

Improving yaw dynamics by feedforward rear wheel steering

Citation for published version (APA):

Besselink, I. J. M., Veldhuizen, T. J., & Nijmeijer, H. (2008). Improving yaw dynamics by feedforward rear wheel steering. In *Proceedings of the Intelligent Vehicles Symposium, 2008 : Eindhoven, Netherlands, 4 - 6 June 2008* (pp. 246-250). Institute of Electrical and Electronics Engineers. <https://doi.org/10.1109/IVS.2008.4621314>

DOI:

[10.1109/IVS.2008.4621314](https://doi.org/10.1109/IVS.2008.4621314)

Document status and date:

Published: 01/01/2008

Document Version:

Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

- A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
- The final author version and the galley proof are versions of the publication after peer review.
- The final published version features the final layout of the paper including the volume, issue and page numbers.

[Link to publication](#)

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal.

If the publication is distributed under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license above, please follow below link for the End User Agreement:

www.tue.nl/taverne

Take down policy

If you believe that this document breaches copyright please contact us at:

openaccess@tue.nl

providing details and we will investigate your claim.

Improving Yaw Dynamics by Feedforward Rear Wheel Steering

Igo Besselink, Tjalling Veldhuizen and Henk Nijmeijer

Abstract—Active rear wheel steering can be applied to improve vehicle yaw dynamics. In this paper two possible control algorithms are discussed. The first method is a yaw rate feedback controller with a reference model, which has been reported in a similar form previously in literature. The second controller is a feedforward controller, which only requires the front wheel steering angle and vehicle forward velocity. It has a similar performance as the feedback controller. Both controllers are evaluated using an enhanced bicycle model, which includes tyre relaxation behaviour and suspension steering compliance.

I. INTRODUCTION

Since the 1980's active rear wheel steering has caught the attention of the vehicle industry and research institutions. Controlling the steering angle of the rear wheels can improve a vehicle's handling characteristics and ultimately increase vehicle safety. By steering both the front and rear wheels at the same time the lateral acceleration can be built up more quickly and the side slip angle of the vehicle body is reduced. Another possible advantage is a reduction of the yaw oscillations during transient manoeuvres and the stability of the vehicle is improved. Also during low speed driving the turning radius of a vehicle can be reduced by steering the front and rear wheels in the opposite direction.

To achieve these goals a wide variety of controllers has been proposed, an early overview can be found in [1]. Generally speaking the initial papers focus on feedforward control aiming to minimise the side slip angle of the vehicle. Later a feedback loop for the vehicle's yaw rate is added to increase stability for external disturbances. More recent papers use a reference model to describe the desired steering response, as described for example in [2]. Another more recent development is that the active rear wheel steering is becoming an element in an integrated "global chassis controller", and is combined for example with active front steering [3]. In this paper the discussion will nevertheless be limited to a set-up where a conventional vehicle is equipped with active rear wheel steering.

This paper is organized as follows. In section 2 vehicle modelling will be discussed. The bicycle model is used very often in the development of rear wheel steering control systems. Since the aim is to improve the dynamics, some enhancements will be introduced which make the model

more accurate for frequencies in excess of 0.5 Hz. Section 3 deals with the design of a control system using yaw rate feedback and a first order reference model. The results obtained with the feedback controller marked the starting point of the development of a simple and straightforward feedforward controller, which is discussed in section 4. The performance of this controller is checked by simulations using the enhanced bicycle model and appears to be promising. Finally some concluding remarks are given in section 5.

II. ENHANCED BICYCLE MODEL

A. Equations of motions

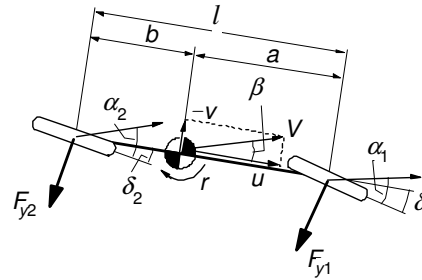


Fig. 1 The bicycle model with rear wheel steering

The well known bicycle model as shown in figure 1 is used. The linearised equations of motion read:

$$m(\dot{v} + ur) = F_{y1} + F_{y2} \quad (1)$$

$$I_{zz}\dot{r} = aF_{y1} - bF_{y2} \quad (2)$$

Where m equals the mass of the vehicle, I_{zz} is the yaw moment of inertia, u the forward velocity, v the lateral velocity and r the yaw velocity of the centre of gravity. The distance of the front and rear tyre to the centre of gravity is given by the distances a and b respectively. The lateral tyre forces at the front and rear are given by:

$$F_{y1} = C_{F\alpha,1}\alpha'_1 \quad \text{and} \quad F_{y2} = C_{F\alpha,2}\alpha'_2 \quad (3)$$

Linear tyre behaviour is assumed, $C_{F\alpha}$ represents the tyre cornering stiffness, the index 1 and 2 refer to the front and rear tyre respectively. The dynamic tyre side slip angles α' at the front and rear are calculated using the following differential equations and include the effect of both the tyre relaxation length σ and steering compliance c :

$$\frac{\sigma_1}{u}\dot{\alpha}'_1 + \alpha'_1 = \delta_1 - \frac{v + ar}{u} - C_{F\alpha,1}\alpha'_1 c_1 \quad (4)$$

$$\frac{\sigma_2}{u}\dot{\alpha}'_2 + \alpha'_2 = \delta_2 - \frac{v - br}{u} - C_{F\alpha,2}\alpha'_2 c_2 \quad (5)$$

Manuscript received March 25, 2008. Eindhoven University of Technology, Department of Mechanical Engineering, section Dynamics and Control. Igo Besselink; e-mail: i.j.m.besselink@tue.nl. Henk Nijmeijer; e-mail: h.nijmeijer@tue.nl. Tjalling Veldhuizen graduated on the subject of active rear wheel steering.

The motivation for taking into account the relaxation length is that it affects the dynamics of the vehicle at already relatively low frequencies (> 0.5 Hz) as is shown in [4]. Please note that there is a difference between the kinematic side slip angle of the tyre α (as shown in figure 1) and the dynamic tyre side slip angle α' . For the front tyre the following relation holds:

$$\alpha_1 = \delta_1 - \frac{v + ar}{u} \quad (6)$$

The steering compliance c gives the additional steering angle as a result of a lateral force, the unit is typically deg./kN. Including the steering compliance of the suspension enables us to distinguish between the characteristics of the tyre and axle. The linear tyre characteristics (relaxation length and cornering stiffness) have been measured using the flat plank tyre test facility of the Eindhoven University of Technology and can be directly incorporated into the simulation model, for more details see [5].

B. Model validation

Driving tests are available for an upper class rear wheel driven sedan to validate the enhanced bicycle model. Steady state cornering is shown in figure 2. The enhanced bicycle model is a linear model, an assumption which is valid up to lateral accelerations of approximately 4 m/s^2 . Random steering tests have been executed at 100 km/h and the accompanying transfer functions have been determined, as shown in figure 3. The results shown are for a loaded vehicle, results for the same vehicle, but in an unloaded condition are given in [5]. A full listing of the vehicle model parameters can be found in the appendix.

As can be seen from the figures, the steady-state behaviour is captured very well, the dynamics are described fairly well keeping in mind the simplicity of the model. Certainly by “tuning” the model parameters a better agreement can perhaps be obtained, but we tried use measured quantities as much as possible (like e.g. the tyre parameters).

C. Step steer dynamics

The dynamic response of a vehicle on steering input, can be illustrated quite well using a step steer input. Using the model as developed and validated in the previous paragraph, the step steer response of the loaded vehicle has been calculated at four different forward velocities, see figure 4. Figure 4 makes clear that in particular the yaw velocity shows a large overshoot, especially at high forward velocities. As will be shown in the next sections active rear wheel steering can be applied to suppress this phenomenon.

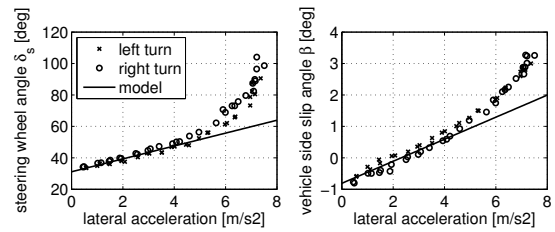


Fig. 2 Comparison of measurements and model for steady-state circular testing (loaded vehicle, corner radius 100 m).

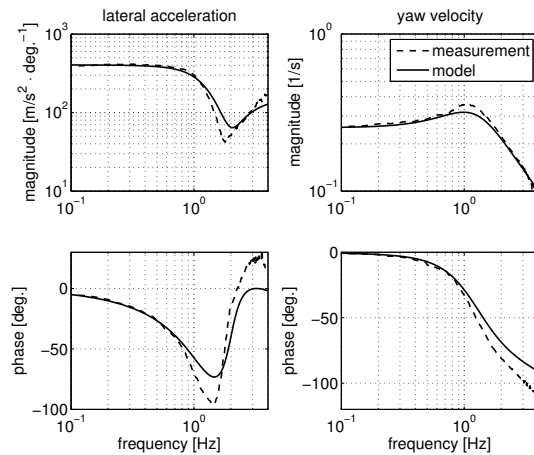


Fig. 3 Comparison of measurements and model for the random steering test (loaded vehicle, forward velocity: 100 km/h).

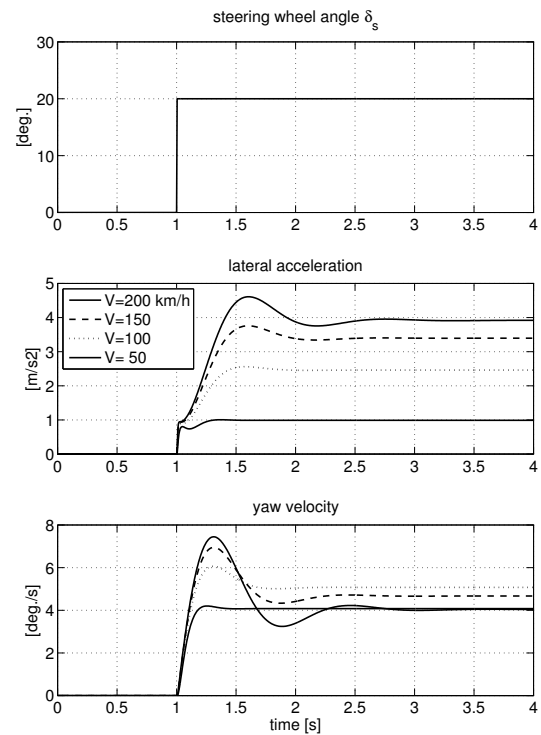


Fig. 4 Simulated step response of a conventional vehicle (loaded condition).

III. YAW RATE FEEDBACK CONTROLLER

A. Design goals for the controller

Based on a literature review the following conclusions were drawn with respect to the design of the active rear wheel steering controller [6]:

- The control scheme should include a reference model which describes the desired vehicle response depending on certain inputs as steering angle of the front wheels and vehicle speed. The reference model should have some tuning capabilities so that a desirable vehicle response can be obtained.
- The lateral dynamics are governed by the yaw rate and vehicle side slip angle. Since only one additional degree of freedom is available, the rear wheel steering angle, only one of these two quantities can be controlled.
- Early papers on active rear steering focus on reducing the vehicle side slip angle, more recent papers focus on the controlling the yaw motion. The overshoot in yaw velocity (“fishtailing”) as shown in figure 4 is undesirable and leads to an increased workload for the driver.

So based on these considerations a controller was developed with the goal of minimizing the yaw rate overshoot.

B. Controller development

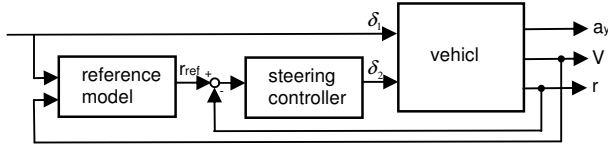


Fig. 5 Structure of the rear wheel steering controller using yaw rate feedback [6].

The structure of the controller is shown in figure 4. Two main items can be distinguished: a reference model and rear wheel steering controller. The task of the reference model is to provide a reference yaw rate. Inspired by the work of Song [2], the choice was made to use a first order differential equation with the steering angle as input.

$$H_{r_{ref}, \delta_1}(s) = H_{r_0}(V) \frac{1}{\tau_r s + 1} \quad (7)$$

In this equation $H_{r_0}(V)$ is the steady state yaw velocity gain, which is dependent on the forward velocity V . For a vehicle having an understeer gradient η and a wheelbase l the following relation holds [7]:

$$H_{r_0}(V) = \frac{V}{l + \frac{\eta}{g} V^2} \quad (8)$$

In this equation g is the gravitational constant. If the actual understeer gradient of the controlled vehicle is equal to the understeer gradient of the reference model no additional rear wheel steering angle will be required for stationary cornering. This approach will compensate for changes in the

understeer gradient, e.g. when the vehicle is loaded: the controlled vehicle would maintain the same stationary handling characteristics as the reference model.

Another choice which has to be made is the time constant τ_r in the first order reference differential equation. From the step response of a bicycle vehicle model (without relaxation behaviour and rear wheel steering) an equivalent time constant τ for the yaw rate can be defined by the ratio between the steady state yaw rate (r_{ss}) and the derivative of the yaw rate at $t=0$ ($\dot{r}(0)$), it is given by [7]:

$$\tau_r = \frac{r_{ss}}{\dot{r}(0)} = \frac{I_{zz} V}{a C_1 l \left(1 + \frac{\eta}{g l} V^2 \right)} = \frac{I_{zz}}{a C_1} H_{r_0}(V) \quad (9)$$

Where C_1 is the cornering stiffness of the front axle. So the idea is that in case of a step steer application that the initial rise in yaw velocity will be the same as for a conventional vehicle without rear wheel steering.

The rear wheel steering controller is developed through loopshaping, it is designed to have maximum bandwidth with sufficient robustness. Though the controller is designed for a vehicle speed of 120 km/h, it also proved to perform as expected at 200 km/h. No dependency on the forward velocity has to be included. For more details on the controller design, reference is made to [6].

C. Simulation results

The response of the controlled vehicle to a step steer input is shown in figure 6. For reasons of clarity only one forward velocity is chosen, but a very similar response is obtained at other velocities. Obviously the yaw rate overshoot is very effectively suppressed by the active rear wheel steering system. The yaw rate reference signal is not shown in figure 6, because the difference with the actual yaw rate is very small. It can be clearly observed that both the steady-state value and initial rise of the yaw velocity are the same as for the conventional vehicle, only the overshoot is effectively eliminated.

To achieve these results on a real vehicle a very accurate yaw rate signal should be available. Practical experience has shown that the yaw rate signal from an ESP sensor does not meet this requirement as it contains too much noise, resulting in a highly degraded controller performance [6].

IV. FEEDFORWARD CONTROLLER

A. Observations, controller layout

An important observation can be made when looking at figure 6: the required rear wheel steering angle to eliminate the yaw velocity oscillation is a rather smooth signal, which does not seem to be related to the frequency of the original yaw oscillation. So apparently it is not necessary to actively apply counter steering at the rear wheels depending on the yaw velocity oscillation.

Furthermore figure 6 shows the step response of the rear wheel steering angle in relation to the front steering angle.

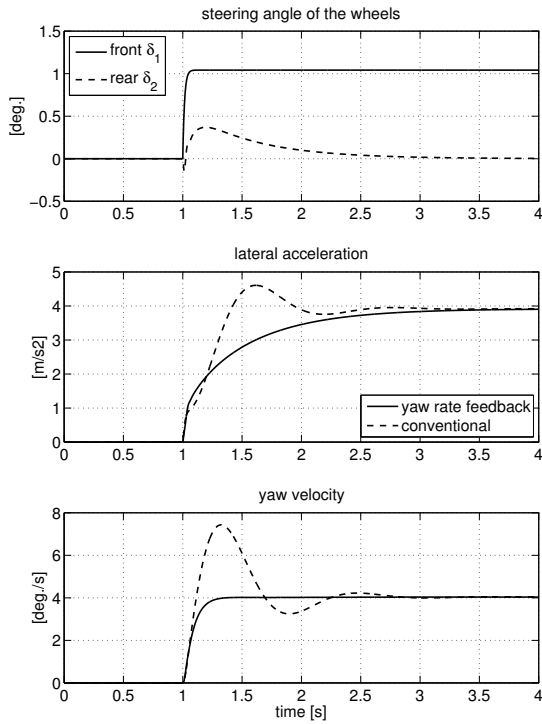


Fig. 6 Simulated step response at 200 km/h for the loaded vehicle, with and without rear wheel steering using yaw rate feedback.

Not taking into account the initial negative rear wheel steering angle, the following transfer function is proposed to relate the steering angle of the rear wheels to the steering angle of the front wheels:

$$H_{\delta_2, \delta_1}(s) = \frac{K(\tau_1 - \tau_2)s}{(\tau_1 s + 1)(\tau_2 s + 1)} \quad (10)$$

In this equation K equals the gain, τ_1 and τ_2 are two time constants. The response for a unit step at $t=1$ s is shown in figure 7.

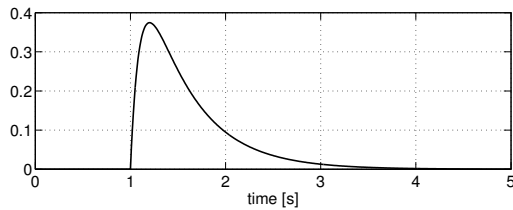


Fig. 7 Unit step response of the transfer function given in equation (10) ($K=0.7$, $\tau_1=0.5$ s., $\tau_2=0.1$ s.).

The structure of the feedforward rear wheel steering controller is shown in figure 8. For a given forward velocity the controller parameters are determined by optimizing the yaw velocity step response of the simulation model using numerical optimisation techniques. The controller settings are only dependent on the forward velocity, but no difference exists for the loaded and unloaded condition of the vehicle, both are included in the optimisation.

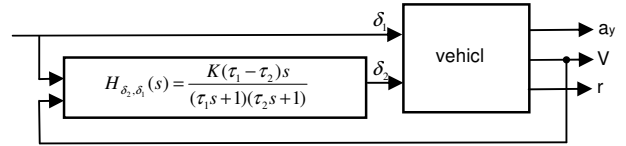


Fig. 8 Feedforward rear wheel steering controller.

B. Simulation results

With an appropriate selection of the controller parameters, the feedforward controller is also very well capable of eliminating the yaw velocity overshoot and oscillations, as is shown in figure 9. This simulation was done for exactly the same conditions as shown in figure 6, only the controller was changed. So it can be seen that for this condition the performance of the feedforward controller is very similar to the feedback controller.

Similar to figure 4, the step response for different forward velocities is shown in figure 10. The optimization process shows that below approximately 70 km/h no rear wheel steering assistance is required. Figure 10 also illustrates that at higher forward velocities the required rear wheel steering angle has to increase and is needed for a longer period of time. On the other hand it also might be feasible to fix the time constants τ_1 and τ_2 and only adjust the gain depending on the forward velocity, this has not been investigated further at this point.

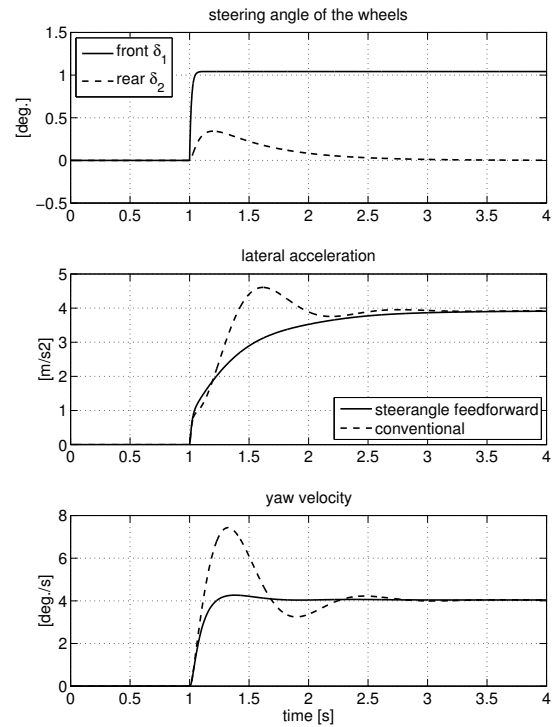


Fig. 9 Simulated step response at 200 km/h for the loaded vehicle, with and without rear wheel steering using feedforward.

V. CONCLUDING REMARKS

In this research we set ourselves the task of improving the yaw dynamics of a road vehicle using rear wheel steering. After first having evaluated a more complex controller, it appears that this goal can be achieved by a simple feedforward control law, which eliminates the need for an accurate yaw rate sensor. Since this is a simulation exercise, it remains to be seen how a real driver would react to a vehicle equipped with this controller. Driving tests would reveal if we have struck the right compromise or not. Furthermore we have restricted ourselves to handling only (no combined steering/braking), use a relatively simple vehicle model and stay within the linear range of vehicle behaviour ($< 4 \text{ m/s}^2$). It is also clear that on a real vehicle the feedforward controller might perform less well and that still additional sensors are required combined with a feedback controller to obtain the desired performance.

Nevertheless feedforward rear wheel steering could be a viable solution. The new Renault Laguna coupe seems to be equipped with this option, as described in [8]. According to [8] the choice for an open loop system was made because it is very robust and will behave predictably even when something is wrong with the car (like a puncture). Further details of the Renault system are not available to date and the feedforward controller presented here has no relation to the system developed by Renault whatsoever.

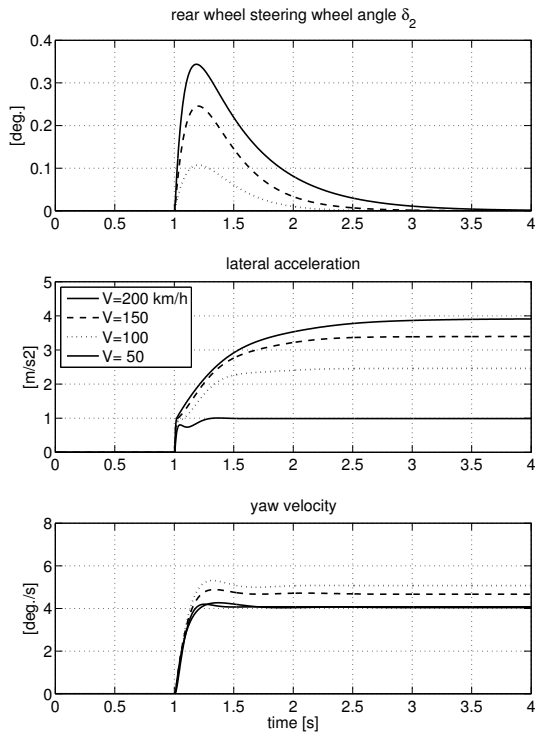


Fig. 10 Simulated step response of a vehicle (loaded condition) using feedforward.

Finally the transfer functions of the vehicle at 150 km/h are shown in figure 11. For low frequencies the phase delay increases, but around 1-2 Hz a clear phase lead is obtained compared to the conventional vehicle. What is not shown in figure 11, is that the transfer functions of the vehicle equipped with the feedforward controller are much less dependent on forward velocity compared to a conventional vehicle, which is considered to be a clear advantage.

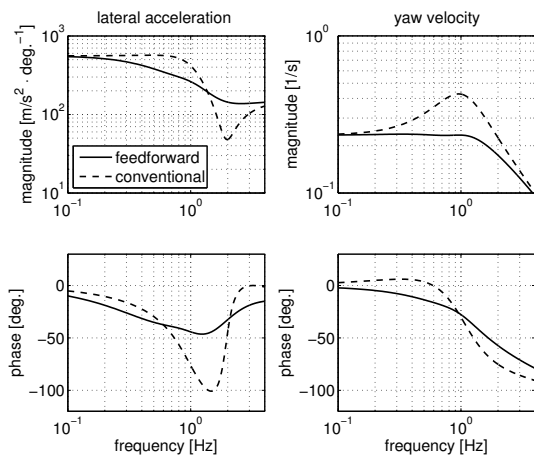


Fig. 11 Transfer functions with respect to steering input, loaded vehicle, 150 km/h.

APPENDIX

vehicle parameter	symbol	unloaded/loaded
vehicle mass	m	1659/1954 kg
yaw moment of inertia	I_{zz}	2259/2960 kgm^2
wheelbase	l	2.83 m
distance CG to front	a	1.42/1.63 m
cornering stiffness front	$C_{Fa,1}$	3030/3030 N/deg (2 tyres)
cornering stiffness rear	$C_{Fa,2}$	3038/3820 N/deg (2 tyres)
relaxation length front	σ_1	0.45/0.45 m
relaxation length rear	σ_2	0.45/0.56 m
steering compliance front	c_1	0.35 deg/kN
steering compliance rear	c_2	0.05 deg/kN
steering ratio	i_s	19.2

REFERENCES

- [1] Y. Furukawa, N. Yuhara, S. Sano, H. Takeda, "A Review of Four-Wheel Steering Studies from the Viewpoint of Vehicle Dynamics and Control", *Vehicle System Dynamics* vol. 18, pp. 151-186, 1989.
- [2] J. Song, Y. Yoon, "Feedback control of four-wheel steering using time delay control", *Int. J. of Vehicle Design*, vol. 19, 1998.
- [3] R. Marino; S. Scalzi; F. Cinili, "Nonlinear PI front and rear steering control in four wheel steering vehicles", *Vehicle System Dynamics*, vol. 45, pp. 1149 – 1168, 2007.
- [4] I.J.M. Besselink, "Lecture notes Vehicle Dynamics 4L150", Eindhoven University of Technology, 2005.
- [5] I.J.M. Besselink, L.W.L.Houben, I.B.A. op het Veld, A.J.C. Schmeitz, "Run-flat versus conventional tyres: an experimental and model based comparison", *VDI-Berichte Nr. 2014*, pp. 185-202, 2007.
- [6] T.J. Veldhuizen, "Yaw rate feedback by active rear wheel steering", Masters' thesis, Eindhoven University of Technology, DCT 2007.80, 2007.
- [7] H.B. Pacejka, "Tyres and Vehicle Dynamics", Butterworth-Heinemann, 2005.
- [8] Automotive Engineer, "Renault steers Laguna upmarket", October 2007, page 53, Professional Engineering Publishing Ltd., Londen.