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Citation for published version (APA):

**DOI:**
10.1088/0022-3735/11/10/017

**Document status and date:**
Published: 01/01/1978

**Document Version:**
Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

**Please check the document version of this publication:**

- A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
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Download date: 02. Nov. 2023
A phase quadrature feedback interferometer using a two-mode He-Ne laser

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Received 23 March 1978, in final form 4 July 1978

Abstract A new feedback interferometer is developed with which not only the magnitude but also the sign of a phase change caused by a variation in the optical path can be measured. This is an important feature, for example, in plasma diagnostics.

Phase quadrature is obtained by using two axial modes of a He-Ne laser. The two interference signals can be separately detected by virtue of the orthogonal polarisation of the two axial modes of the laser without Brewster windows. The result is a versatile, stable and accurate instrument. In the experimental test the sensitivity proved to be approximately 0.1 pm. In plasma applications the instrument would have a sensitivity of several pm. Several methods have been proposed to improve e.g. the sensitivity of the interferometer (Clunie and Rock 1964, Fielding 1972). In particular, the two-wavelength FBI (i.e. 632.8 and 3390 nm, to be distinguished from the two-mode FBI in this paper) proved to be a sensitive and yet simple diagnostic tool for the measurement of electron density in plasmas.

The main difficulty in using the FBI is that only one phase datum is obtained. Where the phase change is larger than π it may be impossible to determine the sign of the phase change from the observed fringe pattern. It is necessary to obtain two phase data which are at least at a certain degree independent to solve this problem; if possible one should measure the phase in quadrature (sin ϕ, cos ϕ). Several methods have been proposed to avoid the ambiguity associated with the use of a single phase datum. The methods, also applied in other types of interferometer, are based on one of the following two principal solutions.

The first method is the use of time modulation of the optical path superimposed on the change to be measured. For the FBI this can be achieved e.g. by uniform translation of the external mirror (Belousova and Zaprıyagaev 1974, Baker et al 1965, Kricker and Smith 1965). Naturally, the rate of change of the optical path should be larger than, or at least equal to, the rate of change of the phase to be measured. This method has the disadvantage that it requires a fast modulation of the location of at least one optical component.

The other method is to obtain two interference signals at the same time, instead of one; this can be achieved e.g. by the use of two reference beams which are 90° out of phase. The two interference signals are separately detected in the form of sin ϕ, cos ϕ.

One example of this type of quadrature interferometer is the Mach–Zehnder interferometer of Buchenauer and Jacobson (1977). They employed a linearly polarised measuring beam and a circularly polarised reference beam of the same frequency. Two interference signals were generated, one proportional to the cosine and the other to the sine of the phase shift ϕ caused by the plasma.

In this paper we will describe a phase quadrature FBI according to the second method. We used two adjacent axial modes of a He–Ne laser as the ‘independent’ beams to obtain two ‘independent’ phase shift data. The frequencies of the modes differ by a small amount and consequently the two phase data will generally differ. We will show that for specific ratios of the external-cavity and laser-cavity lengths the two phase data are in quadrature, i.e. in the form sin ϕ and cos ϕ. For convenience of detection it is advantageous to use a laser without Brewster windows. Adjacent axial modes of such a laser appear to be orthogonally polarised in fixed directions (Bennett et al 1973, Battorn et al 1972). If a two-axial-mode laser without Brewster windows is used, the two interference signals can easily be separated with the aid of polarisers.
2 Principle of the phase quadrature feedback interferometer

If a He–Ne laser with two axial modes is used in an FBI, one has in principle two interferometers in one configuration provided that the modes are not coupled. This appears to be a reasonable assumption (Peek et al. 1967). The two signals can easily be separated by means of a beam splitter BS and polarisers P1 and P2 (figure 1).

The relation of the two phase data $\psi_1$ and $\psi_2$ depends on the ratio of the external-cavity length $L_{ext}$ to the length of the laser cavity, $L_1$. This relation will be calculated under two assumptions: firstly that the two modes are uncoupled, and secondly that $M_3$ is so orientated that it does not couple the two polarisation directions. In that case the two interferometers are independent. Only the frequencies of the two oscillations are related by the fact that two adjacent axial cavity modes are used (see $\S3$):

$$f_1 = \frac{c}{\lambda_1} = \frac{m}{2L_1}, \quad f_2 = \frac{c}{\lambda_2} = \frac{m+1}{2L_1},$$

where $f_2 > f_1$ and $m$ is an integer. The wavenumbers differ by an amount

$$\Delta k = k_2 - k_1 = \pi/L_1.$$

Here it is assumed that the difference in the frequency pulling of the two axial modes towards the atomic transition frequency can be neglected with respect to $\Delta f = c/2L_1$, the mode spacing. This assumption has been verified during the experiments by monitoring the frequency difference, which remained constant.

The phase shift experienced in the up and down path in the external cavity is expressed as

$$\psi_{1,2} = 2\pi \frac{2L_{ext}}{\lambda_{1,2}} = 2L_{ext}f_{1,2},$$

and the difference between $\psi_1$ and $\psi_2$ is, with (2),

$$\Delta \psi = \psi_2 - \psi_1 = 2L_{ext}(k_2 - k_1) = (L_{ext}/L_1) 2\pi.$$

This relation between the two independent phase data depends on the ratio $L_{ext}/L_1$. Several cases can be discerned.

(a) $L_{ext}/L_1 = i$, $\Delta \psi = 2m\pi$,

(b) $L_{ext}/L_1 = i + \frac{1}{2}$, $\Delta \psi = (2i+1)\pi$,

(c) $L_{ext}/L_1 = i \pm \frac{1}{2}$, $\Delta \psi = 2i\pi \pm \frac{1}{2}\pi$,

where the phase data differ by $\frac{1}{2}\pi$. In principle this offers the possibility of phase quadrature.

When only a small part of the light is reflected into the laser cavity (i.e. the reflection coefficient $R_3$ of mirror $M_3$ is much less than unity), the modulation of the laser output intensity of the two modes $\Delta I_1, \Delta I_2$ depends harmonically on the phase shift $\psi_{1,2}$:

$$\Delta I_1, \Delta I_2(\psi_{1,2}) = C_1 + C_2 \cos \psi_{1,2}.$$

This is not true if $R_3$ is not small; in that case, if $|1 - R_3| < 1$, one can derive (Peek et al. 1967)

$$\Delta I_1, \Delta I_2(\psi_{1,2}) \approx C_1 + \frac{C_2}{1 + C_3 \cos \psi_{1,2}}.$$

The coefficients $C_1, C_2, C_3$ contain the gain, saturation factor and reflectivities of the laser and the reflectivity of the end-mirror.

| Table 1 Relation between the two phase data $\psi_{1,2}$ and the modulated laser output $\Delta I_1, \Delta I_2$ |
|---------------------------------|---------------------------------|---------------------------------|-----------------|-----------------|
| $L_{ext}/L_1$ | $\psi_{1,2}$ | $\Delta I_1$ | $\Delta I_2$ | Lissajou figure |
| (a) $i$ | $2\pi i$ | $C_2 \cos \psi_1$ | $C_2 \cos \psi_1$ | $\bigcirc$ |
| (b) $i + \frac{1}{2}$ | $2\pi i + \pi$ | $C_2 \cos \psi_1$ | $-C_2 \cos \psi_1$ | $\bigcirc$ |
| (c) $i \pm \frac{1}{2}$ | $2\pi i \pm \frac{1}{2}\pi$ | $C_2 \cos \psi_1 \pm C_2 \sin \psi_1$ | $\bigcirc$ |

It is clear that even for a moderate reflection coefficient $R_3$, (10–30%), the anharmonicity of equation (9) cannot be neglected.

If equation (8) is valid ($R_3 < 1$), the performance of the two-mode FBI should be as shown in table 1. In the last column $\Delta I_1$ and $\Delta I_2$ have been represented as a Lissajous pattern, which can be visualised on an $X$–$Y$ oscilloscope, as discussed in $\S3$. In cases (a) and (b) the two phase data are the same (apart from a sign difference) and there is no need to use two modes. However, in case (c) the two signals are in quadrature.

3 Experimental results

In the actual configuration of the two-mode FBI a Spectra Physics 133 laser with a mode spacing of 570 MHz was used. This laser has no Brewster windows and exhibits two or three axial modes. We used no stabilisation, but only adjusted the input power of the power supply in such a way that the laser produced two modes, which were measured. Experiments with a Hughes 3121H laser, with only two axial modes, are in preparation.

A periodic variation of the optical path length in the external cavity is obtained by modulating the position of $M_3$ by the application of an AC voltage to the piezo-element $P$ on which $M_3$ is fastened. This is equivalent to a variation in the optical path by a change in refractive index of the plasma. The two interferometric signals from detectors $D_1$ and $D_2$ (figure 1(a)) are recorded on an $X$–$Y$ oscilloscope to produce the resulting Lissajous pattern. The results are shown in figure 1(b). For equal lengths (case (a)), a straight line is indeed obtained. At half-length positions, an approximately straight line with a negative slope is found, in agreement with the expectation for case (b) (cf table 1). At the position $c'$ (figure 1) the Lissajous figures are circles; in this case the two-mode FBI yields a linear chi and cos $\psi$ as well. However, the position $c'$ is not in agreement with the condition $L_{ext}/L_1 = \pm \frac{1}{2}$ (table 1, case (c)). This can partly be explained by using the correct equation for the relation between $\psi_1$ and $\psi_2$ and the variations in the intensities $I_1$ and $I_2$. Also other effects may contribute to a deviation of the phase quadrature position from the predicted value: mode coupling in the laser will invalidate the assumption of two independent signals, and anisotropy of e.g. the end-mirror may cause a coupling between the polarisation of each mode. Nevertheless, the two-mode FBI appears to give two phase data in quadrature.

By application of DC voltages to the piezo-element $P$ it has been checked that the relation between the position of $M_3$ and the measured phase is approximately linear. An accuracy of $\frac{1}{2}$ to $\frac{1}{4}$ fringe is obtained in which the uncertainty is mainly due to the mechanical instability in this test experiment.
A phase quadrature feedback interferometer

Figure 1(a) Experimental arrangement of the FBI with the various positions of mirror M3 along the optical axis.

Position a

Position b

Position c

Position c'

Figure 1(b) The corresponding Lissajous patterns on the oscilloscope; position c' corresponds to phase quadrature.

4 Conclusions
The two-mode feedback interferometer can be used to measure accurately the phase variation in phase quadrature caused by changes in the optical path. The accuracy of the instrument in the test arrangement proved to be \( \frac{1}{8} \) to \( \frac{1}{4} \) fringe which, at

\[ \lambda = 632.8 \text{ nm}, \]

is equivalent to a length variation of approximately 0.1 \( \mu \text{m} \). In plasma experiments this would mean a sensitivity of

\[ n_e L > 2 \times 10^{20} \text{ m}^{-2}. \]

It is clear that the sensitivity can be improved by using the proposed principle of two-mode operation at longer wavelengths, i.e. with a 3390 nm He-Ne laser.

Acknowledgments
The authors wish to express their appreciation to the head of the Metrology Laboratory of the Eindhoven Technical University, Dr J Koning, for the use of his lasers and optical apparatus and for helpful discussions.

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