In this paper, we analyze the impact of mergers of consumer cooperatives on consumers. We build a game-theoretic model of a supply chain where multiple suppliers produce a product to maximize their profits subject to capacity constraints, and multiple consumer cooperatives procure this product at the quantity that maximizes their member-consumers’ utilities. We show that as consumer cooperatives merge and the number of cooperatives decreases, each cooperative’s buyer power increases, and hence the market price decreases. Despite driving the market price down, interestingly, mergers of consumer cooperatives do not always benefit consumers. Specifically, consumer utility is unimodal in the number of cooperatives, so there is an optimal number of cooperatives. This is because when the number of cooperatives is below a certain threshold, suppliers reduce production too much, and this in turn harms consumers. Therefore, although consumer cooperatives aim to benefit consumers, they may end up harming consumers when they merge “too much.” The only exception to this result is when suppliers have tight capacities, in which case consumers always benefit from mergers of cooperatives. More generally, we show that the optimal number of cooperatives is non-decreasing in suppliers’ production capacity.

Key words: Supply Chain Management; Competition; Game Theory; Not-for-Profit.
Retail Services have merged in the UK in 2002, Royal Friesland Foods and Campina have merged in the Netherlands in 2008, and Farmers Ranchers Coop and Central Valley Ag have merged in Nebraska in 2020. These mergers have motivated us to analyze the impact of cooperative mergers on their member-consumers’ utilities.

For this analysis, we develop a game-theoretic model where suppliers maximize their profits subject to capacity constraints, and cooperatives maximize their member-consumers’ utilities. We model the market between suppliers and cooperatives using the market-game mechanism proposed by Korpeoglu et al. (2020a). The market-game mechanism captures each supplier’s seller power and each cooperative’s buyer power where the market price is determined based on production decisions of suppliers and procurement decisions of cooperatives. By using our market-game model, we ask the following two questions. First, what is the impact of the number of cooperatives on consumer utility? The answer to this question is important because as cooperatives merge, the number of cooperatives decreases. Second, how does the answer to the first question change based on the suppliers’ characteristics; specifically, whether the production capacity is tight or not?

We show that as cooperatives merge and the number of cooperatives decreases, the market price decreases. This result is somewhat intuitive because mergers of cooperatives increase cooperatives’ buyer power, which drives down the market price. When the market price is below a certain threshold, suppliers reduce production because the business is not as lucrative as it was before. This reduction in production in turn harms consumers. Therefore, we show that consumer utility is unimodal in the number of cooperatives, so consumer utility first increases and then decreases as cooperatives merge. This result shows that although consumer cooperatives aim to maximize consumer utility, they may end up harming consumer utility when they merge “too much.” Thus, there must be a healthy balance between cooperation and competition among consumer cooperatives. We show that the only exception to this result is when suppliers have tight capacities, in which case consumers always benefit from mergers of cooperatives.

Related Literature

Our paper is related to cooperative studies in the socially responsible operations literature, the literature on mergers in supply chains, and the market-game literature.

Cooperative studies in the socially responsible operations literature mainly focus on farmer cooperatives. For example, Chen et al. (2013) study a farmer cooperative where farmers share information about crop prices and weather forecast with other farmers. An et al. (2015) analyze whether aggregating smallholder farmers is beneficial to farmers in terms of reducing production costs, price uncertainty, increasing yield, and brand awareness. Qian and Olsen (2020) examine the coordination of operational and financial decisions of a farmer cooperative in New Zealand.
While these papers study farmer cooperatives and consider a single cooperative, we study consumer cooperatives and allow for multiple cooperatives. The exception to the above papers and the closest study to ours is by Korpeoglu et al. (2020b) who study the entry of a not-for-profit food cooperative and its competition with a for-profit retailer. While Korpeoglu et al. (2020b) abstracts away from production and considers a single cooperative, we consider production and the strategic interactions between multiple consumer cooperatives and multiple suppliers.2

Another stream of related literature is the literature on mergers in supply chains (e.g., Gupta and Gerchak 2002, Lin et al. 2014, Xiao 2020). The closest paper in this literature to ours is Cho (2014) who studies horizontal merger of two firms that engage in quantity competition and face deterministic demand. While this paper focuses on profit-maximizing firms, we consider consumer cooperatives that aim to maximize consumer utility, and we also consider capacity constraints for suppliers. Similar to Cho (2014), cooperatives in our model face deterministic demand and they engage in quantity competition because quantity competition is a good fit for many industries including food (Corbett and Karmarkar 2001, Cabral 2000, Korpeoglu et al. 2020a).3 Yet, while Cho (2014) employs the Cournot model to capture quantity competition, we employ the market-game model because the market-game model captures both suppliers’ seller power and cooperatives’ buyer power, whereas the Cournot model captures only suppliers’ seller power (cf. Korpeoglu et al. 2020a).

Our paper is also related to the market-game literature. This literature focuses on interactions among traders without considering production (e.g., Peck et al. 1992, Goenka 2003) with a few exceptions. Dubey and Shubik (1977) show the existence of an equilibrium under the market-game model with production, and Chen et al. (2017) show that a firm’s market share and profit margin are positively correlated. The closest studies to our paper are Spear (2003) and Korpeoglu et al. (2020a). Spear (2003) analyzes the electricity market between electricity suppliers and consumers without considering consumer cooperatives. Korpeoglu et al. (2020a) build a market-game framework for supply chains and consider profit-maximizing retailers, whereas we consider non-profit cooperatives. Moreover, while suppliers in their model do not face capacity constraints, suppliers in our model face capacity constraints. Our results show that these modeling differences play a crucial role in determining policy insights. We contribute to the market-game literature by studying non-profit consumer cooperatives and by explicitly modeling production decisions of suppliers.

2 Our paper is also broadly related to group-buying literature (see, for example, Hu et al. 2013). While group buying papers consider a profit-maximizing firm that sells to consumers only when the total number of committed purchases exceeds a certain threshold, our paper considers non-profit cooperatives that do not require the number of committed purchases exceed a certain threshold.

3 Other industries that are good fit for quantity competition are automotive, apparel, consumer appliances, electronic equipment (Corbett and Karmarkar 2001), wheat, cement, steel, cars, computers, video games, airlines, and microchips (Cabral 2000).
2. Model

Consider a model where $C$ consumer cooperatives procure a product from $P$ suppliers on behalf of their member-consumers. Below, we elaborate on our model of suppliers, consumers, and cooperatives.

**Suppliers.** Each supplier $p \in \{1, 2, \ldots, P\}$ produces $o_p$ units of output (hereafter, production quantity) by incurring a cost $\psi(o_p)$, which is increasing and convex. We assume that $\psi'(o_p)o_p$ is also an increasing and convex function. This assumption is satisfied by many convex functions such as power functions (i.e., $\psi(o_p) = o_p^a$ for $a \geq 1$) and exponential functions (i.e., $\psi(o_p) = \exp\{ao_p\}$ for $a > 0$). Each supplier $p$ charges a price $r$ for each unit he sells, and hence its profit is $\pi(o_p) = ro_p - \psi(o_p)$. Each supplier $p$ faces a capacity constraint that restricts its production quantity $o_p$ to a capacity $\kappa$; i.e., $o_p \leq \kappa$.

**Consumers and cooperatives.** There is a continuum of consumers with measure normalized to 1. Each cooperative $c \in \{1, 2, \ldots, C\}$ contains $N(= 1/C)$ measure of member-consumers and places an order $q_c$ (hereafter, order quantity) to procure on behalf of its members. We refer to $N$ as the cooperative size. Each cooperative $c$ equally allocates its order quantity $q_c$ among her members, and pays $b_c$ (hereafter, *procurement budget* as defined in Korpeoglu et al. 2020a) for her order. Each consumer has a fixed income $I$, and receives utility $u_1$ from the consumption of the product sold by suppliers and receives utility $u_2$ from the remaining income which she uses to procure other products. Utility functions $u_1$ and $u_2$ are increasing and concave. Thus, the utility of a consumer in cooperative $c$ is $U_c = u_1\left(\frac{q_c}{N}\right) + u_2\left(I - \frac{b_c}{N}\right)$, where $\frac{q_c}{N}$ denotes a consumer’s consumption of the product sold by suppliers, and $I - \frac{b_c}{N}$ denotes her remaining income.$^4$

**Market game.** To capture the market between suppliers and cooperatives, we utilize the market-game framework of Korpeoglu et al. (2020a). In the market game, the market-clearing price $r$ is determined based on simultaneous actions of consumer cooperatives and suppliers. Specifically, the price $r$ is determined by the ratio of the total procurement budget $B = \sum_{c=1}^{C} b_c$ to the total production quantity $O = \sum_{p=1}^{P} o_p$:

$$
r \equiv \frac{B}{O} = \frac{\sum_{c=1}^{C} b_c}{\sum_{p=1}^{P} o_p}.
$$

Hence, the market-clearing price $r$ increases with procurement budgets, and decreases with production quantities. Note that in the market-game mechanism, all suppliers sell and all cooperatives procure at the same market-clearing price $r$ because there is a single product, and all suppliers and cooperatives are identical.

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$^4$ Here, $u_2$ can be considered as an indirect utility function of a bundle of products that a consumer consumes given her remaining income $I - \frac{b_c}{N}$. When a consumer’s utility function is additively separable, increasing, and concave in each of the products in the bundle, the resulting indirect utility function is also increasing and concave in $I - \frac{b_c}{N}$.
Each cooperative \( c \)'s order quantity \( q_c \) is determined as the ratio of its procurement budget \( b_c \) to the price \( r \) as follows:

\[
q_c = \frac{b_c}{r} = \frac{b_c}{B O} = \frac{b_c}{b_c + B_{-c} O},
\]

(2)

where \( B_{-c} \) denotes the total procurement budgets of cooperatives other than cooperative \( c \). Similarly, \( O_{-p} \) denotes the total production quantity of suppliers other than supplier \( p \). Note that each cooperative \( c \) can place an order \( q_c \) by paying \( q_c B_{-c} - q_c O \). Thus, for any given \( B_{-c} \) and \( O \), there is a one-to-one mapping between \( q_c \) and \( b_c \), and hence we can represent each cooperative \( c \)'s action in terms of her procurement budget \( b_c \). Finally, because the total order quantity is equal to the total production quantity, the market between suppliers and cooperatives clears as follows:

\[
Q = \sum_{c=1}^{C} q_c = \sum_{c=1}^{C} \frac{b_c}{BO} = O = \sum_{p=1}^{P} o_p.
\]

(3)

The sequence of events is as follows. Cooperatives determine their procurement budgets, and suppliers determine their production quantities simultaneously.\(^5\) Then, cooperatives receive their order quantities, and suppliers receive their payments.

**Equilibrium.** We derive Nash equilibrium of this game by solving each cooperative’s and supplier’s optimization problems and by deriving their best-response functions. For the rest of the paper, we denote equilibrium (and best-response) values by “\( \hat{\cdot} \”). For example, we denote cooperative \( c \)'s equilibrium procurement budget by \( \hat{b}_c \), and supplier \( p \)'s equilibrium production quantity by \( \hat{o}_p \).

Given the total equilibrium procurement budget \( \hat{B}_{-c} \) of cooperatives other than cooperative \( c \) and the total equilibrium production quantity \( \hat{O} \), each cooperative \( c \) chooses her procurement budget \( b_c \) to maximize the total utility of its members given as:

\[
NU(b_c) = N \left[ u_1 \left( \frac{q_c}{N} \right) + u_2 \left( I - \frac{b_c}{N} \right) \right] = N \left[ u_1 \left( \frac{1}{N} \frac{b_c}{b_c + \hat{B}_{-c} \hat{O}} \right) + u_2 \left( I - \frac{b_c}{N} \right) \right],
\]

(4)

where the second equality follows from the order quantity \( q_c = \frac{b_c}{b_c + \hat{B}_{-c} \hat{O}} \). Similarly, given the total equilibrium production quantity of suppliers \( \hat{O}_{-p} \) other than supplier \( p \), and the total equilibrium procurement budget \( \hat{B} \), each supplier \( p \) chooses its production quantity \( o_p \) subject to capacity constraint \( o_p \leq \kappa \) so as to maximize its profit given as:

\[
\pi(o_p) = ro_p - \psi(o_p) = \frac{o_p \hat{B}}{o_p + \hat{O}_{-p}} - \psi(o_p),
\]

(5)

where the second equality follows from the price \( r = \frac{\hat{B}}{o_p + \hat{O}_{-p}} \).

Because the equilibrium depends on whether suppliers’ capacity constraints are tight or not, we define two types of equilibria. We define an uncapacitated Nash equilibrium as a set of actions

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\(^5\) Korpeoglu et al. (2020a) show that the simultaneous-move market game and the Stackelberg market game lead to qualitatively similar results. We choose the simultaneous-move market game to illustrate our findings because it produces cleaner results. Our main results extend to the Stackelberg market game as well.
\( \{\tilde{b}_1, \tilde{b}_2, \ldots, \tilde{b}_C, \tilde{o}_1, \tilde{o}_2, \ldots, \tilde{o}_P \} \) such that each cooperative \( c \)'s procurement budget \( \tilde{b}_c \) maximizes (4) given \( \tilde{B}_c = \sum_{i \neq c} \tilde{b}_i \) and \( \tilde{O} = \sum_{p=1}^P \tilde{o}_p \); and each supplier \( p \)'s production quantity \( \tilde{o}_p \) maximizes (5) given \( \tilde{O}_p = \sum_{j \neq p} \tilde{o}_j \) and \( \tilde{B} = \sum_{c=1}^C \tilde{b}_c \). Similarly, a capacitated Nash equilibrium is a set of actions \( \{\hat{b}_1, \hat{b}_2, \ldots, \hat{b}_C, \hat{o}_1, \hat{o}_2, \ldots, \hat{o}_P \} \) such that each cooperative \( c \)'s procurement budget \( \hat{b}_c \) maximizes (4) given \( \hat{B}_c = \sum_{i \neq c} \hat{b}_i \) and \( \hat{O} = \sum_{p=1}^P \hat{o}_p \); and each supplier \( p \)'s production quantity \( \hat{o}_p \) maximizes (5) subject to the capacity constraint \( \hat{o}_p \leq \kappa \) given \( \hat{O}_p = \sum_{j \neq p} \hat{o}_j \) and \( \hat{B} = \sum_{c=1}^C \hat{b}_c \).

3. Analysis of Mergers of Consumer Cooperatives

This section proceeds as follows. We first show the existence, uniqueness, and symmetry of a Nash equilibrium and characterize the capacitated and uncapacitated equilibria. We then analyze how consumer utility changes with the number of cooperatives. We discuss the practical implications of our results at the end of the section.

**Proposition 1.** There exists a unique pure-strategy uncapacitated Nash equilibrium, and the unique equilibrium is symmetric where each supplier produces the uncapacitated production quantity \( \tilde{o} \), and each cooperative has the uncapacitated procurement budget \( \tilde{b} \) that satisfy the following characterizing equations:

\[
\begin{align*}
    \frac{u_1'(P\tilde{o})}{C-1} & \left( \frac{P\tilde{o}}{b} \right) - u_2' \left( I - C\tilde{b} \right) = 0, \\
    \tilde{b} &= \frac{P^2}{C(P-1)} \tilde{o} \psi'(\tilde{o}).
\end{align*}
\]

Furthermore, there exists a unique pure-strategy capacitated Nash equilibrium, and the unique equilibrium is symmetric where each supplier produces the capacitated production quantity \( \hat{o} \), and each cooperative has the capacitated procurement budget \( \hat{b} \). When \( \tilde{o} \leq \kappa \), \( \hat{o} = \tilde{o} \) and \( \hat{b} = \tilde{b} \), and when \( \tilde{o} > \kappa \), \( \hat{o} = \kappa \) and \( \hat{b} \) solves:

\[
\begin{align*}
    \frac{u_1'(P\kappa)}{C^2} & \left( \frac{P\kappa}{b} \right) - u_2' \left( I - C\hat{b} \right) = 0.
\end{align*}
\]

Proposition 1 shows that when the uncapacitated equilibrium production quantity satisfies the capacity constraint, the uncapacitated equilibrium is the same as the capacitated equilibrium. When the capacity constraint is violated under the uncapacitated equilibrium, on the other hand, the capacitated production quantity is equal to the production capacity. This intuitive result helps us perform comparative statistics on the uncapacitated equilibrium and infer comparative statics on the capacitated equilibrium in the following proposition.

**Proposition 2.** The uncapacitated production quantity \( \tilde{o} \) is increasing, the capacitated production quantity \( \hat{o} \) is non-decreasing, and both uncapacitated price \( \tilde{r} \) and capacitated price \( \hat{r} \) are increasing in the number of cooperatives \( C \).
Proposition 2 shows that the uncapacitated production quantity \( \tilde{o} \) is increasing in the number of cooperatives \( C \). The intuition is as follows. When the number of cooperatives increases, a larger number of cooperatives compete for the total production quantity of suppliers. This more intense competition induces each cooperative to increase its procurement budget per consumer, which leads to a higher total procurement budget. By anticipating a higher total procurement budget, each supplier increases its production quantity to get a larger share from the higher total procurement budget. This leads to an increase in the uncapacitated production quantity; see Figure 1(a). As Figure 1(b) illustrates, if the uncapacitated production quantity \( \tilde{o} \) is below the production capacity \( \kappa \), the supplier increases the equilibrium (capacitated) production quantity \( \hat{o} \); and if \( \tilde{o} \) is larger than \( \kappa \), then the supplier keeps the equilibrium production quantity \( \hat{o} \) at the production capacity \( \kappa \). Thus, the equilibrium production quantity \( \hat{o} \) is non-decreasing in \( C \).

Proposition 2 further shows that the equilibrium price \( \hat{r} \) is increasing in the number of cooperatives \( C \). As discussed above, the total procurement budget by consumer cooperatives increases as \( C \) increases. Also, as discussed above, the equilibrium production quantity \( \hat{o} \) is non-decreasing in \( C \), so the total production quantity is also non-decreasing in \( C \). The total procurement budget increases more than the total production quantity, so the equilibrium price \( \hat{r} \) increases with the number of cooperatives \( C \); see Figure 1(c). This is somewhat intuitive because as more cooperatives compete for the total production quantity, each cooperative’s buyer power decreases, which drives the equilibrium price up. On the flip side, as cooperatives merge and the number of cooperatives decreases, each cooperative’s buyer power increases, and hence the equilibrium price decreases.

Proposition 2 shows that increasing the number of cooperatives \( C \) has two opposing effects on consumer utility: a larger production quantity by suppliers increases the consumption of the product per consumer, but larger procurement budget per consumer decreases the leftover income for the consumption of other goods. Theorem 1 shows that due to these two opposing effects,
consumer utility is unimodal in the number of cooperatives $C$, i.e., there is an optimal number of cooperatives $C^\ast$. We next present our main result.

**Theorem 1.** (a) There exists $C^\ast$ such that $\frac{\partial \hat{U}_m}{\partial C} > 0$ for all $C < C^\ast$ and $\frac{\partial \hat{U}_m}{\partial C} < 0$ for all $C > C^\ast$. (b) $C^\ast$ is non-decreasing in the production capacity $\kappa$. Furthermore, there exists $\kappa^\ast$ such that when the production capacity $\kappa < \kappa^\ast$, consumer utility is monotonically decreasing in $C$.

Theorem 1(a) shows that consumer utility is unimodal in the number of cooperatives, so there is an optimal number of cooperatives $C^\ast$; see Figure 2(a), where $C^\ast = 11$. The intuition is as follows. Mergers of cooperatives decrease the number of cooperatives, and increase each cooperative’s buyer power, and drive the equilibrium price down. By anticipating the reduction in price, suppliers reduce their production quantities. When the number of cooperatives is below a certain threshold, suppliers reduce production quantities “too much,” and this underproduction in turn harms consumers. Therefore, although consumer cooperatives aim to maximize consumer utility, they may end up harming consumer utility when they merge “too much.”

Theorem 1(b) shows that the optimal number of cooperatives $C^\ast$ is non-decreasing in suppliers’ production capacity $\kappa$; see Figure 2(b). Thus, when the production capacity $\kappa$ is tight, mergers of cooperatives always benefit consumers. The intuition is as follows. As explained in the intuition of Proposition 2, increasing the number of cooperatives has two opposing effects on consumer utility. First, a larger production quantity by suppliers increases the consumption of the product per consumer. Second, a larger procurement budget per consumer decreases the leftover income for the consumption of other goods. When the production capacity $\kappa$ is tight, as Figure 1(b) shows, the equilibrium production quantity $\hat{o}$ is equal to the production capacity $\kappa$, so the equilibrium production capacity does not change. Hence, a consumer’s consumption of the product sold by suppliers does not change. On the other hand, each cooperative’s procurement budget per consumer increases, which leaves a smaller budget for procurement of other goods. Thus, when the production capacity is sufficiently tight (i.e., $\kappa < \kappa^\ast$), decreasing the number of cooperatives is
always beneficial to consumers. Therefore, mergers of cooperatives always benefit consumers when the production capacity is tight. Theorem 1(a) and Theorem 1(b) together imply that in industries where suppliers have loose capacities, consumers benefit from a healthy balance between cooperation and competition among cooperatives; yet in industries where suppliers have tight capacities, consumers always benefit from mergers of cooperatives.

4. Conclusion

Consumer cooperatives constitute a sizable portion of the world’s GDP, and they are active in various industries. Consumer cooperatives in various countries including Switzerland, the Netherlands, the UK, and the USA merge to increase consumers’ buyer power and hence their utilities. This observation has motivated us to study the impact of mergers among consumer cooperatives on consumer utility. To study this problem, we build a model of supply chains with multiple suppliers that face capacity constraints and multiple cooperatives that aim to maximize their member-consumers’ utilities. We find that although mergers of consumer cooperatives achieve higher buyer power and drive down the market price, they may end up harming consumer utility when they merge too much. This is because when cooperatives merge too much, and hence the number of cooperatives is below a certain threshold, suppliers reduce production quantities too much, and this underproduction in turn reduces consumer utility. Therefore, consumers benefit from a healthy balance between cooperation and competition among cooperatives. The only exception to this result is when suppliers’ production capacity is tight, in which case consumers always benefit from cooperative mergers.

As a first step towards understanding the impact of cooperative mergers, we have made some simplifying assumptions. First, we assume identical cooperatives and identical suppliers for tractability, but it might be of interest to extend our results to heterogenous cooperatives and heterogenous suppliers. Second, we assume that cooperatives can gauge preferences of their members. Although this assumption is reasonable as long as cooperatives are managed by a healthy and representative portion of their members, it may be harder to achieve when cooperatives are very large. Thus, an interesting future research direction is to analyze a case where cooperatives have some uncertainty about preferences of their members. Our conjecture is that our main results would be directionally the same, but incorporating uncertainty about members’ preferences may lead to other interesting dynamics and new research questions.

References


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Appendix. Proofs

Proof of Proposition 1. Existence of equilibrium. Theorem 1.2 in Fudenberg and Tirole (1991) on page 34 proves that there exists a pure-strategy Nash equilibrium if (i) each cooperative’s and supplier’s action space is nonempty, compact, and convex subset of Euclidean space and (ii) each cooperative’s utility function is continuous and quasi-concave in its procurement budget and each supplier’s profit function is continuous and quasi-concave in its production quantity. We will prove each condition for cooperatives and suppliers, respectively.

First, each cooperative $c$’s action is its procurement budget $b_c \in \mathbb{C}_+$, which is constrained by the cooperative’s total income $NI$. Thus, its action set is $[0, NI]$, which is a non-empty, compact, and convex subset of Euclidean space. Each cooperative $c$’s utility function $NU_c$ given in (4) is continuous in $b_c$. The first derivative of $NU_c$ with respect to $b_c$ is

$$\frac{\partial (NU_c)}{\partial b_c} = u'_1 \left( \frac{1}{N} \frac{b_c}{b_c + B_c} \hat{O} \right) \frac{\hat{B} - c \hat{O}}{(b_c + B_c)^2} \hat{O} - u'_2 \left( I - \frac{b_c}{N} \right).$$

Utility functions $u_1$ and $u_2$ are concave, so $u'_1$ and $u'_2$ are decreasing functions. Moreover, $\frac{1}{N} \frac{b_c}{b_c + B_c} \hat{O}$ is increasing in $b_c$, $\frac{\hat{B} - c \hat{O}}{(b_c + B_c)^2}$ is decreasing in $b_c$, and $I - \frac{b_c}{N}$ is decreasing in $b_c$. Thus, $\frac{\partial (NU_c)}{\partial b_c}$ is also quasi-concave in $b_c$.

Second, the action of each supplier $p$ is its production quantity $o_p \in \mathbb{R}_+$, and the capacity constraint requires that $o_p \leq \kappa$. Thus, the action set of supplier $p$ is $[0, \kappa]$, and this set is a non-empty, compact, and convex subset of Euclidean space. Each supplier $p$’s profit function $\pi(o_p)$ given in (5) is continuous in $o_p$. The first derivative of $\pi(o_p)$ with respect to $o_p$ is

$$\frac{\partial \pi(o_p)}{\partial o_p} = \frac{\hat{O}_p \hat{B}}{(o_p + \hat{O}_p)^2} - \psi'(o_p).$$

Because cost function $\psi$ is convex, $\psi'$ is an increasing function, and $\frac{\hat{O}_p \hat{B}}{(o_p + \hat{O}_p)^2}$ is decreasing in $o_p$, so $\frac{\partial \pi(o_p)}{\partial o_p}$ is decreasing in $o_p$. Then, $\pi(o_p)$ is concave in $o_p$, and hence it is also quasi-concave in $o_p$. Because all conditions of Theorem 1.2 in Fudenberg and Tirole (1991) are satisfied, there exists a pure-strategy Nash equilibrium. Moreover, because each cooperative $c$ (resp., supplier $p$) has a concave utility (resp., profit) function, the Kuhn-Tucker conditions of each cooperative $c$’s and
supplier \( p \)'s optimization problems are sufficient for equilibrium. Under the standard assumptions on the consumer's utility functions, given \( \hat{O} > 0 \), each cooperative's procurement budget should be greater than zero. Thus, the first-order condition of each cooperative \( c \)'s \((\in \{1, 2, \ldots, C\})\) utility-maximization problem given in (4) is

\[
\frac{\partial (NU_c)}{\partial b_c} = u'_1 \left( \frac{1}{N} \frac{\hat{b}_c \hat{O}}{B} \right) \left( \frac{\hat{B} - \hat{b}_c \hat{O}}{B^2} \right) - u'_2 \left( \frac{1}{N} \frac{\hat{b}_c \hat{O}}{B} \right) = 0. \tag{9}
\]

The Lagrangian function of each supplier \( p \)'s \((\in \{1, 2, \ldots, P\})\) profit-maximization problem is

\[
L(o_p, \mu_p, \lambda_p) = \frac{o_p \hat{B}}{o_p + \hat{O}_p} - \psi(o_p) + \mu_p(\kappa - o_p) + \lambda_p o_p,
\]

where \( \mu_p \geq 0 \) is the Lagrange multiplier of the capacity constraint and \( \lambda_p \geq 0 \) is the Lagrange multiplier of the non-negativity constraint \( o_p \geq 0 \). The Kuhn-Tucker conditions are

\[
\begin{align*}
\frac{\partial L}{\partial o_p} &= \hat{O}_p \frac{\hat{B}}{\hat{O}^2} - \psi'(\hat{o}_p) - \mu_p + \lambda_p = 0, \forall p \in \{1, 2, \ldots, P\} \tag{10} \\
\frac{\partial L}{\partial \mu_p} &= \kappa - \hat{o}_p \geq 0, \forall p \in \{1, 2, \ldots, P\} \tag{11} \\
\frac{\partial L}{\partial \lambda_p} &= \hat{o}_p \geq 0, \forall p \in \{1, 2, \ldots, P\} \tag{12} \\
\hat{\mu}_p(\kappa - \hat{o}_p) &= 0, \forall p \in \{1, 2, \ldots, P\} \tag{13} \\
\hat{\lambda}_p \hat{o}_p &= 0, \forall p \in \{1, 2, \ldots, P\}. \tag{14}
\end{align*}
\]

**Symmetry of equilibrium.** We will prove that no asymmetric equilibrium can exist. Suppose to the contrary that there exists an asymmetric equilibrium. Then, either cooperatives’ equilibrium procurement budgets are asymmetric or suppliers’ equilibrium production quantities are asymmetric.

First, suppose that cooperatives’ equilibrium procurement budgets are asymmetric. Then, there must be at least two cooperatives (cooperatives 1 and 2 without loss of generality) such that \( \hat{b}_1 > \hat{b}_2 \).

Then, we obtain the following strict inequality

\[
u'_1 \left( \frac{1}{N} \frac{\hat{b}_1 \hat{O}}{B} \right) \left( \frac{\hat{B} - \hat{b}_1 \hat{O}}{B^2} \right) - u'_2 \left( \frac{1}{N} \frac{\hat{b}_1 \hat{O}}{B} \right) < u'_1 \left( \frac{1}{N} \frac{\hat{b}_2 \hat{O}}{B} \right) \left( \frac{\hat{B} - \hat{b}_2 \hat{O}}{B^2} \right) - u'_2 \left( \frac{1}{N} \frac{\hat{b}_2 \hat{O}}{B} \right),
\]

where \( u'_1 \) and \( u'_2 \) are decreasing functions. Thus, either \( \hat{b}_1 \) or \( \hat{b}_2 \) cannot satisfy (9), and hence cannot be an equilibrium procurement budget, which is a contradiction.

Second, suppose that suppliers’ equilibrium production quantities are asymmetric. Then, there must be at least two suppliers (suppliers 1 and 2 without loss of generality) such that \( \hat{o}_1 > \hat{o}_2 \).

In this case, (11) implies that \( \hat{o}_2 < \kappa \), so we have \( \hat{\mu}_2 = 0 \) from (13). Because \( \hat{\lambda}_2 \geq 0 \), we have \( \frac{\partial \hat{\mu}_2 \hat{\hat{B}}}{\partial o_2} - \psi'(\hat{o}_2) \leq 0 \). Similarly, because \( \hat{o}_1 > \hat{o}_2 \), we have \( \hat{\lambda}_1 > 0 \) from (12), which implies that \( \hat{\lambda}_1 = 0 \). Because \( \hat{\mu}_1 \geq 0 \), we have \( \frac{\partial \hat{\mu}_1 \hat{\hat{B}}}{\partial o_1} - \psi'(\hat{o}_1) \geq 0 \). Moreover, \( \frac{\partial \hat{\mu}_1 \hat{\hat{B}}}{\partial o_1} - \psi'(\hat{o}_p) \) is strictly decreasing in \( \hat{o}_p \),
so $\tilde{o}_1 > \tilde{o}_2$ implies that $\frac{\tilde{o}_1 - \tilde{o}_2}{\tilde{o}_1^2} - \psi'(\tilde{o}_1) < \frac{\tilde{o}_1 - \tilde{o}_2}{\tilde{o}_2^2} - \psi'(\tilde{o}_2) \leq 0$. However, $\frac{\tilde{o}_1 - \tilde{o}_2}{\tilde{o}_2^2} - \psi'(\tilde{o}_1) < 0$ contradicts with $\frac{\tilde{o}_1 - \tilde{o}_2}{\tilde{o}_1^2} - \psi'(\tilde{o}_1) \geq 0$. Therefore, the equilibrium should be symmetric.

**Uniqueness of equilibrium.** Let $\tilde{b}$ denote each cooperative’s equilibrium procurement budget and $\tilde{o}$ denote each supplier’s equilibrium production quantity. Evaluating (9) - (14) at the symmetric equilibrium (i.e., $\tilde{b}_c = \tilde{b}, \tilde{o}_p = \tilde{o}$, $\tilde{B} = C\tilde{b}, \tilde{O} = P\tilde{O}, \tilde{b}_c = (C - 1)\tilde{b}, \tilde{O} = (P - 1)\tilde{o}$) yields the following simultaneous equations for the equilibrium procurement budget $\tilde{b}$ and production quantity $\tilde{o}$

\[
\begin{align*}
\quad u'_1 (P\tilde{O}) \frac{(C - 1)\tilde{P}}{C\tilde{B}} - u'_2 \left( I - \tilde{B} \right) &= 0, \\
\frac{(P - 1)\tilde{B}}{P^2\tilde{\sigma}} - \psi'(\tilde{o}) - \hat{\mu} + \hat{\lambda} &= 0, \\
\kappa - \tilde{o} &\geq 0, \\
\tilde{o} &\geq 0, \quad \hat{\mu}(\kappa - \tilde{o}) = 0, \\
\hat{\lambda} \tilde{o} &= 0.
\end{align*}
\]

To satisfy (15), we need $\tilde{B} > 0$. Thus, from (16), as $\tilde{o}$ approaches zero, the left-hand side of (16) approaches infinity so in order for (16) to be satisfied, we need $\tilde{o} > 0$, and hence $\hat{\lambda} = 0$. In this case, the uncapacitated equilibrium will be the solution to (15) and (16) given $\hat{\mu} = \hat{\lambda} = 0$. When $\tilde{o}$ satisfies capacity constraint, i.e., $\tilde{o} \leq \kappa$, clearly the uncapacitated equilibrium solves (15) - (18).

We will show that in that case $\tilde{o} = \kappa$ cannot be an equilibrium production quantity, so the unique equilibrium production quantity $\tilde{o} = \tilde{\sigma}$. Suppose to the contrary that both $\tilde{o}_1 = \kappa$ and $\tilde{o}_2 = \tilde{\sigma}$ are equilibrium production quantities, which means that they both satisfy (15) - (18). Because $\tilde{o}_2 = \tilde{\sigma} > 0$, we have $\tilde{\lambda}_2 = 0$ from $\hat{\lambda}\tilde{o} = 0$. (17) implies that $\tilde{o}_2 = \tilde{o} < \kappa$, and hence $\hat{\mu}_2 = 0$ from $\hat{\mu}(\kappa - \tilde{o}) = 0$. Then, $\frac{(P - 1)\tilde{B}}{P^2\tilde{o}^2} - \psi'(\tilde{o}) = 0$ from (16). Similarly, since $\tilde{o}_1 = \kappa > 0$, we have $\tilde{\lambda}_1 = 0$ from $\hat{\lambda}\tilde{o} = 0$ and $\hat{\mu}_1 \geq 0$ from $\hat{\mu}(\kappa - \tilde{o}) = 0$. Then, $\frac{(P - 1)\tilde{B}}{P^2\tilde{o}^2} - \psi'(\tilde{o}_1) \geq 0$ from (16). Because $\frac{(P - 1)\tilde{B}}{P^2\tilde{o}^2} - \psi'(\tilde{o})$ is strictly decreasing in $\tilde{o}$, $\tilde{o}_2 = \tilde{o} < \kappa = \tilde{o}_1$ implies that $\frac{(P - 1)\tilde{B}}{P^2\tilde{o}_1^2} - \psi'(\tilde{o}_1) < \frac{(P - 1)\tilde{B}}{P^2\tilde{o}_2^2} - \psi'(\tilde{o}_2) = 0$. Yet, $\frac{(P - 1)\tilde{B}}{P^2\tilde{o}_1^2} - \psi'(\tilde{o}_1) < 0$ contradicts $\frac{(P - 1)\tilde{B}}{P^2\tilde{o}_1^2} - \psi'(\tilde{o}_1) \geq 0$. Thus, if $\tilde{o} \leq \kappa$, the uncapacitated equilibrium is the unique capacitated equilibrium.

Suppose $\tilde{o} > \kappa$. Then, because $\frac{(P - 1)\tilde{B}}{P^2\tilde{o}_1^2} - \psi'(\tilde{o})$ is strictly decreasing in $\tilde{o}$, $\frac{(P - 1)\tilde{B}}{P^2\tilde{o}_1^2} - \psi'(\tilde{o}) < \frac{(P - 1)\tilde{B}}{P^2\kappa^2} - \psi'(\kappa)$. Thus, $\tilde{o} = \kappa$ satisfies (16) for $\hat{\mu} > 0$, $\tilde{o} = \kappa$ is the unique production quantity in equilibrium. Because (15) yields a unique $\tilde{b}$ given $\tilde{o}$, the equilibrium with $\tilde{o} = \kappa$ is the unique equilibrium. ■

**Proof of Proposition 2.** The uncapacitated equilibrium solves the following equations derived from (15) and (16):

\[
\begin{align*}
u'_1 (P\tilde{O}) \frac{(C - 1)\tilde{P}}{C\tilde{B}} - u'_2 \left( I - \tilde{B} \right) &= 0, \\
\frac{P^2}{P - 1}\tilde{B} &= \tilde{\sigma} \psi'(\tilde{o}).
\end{align*}
\]
Plugging (20) into (21) yields:

$$\Omega(\overline{c}, C, P) \equiv u'_1(P\overline{c})(C-1)(P-1)PC\psi'(\overline{c}) - u'_2\left(I - \frac{P^2\overline{c}\psi'(\overline{c})}{P-1}\right) = 0.$$  \hfill (21)

By Implicit Function Theorem, $\frac{\partial \Omega}{\partial \overline{c}} = -\frac{\partial u'_1}{\partial \overline{c}}$. Since $u'_1 > 0$, $\frac{\partial \Omega}{\partial \overline{c}} = u'_1(P\overline{c})\frac{(P-1)}{PC\psi'(\overline{c})} > 0$. Moreover,

$$\frac{\partial \Omega}{\partial \overline{c}} = u''_1(P\overline{c})\frac{(C-1)(P-1)}{PC\psi'(\overline{c})} - u'_1(P\overline{c})\frac{(C-1)(P-1)}{PC\psi'(\overline{c})^2} + u'_2\left(I - \frac{P^2\overline{c}\psi'(\overline{c})}{P-1}\right)\frac{P^2[\psi'(\overline{c}) + \overline{c}\psi''(\overline{c})]}{P-1} < 0,$$

because $u'_1 > 0$, $u''_1 < 0$, $u''_2 < 0$, $\psi' > 0$, and $\psi'' > 0$. Thus, $\frac{\partial \Omega}{\partial \overline{c}} > 0$. This will imply that if $\overline{c} < \kappa$, $\overline{c}$ is also increasing in $C$. If $\overline{c} > \kappa$, because $\overline{c}$ increases with $C$ and stays infeasible, $\overline{c} = \kappa$ as $C$ increases. Thus, $\overline{c}$ is non-decreasing in $C$.

When $\overline{c} < \kappa$, $\overline{c} = \overline{c}$ and $\overline{b} = \tilde{b}$. Thus, from (21), $\tilde{r} = \overline{b} / \overline{c} = \overline{b} / \overline{c} = P_{-1}\psi'(\overline{c})$ is increasing in $C$ because $\overline{c}$ is increasing in $C$. When $\overline{c} \geq \kappa$, $\overline{c} = \kappa$, so from (15), $\tilde{r} = \overline{b} / \overline{c}$ satisfies:

$$u'_1(P\kappa)\frac{(C-1)}{C} = \tilde{r}u'_2\left(I - \tilde{r}P\kappa\right).$$  \hfill (22)

Since $u'_1(P\kappa) > 0$ is constant in $C$, $\frac{C-1}{C}$ is increasing in $C$, and $\tilde{r}u'_2\left(I - \tilde{r}P\kappa\right)$ is increasing in $\tilde{r}$, we have $\tilde{r}$ increasing in $C$. $\blacksquare$

**Proof of Theorem 1.** In equilibrium, consumer utility $\hat{U}_c = u'_1(P\overline{c}) + u'_2\left(I - \hat{B}\right)$. We first show that $U_c$ is unimodal in $C$, i.e., that there exists $C^*$ such that $\frac{\partial U_c}{\partial C} > 0$ for all $C < C^*$ and $\frac{\partial U_c}{\partial C} < 0$ for all $C > C^*$. When $\overline{c} \geq \kappa$, $\overline{c} = \kappa$ is constant in $C$ and $\hat{B} = \tilde{r}P\kappa$ is increasing in $C$, so $\hat{U}_c$ decreases with $C$. If $\overline{c} \geq \kappa$ even for $C = 2$, then because $\overline{c}$ increases with $C$, $\overline{c} = 2$. Suppose that $\overline{c} < \kappa$ for $C = 2$. Then because $\overline{c}$ is increasing in $C$, there exists $C_0 \in (2, +\infty)$ such that for all $C < C_0$, we have $\overline{c} < \kappa$ and for all $C \geq C_0$, we have $\overline{c} \geq \kappa$. For $C < C_0$, $\overline{c} < \kappa$, so $\overline{c} = \overline{c}$ and $\overline{b} = \tilde{b}$. Thus, from (20), $\tilde{U}_c = u'_1(P\overline{c}) + u'_2\left(I - \frac{P^2\overline{c}\psi'(\overline{c})}{P-1}\right)$. The derivative of $\tilde{U}_c$ with respect to $C$ is $\frac{\partial \tilde{U}_c}{\partial C} = \phi(\overline{c})\frac{\partial \overline{c}}{\partial C}$, where $\phi(\overline{c}) \equiv u'_1(P\overline{c})P - u'_2\left(I - \frac{P^2\overline{c}\psi'(\overline{c})}{P-1}\right)p^2[1 + \psi'(\overline{c})]$. Noting that $\overline{c}\psi'(\overline{c})$ is convex, $\psi'(\overline{c}) + \overline{c}\psi''(\overline{c})$ is increasing in $\overline{c}$. Also $u'_1(P\overline{c})P$ is decreasing in $\overline{c}$ and $u'_2\left(I - \frac{P^2\overline{c}\psi'(\overline{c})}{P-1}\right)$ is increasing in $\overline{c}$. Thus, $\phi(\overline{c})$ is decreasing in $\overline{c}$ and hence $C$. We have three cases. First, if $\phi(\overline{c}) \leq 0$ even under $C = 2$, then because $\phi(\overline{c})$ is decreasing in $C$, $\phi(\overline{c}) < 0$ for all $C > 2$. Thus, $C^* = 2$. Second, if $\phi(\overline{c}) \geq 0$ under $C = C_0$, because $\phi(\overline{c})$ is increasing in $C$, $\phi(\overline{c}) > 0$ for all $C < C_0$. Thus, $C^* = C_0$. Finally, if $\phi(\overline{c}) > 0$ even under $C = 2$ and $\phi(\overline{c}) < 0$ under $C = C_0$, because $\phi(\overline{c})$ is continuous and decreasing in $C$, there should exist $C^* \in (2, C_0)$ such that $\phi(\overline{c}) > 0$ for all $C < C^*$ and $\phi(\overline{c}) < 0$ for all $C > C^*$. Because $\frac{\partial \hat{U}_c}{\partial c} = \phi(\overline{c})\frac{\partial \overline{c}}{\partial C}$ and $\frac{\partial \overline{c}}{\partial C} > 0$, we have $\frac{\partial \hat{U}_c}{\partial C} > 0$ for all $C < C^*$ and $\frac{\partial \hat{U}_c}{\partial C} < 0$ for all $C > C^*$.

Note that the uncapacitated equilibrium does not depend on the capacity $\kappa$. When $\kappa$ increases, it is more likely that $\overline{c} < \kappa$ so $C_0$ is non-decreasing in $\kappa$. In the first and third cases discussed above,
$C^*$ is less than $C_0$ so it does not change with $\kappa$. In the second case discussed above, $C^* = C_0$ which is non-decreasing in $\kappa$. Thus, $C^*$ is non-decreasing in $\kappa$. Finally, when $\kappa$ is sufficiently small, $\tilde{\kappa} \geq \kappa$, even under $C = 2$. Thus, as discussed above $C^* = 2$ in that case. ■