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Quad Active-Bridge Single-Stage Bidirectional Three-phase AC-DC Converter with Isolation: Introduction and Optimized Modulation

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Abstract—This paper introduces the quad active-bridge (QAB) ac-dc converter, which provides isolation and bidirectional power flow in a single-stage topology. While Matrix-based topologies commonly use bipolar voltage switches, the proposed solution allows the use of unipolar voltage switches. Compared to a six-switch PFC, the average voltage stress is reduced by a factor two. A modulation scheme is derived for this converter, that is analytically optimized towards minimum conduction losses to improve the conversion efficiency. The modulation scheme consists of closed-form algebraic solutions such that no look-up tables are required for implementation. Considerations for converter design and the main component selection procedures are given. The proposed topology and modulation strategy is implemented in a 20 kW prototype. Measurements are conducted and show that the methods operate as expected. Experimental results show that the load step is stable, the power factor is close to unity, and the phase current THD is than 5%. Lastly, it is shown that the circulating current can be controlled independent of the output power. The methods and tools provided can be used for the design and analysis of ac-dc power converters based on the QAB-topology.

Index Terms—power conversion, ac-dc converters, three-phase electric power

I. INTRODUCTION

HIGH-performance ac-dc power converters are required in a broad range of applications to allow, for example, electric cars to be recharged, solar panels and wind turbines to deliver energy to the grid, and high-voltage dc grids to operate. Various topologies exist for these applications [1]–[7]. In some cases, for example in a battery charger for an electric car, the power can even flow in both directions: charging using solar energy during daytime and powering the household during the night, when the solar panels are inactive. To ensure safety and low leakage currents, galvanic isolation between energy sources is commonly desired, but often is not implemented due to the additional cost and complexity.

The quad active-bridge (QAB) converter for three-phase ac-dc conversion allows bi-directional power flow and galvanic isolation [8]. No intermediate high-voltage bus is used, and no bipolar voltage switches are required. Furthermore, the average voltage stress is a factor two lower when compared to a regular six-switch inverter. This makes the converter highly competitive for applications where high-power, compact, and low-cost solutions are desired. While the existing phase-shift modulation scheme does result in soft-switching operation, the circulating currents can be high and significantly limit the conversion efficiency.

This paper describes the power-balance control method for the QAB topology. This method employs an algebraically optimized modulation scheme to minimize the circulating current. Moreover, it is made possible to scale the circulating current independent from the AB output currents to, for instance, actively control the thermal stress of active and passive components.

Outline

This paper focuses on introducing the quad active-bridge ac-dc converter topology, proposes a modulation strategy and contains experimental verification. The paper is structured as follows. Firstly, the topology is introduced and the neutral-voltage lift concept is described. Then the modeling technique and the modulation strategy are discussed, followed by the simulation and measurement results. Lastly, the conclusions are presented.

II. INTRODUCING THE QAB TOPOLOGY

The proposed QAB-topology is shown in Fig. 1. The full bridges on the mains side are connected to the primary side of transformers $T_R$, $T_S$ and $T_T$ (see Fig. 1). The secondary terminals of the transformers are series-connected. The voltages are therefore summed on the secondary side. An inductor, $L_{NVL}$, is used to determine the maximum power flow and the amount of circulating current, similar to the situation in a DAB topology [9]–[17]. In this analysis, the transformers are assumed to have 1:1 winding ratio. As can be seen, a capacitor $C_{NVL}$ is added to the neutral line. The function of this capacitor is to create an offset in the neutral, hence the name: neutral-voltage lift capacitor.

Due to the fact that in a balanced three-phase system ($i_{CM}(t) = 0$), the current through capacitor $C_{NVL}$ is zero as well, and its voltage remains constant. This voltage, $v_{NVL}(t)$, can therefore be controlled by the common-mode current through all three phases, defined by

$$i_{CM}(t) = -i_{C,NVL}(t) = ig(t) + is(t) + ir(t).$$

By ensuring that $v_{NVL}(t) > -\min(v_{NVL}(t))$ with $n \in \{R,S,T\}$ always holds, the voltages across all switching legs are positive, as shown in Fig. 2. That is to say
A. Neutral-voltage lift

To implement NVL, some points require attention, which are the power-up sequence and in-rush current limiting of the converter, as well as the control of \( V_{\text{NVL}} \). When the converter is powered on, a pre-charge period is required in order to charge the capacitor \( C_{\text{NVL}} \) and to prevent high in-rush currents. The pre-charging is achieved by temporarily placing a (PTC) resistor in series with the mains connections, as shown in Fig. 4. As can be seen, the capacitor is then charged via the semiconductor body diodes and these resistors. After the capacitor voltage has reached a sufficiently high value, the pre-charge resistors are bypassed by switches that are connected in parallel to these resistors. Then, the NVL voltage control loop is activated in order to further charge the capacitor to its desired value. This process is shown in Fig. 5. Here, \( t_1 \) indicates the moment where the converter is connected to the mains grid, \( t_2 \) indicates the moment where the voltage approaches the mains peak voltage \( v_{\text{mains}} \), and the bypass relays are closed. Then, the control loops are activated to further charge the capacitor towards the desired voltage \( V_{\text{NVL,ref}} \). Finally, \( t_3 \) indicates the moment where the converter starts operating normally. From this point onwards, the common-
to stabilize its voltage and can provide sufficient disturbance rejection. Secondly, the magnitude of the disturbances, that are present in the capacitor current, are dependent on the disturbance rejection of the converter phase current controllers. An estimation of the capacitor value can be done as follows. By knowing the disturbance rejection \( S_i(j\omega) \) of the converter current control loop at the mains frequency \( \omega_{\text{mains}} \), and the RMS phase current \( I_i \), an estimation of the disturbance current \( I_{\text{CM,d}} \) can be made by

\[
I_{\text{CM,d}} = 3I_i S_i(j\omega_{\text{mains}}). \tag{2}
\]

Then, by defining a desired voltage swing on the capacitor \( V_{\text{NVL,d}} \) and a given disturbance rejection \( S_{\text{NVL}(s)} \) of the capacitor voltage control loop, the capacitor value can be found by

\[
C_{\text{NVL}} = \frac{I_{\text{CM,d}} S_{\text{NVL}}(j\omega_{\text{mains}})}{\omega_{\text{mains}} V_{\text{NVL,d}}}. \tag{3}
\]

For instance, with \( S_i(j\omega_{\text{mains}}) = -30 \) dB disturbance rejection of \( I_i = 30 \) A RMS at 50 Hz mains frequency, an allowed voltage swing of \( V_{\text{NVL,d}} = 10 \) V RMS, and \( S_{\text{NVL}}(j\omega_{\text{mains}}) = -20 \) dB disturbance rejection of the capacitor voltage control loop, the capacitor value must be approximately

\[
C_{\text{NVL}} \approx 88 \ \mu\text{F}. \tag{4}
\]

### III. Converter Fourier-series model

This section introduces a modeling method for the QAB converter. For this purpose, a voltage source model is derived as follows. The ABs are all reflected to the primary side, and the secondary sides of all transformers are connected in series. The resulting schematic is shown in III. It must be noted that reflecting a source from the secondary side to the primary side equals reversing its polarity. In the following, the parameter \( i \) indicates the number of ABs, dictated by the QAB-converter topology to be equal to \( i = 4 \). The parameters \( n \) and \( m \) refer to an arbitrary AB in the topology, therefore \( n, m \in \{R, S, T, dc\} \).

In this model, each AB generates a voltage \( u_i \) across a transformer \( T_n \). The transformer primary side voltage equals the

---

**Fig. 3.** Proposed control layout, consisting of a PLL, a current controller \( \Gamma_i \), a system decoupler, a common-mode current controller \( \Gamma_{\text{NVL}} \), and a pulse-width modulator.

**Fig. 4.** Pre-charging of NVL capacitor via resistors. The red line indicates one of the possible current paths. When pre-charging has completed, the switches are closed to bypass the resistors.

**Fig. 5.** Waveforms while pre-charging the NVL capacitor.
secondary side voltage because all transformers are assumed to be ideal, which means there is no magnetizing inductance and no leakage inductance, and the winding ratio is 1:1. A separate inductor \( L_{\sigma} \) is added that represents the combined leakage inductance of all transformers as follows

\[
L_{\sigma} = \sum_{n=1}^{i} L_{n},
\]

where \( L_{n} \) is the supply of the AB. Finally, all transformer voltages \( u_{n} \) can be represented by voltage waveforms produced by voltage sources, as shown in III.

The block-shaped waveforms \( u_{n} \) produced by the ABs have an amplitude, a duty cycle, a period time and a phase shift, as shown in Fig. 7(a). There, the gate signals of switches \( S_{nn} \) are indicated by black bars. As suggested in the figure, it holds that

\[
u_{n}(t) = u_{n1}(t) - u_{n3}(t),
\]

where \( u_{n}(t) \) is the voltage across the transformer \( T_{n} \) secondary side, and \( v_{n} \) the supply of the AB \( n \) and thereby defines the amplitude of the voltage over the transformer \( u_{n}(t) \). The waveforms are transformed to Fourier-series, which are subsequently used to derive the current through the inductor \( L_{\sigma} \) in the circuit. Finally, the output powers of each AB can be found by using the Fourier-series of the current and voltages, resulting in an analytical converter model.

It is assumed that the switching frequency of the ABs, \( \omega_{sw} \), is much higher than the frequencies present in the supply voltage of the AB, \( v_{n}(t) \), such that may be assumed that

\[
v_{n}(t) = v_{n}(t + T_{sw})
\]

with \( T_{sw} = \frac{2\pi}{\omega_{sw}} \). Variables that use the above assumption are indicated by the ‘-symbol.

In view of the quantities in Fig. 7(a), the following arrays of variables are defined:

\[
\mathbf{u}'(t) = \begin{bmatrix} u'_{n1}(t) \\ u'_{n3}(t) \\ u'_{n4}(t) \end{bmatrix}, \quad \mathbf{v}' = \begin{bmatrix} v'_{R} \\ v'_{S} \\ v'_{T} \end{bmatrix}, \quad \mathbf{d} = \begin{bmatrix} d_{R} \\ d_{S} \\ d_{T} \end{bmatrix}, \quad \mathbf{\Phi} = \begin{bmatrix} \phi_{R} \\ \phi_{S} \\ \phi_{T} \end{bmatrix}, \quad \mathbf{\Phi} = \begin{bmatrix} \phi_{dc} \end{bmatrix}
\]

where \( \mathbf{u}'(t) \) contains the set of block-shaped voltages \( u'_{n}(t) \) at the secondary side of the high-frequency transformers.

The set of active-bridge bus voltages is combined in \( \mathbf{v}' \). The set of phase-shifts between the full-bridge switching legs is represented by \( \mathbf{\Phi} \), and \( \mathbf{d} \) is the set of duty-cycles for \( u'_{n}(t) \).

Then, the function \( \mathbf{u}'(t) \) can be written in the orthogonal Fourier-series representation

\[
\mathbf{u}'(t) = \sum_{k=1}^{\infty} \mathbf{u}a'(k) \cos((2k - 1)\omega_{sw}t) + \mathbf{u}b'(k) \sin((2k - 1)\omega_{sw}t)
\]

with

\[
\mathbf{u}a'(k) = \mathbf{U}'(k) \left( \sin((2k - 1)(\Phi + 0.5\pi d)) \right),
\]

\[
\mathbf{u}b'(k) = \mathbf{U}'(k) \left( \cos((2k - 1)(\Phi + 0.5\pi d)) \right)
\]

and \( \mathbf{U}'(k) = \frac{\mathbf{U}}{N_{sw}/2 \pi} \). The current contribution through \( L_{\sigma} \) of source voltage \( u'_{n}(t) \) is equal to the integral of the sum of the inductor voltage divided by the reactance of the inductor, being

\[
I'_{\sigma}(t) = \int \frac{\mathbf{u}'(t) dt}{L_{\sigma}} = \sum_{k=1}^{\infty} -\mathbf{u}b'(k) \cos((2k - 1)\omega_{sw}t)
\]

\[
\frac{(2k - 1)\omega_{sw}L_{\sigma}}{(2k - 1)\omega_{sw}L_{\sigma}}
\]
The AB input current can be found by
\[ I_\text{in}'(t) = \sum_{k=1}^{\infty} I_k' \cos((2k-1)\omega_m t) + I_{\text{rb}}(k) \sin((2k-1)\omega_m t) \] (12)

Hence, the total inductor current is equal to the superposition of all currents, and therefore found to be
\[ I'(t) = \sum_{k=1}^{\infty} I'_k \cos((2k-1)\omega_m t) \] (13)

with the Fourier-coefficients
\[ I'_k = \frac{1}{(2k-1)\omega_m L_r} \sum_{n=1}^{\infty} U_{n,k} \cos((2k-1)\omega_m t), \quad I'_\text{rb}(k) = \frac{1}{(2k-1)\omega_m L_r} \sum_{n=1}^{\infty} U_{n,k} \sin((2k-1)\omega_m t). \] (14)

To obtain a relatively simple expression that can be used for optimization purposes, only the first harmonic is used (\(k_{\text{max}} = 1\)). By doing this, the coefficients become
\[ U_{k} = 0.5 U_k \sin(\phi_k - 0.5 \pi d) - U_k \sin(\phi_k - 0.5 \pi d) \]
\[ = 2 U' \cos(\phi_k) \sin(0.5 \pi d) \] (15)
\[ U_{k} = 0.5 U_k \cos(\phi_k - 0.5 \pi d) \]
\[ = -2 U' \sin(\phi_k) \sin(0.5 \pi d). \] (16)

with \( U' = 2 \sqrt{\pi} \). This is an approximation, similarly to [11], [12]. Now, by defining
\[ X = \sin(0.5 \pi d) \] (17)

the power of source \( n \) when using only the first harmonic is found by
\[ P'_n = 0.5 I'_{\text{in}}(1) U_n(1) + 0.5 I'_{\text{rb}}(1) U_n(1) \]
\[ = \frac{1}{\omega_m L_r} \sqrt{\frac{1}{2}} v_n X_0 \left( v_n X_1 \sin(\phi_1 - \phi_0) + \ldots + v_n X_i \sin(\phi_i - \phi_0) \right) \] (18)

and the current through \( L_r \) is found by
\[ I' = \sqrt{0.5 \sum_{n=1}^{\infty} \frac{4 v_n X_n \sin(\phi_n)}{\pi \omega_m L_r} + 0.5 \sum_{n=1}^{\infty} \frac{4 v_n X_n \cos(\phi_n)}{\pi \omega_m L_r} \} \] (19)

The AB input current can be found by
\[ I'_n = \frac{P'_n}{V_n}. \] (20)

When assuming ideal efficiency, the sum of the powers of ABs delivering power (\( P_{\text{pos}} \)) is equal to the sum of the powers of ABs receiving power (\( P_{\text{neg}} \)), as described by
\[ P'_{\text{pos, set}} = \sum_{n=1}^{\infty} \left\{ P'_{n, \text{set}} : P_{n, \text{set}} \geq 0 \right\} \]
\[ P'_{\text{neg, set}} = \sum_{n=1}^{\infty} \left\{ P'_{n, \text{set}} : P_{n, \text{set}} < 0 \right\} \]
\[ P'_{\text{pos, set}} = P'_{\text{neg, set}}. \] (21)

This is similar to an \( i = 2 \) situation where all positive power is processed by only one source \( U_{\text{pos}} = v_{\text{pos}} X_{\text{pos}}, \) as well as the negative power is processed by another source \( U_{\text{neg}} = v_{\text{neg}} X_{\text{neg}}. \)

The following therefore assumes a two-sources problem. Further on, the model is extended to distribute the power across all ABs of the QAB-converter.

The RMS value of the current through the leakage inductance \( L_r \) determines the resistive losses in the converter. To minimize the RMS current through the inductor, the following optimization problem should be solved:
\[ \text{minimize } I'_n \]
\[ \text{with } \left\{ P'_{\text{pos, set}} \right\} = \left\{ P'_{\text{neg, set}} \right\}. \] (24)

with \( P'_n \) the active power of AB \( n, \) resulting from the first-harmonic approximation (\( k_{\text{max}} = 1 \)) given by (18). \( P_{\text{set}} \) is the set-point of the active power of AB \( n. \) Here, the optimization algorithm selects the duty cycles \( d_n \) and phase shifts \( \phi_n \) to have the desired output power with the minimum amount of (RMS) current through the converter.

It is assumed that the first harmonic is dominant in causing circulating currents. The cost function \( W \) is simplified by minimizing
\[ W = 2 \left( \frac{\pi \omega_m L_r I'_n}{4} \right)^2, \] (26)

which yields the same \( \phi \) and \( d \) because the parameters \( \omega_m \) and \( L_r \) are both kept constant in the optimization procedure. By substitution of (19) into (26), the cost function \( W \) is found to be
\[ W = \left( \sum_{n=1}^{\infty} v_n X_0 \sin(\phi_n) \right)^2 + \left( \sum_{n=1}^{\infty} v_n X_0 \cos(\phi_n) \right)^2. \] (27)

While the minimization of \( W \) results in the lowest circulating currents, it does not guarantee the output power equals the power set-point. This is, however, a constraint which needs to be met. Therefore
\[ \sum_{n=1}^{\infty} \left\{ P'_n - P'_{n, \text{set}} \right\}^2 = 0, \] (28)

to ensure all outputs have the desired power. (28) is valid when \( |\phi_m - \phi_n| \ll \pi/2, \) such that \( \sin(\phi_m - \phi_n) \approx \phi_m - \phi_n \) \( \forall \) \( m, n \) where \( \{ n, m \} \in \{ n \leq i, m \leq j \} \) holds, as this linearisation is used to calculate the phase shifts. It can be shown that a strong
approximation of the criterion in (28) can be accomplished when
\[ (\phi_{\text{pos}} - \phi_{\text{neg}})^2 = \alpha^2, \quad (29) \]
where \( \alpha \) is a parameter introduced to fix the maximum cross-
difference between the phase shifts, to provide a trade-off between model accuracy (linearisation error) and the amount of circulating current. The parameter should be in the linear range of the model, bounded to \( 0 < \alpha < \frac{\pi}{2} \). Note that, when (29) is met, the desired output powers are guaranteed to be within the linearisation errors, and the error becomes smaller when \( \alpha \) is smaller.

Using the method of Lagrange multipliers [24], the constraint (29) is taken into account together with the cost function (27) as follows:
\[
\text{minimize} \quad H = W + \lambda \left( (\phi_{\text{pos}} - \phi_{\text{neg}})^2 - \alpha^2 \right). \quad (30)
\]

Now, the optimum is found when for each parameter \((X_n, \phi_n)\) the derivatives of \( H \) are equal to zero.

First, the cost function (27) can be simplified by using the approximation
\[
W = \left( \sum_{n=1}^{5} v_n^2 X_n \sin(\phi_n) \right)^2 + \left( \sum_{n=1}^{5} v_n^2 X_n \cos(\phi_n) \right)^2 
= (v_1^2 X_1)^2 + (v_2^2 X_2)^2 + 2(v_1^2 X_1)(v_2^2 X_2) \cos(\phi_1 - \phi_2) \quad (31)
\approx U_{\text{pos}}^2 + U_{\text{neg}}^2,
\]
which is valid because \( \cos(\phi_{\text{pos}} - \phi_{\text{neg}}) \to 0 \) as the phase shift is selected to be large to optimize efficiency, as described in the next paragraph.

Newton’s method is used to find the optimum for \( i = 2 \), where each term of the Jacobian of \( H \) is set to zero. The solution is obtained from
\[
\text{minimize} \quad H = (U'_{\text{pos}})^2 + (U'_{\text{neg}})^2 + \lambda \left( (\phi_1 - \phi_2)^2 - \alpha^2 \right) \quad (32)
\]
After substitution of \( \phi_n \) from (18) (similar to phase-shift control from [12]) and assuming \( \phi_1 = 0 \) yields the following optimization assignment:
\[
\text{minimize} \quad H = (U'_{\text{pos}})^2 + (U'_{\text{neg}})^2 \quad (33)
\]
\[
+ \lambda \left( \arcsin \left( \frac{\omega_{\text{Lcr}} \pi^2}{8(U'_{\text{pos}} + U'_{\text{neg}})} \right) \left( \frac{P'_{\text{pos}}}{U'_{\text{pos}}} - \frac{P'_{\text{neg}}}{U'_{\text{neg}}} \right)^2 - \alpha^2 \right). \quad (34)
\]
By power conservation it holds that \( P_2 = -P_1 \), such that this equation can be rewritten into
\[
\text{minimize} \quad H = (U'_{\text{pos}})^2 + (U'_{\text{neg}})^2 \quad (35)
\]
\[
+ \lambda \left( \arcsin \left( \frac{\omega_{\text{Lcr}} \pi^2}{8(U'_{\text{pos}} + U'_{\text{neg}})} \right) \left( \frac{P'_{\text{pos}}}{U'_{\text{pos}}} + \frac{P'_{\text{neg}}}{U'_{\text{neg}}} \right)^2 - \alpha^2 \right). \quad (36)
\]
This optimization is carried out by equating the Jacobian of \( H \) to zero, which is equal to
\[
J = \begin{pmatrix}
2U'_{\text{pos}} \\
2U'_{\text{neg}}
\end{pmatrix}
- \begin{pmatrix}
\frac{2\omega_{\text{Lcr}} \sin(\phi_{\text{pos}})}{\sqrt{1 - \left( \frac{\omega_{\text{Lcr}} \pi^2}{8U'_{\text{pos}} + U'_{\text{neg}}} \right)^2}} \\
\frac{2\omega_{\text{Lcr}} \sin(\phi_{\text{neg}})}{\sqrt{1 - \left( \frac{\omega_{\text{Lcr}} \pi^2}{8U'_{\text{pos}} + U'_{\text{neg}}} \right)^2}}
\end{pmatrix}
\]
\[
\text{arcsin}^2 \left( \frac{\sigma P'_{\text{pos}}}{U'_{\text{pos}}(U'_{\text{pos}} + U'_{\text{neg}})} - \alpha^2 \right)
\]
with
\[
J = \begin{pmatrix}
\frac{\partial H}{\partial (U'_{\text{pos}})} \\
\frac{\partial H}{\partial (U'_{\text{neg}})}
\end{pmatrix}, \quad \sigma = \frac{\omega_{\text{Lcr}} \pi^2}{8}. \quad (37)
\]
Solving \( J = 0 \) for \( U'_{\text{pos}} \) and \( U'_{\text{neg}} \) with \( \alpha = \pi/2 \) results in
\[
U'_{\text{pos}} = v'_{\text{pos}} X_{\text{pos}} = \frac{\pi \sqrt{P'_{\text{pos,set}} \omega_{\text{Lcr}} L_{\text{cr}}}}{2 \sqrt{2}} \quad (38)
\]
\[
U'_{\text{neg}} = v'_{\text{neg}} X_{\text{neg}} = \frac{\pi \sqrt{P'_{\text{neg,set}} \omega_{\text{Lcr}} L_{\text{cr}}}}{2 \sqrt{2}} \quad (39)
\]
with \( P'_{\text{pos,set}} \) the sum of the power set-points of all positive sources, and \( v'_{\text{pos}} X_{\text{pos}} \) the sum of all phasor lengths of all positive sources. Furthermore, because it holds that \( P'_{\text{pos,set}} = P'_{\text{neg,set}} \), the phasor lengths are equal, being
\[
U'_{\text{neg}} = U'_{\text{pos}}. \quad (40)
\]
with \( \alpha = \pi/2 \). Then, calculating the elements of \( d \) and \( \phi \) using (17) and (18) and substitution with \( \alpha = \pi/2 \), assuming \( P'_{\text{pos}} = -P'_{\text{neg}} \geq 0 \) yields, with some manipulations,
\[
\phi_{\text{pos}} = -0.5\alpha = -\pi/4 \quad (41)
\]
\[
\phi_{\text{neg}} = 0.5\alpha = \pi/4, \quad (42)
\]
which meets the constraint \( (\phi_{\text{pos}} - \phi_{\text{neg}})^2 = \alpha^2 \). It must be noted that \( U'_{\text{pos}} = U'_{\text{neg}} \), because the corresponding powers are equal, and so from (39) the maximum output power is achieved when \( \max (X_{\text{pos}}, X_{\text{neg}}) = 1 \).

The two-AB solution is based on a linearized model which is valid as long as \( \sin(\phi_{\text{pos}} - \phi_{\text{neg}}) = \phi_{\text{pos}} - \phi_{\text{neg}} \). This solution can be improved significantly when a description is used which has a broader validity range, as shown in this section. Also, the model is adapted such that it can be applied to the QAB-converter. Furthermore, control over \( U'_{\text{c}} \) is implemented, which allows to manipulate the losses without changing the output powers of the ABSs.

As shown graphically in Fig. 8, for a given \( P'_{\text{pos,set}} \), the following parameters can be defined:
\[
a = U'_{\text{pos}} \sin \left( \frac{\alpha}{2} \right) \quad a^0 = U'_{\text{pos}} \sin \left( \frac{\alpha}{2} \right) \quad (43)
\]
\[
b = U'_{\text{pos}} \cos \left( \frac{\alpha}{2} \right) \quad b^0 = U'_{\text{pos}} \cos \left( \frac{\alpha}{2} \right)
\]
are distributed as follows

\[
P \propto \cos(\alpha)
\]

Thus, the value of \(\alpha\) determines the current through the inductor \(L\), as shown in Fig. 8. Minimizing the voltage equals minimizing the current.

\[
V' = 2b^2\n\]

The value of \(b^2\) determines the voltage across the inductor \(L\), by

\[
V = 2b^2\n\]

and therefore determines the current through the converter. Minimizing \(b^2\) is accomplished by maximizing \(\alpha^2\), given that \(0 \leq \alpha^2 \leq \pi\), while maintaining \(ab = a'b^2\), because the product \(ab\) is proportional to \(P_{n,\text{set}}\) (see (45) below). Therefore, minimization is accomplished by maximizing \(U_{\text{pos}}^{\sigma}\).

To distribute the power across all \(i\) sources proportional to the set-points, the total phasor lengths \(v_{\text{pos}}X_{\text{pos}}\) and \(v_{\text{neg}}X_{\text{neg}}\) are distributed as follows

\[
v_{\alpha}X_{\alpha} = \frac{U_{\text{pos}}'P_{\text{pos,\text{set}}}}{P_{\text{pos,\text{set}}}} \quad (45)
\]

In order to increase the magnitudes of \(U_{\text{pos}}^{\sigma}\) and \(U_{\text{neg}}^{\sigma}\), aiming at reducing the inductor voltage and current, the magnitudes are scaled by

\[
X = \beta X \quad (46)
\]

where \(\beta\) is the scaling factor. The maximum is achieved when one of the duty cycles reaches its limit, at \(max(X) = 1\). The maximum gain is then equal to

\[
\beta_{\text{max}} = \frac{1}{max(X)} \quad (47)
\]

For fixed power \(P_{\text{pos,\text{set}}}'\), it holds that

\[
ab = a'b^2 \quad (48)
\]

thus the value of \(a^2\) is found by

\[
U_{\text{pos}}^2 \sin\left(\frac{\alpha}{2}\right) \cos\left(\frac{\alpha}{2}\right) = U_{\text{pos}}^2 \sin\left(\frac{a^2}{2}\right) \cos\left(\frac{a^2}{2}\right) \quad (49)
\]

With \(\alpha = \pi/2\) it results that, to achieve the minimum voltage across the inductor,

\[
\sin(\alpha^2) = \frac{U_{\text{pos}}^2}{U_{\text{pos}}^2} = \frac{1}{\beta^2} \quad (50)
\]

\[
\alpha_{\text{VL,min}} = \frac{\pi}{2} - \arcsin\left(\frac{1}{\beta_{\text{max}}^2}\right) \quad (51)
\]

and to achieve the maximum voltage across the inductor

\[
\alpha_{\text{VL,max}} = \arcsin\left(\frac{1}{\beta_{\text{max}}^2}\right) \quad (52)
\]

Using (39) it is found that

\[
b_{\text{VL,min}} = U_{\text{pos}}'\beta_{\text{max}} \cos\left(\frac{1}{2}\alpha_{\text{VL,min}}\right) = U_{\text{pos}}'\beta_{\text{max}} \cos\left(\frac{1}{2}\pi - \arcsin\left(\frac{1}{\beta_{\text{max}}^2}\right)\right) \quad (53)
\]

\[
b_{\text{VL,max}} = U_{\text{pos}}'\beta_{\text{max}} \cos\left(\frac{1}{2}\alpha_{\text{VL,max}}\right) = U_{\text{pos}}'\beta_{\text{max}} \cos\left(\frac{1}{2}\arcsin\left(\frac{1}{\beta_{\text{max}}^2}\right)\right) \quad (54)
\]

The converter current can be regulated linearly by changing the voltage across inductor \(L\) between \(b_{\text{VL,min}}^\circ\) and \(b_{\text{VL,max}}^\circ\) by the parameter \(\gamma\), where

\[
b = (1 - \gamma)b_{\text{VL,max}}^\circ + \gamma b_{\text{VL,min}}^\circ \quad (55)
\]

The lowest current is achieved when \(\gamma = 1\), because in that case the voltage over the inductor is the lowest. Then, by (48) and because \(\alpha = \pi/2\), yields

\[
\alpha = \frac{ab}{b^2} = \frac{0.5U_{\text{pos}}^2}{P_{\text{pos,\text{set}},\text{set}}} \quad (56)
\]

\[
\alpha = 2 \arctan\left(\frac{a}{b}\right) \quad (57)
\]

\[
\beta = \frac{1}{\sqrt{\sin(\alpha^2)}} \quad (58)
\]

The phase shifts for all sources are then defined by

\[
\phi_{\text{pos}} = -0.5\alpha^2 \quad (59)
\]

\[
\phi_{\text{neg}} = 0.5\alpha^2 \quad (59)
\]

where \(\phi_{\text{pos}}\) the phase shift assigned to sources processing positive power, and \(\phi_{\text{neg}}\) assigned to sources processing negative power. Consequently, the phase shifts of the ABs are found by

\[
\phi_0 - \phi_1 = \begin{cases} 0 & \text{if } P_{\text{pos,\text{set}},\text{set}} \geq 0 \\ \alpha^2 & \text{otherwise.} \end{cases} \quad (60)
\]

Moreover, by combining (17) and (46) it can be seen that the duty cycle then equals

\[
d = \frac{2}{\pi} \arcsin(X)^\circ \quad (61)
\]

The analysis is based on a first-harmonic approximation. This results in inaccuracies in the analysis, especially when higher harmonics start to play a dominant role in the waveforms that are produced. These higher harmonics become dominant when the maximum gain \(\beta_{\text{max}}\) becomes large, for instance when little current is required by a certain AB, while its bus voltage \(v\) is large. The algorithm will then select a too high duty cycle, which causes unnecessary currents. To alleviate this issue, the maximum gain can be limited by

\[
\beta_{\text{max}} = \frac{\beta_{\text{max,limit}}}{\beta_{\text{max}}} \quad (62)
\]
By empirical evaluation of the circulating current in a large range of scenarios, it is found that $\beta_{\text{max,lim}} = 2.7$ will typically result in minimum circulating current over the full load range. Therefore, this value is used in the evaluation in this paper.

V. COMPONENT DIMENSIONING

Designing the QAB-converter involves selecting the ratings and values of its components. The most important parameters to assist the selection procedure for the main circuit components are given in the following.

The voltage $v_{\text{NVL}}$ can be said to equal

$$v_{\text{NVL}} = 1.15v_{\text{in}}.$$  

(such that sufficient voltage margin is present even when small disturbances occur in the mains grid. Moreover, the value of the capacitor $C_{\text{NVL}}$ is found using (3).

The value of the inductance $L_r$ is determined by (39) such that

$$L_r = \frac{8\left(\tilde{v}_n + \tilde{v}_{\text{NVL}}\right)}{\pi^2 \omega_m i_{\text{pos}}}.$$  

This value is the upper limit, and it should be taken into account that the first-harmonic approximation has inaccuracies, and that a safety margin must be taken into account if the converter will be operated at the currents that are used in the above calculation. Generally, it will be sufficient to take 25% power reserve into account to compensate these two issues, which means that $L_r$ must be reduced by the same amount.

The peak current of this inductor can then be estimated by using (19). The maximum current is reached for a phase shift range of

$$\frac{\pi}{2}.$$  

Then, by using (39) the inductor voltage is found to be

$$V_{\text{dc}} = \frac{2}{\sqrt{2}} U_{\text{pos}} = \frac{\pi}{2} \sqrt{P_{\text{pos,sett}} \omega_m L_r},$$  

with

$$P_{\text{pos,sett}} = \left(\tilde{v}_{\text{NVL}} + \tilde{v}_{\text{n}}\right) i_{\text{mains}},$$  

given that $n \in [R, S, T]$, resulting in

$$i_r = \frac{\pi \sqrt{V_{\text{dc}}^2 + \tilde{v}_{\text{n}}^2}}{2 \sqrt{\omega_m L_r}}.$$  

Moreover, the RMS current is equal to

$$I_n = \frac{\tilde{v}_n}{\sqrt{2}}.$$  

The current $i_r$ flows through all semiconductors and magnetic components, due to the fact that all these components are connected in series. This current therefore also defines the current rating of these parts.

The peak voltages of the semiconductors on the mains side and dc side are equal to

$$\tilde{v}_n = \begin{cases} v_{\text{NVL}} + \tilde{v}_{\text{n}} & \text{for } n \in [R, S, T] \\ v_{\text{dc}} & \text{for } n \in \{\text{dc}\}. \end{cases}$$  

The average semiconductor voltage, on the other hand, is defined by

$$\langle v_n \rangle = \begin{cases} \langle v_{\text{NVL}} \rangle(t) & \text{for } n \in [R, S, T] \\ v_{\text{dc}} & \text{for } n \in \{\text{dc}\}. \end{cases}$$

The switching frequency $\omega_m$ must be established to the find results of the above equations. Because many a thing determine its optimal value, one can do an optimization which uses, for instance, the resulting converter losses and cost. This optimization procedure is, however, not described in this paper.

The analysis that is carried out assumes that the transformers are ideal and have a 1:1 winding ratio. This means, however, that the output voltage must be in a certain range for the converter to perform properly. If a higher or lower output voltage is desired, the winding ratios of the transformers should be changed. To add the transformer winding ratio, $n_{\text{ratio}}$, the values for analysis are found according to

$$u_{\text{dc}} = u_{\text{dc},s}, \quad v_{\text{dc}} = v_{\text{dc},s}, \quad i_{\text{dc}} = n_{\text{ratio}} i_{\text{dc},s}, \quad i_r = \frac{i_r, s}{n_{\text{ratio}}},$$  

$$L_{\text{sr}}, s = n_{\text{ratio}}^2 L_r.$$  

where $u_{\text{dc},s}$, $v_{\text{dc},s}$, $i_{\text{dc},s}$, and $L_{\text{sr}, s}$ are the secondary-side values. The transformer winding ratio can be used to minimize circulating currents in a three-phase application. To achieve maximum utilization of the available voltage, hence minimize the current required to transfer a certain power, unity duty cycle must be obtained, and it follows from (45) that therefore

$$X_{\text{dc}} = 1 = \frac{U_{\text{pos}} P_{\text{dc}}}{P_{\text{pos}} v_{\text{dc}}}.$$  

where

$$U_{\text{pos}} = \frac{\pi}{2} \sqrt{P_{\text{pos,sett}} \omega_m L_r}$$

which, in the case that one of the three mains phase voltages are maximum and by using (64), equals

$$U_{\text{pos}} = \frac{\pi}{2} \sqrt{\left(\tilde{v}_{\text{n}} + \tilde{v}_{\text{NVL}}\right)^2 \omega_m \left(\tilde{v}_{\text{NVL}} + \tilde{v}_{\text{n}}\right)}$$  

such that, when using power conservation according to

$$P_{\text{dc}} = \frac{3\tilde{v}_{\text{n}} v_{\text{dc}}}{2},$$

it follows that

$$\frac{3\tilde{v}_{\text{n}} v_{\text{dc}}}{2} = \frac{3 \tilde{v}_{\text{n}} n_{\text{ratio}}}{2 v_{\text{dc},s} \tilde{v}_{\text{n}}} = 1$$

$$n_{\text{ratio}} = \frac{2 v_{\text{dc},s}}{3 \tilde{v}_{\text{n}}},$$

This ratio is the number of turns on the secondary side divided by the number of turns on the primary side of each transformer.

Remark that, in practical applications, and in the analysis that is used, the inductor $L_r$ is located on the secondary side. Hence, when $n_{\text{ratio}} \neq 1$, $L_{\text{sr}, s}$ must be used. Moreover, in
TABLE I
SIMULATION AND MEASUREMENT PARAMETERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
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<td>$\omega_{sw}$</td>
<td>2\pi 45</td>
<td>krad/s</td>
</tr>
<tr>
<td>$V_{EN}$</td>
<td>230</td>
<td>V ac RMS</td>
</tr>
<tr>
<td>$V_{dc}$</td>
<td>500</td>
<td>V</td>
</tr>
<tr>
<td>$C_{NVL}$</td>
<td>220</td>
<td>$\mu$F</td>
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<tr>
<td>$C_{NVL}$ voltage</td>
<td>375</td>
<td>V</td>
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<tr>
<td>$L_{p}$</td>
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<tr>
<td>$V_{nN}$</td>
<td>230</td>
<td>V ac RMS</td>
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<tr>
<td>$V_{dc}$</td>
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<td>28</td>
<td>$\mu$H</td>
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</tbody>
</table>

VI. RESULTS

Measurement and simulation parameters are specified in Table I. Simulations have been carried out using MATLAB and the Plexim PLECS toolbox to verify the converter operation and the proposed control method. Moreover, to verify the converter operation, measurements were carried out on a 20 kW QAB-converter prototype with SiC MOSFET modules.

For the measurements, a setup is used which consists of a power analyzer, two sources, and a controlling computer, as shown in Fig. 9. The dc-side power source comprises a load resistor and a dc power supply, such that both current sourcing and current sinking on the dc-side is possible. The ac-side power source consists of three four-quadrant power amplifiers which are combined to form a three-phase ac source. Amidst the sources and the QAB-converter prototype is a power analyzer to measure the performance of the converter. Table II shows a summary of all equipment that is used. A photo of the setup is shown in Fig. 10.

Firstly, the initialization of the neutral-voltage lift is verified and shown in Fig. 11. As can be seen, it pre-charges via the body diodes and series resistors. Then, when the gate drivers and the control loop are initialized, the voltage is lifted further, up to the working voltage. This procedure is according to Fig. 5.

Next, waveforms of the ac- and dc-side were measured at 10 kW in rectifier mode. These are shown in Fig. 12. Some distortions are present near the peaks of the current. These irregularities worsen significantly if the dead time is increased from 400 ns to 800 ns.

A load step is measured from -5 kW to +5 kW, which is

**FIG. 10.** Overview photo of prototype QAB-converter in laboratory set-up.

**FIG. 11.** Measurement of neutral-voltage lift (blue, 50 V/div) and common-mode current (yellow, 1 A/div) during the initialization phase of the converter.
can be seen, the current can be varied while maintaining the same output power. The significant difference that appears to be present between simulation and measurements results for lower values of $\gamma$ might be caused by resistive losses, semiconductor dead time effects, or errors in matching the actual magnetics with the models, for instance. For $\gamma$ close to 1 the predictions are correct, and the trend of the response is correct as well. The mismatch does not cause any other unexpected behavior, and it is therefore anticipated that it does not preclude thermal stress control to be used effectively.

Finally, some measurements are taken to show the transformer voltages $u_n$ with $n \in [R, S, T, dc]$ in both the rectifier mode and the inverter mode, as shown in Fig. 16 and Fig. 17 respectively. Here, it can be clearly seen that each transformer is modulated according to its own setpoint, and that the reactive currents are low, as governed by the PBC method. Moreover, it can be seen that almost no ringing and other parasitic effects are present.

Fig. 14. Power factor of QAB-converter prototype operating as rectifier (black) and inverter (grey dashed).

Then, at over 14 kW, which is the maximum output power of the ac-source, detailed information of the input and output is given in Table III. As can be seen, the power factor is high, and the current waveforms have a relatively low total harmonic distortion.

The power factor has been measured in both inverter and rectifier mode, which is shown in Fig. 14. It is demonstrated that its value is higher than 0.98 from 10% output power onwards.

Then, the parameter $\gamma$ was varied while measuring the RMS current $i_\sigma$, as shown in Fig. 15. This is done for a range of input powers and compared with simulation results. As shown in Fig. 13. The behavior is as expected.

### Table III

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Mains voltage</td>
<td>229.5 V RMS</td>
</tr>
<tr>
<td>Mains current</td>
<td>20.65 A RMS</td>
</tr>
<tr>
<td>Mains power</td>
<td>14.17 kW</td>
</tr>
<tr>
<td>Mains power factor</td>
<td>0.9967</td>
</tr>
<tr>
<td>DC voltage</td>
<td>501.5 V</td>
</tr>
<tr>
<td>DC current</td>
<td>26.21 A</td>
</tr>
<tr>
<td>DC power</td>
<td>13.14 kW</td>
</tr>
</tbody>
</table>

Fig. 15. Measured effect of parameter $\gamma$ on RMS current of $i_\sigma$ averaged over a full ac mains cycle at various input powers and compared with simulations, with $\gamma \in [1, 0.9, 0.8, 0.7]$, respectively.
The analysis methods, modulation strategy, and component dimensioning guidelines can be used for the design and analysis of ac-dc power converters based on the QAB-topology. The methods are validated to be accurate, and it is demonstrated that the topology operates as expected. Additionally, the features of this converter make it an attractive alternative to common two-stage topologies.

**References**


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