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K.W. Verkerk
Department of Electrical Engineering - Control Systems group
Eindhoven University of Technology
P.O. box 513, 5600 MB Eindhoven, The Netherlands
k.w.verkerk@tue.nl

1 Introduction

An observer can be used in high precision control of flexible systems to increase the achievable feedback control bandwidth [1]. As the real flexible system contains a large amount of modes, it is required to truncate the model used in the observer. This truncation will introduce model mismatch which increases the estimation error, specifically during acceleration trajectories. This abstract describes a method to reduce the model mismatch in the frequency region of interest with a minimum increase in the order of the observer model.

The flexible systems considered are assumed to be in modal state space form,

$$\Sigma \left\{ \begin{array}{rcl} \dot{\eta} & = & \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\Omega^2 & -2\zeta\Omega \end{bmatrix} \eta + \begin{bmatrix} \mathbf{0} \\ \Phi_a^T \end{bmatrix} u \\ y & = & \begin{bmatrix} \Phi_s & \mathbf{0} \end{bmatrix} \eta \end{array} \right. \tag{1}$$

Here, $\eta = \begin{bmatrix} \eta_1 & \dots & \eta_N & \dot{\eta}_1 & \dots & \dot{\eta}_N \end{bmatrix}^T$ denotes the modal state vector, Ω is a diagonal matrix containing the N modal frequencies in ascending order, $\Phi_s \in \mathbb{R}^{p\times N}$ describes the mapping from each mode to each of the p outputs, $\Phi_a \in \mathbb{R}^{m\times N}$ describes the mapping from each of the m inputs to each mode, $\zeta \in \mathbb{R}^{N\times N}$ is the modal damping matrix, and $\mathbf{0}$ and $\mathbf{I}$ are respectively zero and identity matrices of appropriate dimensions.

The limiting factor on feedback bandwidth stems from the first few flexible modes and thus one would like to use an observer model containing only $n < N$ modes. The structure of (1) can be easily truncated to the appropriate number of modes. Unfortunately the higher modes, that are discarded by the truncation, do affect the system transfer function at low frequencies too, thereby introducing model error in the observer. By suitably selecting augmentation dynamics for the truncated model this mismatch can be reduced.

2 Augmentation dynamics

For low frequencies the $n_d = N - n$ discarded modes can be described by a constant gain. The dynamics of the discarded modes are given by,

$$\Sigma_d \left\{ \begin{array}{rcl} \dot{\eta}_d & = & \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\Omega_d^2 & -2\zeta_d\Omega_d \end{bmatrix} \eta_d + \begin{bmatrix} \mathbf{0} \\ \Phi_{ad}^T \end{bmatrix} u \\ y_d & = & \begin{bmatrix} \Phi_{sd} & \mathbf{0} \end{bmatrix} \eta_d \end{array} \right. \tag{2}$$

where all matrices are truncations of the system matrices in (1). The DC gain, K, of (2) is given by,

$$\begin{aligned} K &= \begin{bmatrix} \Phi_{sd} & \mathbf{0} \end{bmatrix} \left(- \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\Omega_d^2 & -2\zeta_d\Omega_d \end{bmatrix} \right)^{-1} \begin{bmatrix} \mathbf{0} \\ \Phi_{ad}^T \end{bmatrix} \\ &= \begin{bmatrix} \Phi_{sd} & 0 \end{bmatrix} \begin{bmatrix} 2\left(\Omega_d^2\right)^{-1}\zeta_d\Omega_d & \left(\Omega_d^2\right)^{-1} \\ -I & 0 \end{bmatrix} \begin{bmatrix} 0 \\ \Phi_{ad}^T \end{bmatrix} \\ &= \Phi_{sd}\left(\Omega_d^2\right)^{-1}\Phi_{ad}^T \end{aligned} \tag{3}$$

where $\Phi_{sd} \in \mathbb{R}^{p\times n_d}$, $\Phi_{ad} \in \mathbb{R}^{m\times n_d}$, and $\Omega_d \in \mathbb{R}^{n_d\times n_d}$.

The matrix K can be added as a direct feedthrough term to the truncated model. Alternatively, a low pass structure can be used to keep the observer model strictly proper. From the matrix dimensions in (3) it follows that rank $(K) = r \leq \min(n_d, m, p)$. By taking the singular value decomposition (SVD) of K one obtains $K = U\Sigma V^T$, where $U \in \mathbb{R}^{p\times r}$, $V \in \mathbb{R}^{r\times m}$ are both unitary and $\Sigma \in \mathbb{R}^{r\times r}$ is a diagonal matrix containing the singular values. The resulting low pass augmentation dynamics of order $r \ll 2n_d$ are then given by,

$$\Sigma_z \left\{ \begin{array}{rcl} \dot{z} & = & -\omega_c\mathbf{I}z + \omega_c V^T u \\ y_z & = & U\Sigma z \end{array} \right. \tag{4}$$

where $\omega_c > \omega_n$ is the cut-off frequency of the low pass filters.

When parameter dependency is present in the system (2) this method can still be applied. The DC gain (3) will become parameter dependent as well. By grouping the known parameter dependencies at the input or output matrices it is possible to take the SVD and obtain parameter independent low pass augmentation dynamics that are pre or post multiplied by a parameter dependent matrix.

References

[1] K.W. Verkerk, H. Butler, P.P.J. v.d. Bosch, “Improved tracking accuracy for high precision systems through estimation of the flexible dynamics”, submitted for the European Control Conference, Linz 2015