Comparison of linear theory with wave propagation experiment in flexible vessels with wall thickness variation and geometric tapering

COMPARISON OF LINEAR THEORY WITH WAVE PROPAGATION EXPERIMENT IN FLEXIBLE VESSELS WITH WALL THICKNESS VARIATION AND GEOMETRIC TAPERING

Christina G. Giannopapa(1), Marcel C. M. Rutten(2), George Papadakis(3), Frans N. van de Vosse(2), Arris S. Tijsseling(1)

(1) Mathematics and Computer Science
Eindhoven University of Technology
PO Box 513, 5600 MB Eindhoven
The Netherlands

(2) Biomedical Engineering
Eindhoven University of Technology
PO Box 513, 5600 MB Eindhoven
The Netherlands

(3) Mechanical Engineering
King's College London
London WC2R 2LS
United Kingdom

INTRODUCTION

The study of the wave propagation phenomenon in fluid-filled flexible vessels is often motivated by the need to understand arterial blood flow. Even though the general principles for wave propagation in flexible vessels are known [1,2] there is lack in the literature of well-defined experiments taking into consideration the wall thickness variation and the geometric tapering that characterizes the human vessels, in particular the aorta. In-vitro laboratory experiments in mechanically and constitutively well-defined systems are needed for the validation of numerical and analytical models.

A set of tubes has been designed and manufactured according to aortic specifications, such that the wave speed of the traveling waves is similar to that in the aorta. A series of pulse propagation experiments has been performed and compared with predictions from linear theory.

LINEAR THEORY

A thorough theoretical investigation of the propagation of pressure disturbances in straight viscoelastic tubes with constant wall thickness is given in [3]. The wave speed in the fluid contained in an axially tethered thin-walled elastic tube is given by:

$$c_0 = \sqrt{\frac{Eh}{\left(1-\nu^2\right)2r\rho_f}} \tag{1}$$

where E is Young's modulus, h is the tube's wall thickness, ν is Poisson's ratio, r is the tube radius and ρ_f is the fluid density.

Multiple reflections occur from the tube's ends. When N reflections are taken into account, the pressure p, the flow rate Q and the wall distension D can be determined from [3,4,5]:

$$p(\omega,z,t) = \hat{p}(\omega,0)e^{-ikz}\prod_{\lambda=1}^{N}\left[1+\Gamma_{0L}e^{(-1)^{\lambda}2ik(L-z)}\right]e^{i\omega t} \tag{2}$$

$$Q(\omega,z,t) = \hat{Q}(\omega,0)e^{-ikz}\prod_{\lambda=1}^{N}\left[1-\Gamma_{0L}e^{(-1)^{\lambda}2ik(L-z)}\right]e^{i\omega t} \tag{3}$$

$$D(\omega,z,t) = \hat{D}(\omega,0)e^{-ikz}\prod_{\lambda=1}^{N}\left[1+\Gamma_{0L}e^{(-1)^{\lambda}2ik(L-z)}\right]e^{i\omega t} \tag{4}$$

where ω is the angular frequency, $\hat{p}(\omega,0), \hat{Q}(\omega,0), \hat{D}(\omega,0)$ are the complex wave amplitudes measured at location z=0, k is the propagation coefficient and Γ_{0L} is the reflection coefficient. The properties used for the calculation are given in Table 1, where Γ is the gamma function.

Properties		Units	Value
Fluid density	ρ_f	kg/m^3	998
Dynamic viscosity	η_f	Ns/m^2	0.001
Poisson's ratio	ν	-	0.5
Viscoelastic coefficient	c	MPa/s^{-n}	1.3
Viscoelastic coefficient	n	-	0.065
Viscoelastic modulus	$\hat{E}_s$	MPa	$c\Gamma(1-n)\omega^n e^{in\pi/2}$

Table 1. Material properties used for the theoretical model

EXPERIMENTAL METHODS

The tubes were manufactured by spin coating and were made from Desmopan 588 polyurethane (Bayer) [4]. The geometric details can be seen in Table 2, where L is the tube length and the taper $Z=\Delta r/L$.

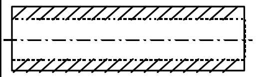
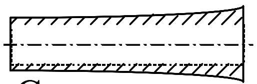
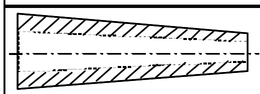
Type	$2r$ [mm]	h ±0.001 [mm]	L [mm]	Slope	$E*h/2r$ [MPa]	c_0 [m/s]
 A	25	0.1	446	0	0.04	6.3
 C	25	0.05-0.1	446	0	0.02-0.04	4.5-6.3
 E	25-12.5	0.05	446	-0.014	0.02-0.04	4.5-6.3
 F	25-12.5	0.1-0.05	446	-0.014	0.04	6.3

Table 2. Geometrical parameters of tubes manufactured

The tubes were designed to be analogues of the human aorta. Tube E has the same stiffness *Eh/2r* as the human aorta measured in [6]. Tube C was manufactured with variable wall thickness such that the theoretical wave speed given by Eq. (1) at any location along the length of the tube is the same as in tube E. Tube F was manufactured with variable wall thickness such that the constant wave speed is the same as in tube A. The tubes were pre-strained axially to 3% to prevent lateral motion. The tube is closed at both ends. A short pulse was applied at one end. The measured quantities were pressure, flow and wall distension at 10 locations along the length of the tube.

RESULTS

The comparison of the analytical results with the experimental measurements in the straight tube A of constant wall thickness can be seen in Fig. 1. The comparison of the normalized pressure, flow and wall distension measurements every 50 mm along the length of the tube against scaled time for tubes A, F and C, E can be seen in Figs. 2, 3 and 4.

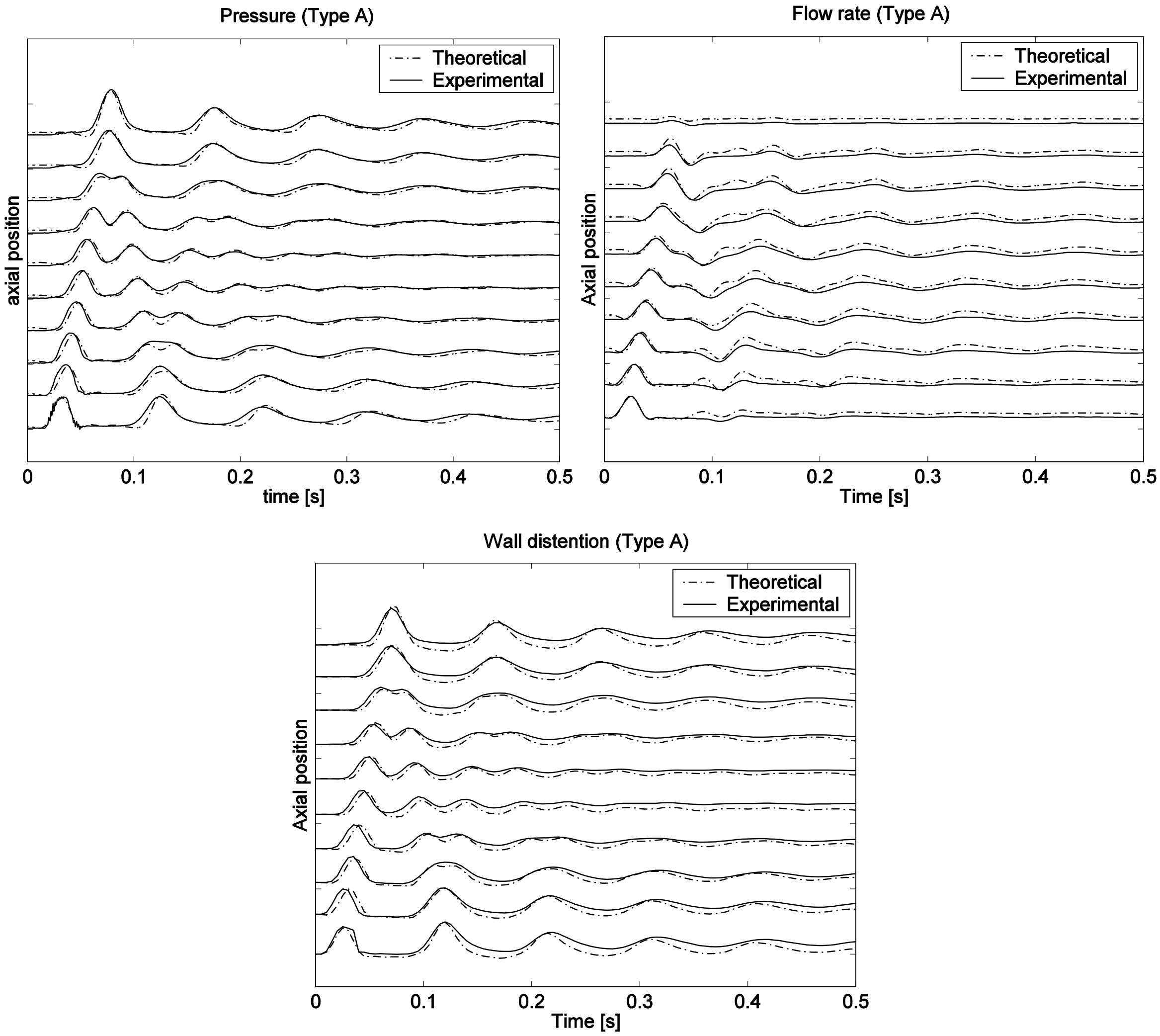


Figure 1. Comparison of experimental measurements in tube A of pressure, flow rate and wall distension with analytical model for a viscoelastic tube.

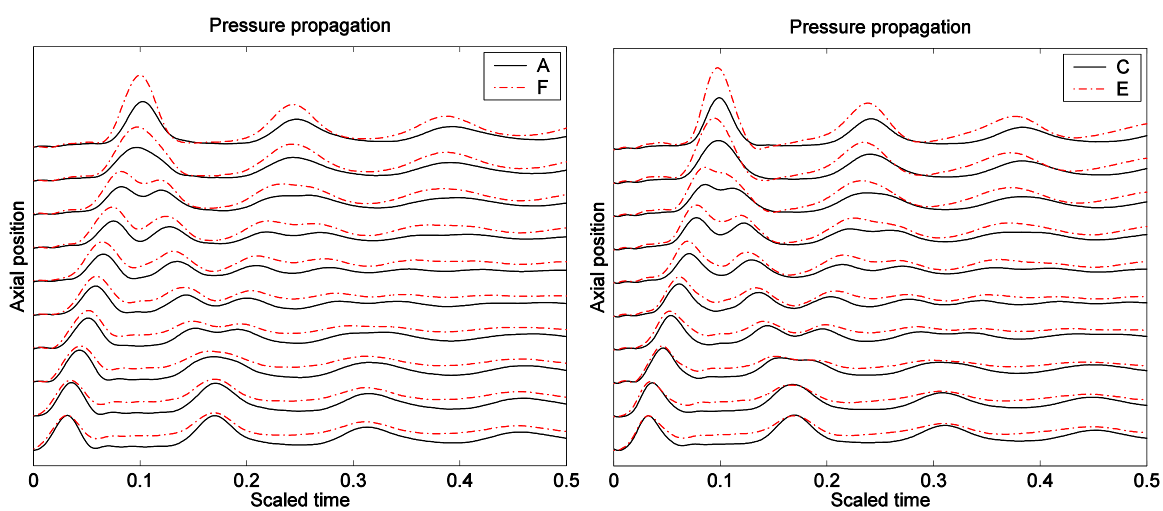


Figure 2. Measured pressures for tubes A, F and C, E.

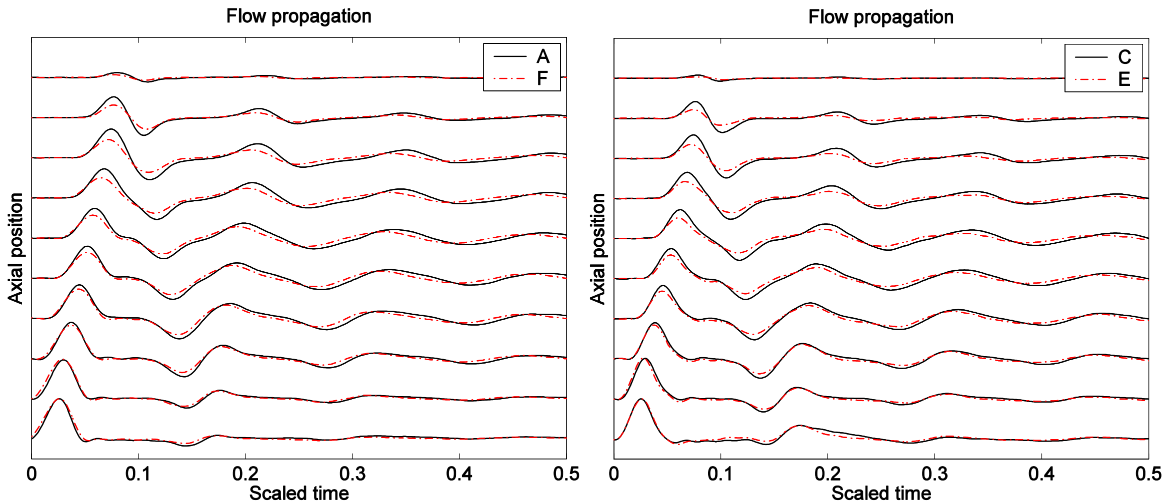


Figure 3. Measured flows for tubes A, F and C, E.

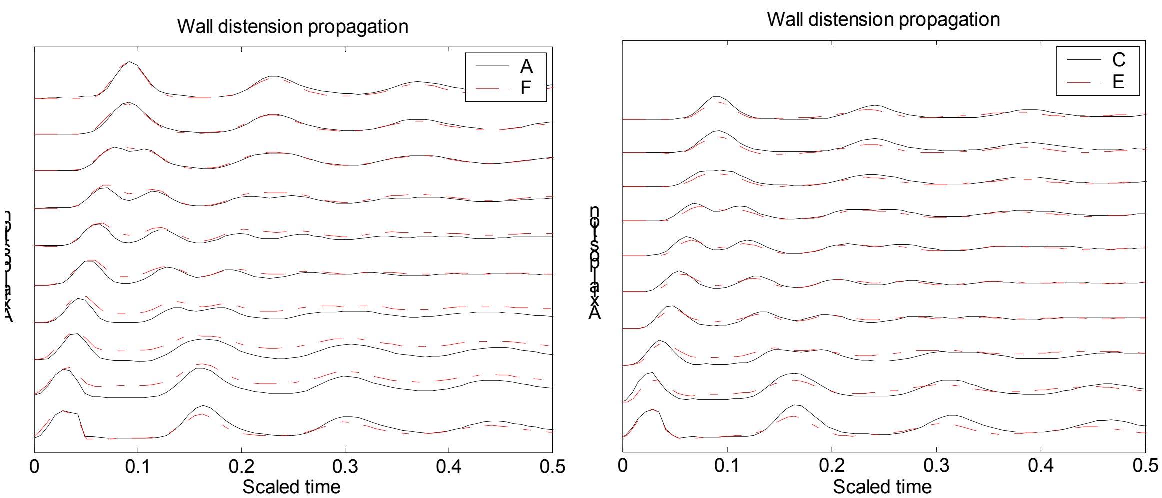


Figure 4. Measured wall distensions for tubes A, F and C, E.

The results have been scaled in amplitude so that the initial pulse has the same amplitude in all tubes. The time has also been scaled using the peak-to-peak value of the incident and reflected wave.

DISCUSSION

The experimental measurements for a straight tube with constant wall thickness are in good agreement with the linear theory for viscoelastic tubes (Fig. 1). From the comparison of the experimental data (Figs. 2-4) in the tubes A and F (and in C and E), it is concluded that, for the pressure wave, the tapering effects are strong and cannot be counterbalanced with wall thickness variation. The tapering leads to higher pressure amplitudes and changed shapes of the pressure pulse. The tapering decreases the amplitude of the associated flow wave. It also reduces the amplitude of the wall distension, which is as expected from the linear theory according to $hED=r^2p$.

The fact that the shape of the pressure pulse is changed, by the geometric tapering and to a lesser degree by the wall thickness variation, is an important observation in cardiovascular research as pressure is often used as a tool for diagnostics. This directly implies that non-linear wave propagation theory needs to be incorporated in modeling the aorta.

ACKNOWLEDGEMENTS

The financial support provided by the EPSRC under the research grant GR/N65769 and by the Marie Curie Fellowships supported by the European Commission is gratefully acknowledged.

REFERENCES

1. Pedley, T. J., 1980, *The fluid mechanics of large blood vessels*, Cambridge University Press, Cambridge, UK.
2. Wiggert, D. C., and Tijsseling, A. S., 2001, "Fluid transients and fluid-structure interaction in flexible liquid-filled piping," ASME Applied Mechanics Reviews, 54, pp. 455-481.
3. Womersley, J. R., 1957, "An elastic tube theory of pulse transmission and oscillatory flow in mammalian arteries," WADC Technical Report TR56-614.
4. Giannopapa, C. G., 2004, "Fluid-structure interaction in flexible vessels," PhD Thesis, University of London.
5. Rutten, M. C. M., 1998, "Fluid-structure interaction in large arteries," PhD Thesis, Eindhoven University of Technology.
6. Westerhof, N., Bosman, F., de Vries, C. J., and Noordergraaf, A., 1969, "Analogue studies of the human systemic arterial tree," J. of Biomechanics, 2, pp. 121-143.