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Deep Learning-based Identification of Koopman Models with Inputs

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1 Introduction

In recent years, there has been a growing interest in the development of global linear embeddings of nonlinear dynamical systems. A possible solution is given by the Koopman framework. The main idea is to lift the nonlinear system to a possibly infinite dimensional, but linear, space where the dynamics are governed by a so-called Koopman operator. In practice, only a limited number of lifting functions (called observables) can be used. However, as the choice is generally ad-hoc, there is no guarantee on the approximation capability. Furthermore, in its original formulation, the Koopman framework only addresses autonomous systems. In the present work, we aim to address these shortcomings.

2 Proposed method

To learn the lifting from data, we employ deep Artificial Neural Networks (deep-ANNs) and develop an encoder based on the concept of reconstructability [1]. The model structure is selected as control affine, leading to a linear form, with a varying input matrix.

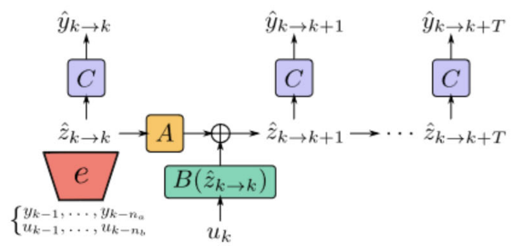

Figure 1: Network structure

The data generating system is considered to have a nonlinear control affine representation:

$$x_{k+1} = f(x_k) + g(x_k)u_k, \qquad y_k = h(x_k) + v_k, \tag{1}$$

where f, g, h are nonlinear maps, $x_k \in \mathbb{R}^{n_x}$ is the state, $u_k \in \mathbb{R}^{n_u}$ is the input, $y_k \in \mathbb{R}^{n_y}$ is the output and $v_k \in \mathbb{R}^{n_y}$ is an additive zero-mean noise. The selected model structure is defined next:

$$\hat{z}_{k+1} = A_\theta \hat{z}_k + B_\theta(\hat{z}_k)u_k, \qquad \hat{y}_k = C_\theta \hat{z}_k, \tag{2}$$

with A being the Koopman matrix, $B(z)$ is a nonlinear function of the lifted state $z \in \mathbb{R}^{n_z}$ and C denotes the linear output map. The control affine model structure is chosen as it offers more flexibility in the learning process, providing a better approximation capability than using a constant input matrix [1]. The subscript θ denotes the parameters (weights and biases) of the ANN. The initial lift to $\hat{z}_{k\to k}$ is achieved using the encoder function e_θ:

$$\hat{z}_{k\to k} := e_\theta(y_{k-n_a:k-1}, u_{k-n_b:k-1}), \tag{3}$$

where e_θ is a combination of the inverse of the constructability map and a set of nonlinear constraints. The indentification approach assumes an Output Error (OE) noise model structure and uses a T-step ahead prediction based identification criterion. The parameters θ are tuned using batch optimization, allowing for the parallelization of the computations.

3 Example

To illustrate the capabilities of the proposed method, we use the Silverbox benchmark [2], which is an electrical implementation of a mass-spring-damper system with a cubic spring nonlinearity, showing a similar behaviour to a forced Duffing oscillator. As can be seen in Fig. 2, when a multisine test input is applied, the identified model can accurately represent the dynamics of the true system. When a linearly increasing filtered Gaussian (arrowhead) input is applied, the error increases towards the end, due to the extrapolation region (where data was not available for training). If this region is discarded, the obtained results are comparable to the state of the art.

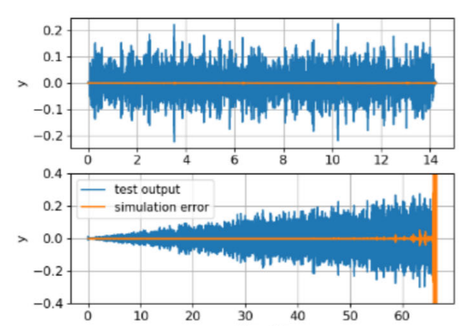

Figure 2: Multisine test (top) and arrowhead (bottom)

References

[1] L. C. Iacob, G. I. Beintema, M. Schoukens and R. Tóth, *Deep Identification of Nonlinear Systems in Koopman Form*, Conference on Decision and Control, 2021

[2] T. Wigren and J. Schoukens, *Three free data sets for development and benchmarking in nonlinear system identification*, European Control Conference, 2013

This work has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreement nr. 714663).