a GH controller. However, the frequency range where the GH controller offers an improvement of the dynamic tire deflection is limited. In [13], a study was performed where a full state-feedback controller was developed to control the tire force. In both simulation and experiment, it is shown that dynamic tire force can be improved by using an adjustable damper. In [14], it is shown that compared to a hard passive damper, only limited improvement in dynamic tire force can be seen.

In this paper, a controller will be developed that does minimise the dynamic tire deflection, and thereby road holding, over the whole frequency range. The development of the controller is aimed at practical implementation, only using limited sensor information and keeping computation demands limited. One of the key features is that the controller does not use the unmeasurable dynamic tire deflection as input. The performance of the controller is also verified by experiments.
The outline of the paper is as follows: The vehicle model used in this paper is discussed in Section 2, which also contains an analysis of the characteristics of the vertical dynamics of a road vehicle. In Section 3, the design of the controller is discussed. Following this, the test vehicle is introduced and the performance of the controller is discussed in Section 4. Finally, conclusions are drawn in Section 5.

2. Vehicle model

A widely used vehicle model for the analysis of vertical dynamics is the quarter car model [15]. Most research on suspension design uses this model for its simplicity, ease of implementation and its accurate representation of the vertical dynamics of one corner vehicle. There are a number of variations of this model concerning tire damping and shock absorber type. In this case, the tire damping has been neglected, since a rolling tire typically has little damping. The shock absorber is of the semi-active type, making the damping force controllable. A schematic representation of this model is displayed in Figure 1.

The model consists of the sprung- and unsprung mass, $m_s$ and $m_u$, respectively. The sprung mass represents the body of the quarter vehicle and the unsprung mass represents the wheel, brake disc, etc. These masses are connected to each other via the suspension system, in this model it consists of a spring and a damper. The spring stiffness is denoted as $k_s$ and the damping coefficient which can be varied over time as $d_s(t)$. The connection with the road is modelled as a linear spring with stiffness $k_t$, which represents the tire stiffness. The symbols $z_s$, $z_u$ and $z_r$ represent the vertical displacements from a static equilibrium for the sprung mass, unsprung mass and road input, respectively. From this model, the following equations of motion can be derived under the assumption of permanent road contact:

$$m_s \ddot{z}_s = -k_s(z_s - z_u) - d_s(t)(\dot{z}_s - \dot{z}_u),$$
$$m_u \ddot{z}_u = -k_t(z_u - z_r) + k_s(z_s - z_u) + d_s(t)(\dot{z}_s - \dot{z}_u).$$

This quarter car model can then be rewritten in a state space form. The generic continuous time-variant form is

$$\dot{x}(t) = A(t)x(t) + Bu(t),$$
$$y(t) = C(t)x(t) + Du(t).$$

![Figure 1. Quarter car vehicle model.](image-url)
With \( u(t) \) the model input, chosen as the rate of change of the road disturbance, \( \dot{z}_r \). Throughout the paper, the integral of \( \dot{z}_r \) will be used, given that it has a more intuitive interpretation than the rate of change of the road disturbance. The desired model outputs are denoted as \( y \) and the state vector \( x \) is chosen as

\[
x(t) = [z_s - z_u, \dot{z}_s, z_s - z_r, \dot{z}_u]^T.
\]

Thus, the first state is the suspension stroke, the second is the sprung mass velocity, the third is the tire deflection and the fourth is the unsprung mass velocity. The matrices \( A(t) \) and \( B \) are defined as

\[
A(t) = \begin{bmatrix}
0 & 1 & 0 & -1 \\
-k_s/m_s & -d_s(t)/m_s & 0 & d_s(t)/m_s \\
0 & 0 & 1 & -1 \\
k_s/m_u & d_s(t)/m_u & -k_t - d_s(t)/m_u & 0
\end{bmatrix}, \quad B = \begin{bmatrix}
0 \\
0 \\
-1 \\
0
\end{bmatrix}.
\]

The matrices \( C(t) \) and \( D(t) \) are dependent on the desired model outputs \( y(t) \). For this research, the output is chosen as

\[
y(t) = [\ddot{z}_s, z_u - z_r, z_s - z_u, \dot{z}_u]^T,
\]

this gives

\[
C(t) = \begin{bmatrix}
-k_s/m_s & -d_s(t)/m_s & 0 & d_s(t)/m_s \\
0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 \\
k_s/m_u & d_s(t)/m_u & -k_t - d_s(t)/m_u & 0
\end{bmatrix}, \quad D = 0.
\]

In this research an active damper is considered, which means that the damping coefficient, \( d_s \), can be influenced by the controller. A graphical explanation of the operating range of the active damper is shown in Figure 2. This shows that there is a lower and upper bound for the achievable damping coefficient, indicated by \( d_{s,\text{min}} \) and \( d_{s,\text{max}} \), respectively. Note that this assumes that both the lower and upper range of the damper are linearly related to the suspension velocity. Additionally, since a positive relative velocity only results in a positive force and a negative relative velocity only gives a negative force, the controlled damper force is only dissipative.

### 2.1. Passive suspension analysis

In order to analyse the vehicle behaviour over the road disturbance frequency range of interest, a number of transfer functions are relevant. For this, the time-varying matrix \( A(t) \) is assumed to be constant, this can be achieved by choosing one damping coefficient. The classic quarter car model is considered, as introduced in (4). The parameters used are based
on the front right corner of the vehicle. An overview of the parameters is given in Table 2.

The transfer functions from road input to sprung mass acceleration, suspension travel, tire deflection and unsprung mass acceleration, respectively, are shown in Figure 3. It is clear that a change in damping coefficient $d_s$ greatly influences the overall response of all four chosen outputs. The natural frequency of the sprung mass is 1.18 Hz. Around this input frequency, a low damping coefficient allows for large sprung mass movements and accelerations. This will have a negative influence on vehicle comfort. Conversely, a high damping coefficient reduces the sprung mass movement around its eigenfrequency. The natural frequency of the unsprung mass is equal to 12.28 Hz, excitations at these frequencies will result in a wheel hop movement. A low value of $d_s$ will result in large unsprung mass oscillations. With respect to the tire deflection, $z_u - z_r$, a low damping has a negative effect at both the sprung and unsprung eigenfrequency. In between those two eigenfrequencies, a hard damping gives worse performance.

Since in practice the dynamic tire deflection, which is the control objective to be minimised, cannot be measured directly, Figure 3 also shows the unsprung mass acceleration in the bottom plot. The influence of a different damping coefficient is most prominent around the unsprung mass eigenfrequency, where a low damping coefficient shows a greater unsprung mass acceleration than a high damping coefficient. Additionally, a difference is visible at the sprung mass eigenfrequency. With a low damping coefficient, the motion of the sprung mass is visible in the unsprung mass acceleration.

### 2.2. Non actuated analysis

It is often assumed [16] that comfort and road holding in a vehicle are conflicting. However, when analysing the transfer function of the quarter car model from road input to either sprung mass acceleration or tire deflection, this trade-off is not true for all frequencies [1].

To analyse this, consider the equations of motion as given in (4). These two equations can be combined since the suspension forces working on the sprung mass are equal to those working on the unsprung mass, this results in

$$m_s\ddot{z}_s + m_u\ddot{z}_u = k_t(z_u - z_r)$$

or in Laplace domain

$$m_sZ_s^2 + m_uZ_u^2 = k_t(Z_u - Z_r).$$
Figure 3. From top to bottom, transfer function from road input, $z_r$, to sprung mass acceleration, $\ddot{z}_s$, suspension travel, $z_s - z_u$, dynamic tire deflection, $z_u - z_r$, and unsprung mass acceleration, $\ddot{z}_u$, for three different damping values.
The transfer function for comfort is defined as

$$H_{\ddot{z}_s/\dot{z}_r}(s) = \frac{s^2 Z_s}{Z_r}. \quad (12)$$

Here, $Z_s$ and $Z_r$ denote the Laplace transform of the sprung mass displacement with respect to a static equilibrium and road displacement, respectively. For tire deflection, the transfer function is expressed as

$$H_{(z_u-z_r)/\dot{z}_r}(s) = \frac{Z_u - Z_r}{Z_r}. \quad (13)$$

Here, $Z_u$ is the Laplace transform of the unsprung mass acceleration. Combining these two transfer functions with (11) gives

$$m_s H_{\ddot{z}_s/\dot{z}_r}(s) + (m_u s^2 + k_t) H_{(z_u-z_r)/\dot{z}_r}(s) = -m_u s^2, \quad (14)$$

which shows that the transfer function of comfort and handling are coupled and their relation is independent of suspension parameters $k_s$ and $d_s$.

To analyse how these transfer functions are related at an arbitrary but fixed frequency, we follow the analysis by Karnopp [1] and therefore introduce the new transfer function

$$H^N_{(z_u-z_r)/\dot{z}_r}(s) = \epsilon H_{(z_u-z_r)/\dot{z}_r}(s), \quad (15)$$

with $0 < \epsilon < 1$ such that $H^N_{(z_u-z_r)/\dot{z}_r}(s)$ represents an arbitrary scaling of the old transfer function $H_{(z_u-z_r)/\dot{z}_r}(s)$. Combining this new transfer function with (14), using $s = j\omega$ and rewriting gives an expression for the comfort as a function of $\epsilon$

$$H^N_{\ddot{z}_s/\dot{z}_r}(j\omega) = \frac{-1}{m_s} \epsilon (m_u \omega^2 - m_s H_{\ddot{z}_s/\dot{z}_r}(j\omega)). \quad (16)$$

For small $\omega$, and thus low frequencies, the term $m_u \omega^2$ vanishes and therefore approximately reduces to

$$H^N_{\ddot{z}_s/\dot{z}_r}(j\omega) = \epsilon H_{\ddot{z}_s/\dot{z}_r}(j\omega), \quad (17)$$

indicating that when at low frequencies a scaling of $\epsilon$ is introduced, it benefits both handling and comfort. This is also graphically shown in Figure 3, which shows that at the sprung resonance frequency around 1 Hz, high damping is beneficial for comfort as well as handling. Beyond this frequency, low damping benefits both. It is only at the wheel hop frequency and beyond approximately 10 Hz that the dynamic tire deflection transfer function is lower with high damping, whereas comfort becomes worse.

Figure 3 also shows that there are certain points in the frequency domain that are independent of the choice of damping. If these points are also independent of suspension stiffness these points are called invariant points. To analyse this, again consider the coupled
transfer function equation, (14), and filling in $s = j\omega$. This gives
\[ m_s H\ddot{z}_s/z_r(j\omega) + (-m_u\omega^2 + k_t)H(z_u - z_r)/z_r(j\omega) = m_u\omega^2. \] (18)

By choosing $\omega = \sqrt{k_t/m_u}$, the term $(-m_u\omega^2 + k_t)$ pre-multiplying $H(z_u - z_r)/z_r(j\omega)$ becomes zero resulting in
\[ H\ddot{z}_s/z_r(j\omega) = \frac{k_t}{m_s}. \] (19)

For this equation, it has to be noted that it is also independent of suspension parameters, i.e. the frequency and amplitude of the invariant point is independent of the choice of $d_s$ and $k_s$ but rather depends on the tire stiffness, unsprung mass and sprung mass. A similar analysis can be performed on the other transfer function relations. A summary of the resulting invariant points is shown in Table 1. Besides invariant points independent of all suspension forces, there are also damping invariant points, as Figure 3 already indicates. In these points, the choice of damping coefficient does not influence the response of the vehicle [17]. For brevity the following symbols have been used in the table which has been taken from [17]
\[
\begin{align*}
\alpha &= \omega_s^2 + \omega_{su}^2 + \frac{\omega_u^2}{2}, \\
\beta &= \frac{m_s + m_u}{2m_u} \omega_s^2 + \frac{m_s + 2m_u}{4(m_s + m_u)} \omega_u^2, \\
\omega_u &= \sqrt{k_t/m_u}, \quad \omega_s = \sqrt{k_s/m_s}, \quad \omega_{su} = \sqrt{k_{su}/m_u}.
\end{align*}
\] (20)

### 3. Controller design

An often used control technique to minimise the dynamic tire deflection is GH control [18,19]. Two implementations of this controller can be found in literature, with one connecting the unsprung mass to an inertial ground and the other effectively increasing the tire damping. For practical application a switched approach is often chosen, however,

| Table 1. Suspension- and damper invariant points, with $\alpha, \beta, \omega_u, \omega_s$ and $\omega_{su}$ given by (20), (21) [17]. |
|---|---|---|
| Transfer function category | Frequency (rad/s) |
| $H_{s_1}/z_r(s)$ | Suspension | $\omega_{s,1} = \sqrt{k_t/m_u}$ |
| $H_{s_2}/z_r(s)$ | Suspension | $\omega_{s,2} = \sqrt{k_t/(m_s + m_u)}$ |
| $H_{s_3}/z_r(s)$ | Suspension | $\omega_{s,3} = 0$ |
| $H_{u_1}/z_r(s)$ | Damper | $\omega_{u,1} = \sqrt{\alpha - \sqrt{\alpha^2 - 2\omega_s^2\omega_u^2}}$ |
| $H_{u_2}/z_r(s)$ | Damper | $\omega_{u,2} = \sqrt{\alpha + \sqrt{\alpha^2 - 2\omega_s^2\omega_u^2}}$ |
| $H_{u_3}/z_r(s)$ | Damper | $\omega_{u,3} = \sqrt{\beta - \sqrt{\beta^2 - \omega_s^2\omega_u^2}}$ |
| $H_{u_4}/z_r(s)$ | Damper | $\omega_{u,4} = \sqrt{\beta + \sqrt{\beta^2 - \omega_s^2\omega_u^2}}$ |
the theoretical ideal case will be discussed first to explain the control concept proposed in this paper. The equations of motion of a quarter car model with GH are given as

\begin{align}
m_s \ddot{z}_s &= -k_s (z_s - z_u) - d_s(t)(\dot{z}_s - \dot{z}_u), \\
m_u \ddot{z}_u &= -k_t (z_u - z_r) + k_s (z_s - z_u) + d_s(t)(\dot{z}_s - \dot{z}_u) - d_g \dot{z}_u. \tag{22}
\end{align}

The frequency response function of the quarter car model with a GH are shown in Figure 4. From this figure, it can be seen that the typical GH controller, indicated by a positive \(d_g\), reduces the dynamic tire deflection, \(z_u - z_r\), compared to no control, beyond the damping invariant point, \(\omega_{D,4}\) and thereby dampens the unsprung resonance. Inversely, when the GH damping is chosen negative, i.e. \(d_g = -1500\) Ns/m, the behaviour for frequencies lower than \(\omega_{D,4}\) is improved. Concluding from this analysis is that when dynamic tire deflection has to be minimised over the whole frequency range, the controller that minimises dynamic tire deflection is a combination of a positive and negative GH coefficient, the total result of such an approach would be a frequency response as shown in Figure 5. Here, the red dash-dot line is a stylised example of the desired frequency response and it is not based on an existing control technique.

In practice an adjustable damper is used to generate the control force that a GH requires. Following from (22), the required control force is given as

\[ F_{GH}(t) = -d_g \dot{z}_u. \tag{23} \]

However, an adjustable damper can only achieve

\[ F_d = d_s(t)(\dot{z}_s - \dot{z}_u), \quad d_{s,\text{min}} \leq d_s(t) \leq d_{s,\text{max}}. \tag{24} \]

An adjustable damper can typically only generate a force opposing the relative velocity. This means that the forces required for the GH controller as introduced in (23) in practice are not always compatible with the capabilities of adjustable damping. Therefore, the switching GH controller is introduced as [19]

\[ d^1_s(t) = d_{GH}(t) = \begin{cases} d_{s,\text{min}}, & \text{if } \dot{z}_u(\dot{z}_s - \dot{z}_u) > 0, \\ d_{s,\text{max}}, & \text{if } \dot{z}_u(\dot{z}_s - \dot{z}_u) \leq 0, \end{cases} \tag{25} \]

where \(d_{s,\text{min}}\) describes the minimum damping coefficient that can be achieved with the adjustable damper and \(d_{s,\text{max}}\) the maximum value. To achieve a negative GH coefficient the switching controller is defined as

\[ d^2_s(t) = d_{IGH}(t) = \begin{cases} d_{s,\text{max}}, & \text{if } \dot{z}_u(\dot{z}_s - \dot{z}_u) > 0, \\ d_{s,\text{min}}, & \text{if } \dot{z}_u(\dot{z}_s - \dot{z}_u) \leq 0, \end{cases} \tag{26} \]

which is now called the inverse GH (IGH) controller.

To achieve minimal dynamic tire load variation, a combination of the IGH controller and passive damping is required. The analysis of the linear system presented in Section 2.1 shows that for frequencies smaller than \(\omega_{D,4}\) the IGH shows the best performance. Additionally, beyond \(\omega_{D,4}\) the lowest dynamic tire deflection is achieved with a GH controller,
Figure 4. From top to bottom, transfer function from road input, $z_r$, to sprung mass acceleration, $\ddot{z}_s$, suspension travel, $z_s - z_u$, dynamic tire deflection, $z_u - z_r$ and unsprung mass acceleration, $\ddot{z}_u$ with a damping of $d_s = 2000 \text{ Ns/m}$ and a GH damping $d_g$ of 1500 and $-1500 \text{ Ns/m}$, respectively.
which is similar to using hard damping. The control rule is therefore formulated as

\[
d_s = \begin{cases} 
    d_{s,\text{max}}, & \text{if } f_{zu} < \omega_{\text{D,A}} \text{ and } \dot{z}_u(\ddot{z}_s - \ddot{z}_u) > 0, \\
    d_{s,\text{min}}, & \text{if } f_{zu} < \omega_{\text{D,A}} \text{ and } \dot{z}_u(\ddot{z}_s - \ddot{z}_u) \leq 0, \\
    d_{s,\text{max}}, & \text{if } f_{zu} > \omega_{\text{D,A}}.
\end{cases}
\] (27)

where \(f_{zu}\) is the estimated frequency of motion of the unsprung mass. Implementing this controller on the quarter car vehicle model and running it through a sinusoidal sweep of road inputs results in the dynamic tire deflection as shown in Figure 6. In addition, the performance of the GH and inverted groundhook controller are shown. From the figure, it becomes clear that the proposed road holding controller manages to combine the benefits of both types of controller and minimise dynamic tire deflection. In terms of comfort, Figure 6 shows that comfort follows the best performing controller type up to \(\omega_{\text{D,A}}\), after which performance becomes as bad as the hard damping case. Note that given the non-linear nature of the switching controller, instead of the transfer function amplitude that was used to analyse the linear system, the standard deviation of the dynamic tire deflection is shown.

The frequency estimation works by finding an upper and lower value in the unsprung mass acceleration signal and calculating the time in between these two points. This time span can then easily be calculated into a frequency by evaluating it as half of a sine function period. The downside of this kind of estimation is that it results in a delay equal to the time span between the lower and upper point. Since it is only used for frequencies above \(\omega_{\text{D,A}}\), the delay will be limited to roughly 50 ms maximum. Compared to [10], the switching frequency is at a higher frequency, therefore, the delays caused by estimating the excitation frequency are less relevant for the dynamics of the quarter car model. Evidently, in practice, there will be noise on the measured wheel acceleration signal. In addition to that the semi-active damper has an internal valve which is capable of changing the damping of the suspension system. When a rapid switch is made from a soft to hard damping or vice-versa, it will result in additional vibrations in both of the masses. Since a frequency estimator is used, these vibrations can influence the outcome of the control rules.
Figure 6. Simulated tire deformation and sprung mass acceleration responses over the frequency range of 0.1 to 50.0 Hz for two passive configurations, in addition to the GH-, IGH controller and the desired controller performance.

when they reach a certain amplitude. This effect is undesired since the control rules function on input caused by road disturbances. The frequency estimation will therefore be filtered with respect to the signal amplitude. If the difference in amplitude between the upper and lower bound is below a certain threshold, $a_{\text{min}}$, the estimated frequency will not be used. An example of the frequency identification on measurement data is shown in Figure 7.

4. Experimental results

To illustrate the functioning of the road holding controller, measurements have been performed. In this section, the test vehicle and road course will first be introduced followed by the measurement results.

4.1. Test vehicle

The test vehicle which is used for the practical experiments presented in this paper is a BMW E90 3-series. Tractive Suspension [20] designed a set of semi-active dampers
which are installed on this vehicle. The damper has a dynamically balanced solenoid valve that can open to allow for a larger oil flow, thereby lowering the damping force. An acceleration sensor is mounted on the lower side of each of these semi-active dampers, measuring the unsprung mass acceleration. Besides that, a two degree of freedom angular rate sensor is installed in the back of the vehicle, measuring the roll and pitch rate. A GPS sensor is installed to log driven speed. The sensors are connected to a control unit developed by Tractive Suspension and runs at sampling rate of 500 Hz. Parallel to the control unit, a logger is used to save the sensor data and valve currents for post-processing analyses. The sprung and unsprung mass velocity are calculated using an observer as discussed in [21,22]. Table 2 presents the vehicle parameters of the test vehicle used for

![Figure 7](image-url)  
**Figure 7.** Frequency estimation of a wheel acceleration signal obtained in a road measurement.

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sprung mass FL</td>
<td>$m_{s,fl}$</td>
<td>364</td>
<td>kg</td>
</tr>
<tr>
<td>Sprung mass FR</td>
<td>$m_{s,fr}$</td>
<td>407</td>
<td>kg</td>
</tr>
<tr>
<td>Sprung mass RL</td>
<td>$m_{s,rl}$</td>
<td>404</td>
<td>kg</td>
</tr>
<tr>
<td>Sprung mass RR</td>
<td>$m_{s,rr}$</td>
<td>378</td>
<td>kg</td>
</tr>
<tr>
<td>Unsprung mass F</td>
<td>$m_{u,f}$</td>
<td>42</td>
<td>kg</td>
</tr>
<tr>
<td>Unsprung mass R</td>
<td>$m_{u,r}$</td>
<td>36</td>
<td>kg</td>
</tr>
<tr>
<td>Spring stiffness front</td>
<td>$k_{s,f}$</td>
<td>24,900</td>
<td>N/m</td>
</tr>
<tr>
<td>Spring stiffness rear</td>
<td>$k_{s,r}$</td>
<td>20,690</td>
<td>N/m</td>
</tr>
<tr>
<td>Tire stiffness front</td>
<td>$k_{t,f}$</td>
<td>225,000</td>
<td>N/m</td>
</tr>
<tr>
<td>Tire stiffness rear</td>
<td>$k_{t,r}$</td>
<td>225,000</td>
<td>N/m</td>
</tr>
</tbody>
</table>
the measurements in this paper. Furthermore, an overview of the hardware is given in Figure 8.

4.2. Test environment

A trajectory has been selected which is displayed in Figure 9. This trajectory is located in Cuijk, the Netherlands and contains multiple road surface qualities and a number of deterministic road inputs. The variety of this trajectory will ensure that the controller functions on a broad and representative road input with respect to Dutch road circumstances. The trajectory is approximately 2.3 km long and will be driven in both directions, this will result in a total length of 4.6 km. There are 7 speed bumps located on this path, these speed bumps vary in length and height. The section labelled as bad road surface has a length of 360 m. Due to traffic and human errors, the forward velocity of the measurements do not match exactly. However, the forward vehicle velocity over time shows a good coherence between the experiments with the different damper settings.
4.3. Measurement results

Recall that the dynamic tire deflection cannot be measured in practice, but the unsprung mass acceleration can. Referring back to Figure 3 that shows the relation between the unsprung mass acceleration and dynamic tire deflection indicates that if the unsprung mass acceleration is low, the dynamic tire deflection is as well. Therefore, the power spectral density of the unsprung mass acceleration of all four corners is presented in Figure 10. In this figure, the unsprung mass acceleration with soft, hard and road holding controller is presented. It can be clearly seen that with the soft damping, the unsprung mass acceleration is low beyond $\omega_{D,3}$, approximately 2 Hz, and that the controlled unsprung mass acceleration is, except for a small difference at 2 Hz, similar. At the wheel hop frequency, the hard damping achieves minimal unsprung mass acceleration, which is matched by the controlled setting. Additionally, for frequencies above the second invariant point ($\omega_{D,4}$) it is similar to the performance of a passive hard damping configuration.
At frequencies lower than the sprung mass eigenfrequency, it can be seen that for the left-hand side the controlled unsprung mass acceleration follows the soft damping and for the right-hand side the hard damping. This is most likely caused by measurement inaccuracy.

Similar to the unsprung mass acceleration as shown in Figure 3, the sprung mass acceleration is not clearly visible in the unsprung mass acceleration, it is therefore impossible to draw conclusions on the dynamic tire deflection at this frequency. Calculating the standard deviation of the unsprung mass acceleration shows a decrease of up to 5% on the front and 11% on the rear compared to the hard damper setting. Additionally, the road holding controller follows the behaviour of the soft damping in between the two invariant points ($\omega_{D3}$ and $\omega_{DA}$). Following the analysis of Section 2.2, it can be expected that the comfort of the vehicle is also better than the comfort that would be achieved with hard damping only.

Figure 11 shows the unsprung mass acceleration of the front left corner for a controlled, soft and hard damping. From the figure, it becomes clear that the controlled vehicle has the lowest peak unsprung mass acceleration. This is achieved by opening the valve in the damper, effectively allowing the tire to move up easily. By making the damping hard
again, oscillations are prevented. The figure also shows that, contrary to the switching rule which explicitly gives only soft and hard damping, the damper valve actually has to move, resulting in an intermediate damping for a short time.

5. Conclusion

The main research objective of this paper was to develop a road holding controller that minimises dynamic tire deflection whilst using a commercially available sensor. To that end a rule-based controller was developed, which switches between soft and hard damping, depending on the frequency of excitation and an IGH control logic. The exact switching frequency is determined by the damper invariant point, which is a function of the vehicle parameters. Simulations show that the designed road holding controller is capable of minimising the dynamic tire deflection, combining the best performance of the hard and soft damping and outperforming existing GH control.

Measurements performed on the open road with a BMW test vehicle equipped with semi-active suspension shows that the unsprung mass acceleration is minimised. At low frequencies the performance is similar to the performance of the soft damper setting, and at the wheel hop frequency the behaviour is similar to that of the hard damper setting. The standard deviation of the unsprung mass acceleration on mixed roads is improved up to 11% over that of the hard damping and the time response also shows a lower unsprung mass acceleration.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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