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Numerical kernel construction for bodies of revolution with high-order Fourier modes

M.C. van Beurden¹, J.A.H.M. Vaessen², and A.G. Tijhuis³

Abstract – Boundary integral equations for bodies of revolution have a Green’s function kernel that can be written as the azimuthal integral over the free-space Green’s function. We show that existing approximation methods for computing this integral suffer from efficiency or stability problems. Improvement of one of these methods leads to controllable accuracy while retaining efficiency.

1 INTRODUCTION

Bodies of revolution (BORs) have received a lot of attention in the EM community over the past 45 years [1,2]. On the one hand, they represent a class of physically interesting scattering objects such as lens systems with a common optical axis. On the other hand, they are mathematically interesting since they allow for the comparison of a large variety of numerical methods with both local and global expansions. This observation has been exploited to construct highly accurate solutions to bridge the gap between classical canonical problems and industrially employed methods, such that the latter can be tested on a more extensive range of problems [3].

We are interested in the scattering by a perfectly conducting BOR. For this situation we formulate an electric-field boundary integral equation (BIE) that can be transformed into a system of decoupled boundary integral equations per Fourier mode in the azimuthal direction, see e.g. [2]. Further, we choose a boundary-element approach to discretize the set of integral equations. The matrix elements of the resulting numerical system are then three-dimensional integrals, one of which is an azimuthal integral over the three-dimensional free-space Green’s function.

An additional complication of the setup we are interested in is the case where the source is close to the BOR and does not exhibit rotational symmetry. For such cases, we need to take a large number of Fourier modes into account, which means that the pertaining integrals of the numerical system need to be computed for large Fourier indices.

2 EXISTING KERNEL RECONSTRUCTION METHODS

The Green’s function kernels of the above mentioned BIEs are given by

$$K(m, \sigma, \sigma') = \int_0^\pi \cos(m\alpha) \frac{\exp[-jkR(\sigma, \sigma')]}{R(\sigma, \sigma')} d\alpha$$

where R denotes the distance between source and observation point, m denotes the index of the azimuthal Fourier mode, and σ and σ' represent points at the generating curve of the BOR. Further, k denotes the wavenumber of the surrounding medium.

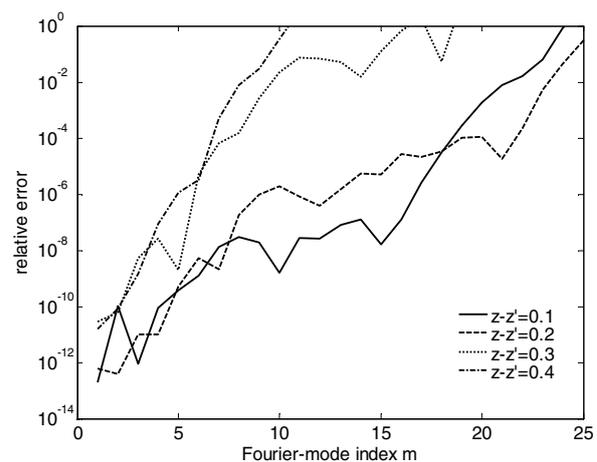


Figure 1: Accuracy of the Bartky transformation as a function of Fourier-mode index, for various distances between source and observation point.

Several quadrature methods have been proposed to approximate this kernel, most of which work well for low values of m . As an example, we discuss the Bartky transformation, see e.g. [4,5,6]. For a circular cylinder with axis of rotation along the z -axis and radius of 0.5 meter, we compute the real part of the above kernel at a wavelength of 1 meter via the Bartky transformation and compare it to the results of an adaptive quadrature

¹ Eindhoven University of Technology, Department of Electrical Engineering, P.O. Box 513, 5600 MB Eindhoven, The Netherlands, m.c.v.beurden@tue.nl, tel. +31 40 247 3976.

² ASML Netherlands B.V., De Run 6501, 5504 DR Veldhoven, The Netherlands, jean-pierre.vaessen@asml.com, tel. +31 40 268 7392.

³ Eindhoven University of Technology, Department of Electrical Engineering, P.O. Box 513, 5600 MB Eindhoven, The Netherlands, a.g.tijhuis@tue.nl, tel. +31 40 247 3800.

rule with high accuracy. Figure 1 shows the accuracy of the Bartky transformation versus Fourier-mode index m , for several distances between source and observation point ($z-z'$). In spite of the fact that we consider points away from pole singularities and source-point singularities, the Bartky transformation performs poorly for increasing Fourier-mode index. For mode indices around 15 to 20 and beyond, the results from the Bartky transformation cease to be useful for numerical implementations of boundary-element methods.

Regarding efficiency, another promising method has been put forward in [7]. It combines an FFT-based algorithm with a singularity-extraction method, where the latter is handled via a recurrence relation. Unfortunately, the stability of this method suffers from severe cancellation errors. Moreover, the coefficients of the expansion series can become very large, which increases the risk of numerical overflow.

We have also assessed several other methods, such as series expansions via special functions (e.g. [8]), with respect to accuracy, stability, and efficiency for higher Fourier-mode indices. In general, we have observed that the approximation methods perform poorly on either stability or efficiency (or both) for Fourier indices larger than 15. Moreover, we have observed that there is a delicate trade-off between the applied type of singularity extraction on one hand, and the stability and efficiency of the kernel approximation on the other.

3 STABILITY AND EFFICIENCY

To combine controllable accuracy with an efficient numerical approach, we follow the general line of thinking as described in [7]. However, we have derived a new stable recurrence relation that does not suffer from cancellation or large coefficients. Further, we employ the FFT on a modified representation [9].

For cases away from pole or source-point singularities, we reach machine precision typically within 20 function evaluations. For cases close to a singularity, we can still achieve 14 to 15 digits of accuracy with a double-precision implementation, albeit at a higher number of function evaluations that is still acceptable (around 100) for most cases.

The most important merit of our method is that we can make a real tradeoff between guaranteed accuracy and efficiency. More details will be given during the presentation.

4 CONCLUSION

We have discussed the performance of existing methods for kernel approximation for bodies of revolution. We have indicated that these methods tend to suffer from loss of accuracy for high Fourier-mode indices and we have outlined an approach to overcome this effectively, without sacrificing efficiency.

Acknowledgments

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References

- [1] Mogens G. Andreasen. Scattering from bodies of revolution. *Antennas and Propagation*, vol. 13, p. 303–310, 1965.
- [2] J.R. Mautz and R.F. Harrington. Radiation and scattering from bodies of revolution. *Appl. Sci.*, vol. 20, p. 405–435, 1969.
- [3] S.S. Vinogradov, P.D. Smith, and E.D. Vinogradova. Canonical problems in scattering and potential theory. CRC Press, London, 2001.
- [4] T.H. Fawzi and K.F. Ali. Development of Bartky's quadrature formulae and applications in BEM analysis. *IEEE Transactions on Magnetics*, vol. 21, p.2355–2358, 1985.
- [5] A.A.K. Mohsen. A fast algorithm for treating EM-scattering by bodies of revolution. *International Journal of Electronics and Communications*, vol. 3, p. 164–170, 2001.
- [6] W. Bartky. Numerical calculation of a generalized complete elliptic integral. *Review of Modern Physics*, vol. 10, p. 264–269, 1938.
- [7] Raj Mittra and Stephen D. Gedney. The use of the FFT for the efficient solution of the problem of electromagnetic scattering by a body of revolution. *IEEE Trans. Ant. Prop.*, vol. 38, p. 313–322, 1990.
- [8] Alaa. K. Abdelmageed. Efficient evaluation of modal Green's functions arising in EM scattering by bodies of revolution. *Progress In Electromagnetics Research*, vol. 27, p. 337–356, 2000.
- [9] J.A.H.M. Vaessen, M.C. van Beurden, and A.G. Tijhuis. Numerically stable kernel integration for bodies of revolution. *In preparation*.