Comment on “Mechanism of Branching in Negative Ionization Fronts”

When the fingers of discharge streamers emerge from a planar ionization front due to a Laplacian instability, their initial spacing is determined by the band of unstable transversal Fourier perturbations and generically dominated by the fastest growing modes. The Letter [1] therefore aims to calculate the temporal growth rate \( s(k) \) of modes with wave number \( k \), when the electric field far ahead of the ionization front is \( E_\infty \). In earlier work [2–4], \( s(k) \) was determined in a pure reaction-drift model for the free electrons, i.e., in the limit of vanishing electron diffusion \( D_e = 0 \). For negative streamers in pure gases like nitrogen or argon, electron diffusion \( D_e > 0 \) should be included into the discharge model. This is attempted in [1] in the limit of large field \( |E_\infty| \) ahead of the front. A different, extensive analysis with different results can be found in [5]. Below we show that the expansion and calculation in [1] are inconsistent, that it does not fit the cross-checked numerical results presented in [5]. Furthermore, we find in [5] that the most unstable wavelength does not scale as \( D_e^{1/3} \) as claimed in [1], but as \( D_e^{1/4} \).

In [1], ionization fronts are only considered in the limit \( |E_\infty| \gg 1 \) ahead of the front which amounts to a saturating impact ionization cross section \( \alpha(E) \rightarrow 1 \). For \( |E_\infty| \gg 1 \), planar fronts obey [[1], Eq. (7)] after all fields are rescaled with \( E_\infty \). For any finite \( E_\infty \), a diffusive layer of width \( 1/\Lambda^* = \sqrt{D_e/|E_\infty| \alpha(E_\infty)|} \) forms in the leading edge of the front [6]. (We denote the diffusion constant \( D \) from [2–6] by \( D_e \), to distinguish it from the \( D = D_e/|E_\infty| \) in [1].) Following the calculation in [1], Eq. (8) reproduces the diffusive layer for large \( |E_\infty| \), but the nonlinear term is incomplete. Then the dispersion relation is calculated by the expansion (11)–(13) about the planar ionization front. Here the expansion of the electron density \( n_e \) starts in order \( \delta^2 \) (where \( \delta \) is the small expansion parameter), while the expansions of ion density \( n_i \) and field \( E \) start in order \( \delta \). The absence of order \( \delta \) in the expansion of \( n_e \) is unexpected, not explained, and in contradiction with the calculation for \( D_e = 0 \) in [4].

Jumping to the result of [1], the dispersion relation in Eq. (21) is given as \( s = |E_\infty k|/[2(1 + |k|)] - D_e k^2 \) in the present notation. The small \( k \) limit \( s = |E_\infty k|/2 + O(k^2) \) of [[1], Eq. (21)] is consistent neither with the limit \( D_e = 0 \), where the asymptote \( s(k) = |E_\infty k| \) for \( |k| \ll \alpha(E_\infty)/2 \) was derived in [4], nor with the case \( D_e > 0 \) where the asymptote \( s = c^*|k|, c^* = Ed_e v^{-1}|E_\infty|, v(E) = |E| + 2\sqrt{D_e |E| \alpha(E)} \) was proposed in [2] and analytically confirmed in [5].

Furthermore, in [5], dispersion curves \( s(k) \) for a range of fields \( E_\infty \) and diffusion constants \( D_e \) are derived as an eigenvalue problem for \( s \); they are plotted in Fig. 1. In one case, the curve is confirmed by numerical solutions of an initial value problem; the curves are also consistent with the analytical small \( k \) asymptote. The results for positive \( s \) are conveniently fitted as \( s(k) = c^*|k|(1 - 4|k|/\Lambda^*)/(1 + a|k|) \) with \( a = 3/\alpha(E_\infty) \) [5]. Figure 1 also shows the prediction from [1] for \( E_\infty = -10 \) and \( D_e = 0.1 \); here the reduced diffusion constant \( D_e/|E_\infty| \) is as small as 0.01, and the assumptions \( |E_\infty| \gg 1 \) and \( D_e/|E_\infty| \ll 1 \) from [1] hold. However, Fig. 1 shows that the data of [5] and the prediction of [1] clearly differ. Therefore also the scaling prediction [[1], Eq. (23)] for the spacing of emergent streamers does not hold; rather our physical arguments and the numerical data in [5] suggest that the fastest growing mode is \( k_{\text{max}} = (\sqrt{1 + \alpha \Lambda^2/4} - 1)/a \approx D_e^{-1/4} \Lambda^* \gg 1 \).

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