FRF Identification Above the Nyquist Frequency

DSPE YOUR PRECISION PORTAL 2023 CONFERENCE ON PRECISION MECHATRONICS 26 & 27 September Sint Michielsgestel www.dspe.nl/conference

FRF Identification Above the Nyquist Frequency

Max van Haren[1], Leonid Mirkin[1,2], Lennart Blanken[1,3] and Tom Oomen[1]
[1] Eindhoven University of Technology, Eindhoven, the Netherlands
[2] Technion - Israel Institute of Technology, Haifa, Israel
[3] Sioux Technologies, Eindhoven, the Netherlands
m.j.v.haren@tue.nl

Extended abstract

Introduction

Fast-sampled models are essential for control design, e.g., to address intersample behavior. Identifying dynamics above the Nyquist frequency of a sensor is not possible with traditional identification techniques, but is very important for systems where the relevant dynamics are above the Nyquist frequency of the sensor, i.e., slowly-sampled systems. Examples of such are vision-in-the-loop systems and chemical processes.

In this paper, an efficient and systematic methodology for identifying the Frequency Response Functions (FRF) of fast-sampled models in a single-experiment using slow-sampled outputs, that disentangles aliased components with arbitrary input signals, is developed. First, an intuitive solution is presented, that leads to a sparse estimate of a fast-sampled model of the slow-sampled system. Second, smoothness assumptions, which is typically at the basis of modern FRF identification, of the fast-sampled system are exploited such that the system is identified for a full frequency spectrum, and aliased components are disentangled from each other.

Problem Setting and Method

In this paper, FRFs of systems are identified above the Nyquist frequency of a slowly-sampled system, where the setting is seen in Figure 1.

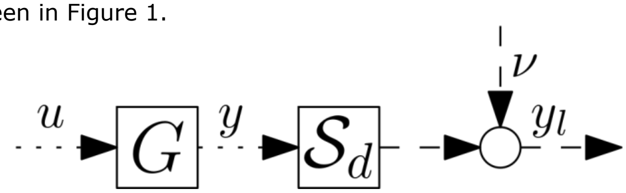

Figure 1: I*dentification setting considered, where a high-rate model of* $G(\Omega_k)$ *is to be identified, using fast actuation* u *and slowly-sampled output* y_l.

The downsampled output in the frequency-domain is a summation of aliased terms, i.e.,

$$Y_l(k) = \frac{1}{F}\sum_{f=0}^{F-1}\Big(G\big(\Omega_{k+Mf}\big)\mathrm{U}(k+Mf)\Big) + V_l(k),$$

with frequency bin k, relating to generalized frequency variable $\Omega_k = \frac{2\pi k}{NT_h}$, with fast sampling time T_h, and $M = \frac{N}{F}$, with N the amount of data of u. Due to the summation, the individual terms cannot be uniquely separated. This is visually illustrated in the left of Figure 2.

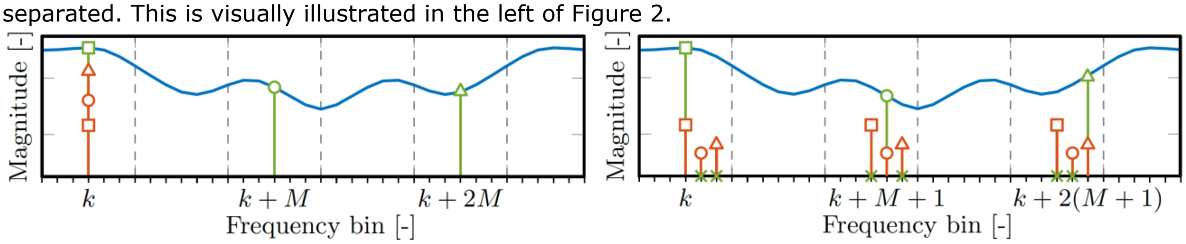

Figure 2: *Left: Frequency response of system* G *(-), that is excited at several frequencies (-), resulting in a summation of terms seen at the output at frequency bin* k *(-). Right: Intuitive solution, where several bins are not excited (x), such that there is no overlap in the output (-) and the system* G *is uniquely found back.*

However, by designing U such that the output Y for a frequency is only influenced by a single frequency of U, the aliased terms can be separated. This is illustrated in the right of Figure 2. This means that at most one out of every F frequencies is excited. This excitation signal closely relates to *zippered* multisines [1], but in contrast does take aliasing into account and hence the aliased contributions can be separated. An example of an excitation signal and the key idea is seen in the right of Figure 1.

Enabling Full Input Spectrum by Local Smoothness Assumptions
The developed method excites at most one out of F frequencies, hence the frequency resolution is significantly reduced. By assuming that the fast-sampled system G is locally smooth, identifying the system for the full frequency-spectrum is enabled. This is visually illustrated in Figure 3.

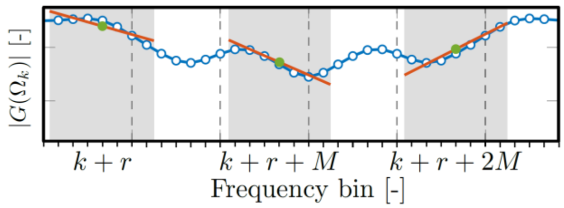

Figure 3: *Illustration of identification of slow-sampled system (•) above the Nyquist that disentangles aliased components by assuming local smoothness (-). The true frequency response is denoted as (-).*

In fact, Local Polynomial Modeling (LPM) [1] is found back as a special case of the developed framework by setting $F = 1$. The method relates to [2].

Experimental Results

The framework has been tested on a mass-spring-damper system. The setup can be seen in Figure 4.

Figure 4: *Experimental setup used for validating the developed framework. It consists of two rotating masses connected via a rubber band. The masses are both actuated by a DC motor. The right mass is virtually suspended to the fixed world by feedback control.*

The settings used during the experiment can be found in Table 1.

Property	Variable	Value
Fast sampling rate	$f_{s,h}$	120 Hz
Slow sampling rate	$f_{s,l}$	30 Hz
Downsampling factor	F	4
Measurement time	T	120 s

Table 1: *Experimental settings.*

The identified model using fast-rate data and slowly-sampled data is seen in the right of Figure 1. The fast and slow sampling rate are 120 and 30 Hz. The results in Figure 5 show that the system, including the resonance, is accurately identified above the Nyquist frequency of the sensor.

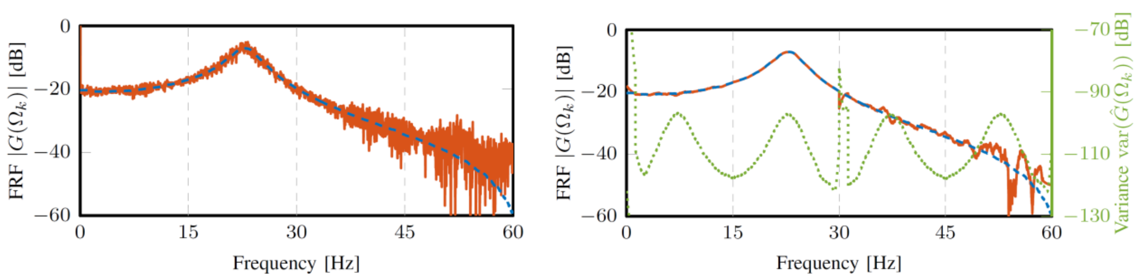

Figure 5: *Left: FRF using the intuitive approach with $\{u, y_l\}$ (-). Right: FRF using the approach with LPM (-) and variance (-). The identified FRF with $\{u, y\}$ is seen in all plots (-).*

Conclusions

The results in this paper enable identifying FRFs of slow-sampled systems where aliasing occurs. First, an intuitive approach is presented by designing a sparse excitation signal. Second, smooth behavior of the system FRF is assumed, that allows to disentangle aliased contributions when exciting the full frequency spectrum. The framework is validated through an experimental example showing promising results.

The approach developed in this paper is a key enabler in facilitating closed-loop, multivariable, and parametric system identification, as well as control design, for slow-sampled systems like vision-in-the-loop systems.

References

[1] Pintelon, R., & Schoukens, J. (2012). System identification: a frequency domain approach.
[2] Van Haren, M., Blanken, L., & Oomen, T. (2022). Frequency Domain Identification of Multirate Systems: A Lifted Local Polynomial Modeling Approach. In 2022 IEEE 61st CDC.

Nyquist frequency, Slow-Sampling, Aliasing, Frequency response function identification