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Analysis of Antennas in the Presence of Large Composite 3-D Structures with Linear Embedding via Green’s Operators (LEGO) and a Modified EFIE

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Abstract—We combine the linear embedding via Green’s operators method with an electric field integral equation (EFIE) to solve the problem of an antenna system which radiates in close proximity of a large 3-D structure. Upon rearranging the relevant equations we include the contribution of the large structure into the EFIE posed over the antenna surface — which results in a “modified” EFIE. The latter can be solved by MoM and direct methods as long as the antenna system is not too large. Therefore, the present approach is superior to the bare MoM, which (for the same problem) would yield a huge matrix to be inverted through iterative methods. We provide validation of the proposed strategy as well as an example of application to a real-life antenna operating nearby a finite frequency selective surface.

I. BACKGROUND AND OVERVIEW

Thanks to the availability of modern powerful and fast computers, nowadays the solution of large (say, ten or more wavelengths in diameter) scattering and radiation problems has become a common task for which indeed many numerical methods have been devised over the past decades. A widely popular approach, based on integral equations, is the Multi-Level Fast Multipole Algorithm (MLFMA) [1], [2]. Used together with the Method of Moments (MoM) [3], the MLFMA allows one to fill the relevant MoM matrix fast and to account efficiently for both large and small scale details of a given object. When applied to large, complex and arbitrarily shaped structures, though, the MLFMA may lead to algebraic systems with millions of unknowns and full matrices, and hence one has to resort to iterative solvers. What’s more, in cases where the structure of interest — albeit large — is mostly comprised of many identical objects arranged in a regular repetitive pattern, the flexibility of the MLFMA is quite an overkill.

For such problems, a far better strategy is one that can take advantage of the inherent “discrete” nature of the structure as well as of its (finite) translational symmetry, if any. Methods following this line of thought are: The Synthetic Function eXpansion (SFX) [4], [5], the Characteristic Basis Function Method (CBFM) [6], [7], the Equivalence Principle Algorithm (EPA) [8] along with its improved version the Tangential- Equivalence Principle Algorithm (T-EPA) [9], and the Linear Embedding via Green’s Operators (LEGO). The latter was first proposed for 2-D electromagnetic band-gap (EBG) devices in [10] and recently fully extended to 3-D problems [11]–[15] by these authors.

Although notable differences exist among the numerical techniques listed above, nevertheless they may be termed domain decomposition methods (DDMs). In fact they all share the basic idea of dividing a large 3-D structure into a number of (possibly interconnected) sub-domains, on which sets of locally entire-domain functions are introduced to expand the unknown. Since normally the total number of functions so defined happens to be far smaller than the original number of sub-domain functions required by the MoM, the corresponding algebraic system matrix can be inverted using direct solvers.

In this paper we address the problem of a perfect electric conducting (PEC) multiport antenna system that is to radiate in the presence of a large 3-D composite structure. The latter, sketched in Fig. 1 along with a two-port antenna, may be a frequency selective surface (FSS), an EBG structure or a high impedance surface, to name but a few practical applications. As anticipated, the electromagnetic (EM) problem in Fig. 1 is more efficiently treated by means of a DDM, such as LEGO, rather than the MLFMA. In LEGO [12] we tackle the EM scattering by an aggregate of $N_D$ bodies (immersed in a homogeneous host medium) by enclosing each object within an arbitrarily-shaped bounded domain $D_k$ (brick), $k = 1, \ldots, N_D$ (see Fig. 1). Then, by invoking Love’s Equivalence Principle [16], we characterize the bricks electromagnetically by means of scattering operators $S_{kk}$, which we subsequently combine in the total inverse scattering operator $S^{-1}$ of the structure. In
the extension to be discussed, we also need to pose an electric field integral equation (EFIE) [3] on the antenna surface \( S_A \) to be solved for the current density \( J_A \).

Unlike the scattering problems we considered in previous papers [12], [15] (where the incident fields were radiated either by sources at infinity or by elemental dipoles) in Fig. 1 a real antenna acts as a source for the structure. However, since we are interested in structures comparatively large with respect to the antenna, we formulate the problem so as to incorporate the effect of the structure into the EFIE. For this reason we call the resulting equation a “modified” EFIE. A key step in our approach is the usage of LEGO and the eigencurrent expansion method (EEM) [12], [15] for determining the contribution of the structure. The overall procedure turns out convenient chiefly because, on the one hand, the contribution of the large structure (the calculation of which may be time consuming) has to be computed only once, whereas, on the other hand, the modified EFIE can be solved with the MoM and \( LU \) factorization, provided the antenna system is not too large.

The rest of the paper is organized as follows. The modified EFIE and its numerical solution are described in Section II and III, respectively. In Section IV we first discuss the validation of the numerical code we developed and then we use it to analyze a two-port antenna system radiating in the presence of a finite FSS. An \( \exp(\text{j} \omega t) \) time variation is assumed for fields and sources, and it is implied throughout.

II. FORMULATION WITH LEGO AND A MODIFIED EFIE

To tackle the EM problem in Fig. 1, we begin by applying LEGO [12] to the large composite structure. As a result, we end up with an integral equation involving the total inverse scattering operator \( S^{-1} \), viz.,

\[
S^{-1} q^i = q^i, \quad (q^s)_k = \hat{q}_k^s, \quad \hat{q}_k^s = \left[ \frac{J_k^i \sqrt{\eta} }{ -M_k^s / \sqrt{\eta} } \right], \tag{1}
\]

where \( \eta = \sqrt{\mu / \varepsilon} \) is the intrinsic impedance of the host medium. The column vectors \( \hat{q}_k^s \) represent (equivalent) scattered and incident current densities on either side of \( \partial \mathcal{D}_k \).

Secondly, on forcing the total tangential electric field to vanish over the antenna surface \( S_A \), we arrive at the EFIE

\[
\begin{bmatrix} E^s + E^s + \sum_{k=1}^{N_D} E^s \end{bmatrix}_{\text{tan}} = 0, \quad \text{on } S_A, \tag{2}
\]

where \( E^s \) is the known incident field due to delta-gap sources [17] existing at each antenna port, \( E^s \) is the scattered field radiated by \( \hat{q}_k^s \) on \( \partial \mathcal{D}_k^s \), and \( E^s \) is the scattered field produced by the current \( J_A \) on \( S_A \), viz.

\[
E^s_{\text{tan}} = \eta \mathcal{L}_A J_A, \tag{3}
\]

with \( \mathcal{L}_A \) the usual EFIE operator [3] normalized to \( \eta \). To specify the EM problem fully we have to supplement (1)-(3) with the coupling relations

\[
q^s_k = \sqrt{\eta} (P_{kk})^{-1} P_{kA} J_A, \quad [E^s]_{\text{tan}} = \sqrt{\eta} P_{sA} q^s_k, \tag{4}
\]

which express the mutual interaction between the antenna and the large structure in Fig. 1. We refer to the operators \( P_{kk} \), \( P_{kA} \) and \( P_{sA} \) as propagators, because in general they link current densities on a surface to tangential fields on another surface. For instance, \( P_{kk} \) (explicitly given in [12, Table II]) relate \( q^s_k \) to the incident tangential fields on \( \partial \mathcal{D}_k^s \). Similarly, \( P_{kA} \) and \( P_{sA} \) represent propagators from \( S_A \) to \( \partial \mathcal{D}_k \) and vice-versa. In particular, \( P_{kA} (P_{sA}) \) is a \( 2 \times 1 \) (\( 1 \times 2 \)) abstract matrix whose elements are integro-differential operators involving the dyadic Green’s function of the background medium [3].

Now, by plugging (3), (4) into (1), (2) and eliminating \( q^i \) we obtain the following modified EFIE

\[
\eta [L_A + L_S] J_A = -[E^s]_{\text{tan}}, \tag{5}
\]

where the operator \( L_S \) rigorously captures the effect of the fixed large structure. By virtue of (1), (4) we can express \( L_S \) formally as

\[
L_S = \Theta_{AS} S T_{SA}, \tag{6}
\]

where \( \Theta_{AS} \) and \( T_{SA} \) are a row and a column vector, respectively, with entries

\[
(\Theta_{AS})_k = P_{kA}, \quad (T_{SA})_k = (P_{kk})^{-1} P_{kA}. \tag{7}
\]

Notice that to derive \( S^{-1} \) in (1) and \( T_{SA} \) in (7), we have to compute \( S_{kk} \) and \( P_{kk} \) only once if the bricks that model the large structure are all identical to one another.

III. SOLUTION THROUGH MoM AND EEM

We solve (5) via standard MoM (in Galerkin’s form) combined with the EEM. To this end, we model the surfaces of the \( N_D \) bodies and the bricks and the antenna by means of 3-D triangular-facet meshes with which we associate suitable sets of Rao-Wilton-Glisson (RWG) [18] vector elements. Specifically, we introduce \( N_O \) (2\( N_F \)) RWG functions on a body’s (a brick’s) surface for representing \( q^s_k \) (\( q^s_A \)) as in [12], plus \( N_A \) functions on \( S_A \) for expanding \( J_A \), viz.

\[
J_A = \sum_{l=1}^{N_A} h_l l_l. \tag{8}
\]

Then, by using (6), plugging (8) into (5) and testing it against \( \{h_l\}_{l=1}^{N_A} \) through a symmetric inner product we obtain

\[
\eta [(L_A + L_S)] [I_A] = -[E^s], \tag{9}
\]

\[
[L_S] = [\Theta_{AS}] [S] [T_{SA}], \tag{10}
\]

where with transparent notation each matrix represents the algebraic counterpart of the corresponding operator, and \( [I_A] \) is a column vector storing \( l_l \). Finally, \( [E^s] \) is a column vector with entries \( \langle [E^s] \rangle_l = \int_{S_l} \bar{d}^2 r \cdot h_l \cdot E^s \), where \( \bar{d}^2 r \) denotes the area element and the integration is carried out on the pair of adjacent triangles which constitute the support of \( h_l \). Once (9) has been solved, we go on to compute the scattered currents coefficients from

\[
[q^s] = \sqrt{\eta} [S] [T_{SA}] [I_A], \tag{11}
\]

which ensues from (1) and the first of (4).
From (10), (11) we see that the calculation of $[L_S]$ and $[q]$ entails the inversion of $[S]^{-1}$, i.e., the algebraic counterpart of $S^{-1}$. As pointed out in [12], $[S]^{-1}$ (of size $2N_F N_D \times 2N_F N_D$) may be relatively large when $N_D \gg 1$. Therefore we employ the EEM to reduce $[S]^{-1}$ to a far smaller matrix which we can invert via LU factorization: We refer the reader to [12, Section IV] for the details. For the sake of clarity we just repeat the result, namely, the expression of $[S]^{-1}$ in the basis of the eigencurrents

$$[\hat{S}]^{-1} = \begin{bmatrix} [\hat{S}_{CC}]^{-1} & [\hat{S}_{CU}]^{-1} \\ [\hat{S}_{UC}]^{-1} & [\hat{S}_{UU}]^{-1} \end{bmatrix} \approx \begin{bmatrix} [\hat{S}_{CC}]^{-1} & [0] \\ [0] & [\Lambda_{UU}]^{-1} \end{bmatrix},$$ (12)

where the subscript C (U) stands for coupled (uncoupled). Only $N_C N_D$ eigencurrents out of the grand total $2N_F N_D$ happen to be coupled and take part in the multiple scattering that occurs amongst the bricks. This contribution is captured by the reduced inverse scattering operator $[\hat{S}_{CC}]^{-1}$ of rank $N_C N_D$. The remaining eigencurrents are uncoupled, as they do not interact with one another nor with any coupled eigencurrent, and hence their contribution amounts to the diagonal matrix $[\Lambda_{UU}]^{-1}$. The latter contains the reciprocal of the eigenvalues required to expand the number of functions $N_F$ required to expand $J_A$ is likely not too large: As a result (9) can be inverted with direct methods.

IV. NUMERICAL RESULTS

We have further extended our code to implement (9). In order to validate the results, we have considered, among others, the simple antenna problem shown in Fig. 2(a). The multi-port antenna consists of $N_P = 2$ half-wavelength dipoles ($\ell = 1$ m, width $w, w/L = 0.1$) which irradiate two nearby PEC spheres (radius $a, a/L = 0.25$). We energize the system by delta-gap sources located at the center of each dipole.

For this problem we obtain a reference solution through a standard EFIE which we solve for the electric current density, say $J$, by the MoM with 717 RWG functions. Secondly, we apply LEGO/EEM by embedding the spheres within $N_D = 2$ cubic bricks, as shown in Fig. 2(b). The number of RWG functions on each sphere is $N_O = 294$, whilst $2N_F = 1800$; the rank of $[L_A] + [L_S]$ in (9) is $N_A = 30$. We set the number of coupled eigencurrents to $N_C = 30$, thereby the size of the reduced inverse scattering operator $[\hat{S}_{CC}]^{-1}$ is $N_D N_C = 60$.

As we force the input voltages and (from $J_A$, $J_A$) we compute the currents flowing in each antenna port, the natural set of network parameters we can derive is the matrix of short-circuit admittances, $Y_A$. From $Y_A$ we pass to the scattering matrix $S_A$ [19] with reference impedances $Z_R = 50 \Omega$ at each port. In Fig. 3 we have plotted (as a function of the electric length $L/\lambda_0$) the reflection ($S_{11}$) and the transmission ($S_{21}$) coefficients computed through LEGO/EEM (*) and the standard EFIE ($\sim \sim \sim$). The comparisons turn out excellent, thus confirming the validity of the approach based on the modified EFIE (9) combined with (14).

As an example of application to real-life structures, we have probed a finite FSS [20] by means of two folded trapezoidal toothed log-periodic antennas (LPAs) [17]. Both the FSS and the LPAs (shown in Fig. 4) are immersed in free-space. The FSS is made of 200 infinitely thin PEC cross-dipoles (length $L = 0.015$ m, width $w, w = L/6$). The latter are distributed over two parallel planes (distance $s/L = 5/12$) on which they
are arranged in a regular 10-by-10 rectangular lattice. Each LPA consists of 12 dipoles arranged in two arms which are tilted to form an angle of 40 degrees. To apply LEGO/EEM we pair adjacent cross-dipoles that lie on different planes tilted to form an angle of 40 degrees. To apply LEGO/EEM are arranged in a regular 10-by-10 rectangular lattice. Each 

![Image of FSS and antennas](image)

Fig. 4. Application example: a finite FSS (comprised of two layers of cross-dipoles) is probed by means of two folded log-periodic antennas.

![Image of close-up of FSS and LEGO brick](image)

Fig. 5. For defining the FSS in Fig. 4: close-up of (a) a pair of cross-dipoles forming the unit cell and (b) the enclosing LEGO brick.

We have computed $S_A$ for the LPAs in the presence of the FSS for 26 frequency samples evenly distributed from 7.5 to 10 GHz and for the separation $d/L \in \{40/3, 8, 20/3\}$ between the antennas. For reference we have also obtained $S_A$ in the case when the FSS is removed. The entries of $S_A$, namely, $S_{11}$ and $S_{21}$ are compared in Fig. 6.

![Graphs of scattering parameters](image)

Fig. 6. Scattering parameters ($|S_{11}|$, $|S_{21}|$) of the system in Fig. 4 versus the cross-dipole electric length $L/\lambda_0$ and for different separations $d$ of the LPAs: ($-$) LPAs without the FSS, ($-$-) LPAs in the presence of the FSS, ($\cdots$) 10% reflected power threshold. Inset: sketch of the FSS and the LPAs.

Apparently, the LPAs — which were designed to be well matched in the frequency band of interest — are not notably affected by the FSS in the range $L/\lambda_0 \in [0.375, 0.425]$, regardless of their reciprocal distance. This suggests that for the very same frequencies the FSS is practically transparent. To some extent, this behavior may be justified upon considering the transmitted and back-scattered radar cross section (RCS) of the FSS shown in Fig. 7. In fact, it is seen that the FSS exhibits a minimum of reflection for $L/\lambda_0 \approx 0.395$.

Conversely, in the upper part of the frequency band, the degree of matching strongly depends on how far the LPAs are located away from the FSS. Contrary to expectation, though, a better matching does not result in an increase in transmitted power, and especially so for the most distant configuration ($d/L = 40/3$). What’s more, a better matching (which means that less power gets reflected back at the LPAs ports) seems at odds with the larger back-scattered RCS of the FSS in the same band (see Fig. 7). This observation, however, may not completely fit to the case at hand, for the LPAs are not in the far field region of the FSS. Therefore, as an explanation to the conspicuous drop in the combined level of $|S_{11}|$, $|S_{21}|$ at odds with the larger back-scattered RCS of the FSS in the same band (see Fig. 7). In fact, it is seen that the FSS exhibits a minimum of reflection for $L/\lambda_0 \approx 0.395$.

We conclude mentioning that determining $S_A$ at a given
frequency for the FSS-LPAs problem required less than 41 minutes on a Linux-based x86_64 workstation endowed with an Intel Xeon 2.66-GHz processor and 8-GB RAM. Of the total computational time only \( \approx 8 \) minutes were used to fill \( [S_{CC}]^{-1} \), thanks to the advantage achieved by exploiting the finite translational symmetry of the FSS, as in [12], [15]. Instead, a sizable fraction of the time was spent in filling \( [\Theta_{AC}] \) and \( [T_{CA}] \), because we had to handle the interaction of each brick (recall \( N_D = 100 \)) with the antennas and vice-versa separately.

V. CONCLUSION

We have discussed a strategy (based on LEGO and a modified EFIE) to deal with antennas radiating in the presence of large 3-D composite structures. The key step is to include the effect of the structure into the EFIE on the antenna surface. From a numerical viewpoint the procedure stands out as a valid alternative to the bare MoM for both flexibility and time requirements. In fact, our approach applies to composite structures comprised of PEC and penetrable bodies as well, while our modified EFIE can be solved by MoM and direct methods. Further extension to structures comprising PEC bodies immersed in a finite dielectric medium is well under way.

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