

Analysis of beat phenomena during transients in pipelines with a trapped air pocket

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ANALYSIS OF BEAT PHENOMENA DURING TRANSIENTS IN PIPELINES WITH A TRAPPED AIR POCKET

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ABSTRACT

Trapped gas pockets may cause severe operational problems in liquid piping systems. The severity of the resulting transients depends on the size and position of the trapped air pocket. Previous numerical simulations by the authors have indicated that a beat is possible to develop for 'medium' size air pockets. This paper investigates the beat phenomenon in detail, both theoretically and experimentally. Trapped air pockets are incorporated as boundary conditions (discrete gas cavities) into two distinct numerical solution schemes: (1) the method of characteristic scheme (MOC) and (2) a conservative solution scheme (CSS). The *classical* discrete gas cavity model (DGCM) allows gas cavities to form at computational sections in the MOC. A discrete gas cavity is governed by the water hammer compatibility equations, the continuity equation for the gas cavity volume, and the equation of state of an ideal gas. A *novel* CSS-based DGCM solves the system of unsteady pipe flow equations and respective state equations for four dependent variables (pressure, density, cross-sectional area, flow velocity) rather than two variables (pressure, flow velocity) in the classical MOC approach. In the MOC-based DGCM, the Courant number is equal to unity. This condition is difficult to fulfil (without using interpolations) in complex pipe networks without modification of wave speeds and/or pipe lengths. The CSS-based DGCM offers flexibility in the selection of computational time and space steps, however, the numerical weighting coefficients in the scheme should be carefully selected. Both models incorporate a convolution-based unsteady friction model. Experimental investigations of beat phenomena have been carried out in the University of Adelaide laboratory apparatus (reservoir-pipeline-valve system). A trapped air pocket is captured at the midpoint of the pipeline in a specially designed device. The transient event is initiated by rapid closure of a side-discharge solenoid valve. Predicted and measured results are compared and discussed. It is shown that the fully-developed beat is strongly attenuated by unsteady friction and not so by steady friction.

KEYWORDS

Discrete gas cavity model; Conservative solution scheme; Trapped air pocket; Beat phenomena; Unsteady friction.

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1. INTRODUCTION

The control of (undesired) gas pockets is one of the major problems in liquid pipeline systems, especially in sewer pumping systems. Air may be found in water pipelines mainly as stationary pockets or moving bubbles of various sizes. Air pockets can develop in a pipeline by bubble entrainment at inflow locations and by gas release as the water pressure falls or the temperature rises. Also, air valves may contain residual air if the air discharge is not properly controlled. Transport of large pockets of air can occur during filling and emptying of pipelines. Air movement along the pipeline can be slow during filling and the air column can become trapped adjacent to a closed valve or at a high point thus separating two water columns.

Air bubbles or pockets in a liquid pipeline system, even in very small volumetric proportions, can not only significantly reduce wave propagation velocity (wave speed), but also cause a major change in the shape and attenuation of the transient pressure waves. The effects of entrapped or entrained air on surge pressures can be either beneficial or detrimental, with the outcome being entirely dependent on the characteristics of the pipeline and its hydraulic machinery, the amount and location of the air, and the nature and cause of the transients. Naturally, the influence of air is most significant in low-pressure systems. Water hammer in a pipeline that contains air may create pressure spikes that are either greater than or less than those that would occur without any air as a result of reflections from the interface between the two-phase and single-phase flows. Burrows and Qiu [1] demonstrated how the presence of air pockets could severely exacerbate surge peaks during pump shut-down in a water pipeline system. The most severe pressure rise occurs during the rapid acceleration of a liquid column towards a volume of air that is completely confined. The pressure peak can be markedly higher than the Joukowsky pressure if the transient is generated rapidly and this may cause damage to the pipeline and its fittings. On the other hand, a large air cavity may act as in effect an accumulator (air cushion) and thereby suppresses the maximum pressure excursions because of the increase in storage capacity. Numerical and experimental examples of high pressure surges associated with the presence of entrapped air in pipelines have been shown in previous research on single-component and two-component two-phase flows [2], [3], [4], [5]. Numerical simulations by the authors [6] have indicated that a beat might develop for 'medium' size air pockets.

This paper investigates the beat phenomenon in detail, both theoretically and experimentally. Trapped air pockets are incorporated as (internal) boundary conditions (discrete gas cavities) into two distinct numerical solution schemes: (1) the method of characteristic scheme (MOC) and (2) the conservative solution scheme (CSS). The *classical* discrete gas cavity model (DGCM) allows gas cavities to form at computational sections in the MOC [7], [8]. A *novel* DGCM that is based on the CSS [9] solves the system of unsteady pipe flow and state equations for four dependent variables (pressure, density, cross-sectional area, flow velocity) rather than two variables (pressure, flow velocity) in the classical MOC approach. Both models incorporate a convolution-based unsteady friction model [10]. This is essential because numerical and experimental investigations herein show that the fully-developed pressure beat is strongly attenuated by unsteady friction. Weak beat phenomena have been observed in the University of Adelaide laboratory apparatus (reservoir-pipeline-valve system) that is comprised of a straight 37.53 m long sloping copper pipe with an internal diameter of 22.1 mm. A trapped air pocket is captured at the midpoint of the pipeline in a specially designed air-pocket device. The transient event is initiated by the rapid closure of a side-discharge solenoid valve with effective valve closure time of 4 milliseconds. The paper reports the comparison of calculated and measured results.

2. TRAPPED AIR POCKET MODELLING IN TRANSIENT LIQUID PIPE FLOW

Trapped air pockets are incorporated as (internal) boundary conditions (discrete gas cavities) into the method of characteristic scheme (MOC) and the conservative solution scheme (CSS).

2.1 Method of Characteristics Scheme

The *classical* discrete gas cavity model (DGCM) is based on simplified water hammer equations (the convective transport terms are neglected). The dependent variables pressure p and flow velocity V (average velocity) are traditionally replaced by piezometric head H and discharge Q . The DGCM allows gas cavities to form at all computational sections in the MOC (internal and end boundaries). A liquid phase with a constant wave speed a is assumed to occupy the reaches connecting the computational sections. A discrete gas cavity is described by the water hammer compatibility equations, the continuity equation for the gas cavity volume, and the ideal gas equation. Their standard finite-difference form within the staggered grid of the method of characteristics is [7]:

- compatibility equation along the C^+ characteristic line ($\Delta x/\Delta t = a$):

$$H_{i,t} - H_{i-1,t-\Delta t} + \frac{a}{gA} ((Q_u)_{i,t} - Q_{i-1,t-\Delta t}) + \frac{f\Delta x}{2gDA^2} (Q_u)_{i,t} |Q_{i-1,t-\Delta t}| = 0 \quad (1)$$

- compatibility equation along the C^- characteristic line ($\Delta x/\Delta t = -a$):

$$H_{i,t} - H_{i+1,t-\Delta t} - \frac{a}{gA} (Q_{i,t} - (Q_u)_{i+1,t-\Delta t}) - \frac{f\Delta x}{2gDA^2} Q_{i,t} |(Q_u)_{i+1,t-\Delta t}| = 0 \quad (2)$$

- continuity equation for the gas cavity volume:

$$(\nabla_g)_{i,t} = (\nabla_g)_{i,t-2\Delta t} + ((1-\psi)(Q_{i,t-2\Delta t} - (Q_u)_{i,t-2\Delta t}) + \psi(Q_{i,t} - (Q_u)_{i,t})) 2\Delta t \quad (3)$$

- ideal gas equation (assuming isothermal conditions):

$$(\nabla_g)_{i,t} (H_{i,t} - z_i - h_v) = (H_0 - z_0 - h_v) \alpha_{g0} A_i \Delta x \quad (4)$$

Section 8 explains the symbols.

At a boundary (reservoir, valve) a device-specific equation replaces one of the compatibility equations. The DGCM model can be successfully used for the simulation of unsteady pure liquid pipe flow by utilizing a low gas void fraction ($\alpha_{g0} \leq 10^{-7}$) [6], [7]. The volume of trapped air pockets at any location should be less than 10% of the pipe reach volume $A_i \Delta x$. It should be noted that in the MOC-based DGCM the Courant number is held equal to unity. This condition is difficult to fulfil in complex pipe networks without applying interpolations or without modification of wave speeds and/or pipe lengths.

2.2 Conservative Solution Scheme

A *novel* DGCM is based on a conservative solution scheme (CSS), namely the Preissmann or box scheme. The gas cavity or cavities are confined at *user* defined locations. The CSS has been developed [9] to improve the accuracy and applicability of transient analysis in liquid and gas pipe networks. The conservation form of the continuity and momentum equations without neglecting convective terms has been used:

$$\frac{\partial}{\partial t}(\rho A) + \frac{\partial}{\partial x}(\rho AV) = 0 \quad (5)$$

$$\frac{\partial}{\partial t}(\rho AV) + \frac{\partial}{\partial x}(\rho AV^2 + pA) + \rho g A \sin \theta + \rho g A h_f = 0 \quad (6)$$

The system of equations is completed by adding the equations of state for fluid compressibility and pipe wall deformability:

$$\frac{d\rho}{\rho} = \frac{dp}{K} \quad \text{and} \quad \frac{dA}{A} = \frac{D}{eE} dp \quad (7) \ \& \ (8)$$

The compressibility of a slightly compressible fluid is introduced with the definition of bulk modulus of elasticity K (Eq. 7). Most transient analysis models use the assumption of linear-elastic behaviour of pipe wall, which is sufficiently accurate for describing hydraulic transients in metal or concrete pipes. The elasticity of the pipe wall and its rate of deformation are a function of pressure only (Eq. 8).

The CSS solves for four dependent variables (pressure p , density ρ , cross-sectional area A , flow velocity V) rather than two variables (piezometric head H , discharge Q) in the classical MOC approach. The CSS method directly calculates the fluid density and pipe wall deformation at every computational time step. Thus, the wave speed is updated at every step and is not used explicitly. This procedure has a significant advantage for analyzing systems with variable wave speeds. An implicit solution algorithm using finite differences (FDM) solves the system of non-linear partial differential equations (5) to (8) based on a Newton-Raphson iterative procedure. The scheme can simulate various liquid and gas transient flows by modifying the state equation for fluid density and adding the energy equation to describe heat transfer. However, the numerical weighting coefficients used in the FDM need to be selected carefully. The presentation of the complete numerical algorithm is beyond the scope of this paper and can be found in [9].

The effect of single or multiple isolated air pockets on transient pipe flows are calculated by using an accumulator boundary condition at *user* defined computational sections. The gas is assumed to follow the reversible polytropic relation [7]:

$$H_A \nabla_g^n = C_A \quad (9)$$

The polytropic exponent n can take values between 1 (isothermal, used herein) and 1.4 (isentropic). C_A is a constant whose value is determined from the initial conditions.

2.3 Unsteady friction

Traditionally the steady (or quasi-steady) friction (skin friction) terms are incorporated into the MOC and CSS algorithms. This assumption is satisfactory for slow transients where the wall shear stress has a quasi-steady behaviour. Previous investigations using the quasi-steady friction approximation for rapid transients have shown significant discrepancies in attenuation, shape and timing of pressure traces when computational results were compared with measurements [11]. A number of unsteady friction models have been proposed in the literature including one-dimensional (1D) and two-dimensional (2D) models. One-dimensional models approximate the actual 2D behaviour and corresponding viscous losses in different ways. The 2D models compute the actual cross-sectional velocity profile continuously during the water hammer event. The correct modelling of transient turbulence is the key issue in all models.

The MOC and CCS based DGCM algorithms in this paper incorporate a convolution-based (with quasi-2-D weighting function) unsteady friction model [10], [11], [12]. The unsteady frictional head loss is formulated by the following term:

$$h_f = \frac{f_0 V |V|}{2gD} + \frac{16\nu}{gD^2} \left(\frac{\partial V}{\partial t} * W_0(t) \right) \quad (10)$$

where "*" represents convolution. The subscript "0" denotes parameter values based on the steady-state condition preceding the transient event. The Darcy-Weisbach relation (first term) defines the steady-state component and the unsteady component (second term) is defined by the

convolution of a weighting function with past accelerations ($\partial V/\partial t$). The weighting function W_0 is based on an assumed steady-state viscosity distribution preceding the transient event, which is kept constant during the event (the “frozen viscosity” assumption). Zielke [10] and Vardy & Brown [12] developed weighting functions for transient laminar (where viscosity is constant anyway) and turbulent flow, respectively. Zielke evaluated Eq.(10) using the full convolution scheme, which is computationally expensive because it involves a complete history of past velocities. An accurate and efficient Vítkovský *et al.* approximate scheme [13] that does not require *convolution* with the complete history of velocities required at each time step is used in this paper.

3. PIPELINE APPARATUS

The experiments presented here provide special cases of localized air cavities at one position in a single pipeline. The layout of the pipeline system is shown in Fig.1. The test pipeline is comprised of a straight 37.53 m long copper pipe with inner diameter of 22.1 mm and a wall thickness of 1.6 mm. The pipeline is rigidly fixed to a foundation plate with a special steel construction at regular intervals along the pipe to prevent vibration during transient events and connects two electronically controlled pressurized tanks (WT and ET in Fig.1). The pressure waves are recorded by four high-resolution flush-mounted strain-gauge pressure transducers at brass blocks (WE, WM, EM, EE), and after triggering are transferred via amplifiers and 16-bit A/D converter to a personal computer with data acquisition interface based on LabVIEW software. The sampling frequency of measured data is 4 kHz. The pressure measured at the valve (WE) is presented in this paper. With a closed service valve at tank WT, transients are generated at location WE by a side-discharge solenoid valve with a fast operating time (effective valve closure time of 4 ms). The initial flow velocities were adjusted by changing the pressure in the compressed-air tank ET.

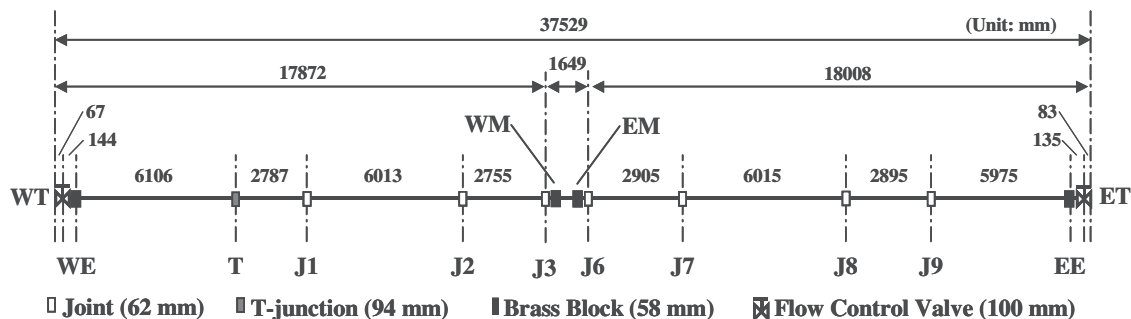


Fig.1 Laboratory pipeline system layout (T-junction is not used)

Fig.2(a) shows three screw bolt type devices for air pockets with relatively small air volumes. The screw bolts have a hole drilled in the middle. These devices were inserted into the brass block at location EM (Fig.1) as shown in Figs.2(b) and 2(c). Therefore, they can be regarded as internal boundaries. The cavity volumes of the air pocket devices were measured by using micro centrifuge tube with a conical bottom. The experimental procedure requires a careful removal of any residual air from the pipeline before the tests. Even a very small volume of air can change the phase and shape of pressure waves. The only air in the system is isolated in the special device.

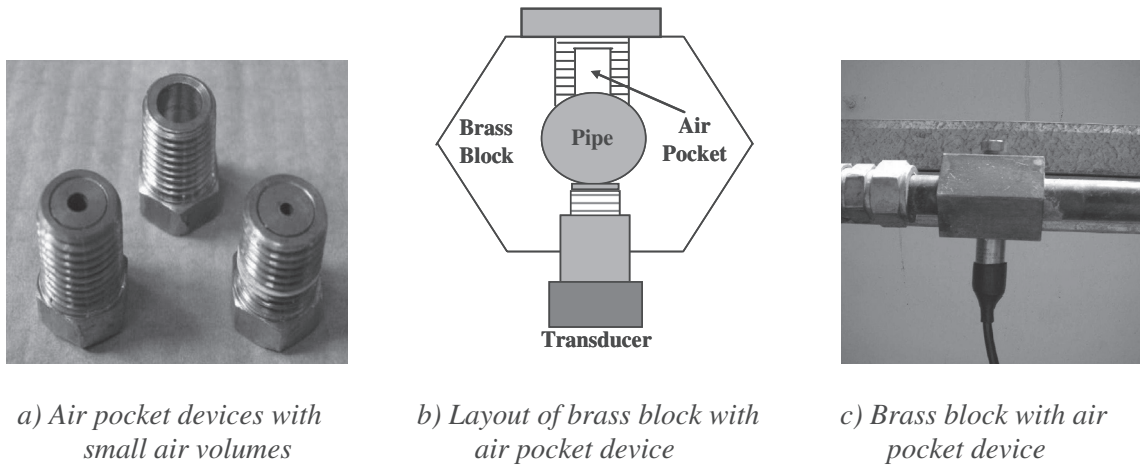


Fig.2 Air pocket devices

4. NUMERICAL AND EXPERIMENTAL INVESTIGATION OF BEAT PHENOMENA

Numerical results from the *classical* and *novel* DGCM are compared with results of laboratory measurements at the valve (WE at brass block in Fig.1). The effect of unsteady friction on the attenuation of the beat phenomenon is investigated. Computational and measured results are presented for the case with initial flow velocity $V_0 = 0.137$ m/s at a constant static head in the pressurized tank ET of $H_{ET} = 49$ m (measured at datum level at the top of the pipe inlet at tank ET) and air pocket volume at atmospheric conditions of $\nabla_{g0,EM} = 3.93 \times 10^{-7}$ m³ (EM brass block in Fig.1). The corresponding air volume is very small in comparison to the total water volume of 0.0144 m³. Water and surrounding temperatures are 21 and 22 °C, respectively. The initial Reynolds number is $Re_0 = 3,050$ ($Re_0 = V_0 D / \nu$) and the respective approximated Vardy-Brown weighting function W_{app} [13] used in the numerical simulations is shown in Fig.3. The measured initial wave speed is $a_0 = 1330$ m/s and the estimated initial steady-state friction coefficient is $f_0 = 0.0044$. The effective valve closure time of $t_{cef} = 0.004$ s is significantly shorter than the water hammer wave reflection time of $2L/a = 0.056$ s.

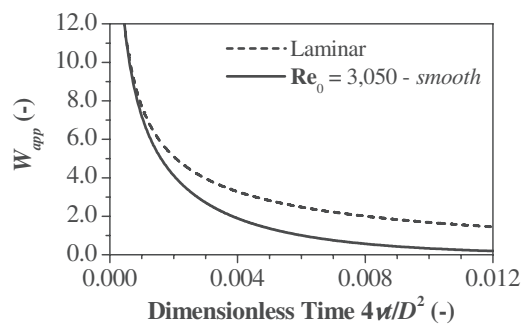


Fig.3 Weighting functions for transient laminar and turbulent flow

The number of pipe reaches for computational runs using the MOC- and CSS-based DGCM is $N = 212$ (213 computational sections) and the time step is $\Delta t = 0.000132$ s and 0.00013 s, respectively. The MOC-based DGCM has been used with a large void fraction $\alpha_{g0,EM} = 5.78 \times 10^{-3}$ ($\alpha_{g0} = \nabla_{g0}/\nabla_{reach}$) at the EM point (trapped air pocket) and much smaller void fractions of $\alpha_{g0} = 10^{-7}$ at the other 212 computational sections (except 0.5×10^{-7} at boundaries). Isothermal behaviour of the gas pocket is assumed in all simulations in both methods. The computational

results are practically the same when either the polytropic or isentropic assumption is used (small air pocket).

First the results from the *classical* and *novel* DGCM using the steady friction model (SF) are compared with the results of measurements - see Fig.4. Both numerical models give practically the same results (Figs.4(a) and 4(b)) and the long-time simulations show that a beat develops. When the time window is shortened, the beginning of a weak experimental beat may be observed in the sense that the short-duration peak values grow. It appears that the steady friction model does not produce sufficient damping both for the bulk pressure traces as for the short-duration pressure peaks. To possibly overcome this deficiency a convolution-based unsteady friction model has been incorporated into both numerical schemes.

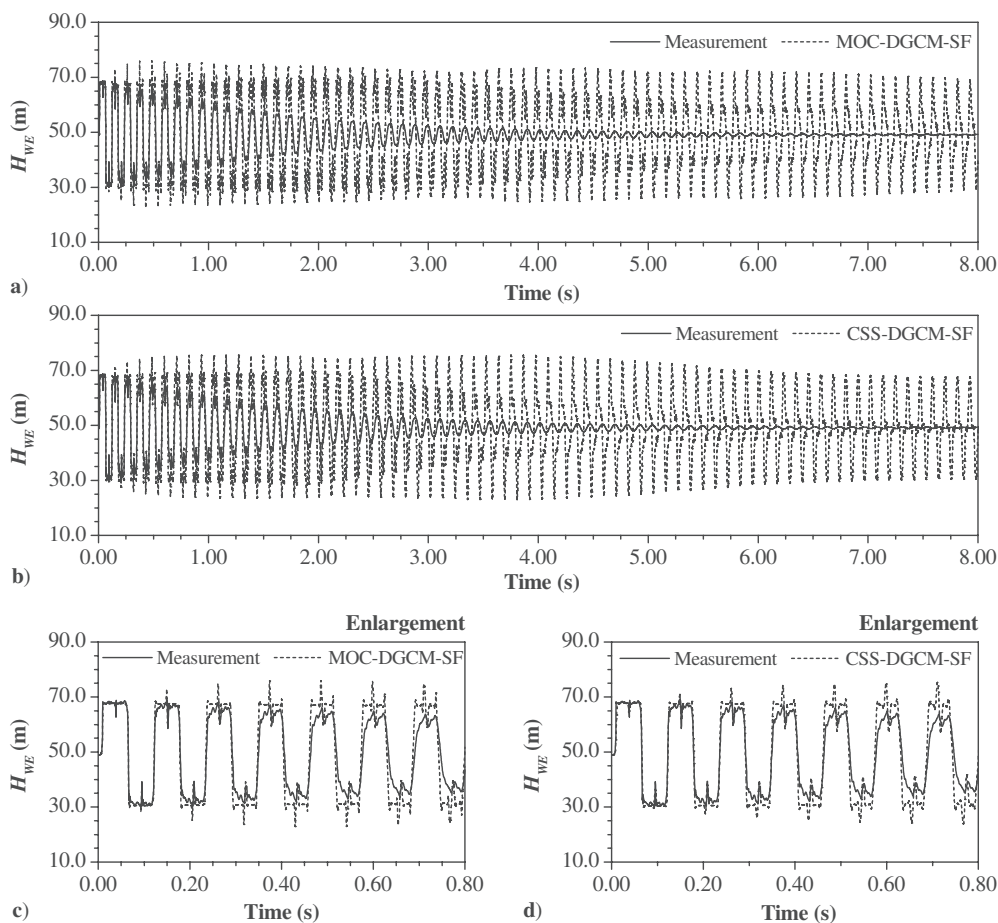


Fig.4 Comparison of head at the valve (H_{WE}): $Re_0 = 3,050$; $V_{g0,EM} = 3.93 \times 10^{-7} m^3$, steady friction model with $f_0 = 0.044$ (SF).

The results from the *classical* and *novel* DGCM using the convolution-based unsteady state friction model (UF) are compared with the results of measurements in Fig.5. Long-time simulations by both methods show that the beat quickly damps out (Figs.5(a) and 5(b)). When the time window is shortened a weak experimental beat may be observed. It appears that for this flow situation, the unsteady state friction model does produce sufficient damping both for bulk pressure traces and short-duration pressure peaks. Again both models give practically the same results. The computed and measured results agree surprisingly well. Note that measured value for wave speed has been used in MOC-DGCM simulations.

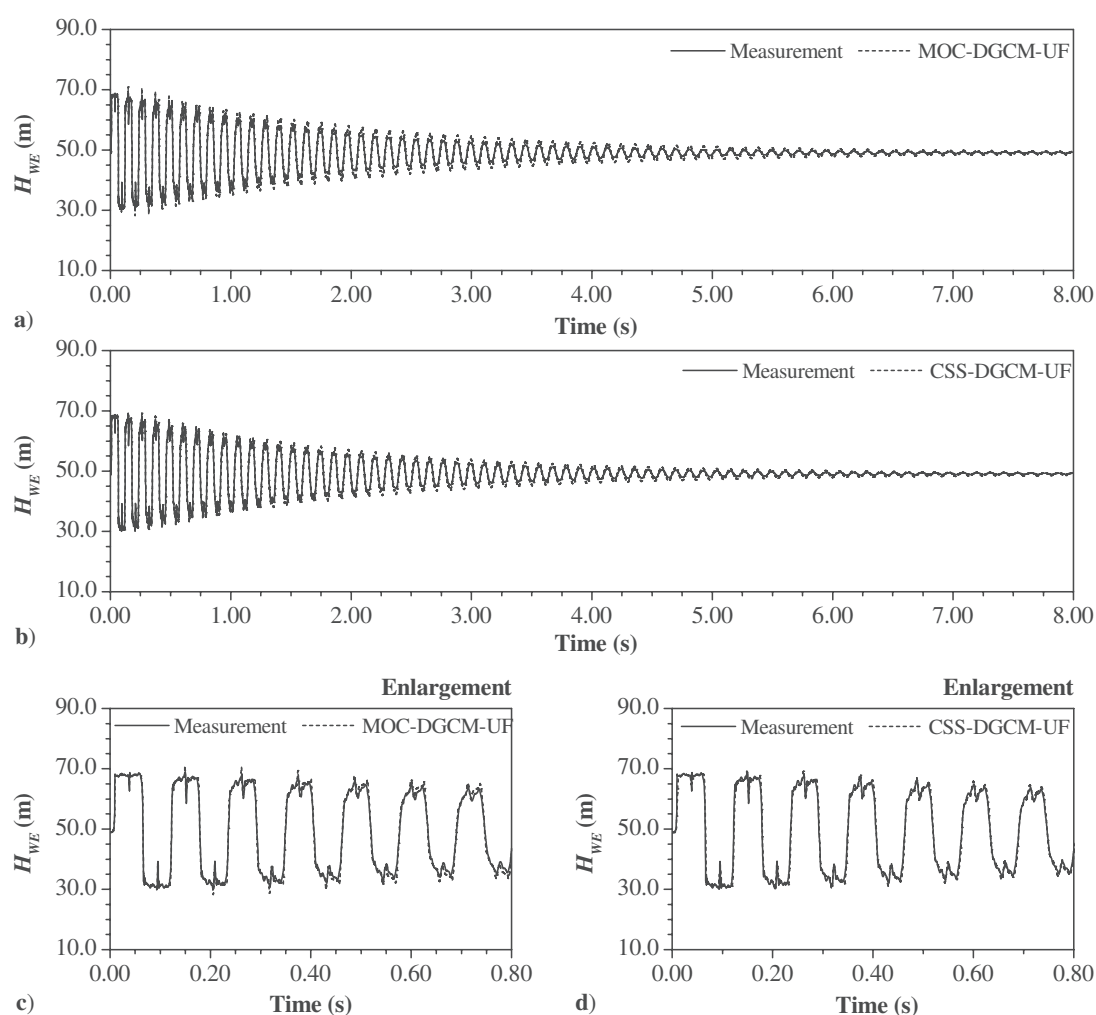


Fig.5 Comparison of head at the valve (H_{WE}): $Re_0 = 3,050$; $V_{g0,EM} = 3.93 \times 10^{-7} m^3$, convolution-based unsteady friction model (UF).

5. CONCLUSIONS

The effect of one small isolated air pocket on pressure waves travelling in a reservoir-pipe-valve system has been investigated theoretically and experimentally. The theoretical investigation concerns a discrete gas cavity model (DGCM) that is incorporated in two different numerical schemes: one based on the method of characteristics and the other on a conservative solution scheme based on the Preissmann or box scheme. Both schemes produced practically the same results. The experimental investigation comprises a specially designed device that holds a small amount of air and that is mounted in the existing pipeline apparatus at the University of Adelaide.

The experimentally observed effect of the small trapped gas pocket is the generation of short-duration pressure peaks that initially grow, but quickly damp out. Numerical models employing steady friction predict that the peaks cumulate into a beat, with maximum pressures much higher than the Joukowsky pressure. The present study demonstrates that applying unsteady friction is sufficient to suppress the development of this potential beat.

6. ACKNOWLEDGEMENTS

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8. NOMENCLATURE

A	(m^2)	pipe area	Q	($m^3 \cdot s^{-1}$)	discharge
a	($m \cdot s^{-1}$)	wave speed	Q_u	($m^3 \cdot s^{-1}$)	upstream-end discharge
C_A	(m^{1+3n})	constant in Eq.(9)	Re	(-)	Reynolds number
C^+, C^-		name of char. Eqs.	t	(s)	time
D	(m)	pipe diameter	t_{cef}	(s)	effective valve closure time
E	(Pa)	modulus of elasticity	V	($m \cdot s^{-1}$)	average flow velocity
e	(m)	pipe-wall thickness	W	(-)	weighting function in UF
f	(-)	friction factor	x	(m)	distance
g	($m \cdot s^{-2}$)	grav. acceleration	z	(m)	elevation
H	(m)	piezometric head	α_g	(-)	gas void fraction
h_f	(-)	unit head loss	Δt	(s)	time step
h_v	(m)	vapour pressure head	Δx	(m)	space step
K	(Pa)	bulk modulus	θ	(rad)	pipe slope
L	(m)	length	ν	($m^2 \cdot s^{-1}$)	kinematic viscosity
N	(-)	number of reaches	ρ	($kg \cdot m^{-3}$)	mass density of fluid
n	(-)	polytropic coeff.	\forall_g	(m^3)	gas cavity volume
p	(Pa)	pressure	\forall_{reach}	(m^3)	pipe reach volume
<i>Subscripts:</i>					
A		absolute	g		gas
app		approximate	i		node number
EM		brass block (Fig.1)	WE		west tank (Fig.1)
ET		east tank (Fig.1)	0		initial conditions
<i>Abbreviations:</i>					
CSS		conser. soln scheme	MOC		method of char.
DGCM		dis. gas cavity model	SF		steady friction
FDM		finite diff. method	UF		unsteady friction