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Pressure wave propagation in a partially water-saturated porous medium

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Reflection and transmission of a stepwise pressure wave incident to a partially water-saturated porous medium is investigated. The strongly dispersive character of the phase velocities and damping due to air bubble resonance causes pressure over- and undershoot in reflection and a fast oscillatory disturbance propagating into the porous medium. Theoretical results are compared with new results from shock tube experiments. A qualitative agreement is found.

The acoustic properties of liquid-saturated porous media are very sensitive to the presence of a small amount of gas in the pores. An extended literature survey has been given by Anderson and Hampton.[1] In recent years, the two-phase Biot theory for wave propagation in porous media has been modified to allow for the presence of a third phase. Garg and Nayfeh[2] and Berryman,Thigpen, and Chin[3] present rather general models, applicable to a wide range of gas volume fractions. Bedford and Stern[4] describe the effect of a small amount of oscillating gas bubbles on the propagation and damping of the two compressional waves. Experimental data are given by Dontsov, Kuznetsov, and Nakoryakov[5] and van der Grinten *et al.*,[6,7] who independently applied a shock tube technique to study wave propagation in a vertical partially liquid-saturated porous column.

In this communication, the linear response of a semi-infinite porous medium to a stepwise pressure wave at normal incidence is discussed, taking into account the rather peculiar frequency dependence of the complex wave number. The results are compared with new experimental data. The experimental setup has been described previously by van der Grinten *et al.*[8,9] A porous cylinder, equipped with pore pressure gages is located in the test section of a vertical shock tube (Fig. 1). The porous column is filled with a water–air bubble mixture. The relevant properties are given in Table I. The preparation procedure is as follows. Water, saturated with dissolved air at a pressure of 2 bar, is flushed at the same pressure through the porous column. Then, the pressure is released, so that air bubbles are formed. The concentration and size of air bubbles is controlled by adding some $Ca(OH)_2$ to the pore liquid. Mean air bubble size and concentration are determined from measurements of pore fluid compressibility. The pore fluid is subjected to a small pressure increase Δp_1 during a time interval τ_1 (typically of the order of several minutes), which is short enough to ensure that almost no air from the bubbles is dissolved in the liquid. From the corresponding change in liquid volume, the "frozen" pore fluid compressibility is found, which is a direct measure for the air volume fraction.

When the pore fluid is subjected to a stepwise pressure change Δp_2 part of the air will gradually dissolve into the liquid in a time interval τ_2; $\tau_2 \gg \tau_1$. This process is diffusion dominated with mean bubble radius and mean bubble distance as important parameters. By repeating the "frozen" compressibility experiments the gradual change of the air volume fraction $(1 - s)$ towards a new equilibrium state can be measured and compared with diffusion theory. The reliability of this method was checked by repeating the experiment in an optically transparant gelatin-water mixture.

A wave experiment proceeds as follows. The shock tube generates a stepwise pressure wave in air, which is transmitted into the water layer on top of the column. This wave partially reflects, partially transmits into the porous medium. This reflection and transmission of a pressure step wave is described theoretically on the basis of the model of Bedford and Stern.[4] They show that the effect of the oscillating air bubbles can be taken into account by introducing a frequency-dependent complex compressibility. We shall as-

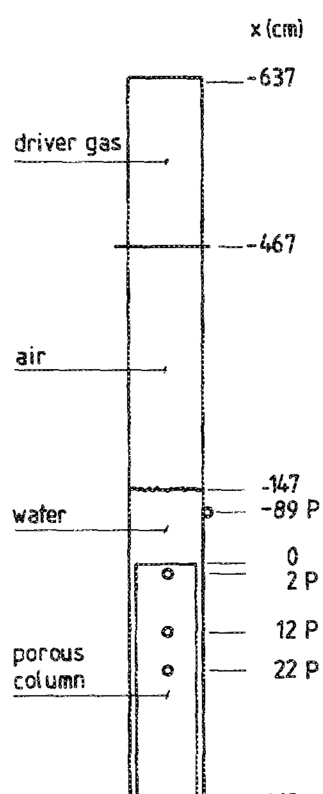

FIG. 1. The vertical shock tube with porous column. p: pressure transducer.

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TABLE I. Relevant properties.

Porous material		Sand particles/epoxy resin
Porosity	n	0.29
Darcy coefficient	k	5.6×10^{-8} kg^{-1} m^3 s
$a=(k\eta)^{-1}$		1.78×10^{10} m^{-2}
Constrained modulus	K_p	4.5 GPa
Bulk modulus of water	K_l	2.2 GPa
Added mass coefficient	α_∞	1.45
Pore radius	R_p	5×10^{-5} m
Viscosity water	η	1.0×10^{-3} kg m^{-1} s^{-1}
Density of the solid	ρ_s	2.65×10^3 kg m^{-3}

sume that the surface tension can be neglected, that the air bubbles are homogeneously distributed over the porous medium, that the bubble radius R is much greater than the pore radius R_p and finally that the air bubbles are compressed isothermally. It can be shown that for our experimental conditions, the most important damping mechanism of the bubble oscillations is due to the Darcy friction of the radial motion of the liquid near the bubbles. In this respect our model is different from that of Bedford and Stern,[4] who considered thermal damping to be the dominant mechanism. Similar to the treatment of wave propagation in bubbly liquids (see e.g., van Wijngaarden[10]), an effective bulk modulus K_g of the gas phase can be defined, that relates the average bubble volume V_g to a change in liquid pressure far away from the bubble: $K_g = V_g^{-1}(\partial V_g/\partial_p)$. Assuming a harmonic variation of liquid pressure p with frequency ω, K_g follows from a solution of the radial linearized momentum equation for the pore liquid surrounding the oscillating bubble. The result is

$$K_g = p_0 - \alpha\rho_l R^2\omega^2/3 + ina\eta\omega F R^2/3, \tag{1}$$

where p_0 is the ambient pressure and F is the frequency-dependent friction factor as discussed by Biot.[11] The other symbols are explained in Table I. Denoting the fraction of the pore volume filled with water by s, the bulk modulus of the pore fluid K_f equals

$$K_f = K_g K_l/[(1-s)K_l + sK_g]\,. \tag{2}$$

Inserting this complex K_f in the conservation laws of mass and momentum of pore fluid and porous material and assuming a $\exp[i(\omega t - kx)]$ dependence of all variables, expressions are obtained for the two-phase velocities $c_j = \omega[\mathrm{Re}(k_j)]^{-1}$ and the damping coefficients $\mathrm{Im}(k_j)$. The results, shown in Fig. 2, are somewhat different from those of Bedford and Stern,[4] because of the different damping mechanism.

For high frequencies the values of the two-phase velocities become approximately equal to the fully saturated ones (dotted lines). This can be understood from Eqs. (1) and (2) since $\omega\to\infty$ implies $K_g\to-\infty$ and $K_f\to K_l/s$.

Comparing the phase velocities for the bubbly pore fluid case with those for the fully saturated case it is interesting to

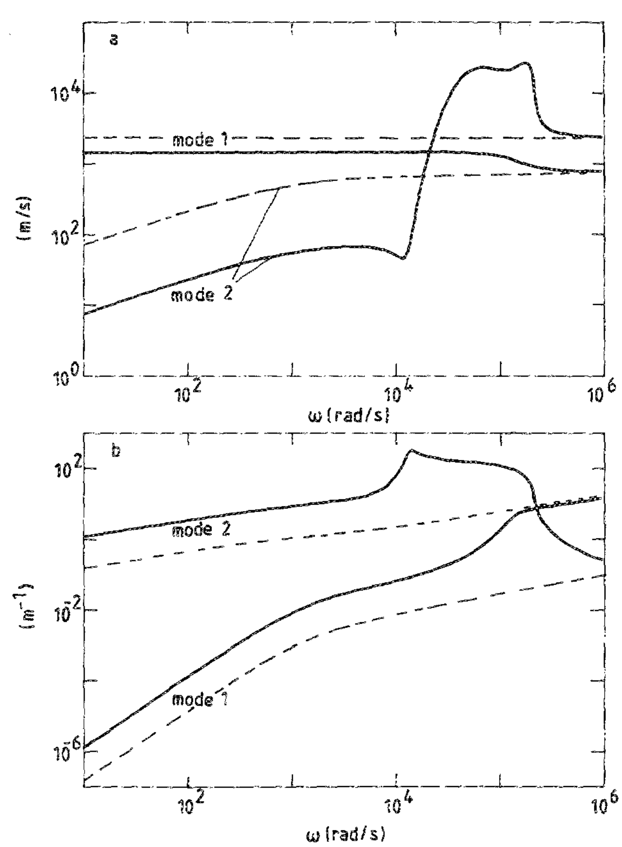

FIG. 2. (a) Phase velocities and (b) attenuation coefficients of compressional waves in a porous medium with water and equally sized air bubbles. Air volume fraction of pores: 0.01; bubble radius: 1 mm. The dotted lines represent the fully saturated case.

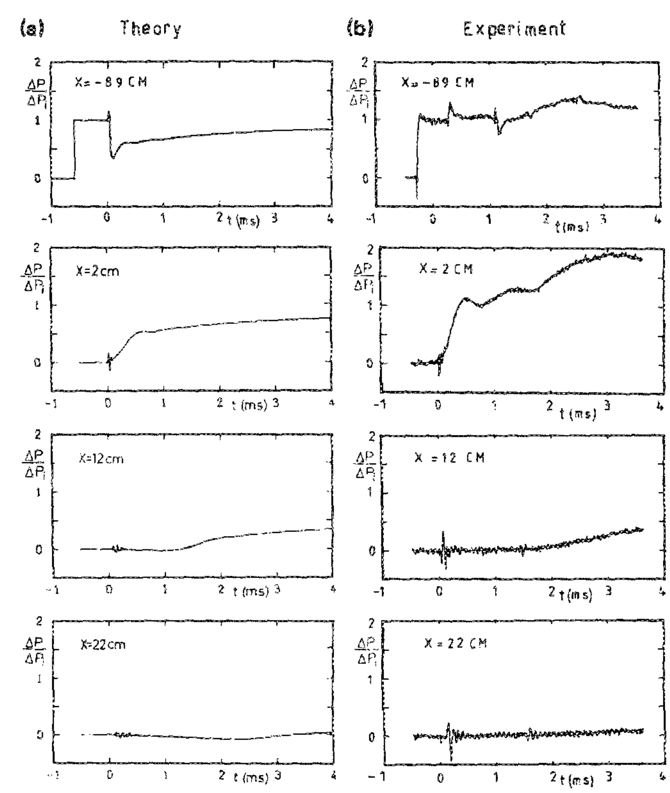

FIG. 3. Response of a porous medium to a stepwise pressure wave at normal incidence. Air volume fraction in pores $(2.3\pm0.2)10^{-3}$, mean bubble radius: 0.8 mm. Pressure time histories above the column (-89 cm), and at distances 2,12, and 22 cm from the surface inside the column.

note that the roles of modes 1 and 2 are interchanged near resonance. This can be understood since resonance may cause the liquid to be less compressible than the porous material in a certain frequency range. A careful analysis did show that there is a critical value of the air volume fraction where this interchange occurs. For our experimental conditions this critical value of $1-s$ is found to be close to 4×10^{-5}.

In the frequency interval between resonance and antiresonance the damping of the second mode is extremely high. The high value of the second mode phase velocity near antiresonance and the corresponding high value of the fluid bulk modulus will lead to a high positive value of the reflection coefficient in this frequency range. The reflection and transmission phenomena caused by an incident pressure step with a finite rise time of 20 μs has been calculated by means of a straightforward Fourier decomposition as described by van der Grinten *et al.*[9] The results are shown in Fig. 3. The reflected wave signal first shows a sharp pressure increase corresponding to antiresonance, followed by a pressure minimum due to bubble resonance. Inside the porous material, at distances of 2, 12, and 22 cm from the top, an oscillatory disturbance is seen to propagate in the porous material with a high velocity, followed by a gradual pressure increase. In Fig. 3(b) experimental results are shown. The agreement with theory still is not satisfactory, yet, the initial pressure overshoot is recognized in the reflection signal and rapid oscillations are observed, indeed, as can be explained on the basis of linear theory.

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An alternative proof of the generalized reciprocity theorem for charge collection

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An alternative proof is given of the generalized reciprocity theorem for charge collection discussed by Misiakos and Lindholm [J. Appl. Phys. **58**, 4743 (1985)]. The present analysis directly yields an equation for the distribution of the charge collection probability in quasi-neutral regions of a semiconductor device and the associated boundary conditions.

A few years ago, the author[1] proved a reciprocity theorem for charge collection by a *p-n* junction in the presence of a unit point source of carriers. The theorem was extended by Misiakos and Lindholm,[2] who included in the transport equation for minority carriers the presence of an electric drift field and the spatial dependence of all semiconductor parameters. Their proof is based on a procedure given by Shockley, Sparks, and Teal[3] and involves both the excess and equilibrium concentrations of minority carriers.

Since the collected current is only dependent on the excess minority-carrier concentration, it seems useful to give a proof that uses only this density. The present analysis explicitly yields the equation that must be solved to find the distribution of the charge collection probability in the device and the related boundary conditions. Other extensions considered in Ref. 2 could be included, but are omitted here for the sake of conciseness.

Let us consider a quasineutral region V of a *p-n* diode (e.g., a nonuniformly doped emitter) in low injection conditions; for easier comparison with the treatment of Ref. 2, the region is assumed to be p type. Let Σ be the junction surface, S the rest of the boundary of V (e.g., the free surface), and v_s the surface recombination velocity at S. If the external injection is represented by a unit point source of carriers at $\mathbf{r}'$, the