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Forces on rapidly growing vapor bubbles on a wall in forced convection with varying angle of inclination

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HIGHLIGHTS

• Shape histories of fast growing bubbles hardly depend on the direction of gravity.
• The inertia forces are responsible for detachment of fast growing bubbles.
• Detachment of boiling bubbles against gravity is explained by inertia of the fluid.
• Wall surface heterogeneities facilitate stopping of the contact line and detachment.

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ABSTRACT

The forces involved in rapid vapor bubble growth are assessed by measurement and analysis of bubble growth in four principal orientations of the boiling surface and channel flow with respect to gravity. The approaching liquid flow is made nearly uniform to warrant the possibility to compute the added mass forces. Almost no difference in shape history is observed between the four cases. This is most remarkable for the case in which the boiling surface is facing downward with respect to gravity. By way of careful shape analysis and extensive force computations, the reason for this remarkable behavior is found to be the crucial role of the inertia force.

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1. Introduction

In order to predict nucleate boiling heat transfer, many mechanistic nucleate flow boiling models have emerged in the past 60 years [1,2]. Various of these mechanistic models attempt to quantify the heat transfer from the wall into the bulk liquid by
It will be shown in this paper that inertia plays a decisive role in boiling bubble detachment under some circumstances. As not only accelerations, but also velocities are important in the inertia force, prediction of detachment requires estimates of the velocities involved. Our measurements show that the bubble foot is expanding and moving, which makes the velocity of expansion independent of those of translational motion. In particular when dynamic viscosity is low and the Jacob number high, both velocities must be estimated in order to assess the inertia force and to predict bubble detachment. Any proper criterion should therefore be based on estimates of these velocities as well. This insight is of course helpful in constructing and examining detachment criterions.

The main goal of this research is to study the forces involved in growing vapor bubbles. The various accelerations involved in this process, for example the acceleration of the interface and the center of mass, induce added mass force components. The added mass coefficients involved are dependent on the shape of the bubble and the distance of its center of mass to the wall and are, therefore, time dependent during bubble growth. For correct assessment of this force, it is crucial to determine the shape dependent added mass coefficients. Expressions for these added mass coefficients have been derived previously for truncated spheres and free spheres in the vicinity of a plane wall [4,5].

In convective boiling, the liquid flow past a growing vapor bubble will induce a drag force on the bubble. However, this drag force should not be approximated using expressions for fully developed flow over a body, as is often mistakenly done [6]. Vapor bubbles in forced convective boiling have a typical lifetime of 5–10 ms from nucleation up to detachment. During this short time, the liquid surrounding the bubble only moves a distance in the order of the bubble diameter (based on a flow velocity of 0.1 m/s near the wall and a bubble diameter of 1 mm). Therefore, a hydrodynamic boundary layer on the bubble has little time to develop and resulting drag force components are much smaller than those calculated from quasi-steady drag expressions [7].

The present study focuses on individual bubbles boiling on a plane wall with a uniform approaching flow velocity. The experimental setup is constructed in a way that approximates this idealized flow condition as closely as experimentally possible. The orientation of the normal direction of the boiling surface with respect to gravity and flow direction can be varied. The force balance calculations in this work are based on an Euler–Lagrange approach as an alternative to the more commonly applied Newtonian method [4,5,8–11]. One of the most important parameters determined in this study is the overpressure in the bubble, which is in fact the only parameter in the force balance that cannot be measured directly.

First, the experimental setup and procedures will be shown, after which the force balance theory based on an Euler–Lagrange approach will briefly be explained. The vapor bubbles of separate configurations of the boiling surface with respect to gravity are presented, analyzed and compared with each other.

2. Experimental setup and experiments

A closed-loop experimental setup was designed and constructed in order to facilitate flow boiling measurements in near saturation conditions. The setup allows for accurate control of flow, temperature and pressure of the bulk fluid, which is de-mineralized water. Furthermore, a custom made de-aerator allows for optimal de-aeration of the liquid, prior to initiation of experiments. The first key feature of the setup is that it can produce individual vapor bubbles at a pre-defined location in a see-through test section. The second key feature is that the vapor bubble nucleation location is approached by a uniform laminar flow. The last key feature is
that rotating the test section can vary the orientation of the normal direction of the surface on which vapor bubbles nucleate.

The flow from the pump enters the inlet section through a custom-made flow conditioner to remove swirl. Next, the flow enters a diffuser and, subsequently, a contraction which gradually transitions to a channel of 5×40 mm² cross section. This development section is 34 hydraulic diameters long, which ensures fully developed flow in the test section. The test section is a stainless steel channel (5×40 mm²) with two transparent polycarbonate windows. The liquid flow in the test section is laminar and fully developed. A so-called bubble generator is positioned in the center of the cross section of the rectangular channel. The base of the bubble generator is a thin (1 mm height) piece of glass with a sharp front edge in order to keep the flow over the bubble generator uniform, see Fig. 1a and c. As close to the front edge of the glass as possible (0.4 mm), a titanium resistor with effective dimensions of 1.25×0.75 mm² is deposited by chemical vapor deposition (the longer dimension being in the direction of the flow). The thickness of the resistor is 200 nm and its resistance is typically 20Ω. By applying an electric current over the resistor, the resistor will act as a local heater through Joule heating. When a liquid flow at saturation conditions passes by the bubble generator, vapor bubble nucleation can be initiated on the titanium layer by applying an electric current. The electric current is directed to the titanium resistor by platinum leads. These platinum leads have a thickness of 2 μm and have a negligible resistance compared to the titanium resistor.

The experiments have two main purposes. First, they serve as a basis for validation of the force balance theory described below, in Section 3. Second, by varying the orientation of the heated surface with respect to gravity, a comparison of bubble shape, deformation evolution and forces can be made between the various cases.

For these purposes, individual vapor bubbles have been created in similar process conditions, but at different inclination angles of the test section. For the inclination angle four principle directions were chosen, as explained in Fig. 2. These inclination angles will from now on be referred to as “Case #”, in combination with small icons (i.e. Case 1: , Case 2: , Case 3: , Case 4: ).

The analysis program consists of three main steps. First, images are transformed (rotated, cropped and resized) and calibrated to transform pixel information to meters. Secondly, the images of each bubble go through an edge detection step in order to accurately determine the bubble contour. Lastly, the information of the first two steps is combined to calculate the geometrical properties of the vapor bubbles during their growth as well as assess the forces involved.

3. Forces acting on growing vapor bubbles; theory

For a growing vapor bubble attached to a wall the force balance normal to the wall can be written as

\[ F_{\text{inertia}} = F_R + F_{\Delta p} + F_{\text{buo}} + F_{\text{drag}} + F_{\text{lift}}. \] (1)

in which the inertia force, \( F_{\text{inertia}} \), encompasses various added mass components. The added mass components are induced by the acceleration of (parts of) the interface, which is actually a deceleration for the largest part of bubble growth, and acceleration of the bubble as a whole. This force has had several names attributed to it in the past, including “bubble growth force” and “expansion force”. Additional hydrodynamic forces induced by both the flowing bulk liquid as well as the velocity originating from the expansion of the bubble itself are accounted for in the drag and lift forces, \( F_{\text{drag}} + F_{\text{lift}} \). Force \( F_{\text{buo}} \) is the well known buoyancy force acting on the center of mass of a vapor bubble in liquid.

By far the largest forces, however, are the capillary force \( F_R \) and the overpressure force \( F_{\Delta p} \); they are larger by several orders of magnitude. The capillary force \( F_R \) pulls the bubble toward the wall. The overpressure force \( F_{\Delta p} \) is a result of the overpressure in the bubble, \( \Delta p = \rho_{\text{vapor}} - \rho_l \) where it should be noted that the relevant pressure in the liquid is the hydrostatic pressure at height of the bubble foot. While the vapor bubble is attached to the wall, it has a contact area with the wall on which this overpressure acts, resulting in a force that pushes the bubble away from the wall. In case the shape is that of a truncated sphere, the sum \( F_R + F_{\Delta p} \) is precisely zero. Any deformations from this spherical shape are a result of the hydrodynamic forces and buoyancy.

It has been shown [8] that expressions for the kinetic energy and generalized forces (including conservative forces) exist for a system containing a growing vapor bubble, through rigorous derivation from the equation of change for the hydrodynamic kinetic energy [12], Ch. 3, p. 81]. The existence of appropriate expressions for the kinetic energy of this system enables the use of the Euler–Lagrangian approach to study dynamics of vapor bubbles during their growth. The advantage of such an approach is that deformations of the bubble interface are accounted for, facilitating the analysis of bubbles with shapes other than truncated spheres. Limitations of this approach have been found mainly in the computation of the drag force related to free vorticity. Modeling of forces on a growing vapor bubble at a plane wall can be appropriately done by inviscid
potential flow theory, even if the approaching flow contains vorticity, as was shown by van der Geld [5]. However, in the current experiments the approaching flow is kept laminar and as uniform as possible in order to yield a negligible lift force due to vorticity and minimize any turbulent fluctuations. Several inertial forces and gravity act on the center of mass, \( x_{CM} \), of the bubble, defined as

\[
\ddot{x}_{CM} = \frac{1}{V} \int_V \ddot{x} dV.
\]

(2)

In which the \( V \) is the volume of the bubble and the mass density of the vapor is assumed constant. It makes sense to define \( h \) as the distance of the center of mass to the wall, see Fig. 3.

The bubble shape can now be described by radius \( R \) as function of angle \( \theta \), measured from the top of the bubble, see Fig. 3, with the origin at the center of mass of the bubble. By defining \( x = \cos(\theta) \), the radial distance \( R \) can be taken as a function of \( x \). Additional important variables are related to the bubble foot, where \( R_f \) is the radial distance to the bubble foot from \( \ddot{x}_{CM} \) with \( \theta \) the angle corresponding to the foot, leading to the definition of \( x_f = \cos(\theta_f) \).

For \( x \in [1, x_f] \), the radial distance can be represented by

\[
R(x) = R_0 + \sum_{n} b_n(t) \cdot P_{n-1}(x).
\]

(3)

with \( P_n \) the \( n \)-order Legendre polynomial. The resulting Legendre coefficients, \( b_n \), serve as generalized coordinates.

In the earlier stages of bubble growth when the bubble has a hemispherical shape, the domain \([1, x_f]\) is far from being the domain \([1, -1]\) for which the Legendre polynomials form an orthogonal basis. This gives the freedom to set the value of \( b_2 \) to zero when fitting the coefficients \( b_n \) in (3) this early growth stage. When bubbles are near the point of detachment, \( x \) is defined over almost the whole interval \([-1, 1]\) and the omission of \( b_2 \) would lead to inaccurate fits. Therefore, fitting of \( b_2 \) is “switched on” when bubbles reach 60% of their total growth time, defined as the time from nucleation up to detachment. In this stage, the value of \( b_2 \) is a few percent of the value of \( h \). Formally, the second generalized coordinate is now equal to \( h + b_2 \). The list of generalized coordinates is now defined as: \( b_1 \), \( h \), \( b_3 \), \( b_4 \),…

The measured bubbles turn out not to be axisymmetric. For this reason, the measured bubbles are split up in two halves to the left and right of the plane perpendicular to both the flow and the wall going through the center of mass of the bubble. Each half will be analyzed independently as the half of an axisymmetric bubble, described by (3). It has been shown that the volumes of the two independent axisymmetric bubbles are nearly the same and that their average is close to the actual volume of the total bubble. This shows that the formal treatment with spherical harmonic functions, replacing (3), is not necessary for the present set of measurements. However, for bubbles exhibiting severe deformations, the application of spherical harmonic functions is inescapable.

The force components related to the isotropic generalized coordinate, \( b_1 \), and the component normal to the wall, \( h + b_2 \), will be assessed. For the isotropic components of the force balance, \( b_1 \), the average of twice the mean curvature \( \langle 2H \rangle \) is seen to be important and is given by [11]

\[
\langle 2H \rangle = \frac{2 \int_0^\pi R^2(\theta) \sin(\theta)Hd\theta}{\int_0^\pi R^2(\theta) \sin(\theta)d\theta}
\]

(4)

The volume of the bubble, \( V_b \), is also expressed in terms of the coordinate system defined in Fig. 3. The expression combines the volume described by angle \( \theta \) and average radius \( R \) and the volume of a cone with height \( h \) and diagonal \( R_f \), yielding

\[
V_b = \frac{2}{3} \pi (1 - x_1)(R^3) + \frac{1}{3} \pi h(R_f^2 - h^2).
\]

(5)

where the \( \langle \rangle \)-brackets indicate the mean value over the contour: \( \int_0^\pi R(x)dx/(1 - x_f) \). The derivatives of the volume to the generalized coordinates are particularly important for the determination of the overpressure force. The derivative with respect to \( b_1 \) yields the surface of the bubble and is seen to reduce to \( 2\pi h(R_f^2 + h) \) for a truncated sphere. The derivative of \( V_b \) with respect to \( h \) is equal to the apparent contact area of the bubble with radius \( R_f \).

The time rate of change of the kinetic energy of the liquid can be written in terms of the sum of the product of forces involved in bubble growth (Eq. (1)) and generalized velocities

\[
\frac{dE_k}{dt} = \sum_n F_n b_n.
\]

(6)

where \( b_n \) are generalized velocities and \( F_n \) expressions for the generalized forces. The kinetic energy \( E_k \) of the liquid in the above expression accounts for turbulence as well as for creeping flow. The velocity field of the incompressible liquid can be written in terms of an irrotational, a vorticity and a mixed component (see [13]). The total force balance is now defined as

\[
-F_n^{\text{inertia}} = -F_n^{\Delta p} + F_n^{\text{buo}} + F_n^{\text{drag}},
\]

(7)

where it should be noted that there are just as many force balances as there are generalized coordinates, numbered \( n \) with maximum \( N \). This approach therefore yields \( N \) independent expressions for the same system, representing the degrees of freedom induced by the deformation of the bubble.

The gravitational force perpendicular to the wall, in \([h + b_2]\) direction, naturally follows from the volume of the sphere, as defined in (5), given by

\[
F_n^{\text{buo}} = \rho g V_b,
\]

(8)

in which the gravitational direction \( g \) differs per case, positive for \( \text{buo} \), negative for \( \text{drag} \) and zero for \( \text{inertia} \) and \( \Delta p \). The buoyancy force in \( b_1 \) direction can be written as

\[
F_1^{\text{buo}} = 2 \pi g \rho (1 - x_f)(h(R_f^2 + \langle xR^3 \rangle)).
\]

(9)

This buoyancy force accounts for the change in potential energy as a result of change in the volume of the bubble.

The overpressure force contains the only quantity which is not measured directly in the experiments, namely the overpressure of the bubble \( \Delta p \). The overpressure is defined as the difference between the homogeneous pressure in the bubble, \( p_b \), minus the pressure on the liquid side, \( p_l \). It should be noted that the definition of \( p_b \) is different per case. For \( \text{buo} \) and \( \text{inertia} \), this pressure is equal to the hydrostatic pressure at the height of the plane wall, whereas for \( \text{drag} \) and \( \Delta p \) this pressure is defined as the hydrostatic pressure at the height of the center of mass, \( x_{CM} \).
The overpressure force, \( F_{n}^{\Delta p} \), for each component \( n \) is described by

\[ F_{n}^{\Delta p} = \Delta p \frac{\partial V_{b}}{\partial b_{n}}. \]  (10)

Each bubble contour fit yields \( N \) force balances, where \( N \) is the degree of the Legendre polynomial, each of which has only one unknown: \( \Delta p \). Nevertheless, only the governing equation for the first generalized coordinate, \( b_{1} \), will be used to assess the overpressure in this manner, which is written as

\[ \Delta p = -\sigma(2H) - \left( \frac{F_{\text{inertia}}^{1} + F_{\text{drag}}^{1} + F_{\text{bu}}^{1}}{\partial V_{b}/\partial b_{1}} \right). \]  (11)

The surface tension force in \( b_{1} \) direction is given by

\[ F_{1}^{\sigma} = \sigma(2H) \frac{\partial V_{b}}{\partial b_{1}}, \]  (12)

where the derivative on the right hand side of the equation follows from (5). Two expressions have been derived [11] for the surface tension force in \( h \)-direction.

The inertial forces, \( F_{\text{inertia}}^{n} \), in Eq. (7) originate by definition from the irrotational component of the kinetic energy of the liquid, \( E_{i}^{n} \). The accompanying added mass coefficients have been derived for potential flows, but have been proven to be equally applicable to situations where viscosity cannot be neglected [5]. The existing expressions for the inertia force have been determined from a potential flow over truncated spheres and deforming spheres in close proximity to a wall [11]. In the case of a deforming bubble near a plane wall, more added mass coefficients are needed. The kinetic energy of the fluid in this case can be written as

\[ E_{k}^{n} = \frac{1}{2} \rho V_{b} \sum_{m,n} \Psi_{m,n} b_{m} b_{n} \]  (13)

where \( \Psi_{m,n} \) is the matrix of added mass coefficients. The analysis of the growing vapor bubbles below has been done using the previously mentioned expressions for hemispheres and free spheres and in one case with computed coefficients \( \Psi_{m,n} \) for a deformed bubble near a wall. The following equation defines the inertial lift, \( F_{\text{lift, inertia}}^{2} \):

\[ F_{\text{lift, inertia}}^{2} = -F_{\text{inertia}}^{n} - \rho V_{b} \left( \Psi_{2,2} \frac{d^{2}(h + b_{2})}{dt^{2}} + \frac{1}{2} \Psi_{1,2} \frac{d^{2}b_{1}}{dt^{2}} \right), \]  (14)

which means that all inertia terms without second order derivatives are considered as lift terms.

The drag force on a growing vapor bubble has been shown to be critically dependent on the time of growth [7]. Even in the case of relatively slowly growing vapor bubbles with a longer lifetime than the bubbles of the present study, \( \Delta t \approx 0.1 \text{s} \), the drag force was shown to be negligible [11]. In the present study, the bubble growth is much faster (\( \Delta t \approx 0.005 \text{s} \)), which means that boundary layers have even less time to develop and the drag force values for motion perpendicular to the wall turn out to be negligible.

A vorticity force exists which can be split into one force due to free vorticity originating from the liquid flow over the frontal edge of the bubble generator and in one force resulting from the free surface zero-stress boundary condition at the bubble surface. This force contribution has been neglected in the current analysis since the flow in the current experiments is predominantly a uniform laminar flow parallel to the wall and since the time for development of (weak) vorticity at the vapor–liquid interface is highly limited. The hydrodynamic boundary layer at the nucleation site of the bubble generator turns out to be 0.31 mm thick in all experiments. Compared to the size of the bubbles, this thickness is at most 30% of the bubble height and, therefore, most of the bubble is growing in a uniform approaching flow. The force balances considered in the present study do not encompass the one on the bubble parallel to the wall; whatever the forces parallel to the wall, they do not affect those in the direction perpendicular to the wall, so the results are general. In the direction normal to the wall, the vorticity boundary layer might affect the lift force, but because of the low bulk liquid velocity the corresponding inertial lift, proportional to \( \nu_{\text{bulk}}^{2} \) in Eq. (14), has been observed to be negligible anyway.

4. Experimental results

The experimental conditions have been kept as constant as possible during and between the various experiments: \( P_{\text{bulk}} = 0.11 \text{ MPa} \), \( T_{\text{bulk}} = 99 \degree C \), subcooling \( T_{\text{sub}} \approx 4 \degree C \) (except for case 2, 3), when the subcooling is 1.6 \degree C; the other values are 3.8 and 4.8 \degree C), bubble generator heat flux \( q_{\text{ac}} = 0.35 \text{ W/mm}^{2} \), bubble release frequency \( f_{b} \approx 25 \text{ Hz} \), bulk liquid velocity is 0.061 m/s. The bulk liquid flow is laminar, \( Re = 1850 \). The velocity at the location of the bubble generator in the center of the channel is determined to be uniform at 0.10 m/s; the velocity profile in the channel has been computed analytically and was validated with PIV measurements [11]. This velocity is named \( v_{\text{bulk}} \) since it is the bulk approaching flow experienced by the bubble.

For each case, images have been recorded at 10,000 fps (with the shutter time at 50 \mu s) in synchronized fashion with the experimental conditions. Around 100 bubbles have been recorded per case. The topological evolution of these bubbles only varies slightly and the analysis of 10 randomly chosen bubbles is seen to be sufficiently accurate. The shape evolution does not appear to vary by a large amount per case (Fig. 4). The first phase of bubble growth is

![Fig. 4](image)

**Fig. 4.** Typical bubble images per for each case. Three points of time in the evolution of the bubble after nucleation are shown, namely after 0.3 ms, half-way to detachment and right before detachment. The images are roughly on the same scale.
characterized by explosive growth and the final phase is much
dependent on the heterogeneity of the bubble generator, as
explained by Baltis and van der Geld [14].

To facilitate the analysis of the data a high accuracy vapor bub-
ble contour analysis is required. The sharpness of bubble contours
in the recorded images as well as the vapor bubble contour anal-
ysis have improved greatly compared to previous work [11]; more
details are given by Baltis [14].

It should be noted that all bubbles are assumed to be circular
from the top of the bubble, this has been verified by observing
the bubble growth through a mirror in the channel. Therefore, the
cross sections of the bubble parallel to the wall are assumed circular.
In combination with the determined contour, a cloud of points in
3D representing the bubble volume is constructed by revolving the
outer contour points around a common axis. The real volume of
the total bubble is then approximated by an alpha shape constructed
from the point cloud by use of a 3D Delaunay triangulation method,
creating an enclosed structure of tetrahedra. The total volume of
these tetrahedra is then the calculated bubble volume. An example
trimetric view of the reconstructed 3D bubble volume overlapping
the original camera image is depicted in Fig. 5.

The bubble volume in case 1 (A) initially increases rapidly while
the growth rate slows down (Fig. 6). After 1.5 ms, the volume starts
to decrease. This observation is a clear indication that the bubble
has exhausted the superheated liquid in its vicinity and is in contact
with the subcooled bulk liquid, \( T_{\text{sub}} \approx 4.8 \) K in case A.

The observed area of the bubble foot, \( A_f \), is determined by taking
the apparent contact points of the bubble at the wall and assuming
a circular foot, \( A_f = \pi r^2 \). In the initial stage, the bubble is a hemi-
sphere which rapidly expands its foot over the wall. After 0.5 ms,
the bubble starts lifting off from the wall and the foot area starts
decreasing again. From just after 1.5 ms, the upstream contact angle
starts increasing again as the bubble is pushed over the edge it is
pinched to, which at the same time causes an additional decrease in
the downstream contact angle (Fig. 6). This behavior leads to the

Fig. 5. Trimetric view of the reconstructed 3D bubble volume using the complete
bubble contour. The plane in the middle shows the original side view image, in which
the solid white line is the central axis of the bubble. The contour of the bubble is
revolved around this white line, resulting in the 3D shape. Both the volume to the
left and to the right of the image are equal.

Fig. 6. Collection of graphs of the geometrical properties for Case 1, A. Ten bubbles were analyzed, the data of which is shown by gray dots (gray crosses for \( \beta_{a,i} \)) and fitted
by a smoothing spline, shown by the solid black line (dashed line for \( \beta_{a,i} \)). Plots for the confidence interval of 95% (twice the standard error) are present but are within the
thickness of the line and are therefore not visible.
For $t < 0.5$ ms the bubble grows as a hemisphere and $(h + b_2)$ rapidly increases, after this stage $(h + b_2)$ appears to increase steadily at 0.17 m/s. However, the increase of $(h + b_2)$ is not actually linear in this stage, but slightly increases over time. The resulting acceleration and deceleration are in the order of two to three times the gravitational acceleration. The radial component, the volume equivalent radius $R_m = \left(3V_b/(4\pi)\right)^{1/3}$, is seen to only decelerate during the full period of growth. The velocity $\Delta r/dt$ starts at a maximum of 0.3 m/s and rapidly decreases to 0.01 m/s after 1 ms (see the gradients of the top right figure of Fig. 6). This trend is similar for the three other cases. The deceleration reaches a maximum of 600 m/s$^2$ around 0.5 ms after nucleation and then rapidly decreases to approximately 20 m/s$^2$ in the last few milliseconds. The importance of these accelerations are clarified below, in Section 5.

The evolution of the distance of the center of mass to the wall, $(h + b_2)$, as well as the shape evolutions presented in Fig. 4 seem to suggest that the behavior of bubbles at different orientations is quite similar. Although for cases $\odot, \circ, \square$, and $\vartriangle$ this may be expected, it is quite remarkable for $\odot$ when gravity acts to press the bubble to the wall. If gravity would be a major influencing factor, there should be a more extensive difference between the four cases.

5. Force analysis

As concluded in the previous section, the spread in geometrical properties in the observed bubbles per case is very small. Therefore, this section will show the averaged results for three bubbles per case, each of which is split in half, as described above, and analyzed as two independent axisymmetric bubbles. This yields six force balance evolutions normal to the wall per case, the components of which are then averaged.

The various forces acting on vapor bubbles during their growth perpendicular to the wall, as described in Section 3, are assessed in this section. Similar to the geometrical analysis, the force analysis initially focuses on $\odot$, after which a comparison between the various cases is made. As concluded in the previous section, the spread in geometrical properties in the observed bubbles per case is very small. Therefore, this section will show the averaged results for three bubbles per case, each of which is split in half, as described at the end of Section 3, and analyzed as two independent axisymmetric bubbles. This yields six force balance evolutions per case, the components of which are then averaged.

All main forces acting on growing vapor bubbles for $\odot$ are summarized in the two graphs in Fig. 7. The graph has been split up for comprehensibility. The black lines indicate the average values of the data of three bubbles, shown as grey markers. Upstream data is indicated by $\square, \times$, and $\vartriangle$, whereas downstream data is indicated by $\odot, \circ, \ldots, \triangle$. The left side of Fig. 7 shows that both the overpressure force, $F_{\text{overp}}^2$, and capillary force, $F_\gamma^2$, exceed the buoyancy force, $F_{\text{buoy}}^2$, at all times. It is important to note that the overpressure $\Delta p$ is determined with Eq. (10) and therefore follows from the first Euler-Lagrange equation, which is fully independent of the second Euler-Lagrange equation that yields the force balance perpendicular to the wall and detailed in Fig. 7. Various ways to determine the overpressure force have been examined in a previous study; see Fig. 10 of [11]. The overpressure force and the other forces of Fig. 7 have here been explained in Section 3. Fig. 7 also shows that the variation between the three bubbles and their two halves is negligible at times prior to 2 ms. From now on only the ensemble averages will be discussed. The sum of the forces of this figure is plotted as a dashed line in the right pane of Fig. 7. All other forces being proven to be negligible, this resultant must represent the inertia force $F_{\text{inertia}}^2$, as defined on the left hand side of Eq. (7). This is validated by comparison with computed values of the inertia force. These computations are performed for two limiting cases, being that of a hemisphere and a free bubble near a plane wall, as well as for a deformed bubble, shown at 1.8 ms in Fig. 7. The hemispherical case is shown as open diamonds at the initial growth stage, $t \leq 0.3$ ms, and the free bubble case is shown as open diamonds in the final growth stage right before the actual detachment, $t \geq 2.1$ ms. For the free bubbles, spheres with radius $R_m$ has been considered and put at a distance $h$ to the wall in order to stay as close to the actual shape as possible.

The observed bubble shapes in these stages of growth correspond well to that of a footed hemisphere and that of a sphere with a narrow foot to the wall. As such, the computed values for the limiting cases described above yield reasonable agreement with $F_{\text{inertia}}^2$ shown by the dashed line. The observed shapes at intermediate times, $0.3 < t < 2.1$ ms, differ from that of a truncated sphere. It is therefore no wonder that if one nonetheless approximates the actual shape by that of a truncated sphere, the computed inertia force differs considerably from the resultant given by the dashed line. As an example, see the crossed out diamond in Fig. 7 at $t = 1.8$ ms. The computed inertia value, $F_{\text{inertia,comp}}^2$, indicated by the filled diamond marker in Fig. 7, has been computed for the actually observed deformed shape. This value at $t = 1.8$ ms shows a good agreement with $F_{\text{inertia}}^2$.

One of the main reasons for the rather large discrepancy between the computed inertia forces for the actual shape, taken to be detached from the wall, and the approximation by a truncated sphere is the rate of change of $\Psi_{2,2}$ with distance $h$, $\partial \Psi_{2,2}/\partial h$. When $h$ diminishes, the added mass of a truncated sphere decreases because less liquid is moved, while the added mass of a detached bubble increases because the same volume comes closer to the wall where the added mass is higher since more liquid has to be accelerated at a higher rate. The actual bubble has a layer of liquid underneath part of its surface that is facing the wall. This part of the bubble may therefore apparently resemble a truncated sphere, while it actually displaces more liquid when moved. This is the physical picture behind the differences in computed inertia forces. This picture is far more obvious at the final stage of growth, when the bubble is actually attached to the wall by a tiny vapor ligament, while $h$ exceeds $R$. Future work will concern the inertia force of a deformed bubble with a foot at a wall.

The force analysis as described above for $\odot$ has also been performed for the other three cases. The inertia calculated as a resultant of the force balance through Eq. (7) is shown for each case in Fig. 8. The same trend is observed for all cases. The inertia force is negative at first, pushing the bubble toward the wall.
while the shape is still comparable to that of a hemisphere. This is in good agreement with the inertia calculated for the limiting case of a hemisphere, $F_{\text{inertia}}^{\text{comp}}$, for which the added mass coefficients are given by van der Geld et al. [11]. As stated in the description of forces for $\mathcal{F}$, the shape of the bubble is only close to that of a hemisphere in the first 0.3 ms. Therefore, the inertia force with added mass coefficients for hemispheres is only calculated for the first 0.3 ms of growth, the result of which is shown by filled markers in this initial time period in Fig. 8. After approximately 1–2 ms, the inertia force is seen to become positive for all cases. The inertia force for the cases with relatively small bubbles does not go over 0.04 mN, however, for $\mathcal{F}$ the inertia force is seen to be much larger. This can be understood from the large differences in volume, which are due to the differences in sub-cooling. Toward the final stages of bubble growth, the shape of the bubble is close to that of a free sphere near a plane wall, although there is a small difference since the observed bubbles still have a foot at the wall. Nevertheless, close agreement is found between $F_{\text{inertia}}^{\text{comp}}$ and $F_{\text{inertia}}^{\text{comp}}$ computed with added mass coefficients for free bubbles near a plane wall, as is shown by the filled markers toward the end of bubble growth for each case in Fig. 8.

The dominant effect of inertia is the reason why the vapor bubbles per case as shown in Fig. 4 do not vary much in shape. In the initial growth stage, the bubble pushes the liquid radially outward from its nucleation point. Newton’s third law of motion dictates that when the bubble pushes its surrounding liquid radially outward, the liquid will exert an equal but opposing force on the bubble. Essentially, this means that the bubble is pushed onto the wall by the surrounding liquid. However, the inertia force is seen to switch sign, see Fig. 8. The explanation for this is found in the large deceleration of the bubble radius, which has a maximum of around 600 m/s$^2$. As the radial velocity of the bubble decelerates, the surrounding liquid is still moving radially outward. Newton’s first law of motion states that an object continues to move at constant velocity, unless an external force acts upon it. The object in the present case is the liquid which is moving away from the vapor bubble at a certain velocity and the external force is the vapor bubble as the radial expansion is decelerating as a result of a decrease in the evaporation rate. In order to decelerate the liquid, the inertia force on the bubble needs to become positive in the isotropic b$_1$ direction and, because of the asymmetry of the bubble caused by the presence of a wall, also in direction b perpendicular to the wall.

Further evidence for this interpretation of the inertia forces is given by the similarity of $\mathcal{F}$ to the other cases. Previous measurements with bubbles growing on a downward facing bubble generator all showed the same results. The slowly growing bubble was found to spread slowly, stick to the wall at all times until it could finally escape via a side of the bubble generator [8]. An upside-down bubble cannot escape against gravity unless another force is in play. The above similarity of all four cases points strongly in the direction of the inertia force as the mechanism that explains bubble detachment in the interesting upside-down case $\mathcal{F}$.

6. Conclusions

The present study describes an investigation of forces involved in rapid vapor bubble growth. Vapor bubble growth is recorded in four principal orientations of the main flow channel. A remarkable observation in the geometrical analysis is seen in the shape histories of the observed bubbles. Almost no difference is observed between the various cases studied. This is most remarkable for the comparison of measurements in which the boiling surface is facing upward and downward with respect to gravity. By way of an extensive force analysis, the reason for this remarkable behavior has been elucidated. As the bubble evaporation rate decreases, the radial expansion decreases and the liquid moving radially outward is decelerated at the same time by the bubble. At this point, the sign of the inertia force becomes positive as the liquid pulls the bubble off the wall, at the same time decreasing the overpressure in the vapor bubble.

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