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A phase quadrature feedback interferometer using a two-mode He-Ne laser

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Abstract A new feedback interferometer is developed with which not only the magnitude but also the sign of a phase change caused by a variation in the optical path can be measured. This is an important feature, for example, in plasma diagnostics.

Phase quadrature is obtained by using two axial modes of a He-Ne laser. The two interference signals can be separately detected by virtue of the orthogonal polarisation of the two axial modes of the laser without Brewster windows. The result is a versatile, stable and accurate instrument. In the experimental test the sensitivity proved to be approximately $\frac{1}{2}$ fringe at a laser wavelength of 632.8 nm, equivalent to 0.1 μm . In plasma applications the instrument would have a sensitivity of $n_e L \approx 2 \times 10^{20} \text{ m}^{-2}$, where n_e is the electron density and L the length of the laser cavity.

1 Introduction

For the measurement of small changes in the optical path caused by small changes of the refractive index, several types of interferometer are in use. Of special interest in this respect is the measurement of the electron density n_e of a plasma.

In 1963 Ashby and Jephcott introduced a relatively simple type of interferometer which is commonly called the feedback interferometer, FBI, or the three-mirror interferometer. The FBI consists of a laser cavity (formed by mirrors M_1 and M_2 , figure 1) coupled with an external cavity (formed by an external mirror M_3 and one of the laser mirrors, M_2). The object whose index of refraction has to be measured is located in the external cavity.

The principle of operation is as follows. The radiation reflected by M_3 re-enters the laser cavity with a phase difference ψ with respect to the phase of the internal laser field. This phase shift is determined by the length of the external cavity and the refractive index of the medium in the cavity. Depending on ψ , positive or negative interference with the laser field occurs, leading to an enhancement or a reduction of the laser output. The phase changes (or 'fringes') can be observed through M_1 , for example. In this set-up the laser is the source as well as the detector, leading to an extremely simple configuration with a minimum of optical components.

The main advantages of the FBI over other types of interferometer (Mach-Zehnder, Michelson) are the simple alignment procedure and the inherent mechanical stability. Only one optical component, M_3 , has to be aligned with the laser; the alignment of the detector is not critical. Since only the positions of the laser and M_3 relative to each other can be affected by mechanical vibrations it is possible to design a stable interferometer. This is of particular interest in experiments in which strong forces occur, such as plasma experiments using strong pulsed magnetic fields. Finally the detection of the phase change (fringes) is simple and insensitive to vibrations.

Since the introduction of the basic design of the FBI by Ashby and Jephcott (1963), several modifications have been proposed to improve e.g. the sensitivity of the interferometer (Clunie and Rock 1964, Fielding 1972). In particular, the two-wavelength FBI (i.e. 632.8 and 3390 nm, to be distinguished from the two-mode FBI in this paper) proved to be a sensitive and yet simple diagnostic tool for the measurement of electron density in plasmas.

The main difficulty in using the FBI is that only *one* phase datum is obtained. Where the phase change is larger than π it may be impossible to determine the sign of the phase change from the observed fringe pattern. It is necessary to obtain *two* phase data which are at least to a certain degree independent to solve this problem; if possible one should measure the phase in quadrature ($\sin \psi$, $\cos \psi$). Several methods have been proposed to avoid the ambiguity associated with the use of a single phase datum. The methods, also applied in other types of interferometer, are based on one of the following two principal solutions.

The first method is the use of time modulation of the optical path superimposed on the change to be measured. For the FBI this can be achieved e.g. by uniform translation of the external mirror (Belousova and Zapryagaev 1974, Baker *et al* 1965, Kricker and Smith 1965). Naturally, the rate of change of the optical path should be larger than, or at least equal to the rate of change of the phase to be measured. This method has the disadvantage that it requires a fast modulation of the location of at least one optical component.

The other method is to obtain *two* interference signals at the same time, instead of one; this can be achieved e.g. by the use of two reference beams which are $\frac{1}{2}\pi$ out of phase. The two interference signals are separately detected in the form of $\sin \psi$, $\cos \psi$.

One example of this type of quadrature interferometer is the Mach-Zehnder interferometer of Buchenauer and Jacobson (1977). They employed a linearly polarised measuring beam and a circularly polarised reference beam of the same frequency. Two interference signals were generated, one proportional to the cosine and the other to the sine of the phase shift ψ caused by the plasma.

In this paper we will describe a phase quadrature FBI according to the second method. We used two adjacent axial modes of a He-Ne laser as the 'independent' beams to obtain two 'independent' phase shift data. The frequencies of the modes differ by a small amount and consequently the two phase data will generally differ. We will show that for specific ratios of the external-cavity and laser-cavity lengths the two phase data are in quadrature, i.e. in the form $\sin \psi$ and $\cos \psi$. For convenience of detection it is advantageous to use a laser without Brewster windows. Adjacent axial modes of such a laser appear to be orthogonally polarised in fixed directions (Bennett *et al* 1973, Balhorn *et al* 1972). If a two-axial-mode laser without Brewster windows is used, the two interference signals can easily be separated with the aid of polarisers.

2 Principle of the phase quadrature feedback interferometer

If a He-Ne laser with two axial modes is used in an FBI, one has in principle two interferometers in one configuration provided that the modes are not coupled. This appears to be a reasonable assumption (Peek *et al* 1967). The two signals can easily be separated by means of a beam splitter BS and polarisers P₁ and P₂ (figure 1).

The relation of the two phase data ψ_1 and ψ_2 depends on the ratio of the external-cavity length L_{ext} to the length of the laser cavity, L_1 . This relation will be calculated under two assumptions: firstly that the two modes are uncoupled, and secondly that M₃ is so orientated that it does not couple the two polarisation directions. In that case the two interferometers are independent. Only the frequencies of the two oscillations are related by the fact that two adjacent axial cavity modes are used (see §3):

$$f_1 = \frac{c}{\lambda_1} = c \frac{m}{2L_1} \quad (1)$$

$$f_2 = \frac{c}{\lambda_2} = c \frac{m+1}{2L_1},$$

where $f_2 > f_1$ and m is an integer. The wavenumbers differ by an amount

$$\Delta k = k_2 - k_1 = \pi/L_1. \quad (2)$$

Here it is assumed that the difference in the frequency pulling of the two axial modes towards the atomic transition frequency can be neglected with respect to $\Delta f = c/2L_1$, the mode spacing. This assumption has been verified during the experiments by monitoring the frequency difference, which remained constant.

The phase shift experienced in the up and down path in the external cavity is expressed as

$$\psi_{1,2} = 2\pi \frac{2L_{\text{ext}}}{\lambda_{1,2}} = 2L_{\text{ext}}k_{1,2} \quad (3)$$

and the difference between ψ_1 and ψ_2 is, with (2),

$$\Delta\psi = \psi_2 - \psi_1 = 2L_{\text{ext}}(k_2 - k_1) = (L_{\text{ext}}/L_1) 2\pi. \quad (4)$$

This relation between the two independent phase data depends on the ratio L_{ext}/L_1 . Several cases can be discerned.

$$(a) \quad L_{\text{ext}}/L_1 = i, \quad \Delta\psi = i2\pi, \quad (5)$$

where i is an integer. The two phase data are equal (apart from $i2\pi$).

$$(b) \quad L_{\text{ext}}/L_1 = i + \frac{1}{2}, \quad \Delta\psi = i2\pi + \pi, \quad (6)$$

where the two phase data are opposite, i.e. if one laser mode is enhanced by the corresponding probing radiation re-entering the laser cavity, the other mode will experience negative interference with its re-entering signal.

$$(c) \quad L_{\text{ext}}/L_1 = i \pm \frac{1}{4}, \quad \Delta\psi = i2\pi \pm \frac{1}{2}\pi \quad (7)$$

where the phase data differ by $\frac{1}{2}\pi$. In principle this offers the possibility of phase quadrature.

When only a small part of the light is reflected into the laser cavity (i.e. the reflection coefficient R_3 of mirror M₃ is much less than unity), the modulation of the laser output intensity of the two modes $\Delta I_{1,2}$ depends harmonically on the phase shift $\psi_{1,2}$:




$$\Delta I_{1,2}(\psi_{1,2}) = C_1' + C_2' \cos \psi_{1,2}. \quad (8)$$

This is not true if R_3 is not small; in that case, if $(1 - R_3) \ll 1$, one can derive (Peek *et al* 1967)

$$\Delta I_{1,2}(\psi_{1,2}) \approx C_1 + \frac{C_2}{1 + C_3 \cos \psi_{1,2}}. \quad (9)$$

The coefficients $C_{1,2,3}$ contain the gain, saturation factor and reflectivities of the laser and the reflectivity of the end-mirror.

Table 1 Relation between the two phase data $\psi_{1,2}$ and the modulated laser output $\Delta I_{1,2}$,

Case	L_{ext}/L_1	$\psi_1 - \psi_2$	ΔI_1	ΔI_2	Lissajous figure
(a)	i	$2\pi i$	$C_2 \cos \psi_1$	$C_2 \cos \psi_1$	
(b)	$i + \frac{1}{2}$	$2\pi i + \pi$	$C_2 \cos \psi_1$	$-C_2 \cos \psi_1$	
(c)	$i \pm \frac{1}{4}$	$2\pi i \pm \frac{1}{2}\pi$	$C_2 \cos \psi_1$	$\pm C_2 \sin \psi_1$	

It is clear that even for a moderate reflection coefficient R_3 , (10–30%), the anharmonicity of equation (9) cannot be neglected.

If equation (8) is valid ($R_3 \ll 1$), the performance of the two-mode FBI should be as shown in table 1. In the last column ΔI_1 and ΔI_2 have been represented as a Lissajous pattern, which can be visualised on an X - Y oscilloscope, as discussed in §3. In cases (a) and (b) the two phase data are the same (apart from a sign difference) and there is no need to use two modes. However, in case (c) the two signals are in quadrature.

3 Experimental results

In the actual configuration of the two-mode FBI a Spectra Physics 133 laser with a mode spacing of 570 MHz was used. This laser has no Brewster windows and exhibits two or three axial modes. We used no stabilisation, but only adjusted the input power of the power supply in such a way that the laser produced two modes, which were measured. Experiments with a Hughes 3121H laser, with only two axial modes, are in preparation.

A periodic variation of the optical path length in the external cavity is obtained by modulating the position of M₃ by the application of an AC voltage to the piezo-element P on which M₃ is fastened. This is equivalent to a variation in the optical path by a change in refractive index of the plasma. The two interferometric signals from detectors D₁ and D₂ (figure 1(a)) are recorded on an X - Y oscilloscope to produce the resulting Lissajous pattern. The results are shown in figure 1(b). For equal lengths (case (a)), a straight line is indeed obtained. At half-length positions, an approximately straight line with a negative slope is found, in agreement with the expectation for case (b) (cf table 1). At the position c' (figure 1) the Lissajous figures are circles; in this case the two-mode FBI yields $\sin \psi$ and $\cos \psi$ as well. However, the position c' is not in agreement with the condition $L_{\text{ext}}/L_1 = i \pm \frac{1}{4}$ (table 1, case (c)). This can partly be explained by using the correct equation for the relation between ψ_1 and ψ_2 and the variations in the intensities I_1 and I_2 . Also other effects may contribute to a deviation of the phase quadrature position from the predicted value: mode coupling in the laser will invalidate the assumption of two independent signals, and anisotropy of e.g. the end-mirror may cause a coupling between the polarisation of each mode. Nevertheless, the two-mode FBI appears to give two phase data in quadrature.

By application of DC voltages to the piezo-element P it has been checked that the relation between the position of M₃ and the measured phase is approximately linear. An accuracy of $\frac{1}{5}$ to $\frac{1}{10}$ fringe is obtained in which the uncertainty is mainly due to the mechanical instability in this test experiment.

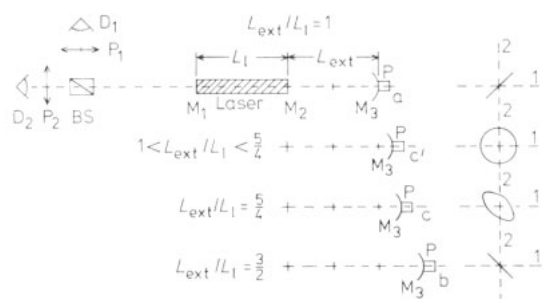
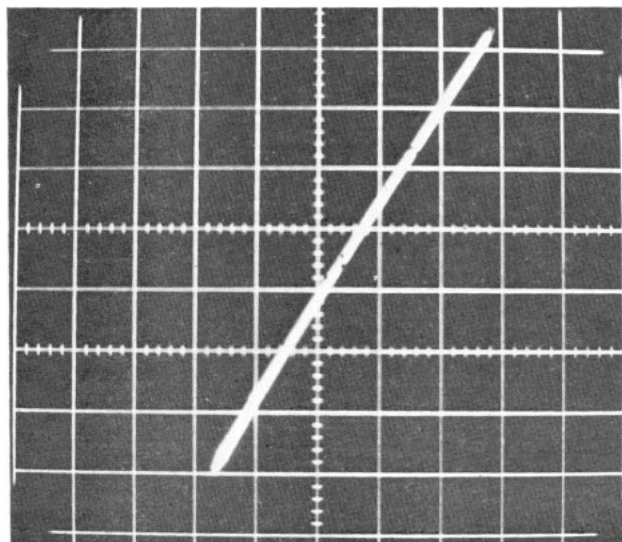
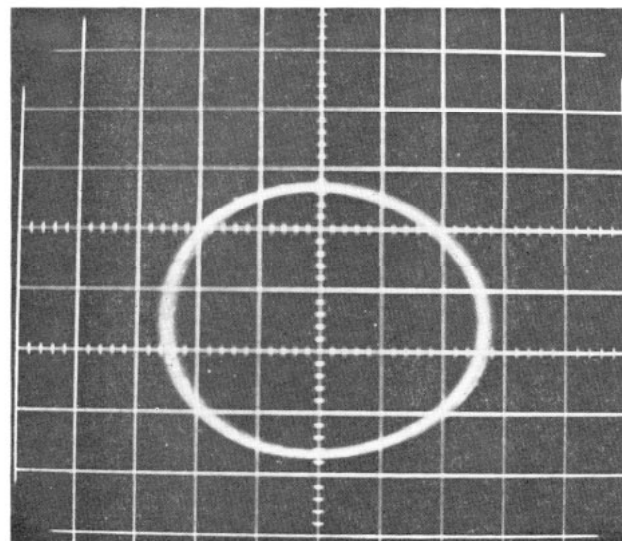


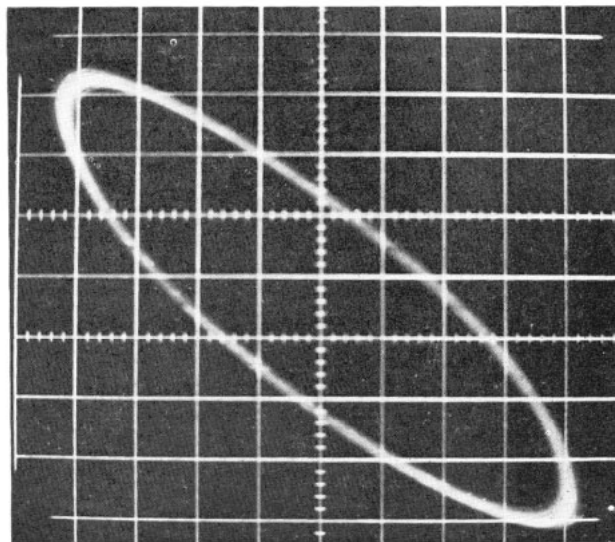
Figure 1(a) Experimental arrangement of the FBI with the various positions of mirror M₃ along the optical axis.



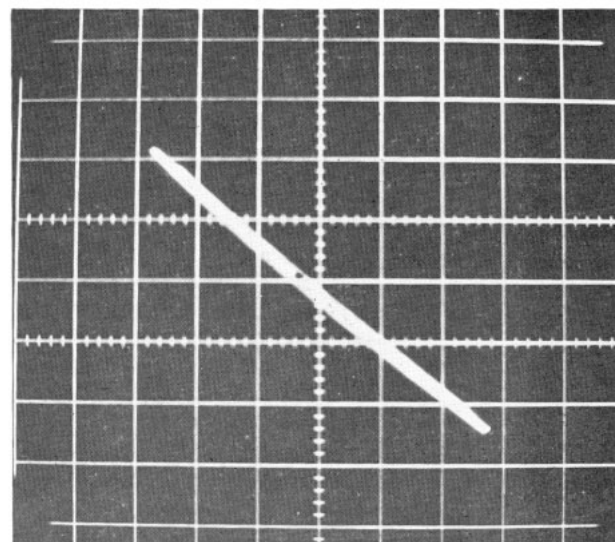
Position a



Position c'



Position c



Position b

Figure 1(b) The corresponding Lissajous patterns on the oscilloscope; position c' corresponds to phase quadrature.

$\lambda = 632.8 \text{ nm}$, is equivalent to a length variation of approximately $0.1 \mu\text{m}$. In plasma experiments this would mean a sensitivity of

$$n_e L > 2 \times 10^{20} \text{ m}^{-2}.$$

It is clear that the sensitivity can be improved by using the proposed principle of two-mode operation at longer wavelengths, i.e. with a 3390 nm He-Ne laser.

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4 Conclusions

The two-mode feedback interferometer can be used to measure accurately the phase variation in phase quadrature caused by changes in the optical path. The accuracy of the instrument in the test arrangement proved to be $\frac{1}{3}$ to $\frac{1}{10}$ fringe which, at

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