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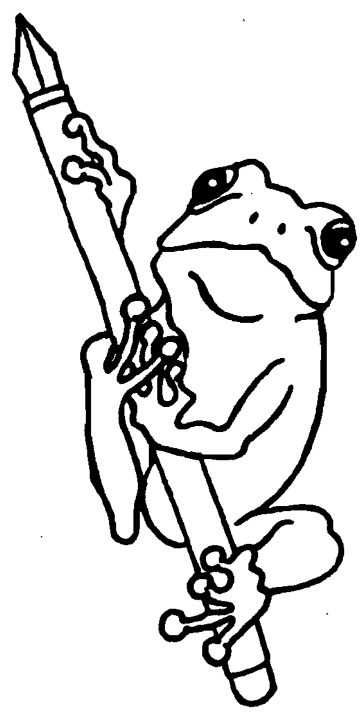

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A Generalized Definition of Flame Stretch for 3D Instationary Premixed Flames

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ABSTRACT - The flame stretch concept is formulated for the general case of 3D instationary flames with finite flame front thickness. It is shown that additional contributions to the stretch rate appear apart from the terms which are usually used in flame studies. These extra terms are associated with variations in the mass density along the flame iso-contours and with variations in flame front thickness in time and space. It is finally shown that the following definition for the stretch rate is generally applicable: $K = \frac{1}{m}\frac{dm}{dt}$, denoting the fractional change of mass in an infinitesimally small flame volume.

Key Words: Flame stretch, 3D instationary flames

1 Introduction

The stretch rate is an important quantity in the understanding of flame phenomena such as extinction and the local structure of turbulent flames. The generally accepted definition of the flame stretch rate reads [1, 3, 5]

$$K_A = \frac{1}{A}\frac{\mathrm{d}A}{\mathrm{d}t}, \tag{1}$$

i.e. K_A is the fractional area change of a small area A in the flame surface, which moves in this surface with a tangential velocity equal to the local tangential fluid velocity. This surface has also a velocity component normal to the local flame surface. In general, this definition is used for the flame surface, which is a sheet that characterizes the location of the flame. Practical expressions for K_A, derived from equation (1), can be found in for instance Refs. [3, 5, 6].

As an improvement to the flame sheet model, we consider a laminar flame to be defined in a small volume between the burnt and unburnt gases, comprising the reaction zone and the upstream preheat and reactant diffusion zones. This region will be referred to as the flame region. The purpose of this paper is to formulate a generalized definition of flame stretch, which is appropriate in this situation for a flame of finite thickness.

2 The Generalized Stretch Definition

Consider the flame region in a premixed gas mixture, defined in terms of a scalar variable Y. Y might be the temperature or the fuel mass fraction; the only restriction on Y being $\nabla Y \neq \mathbf{0}$. We assume that this region is defined by $Y_u \leq Y(\mathbf{x},t) \leq Y_b$, with Y_u and Y_b the values of Y in the unburnt and burnt gas, respectively. In the general three-dimensional case, we can identify surfaces $Y(\mathbf{x},t) = \zeta$ with $Y_u \leq \zeta \leq Y_b$ in the flame region (see Figure 1). Suppose that each of these flame surfaces moves with a local velocity $\mathbf{v}_f$. Consequently, the evolution of these surfaces is governed by the kinematic condition [6]

$$\frac{\mathrm{d}Y}{\mathrm{d}t} = \frac{\partial Y}{\partial t} + (\mathbf{v}_f \cdot \nabla)Y = 0, \tag{2}$$

stating that a point on a flame surface stays on this surface for all t.

Each flame surface can be parametrized by curvilinear, orthogonal coordinates (ξ,η). In this way, we can define the coordinate transformation $\mathbf{x} = (x,y,z) \rightarrow \boldsymbol{\xi} = (\xi,\eta,\zeta)$ in the flame region (see Figure 1). The transformation is time dependent for instationary flames. Associated with the coordinate transformation we can define the scale factors

$$h_\xi = \left| \frac{\partial \mathbf{x}}{\partial \xi} \right|, \; h_\eta = \left| \frac{\partial \mathbf{x}}{\partial \eta} \right|, \; h_\zeta = \left| \frac{\partial \mathbf{x}}{\partial \zeta} \right| \tag{3}$$

and the corresponding unit vectors

$$\mathbf{e}_\xi = \frac{1}{h_\xi}\frac{\partial \mathbf{x}}{\partial \xi}, \; \mathbf{e}_\eta = \frac{1}{h_\eta}\frac{\partial \mathbf{x}}{\partial \eta}, \; \mathbf{e}_\zeta = \frac{1}{h_\zeta}\frac{\partial \mathbf{x}}{\partial \zeta}. \tag{4}$$

All variables can alternatively be expressed in the $\boldsymbol{\xi}$-coordinate system.

By definition of the $\boldsymbol{\xi}$-coordinate system it is obvious that

$$\mathbf{e}_\zeta = \frac{1}{|\nabla Y|}\nabla Y. \tag{5}$$

Let $\mathbf{v}_f = v_{f,\xi}\mathbf{e}_\xi + v_{f,\eta}\mathbf{e}_\eta + v_{f,\zeta}\mathbf{e}_\zeta$ be the velocity of the flame surfaces in the $\boldsymbol{\xi}$-coordinate system. Combining the kinematic condition (2) and equation (5), it is easy to see that

$$v_{f,\zeta} = \frac{-1}{|\nabla Y|}\frac{\partial Y}{\partial t}. \qquad (6)$$

The normal velocity component $v_{f,\zeta} = 0$ for stationary flames, i.e. the flame surfaces are fixed in space. We further assume that the tangential velocity of the flame surfaces is equal to the tangential component of the fluid velocity $\mathbf{v}_t$. Let $\mathbf{v} = v_\xi \mathbf{e}_\xi + v_\eta \mathbf{e}_\eta + v_\zeta \mathbf{e}_\zeta$ be the fluid velocity in the $\boldsymbol{\xi}$-coordinate system, then $\mathbf{v}_f = v_\xi \mathbf{e}_\xi + v_\eta \mathbf{e}_\eta + v_{f,\zeta} \mathbf{e}_\zeta = \mathbf{v}_t + v_{f,\zeta}\mathbf{e}_\zeta$ with $v_{f,\zeta}$ given by equation (6).

Consider an arbitrary control volume $V(t)$ in the flame region moving with velocity $\mathbf{v}_f$ and let $m(t)$ denote the mass contained in $V(t)$, i.e.

$$m(t) = \int_{V(t)} \rho \, \mathrm{d}V, \qquad (7)$$

with ρ the local density of the gas mixture. In the new definition, we want to relate the stretch rate to the rate of change $\frac{\mathrm{d}m}{\mathrm{d}t}$ of m as $V(t)$ moves with velocity $\mathbf{v}_f$ in the flame region. Applying Reynolds' transport theorem [2], it is easy to see that

$$\frac{\mathrm{d}m}{\mathrm{d}t} = \int_{V(t)} \left(\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}_f) \right) \mathrm{d}V. \qquad (8)$$

The new stretch rate K is now defined by

$$\frac{\mathrm{d}m}{\mathrm{d}t} = \int_{V(t)} \rho K \, \mathrm{d}V. \qquad (9)$$

Thus K can be interpreted as the specific rate of change of mass, due to movement of the flame region with velocity $\mathbf{v}_f$. From (8) and (9) it is immediately clear that

$$\rho K = \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}_f). \qquad (10)$$

Using the relation $\mathbf{v} - \mathbf{v}_f = (v_\zeta - v_{f,\zeta})\, \mathbf{e}_\zeta$ and taking into account the continuity equation, equation (10) can also be written as

$$\rho K = -\nabla \cdot (\rho (v_\zeta - v_{f,\zeta}) \mathbf{e}_\zeta), \qquad (11)$$

from which we see that the stretch rate is related to variations of the (normal) mass flux through the flame surfaces.

To obtain more insight in the physical interpretation of this new flame stretch definition, rewrite equation (10) as follows:

$$K = \nabla \cdot \mathbf{v}_f + \frac{1}{\rho}(\frac{\partial \rho}{\partial t} + \mathbf{v}_f \cdot \nabla \rho) = \nabla \cdot \mathbf{v}_t + v_{f,\zeta} \nabla \cdot \mathbf{e}_\zeta + \mathbf{e}_\zeta \cdot \nabla v_{f,\zeta} + \frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}t}. \qquad (12)$$

The first and second term in the last part of this equation are also found by Law [5] and denote the contributions due to stationary flow straining and instationary flame curvature, respectively. These two terms can be expressed in the

$\boldsymbol{\xi}$-coordinate system as follows:

$$\nabla \cdot \mathbf{v}_t = \frac{1}{h_\xi}\frac{\partial v_\xi}{\partial \xi} + \frac{1}{h_\eta}\frac{\partial v_\eta}{\partial \eta} + \frac{1}{h_\xi h_\eta h_\zeta}\left(v_\xi \frac{\partial}{\partial \xi}(h_\eta h_\zeta) + v_\eta \frac{\partial}{\partial \eta}(h_\xi h_\zeta)\right) \tag{13}$$

and

$$v_{f,\zeta}\nabla \cdot \mathbf{e}_\zeta = \frac{v_{f,\zeta}}{h_\xi h_\eta h_\zeta}\frac{\partial}{\partial \zeta}(h_\xi h_\eta). \tag{14}$$

From equation (13) it is clear that velocity variations along flame surfaces and flame thickness variations play a role in the flow straining term, as found by de Goey et al. [4]. The third term in the right-hand side of equation (12) is new and arises from instationary flame thickness variations due to differences in flame velocities $v_{f,\zeta}$ among different flame contours. This term can also be written as:

$$\mathbf{e}_\zeta \cdot \nabla v_{f,\zeta} = \frac{1}{h_\zeta}\frac{\partial}{\partial \zeta}(v_{f,\zeta}). \tag{15}$$

The last term $\frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}t}$ in the right-hand side of equation (12) is also new and denotes the contribution to K due to density variations along the flame iso-contours. It should be noted that this term vanishes when the ρ iso-contours are parallel with the Y flame contours due to equation (2).

Note that equation (12) reduces to:

$$K = \frac{1}{\rho}\nabla \cdot (\rho \mathbf{v}_t) \tag{16}$$

for the case of stationary flames in which $v_{f,\zeta} = 0$. This result has been found recently by de Goey et al. [4] for the case of two-dimensional stationary flames.

Let us finally study the implications of this generalized flame stretch formalism on the definition in equation (1). To that end divide equation (9) by (7) and let the volume $V(t) \to 0$. We then find the generalized flame stretch definition:

$$K = \frac{1}{m}\frac{\mathrm{d}m}{\mathrm{d}t}. \tag{17}$$

This definition is a generalization of the usual definition (1). It should be noted that the volume $V(t)$ should be infinitesimally small, as is the case for the area A given in equation (1).

3 Conclusions

A generalization of the flame stretch formalism to the general case of 3D instationary flames with finite flame front thickness has been presented. The contributions to K, also found by Law [5] are recovered. However, also some new terms have appeared, arising from variations in the mass density along the flame iso-contours and from variations in flame front thickness in time as well as in space. Furthermore, it has been shown that the usual definition for the stretch rate $K_A = \frac{1}{A}\frac{\mathrm{d}A}{\mathrm{d}t}$ should be replaced by the more generally applicable definition $K = \frac{1}{m}\frac{\mathrm{d}m}{\mathrm{d}t}$, denoting the fractional change of mass in an infinitesimal small flame volume. This generalized definition is well suited for studying the effects of flame stretch on the flame behavior in numerical flame studies.

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Figure Captions

Figure 1: Flame surfaces $Y(\boldsymbol{x}, t) = \zeta_1$ and $Y(\boldsymbol{x}, t) = \zeta_2$ defined in terms of the scalar quantity Y and the $\boldsymbol{\xi}$-coordinate system.

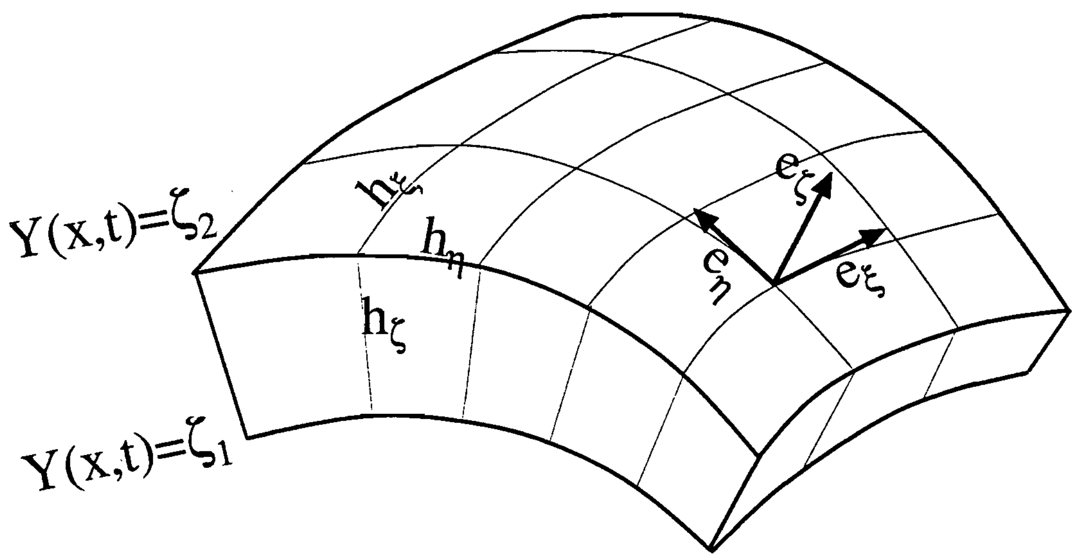

Figure 1, de Goey and ten Thije Boonkkamp