

Poster Abstract: Data-Driven Correct-by-Design Control of Parametric Stochastic Systems

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Poster Abstract: Data-Driven Correct-by-Design Control of Parametric Stochastic Systems*

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ABSTRACT

In this ongoing work, we address data-driven computation of controllers that are correct by design for safety-critical systems and can provably satisfy complex functional requirements. We propose a two-stage approach that decomposes the problem into a data-driven stage and a robust formal controller synthesis stage. The first stage utilizes available Bayesian linear regression methods to compute robust confidence sets for the true parameters of the system. The second stage develops methods for systems subject to both stochastic and parametric uncertainties. We provide simulation relations for enabling control refinement that are founded on coupling uncertainties of stochastic systems via sub-probability measures. Such relations are essential for constructing abstract models that are related to not only one model but to a set of parametric models.

KEYWORDS

Temporal logic control, stochastic systems, parametric uncertainty, data-driven methods

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1 INTRODUCTION

Autonomous cars, smart grids, robotic systems, and medical devices are just a few examples of safety-critical systems that operate in uncertain environments. To guarantee safe operation even if the environment or dynamics are uncertain, it is crucial to synthesize robust controllers such that the controlled system exhibits the desired behavior with the satisfaction being formally verifiable. Therefore, there is an ever-growing demand for so-called correct-by-design approaches, giving formal guarantees on the absence of any undesired behavior of the controlled system. However, this proves to be a very challenging task, even when an exact model of the system is known.

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In this work, we employ abstraction-based techniques, where the abstract model can be independent of the parameter, a lower-order approximation or a finite-state approximation. To give formal guarantees, it is necessary to quantify the similarity between an abstract model and its latent true counterpart. To this end, we relate the systems using approximate simulation relations and refine controllers designed on the abstraction to the respective original model whilst transferring any guarantees to the original system [5].

Formal methods generally start with an exact stochastic model of the system [3], which implies that any guarantees on the correctness of the closed-loop system only hold for that specific model. However, obtaining an exact model of the system is not always possible or expensive and time consuming. To alleviate this issue, data-driven methods [2] try to find the best parameters that minimize an appropriate distance (either in time or frequency domain) between the data and the state trajectories of the identified model.

We are developing an abstraction-based approach suitable for stochastic systems with parameter sets constructed using appropriate data-driven techniques. We learn the parameter set that contains the true parameters with a given confidence level from a finite amount of data and provide closeness guarantees over the whole set of associated models. This allows us to design a controller with quantified guarantees such that the controlled system satisfies a given temporal specification uniformly on this parameter set. The problem considered in this research can be formulated as follows.

Problem 1: *Can we design a controller using only data from the unknown true system such that the controlled system satisfies a given temporal logic specification with at least probability p and confidence $1 - \alpha$?*

In ongoing work, we provide a scheme for answering this question for parametric discrete-time stochastic systems and the class of co-safe linear temporal logic (scLTL) specifications, building upon preliminary work in [4]. In the future, we plan to include this work in the toolbox SySCoRe. The initial version of this tool will be presented at HSCC'23.

2 MODEL CLASS AND SOLUTION APPROACH

Parametric stochastic systems. We consider discrete-time nonlinear systems perturbed by additive stochastic noise under model-parametric uncertainty. This modeling formalism is essential if we can only access an uncertain model of a stochastic system. Consider the models $M(\theta)$ parametrized with θ :

$$M(\theta) : \begin{cases} x_{k+1} &= f(x_k, u_k; \theta) + w_k \\ y_k &= h(x_k), \end{cases} \quad (1)$$

where the system state, input, and observation at the k^{th} time-step are denoted by x_k, u_k, y_k , respectively. The functions f and h specify, respectively, the parametric state evolution of the system and the

observation map. The additive noise is denoted by w_k , which is an independent, identically distributed (i.i.d.) noise sequence with distribution $w_k \sim p_w(\cdot)$. The composition of the controller C with the model $M(\theta)$ is denoted as $C \times M(\theta)$.

Decomposing Problem 1. Consider an unknown system M^* for which we impose a parametric form (1), i.e., $M^* := M(\theta^*)$ where θ^* is unknown. We are interested in designing a controller C to satisfy temporal specifications ψ on the output of the model. This is denoted by $C \times M(\theta^*) \models \psi$. Following Problem 1, since the true parametrization θ^* is unknown, we want to design a controller that does not depend on θ^* and that ensures the satisfaction of ψ with at least probability p_ψ using only data \mathcal{D} from the true system. To address this problem, we decompose it into the following two sub-problems:

Problem 1a: Based on \mathcal{D} , construct a confidence set Θ that contains the true parametrization θ^* with a given confidence $1 - \alpha \in (0, 1)$, i.e., $\mathbb{P}(\theta^* \in \Theta) \geq 1 - \alpha$.

Problem 1b: For a given specification ψ and a threshold $p_\psi \in (0, 1)$, design a controller C for $M(\theta)$ that does not depend on the parameter θ and that guarantees that the following holds:

$$\mathbb{P}(C \times M(\theta) \models \psi) \geq p_\psi, \quad \forall \theta \in \Theta.$$

We address Problem 1a using Bayesian regression methods. Based on the obtained confidence set Θ , we construct a parametric set of models $\{M(\theta) \text{ with } \theta \in \Theta\}$. By designing a controller uniformly valid for the whole parametric set of models, we solve Problem 1b. The controller synthesis for stochastic models is studied in [1] through coupled simulation relations. Although these simulation relations can relate one abstract model to a set of parametric models $M(\theta)$, these relations would lead to a control refinement that is similarly dependent on the true parametrization. Therefore, this approach is unfit to solve Problem 1b, since the required true parameter θ^* is unknown. As one of the main contributions of this work, we start from a parameter-independent control refinement and compute a simulation relation based on a sub-probability coupling to synthesize a single controller that holds for all $\theta \in \Theta$.

3 TWO-STAGE METHODOLOGY

3.1 Method part a: Data-driven parameter estimation

We use a parameter estimation method to identify the unknown true system M^* that can only be observed through samples (u, x, x^+) . We assume that the latent model $M^* := M(\theta^*)$ can be described by (1), where the underlying true parametrization θ^* is unknown. To infer the unknown parametrization, we collect data \mathcal{D} from M^* . For this, we sample a finite number of data points

$$(u, x, x^+)^{(i)}, \text{ where } x^+ = f(x, u; \theta^*) + w,$$

and $i = 1, \dots, N$ is finite. Based on \mathcal{D} , we estimate θ^* with an estimate $\hat{\theta}$ and construct a confidence set Θ which contains the true parameters θ^* with confidence $1 - \alpha \in (0, 1)$, i.e., $\mathbb{P}(\theta^* \in \Theta) \geq 1 - \alpha$ as in Problem 1a. Using Bayesian linear regression, we provide robust results for both linear and nonlinear stochastic systems.

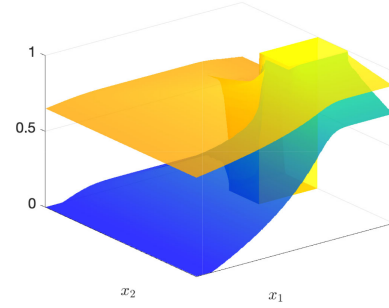


Figure 1: Robust satisfaction probability for all initial states. By obtaining more data, the satisfaction probability is increased from the blue/green curve to the orange/yellow curve.

3.2 Method part b: Sub-similarity relations

Consider a set of models $\{M(\theta) \text{ with } \theta \in \Theta\}$ and suppose that we have chosen an abstract (nominal) model \hat{M} defined as

$$\hat{M} : \begin{cases} \hat{x}_{k+1} &= \hat{f}(\hat{x}_k, \hat{u}_k; \hat{\theta}) + \hat{w}_k, \\ \hat{y}_k &= \hat{h}(\hat{x}_k). \end{cases} \quad \hat{w}_k \sim \hat{p}_{\hat{w}}(\cdot),$$

Based on this abstract model, we design a *single* controller and quantify the satisfaction probability over all models $M(\theta)$ in the set of models. To this end, we formalize the notion of a *state mapping* and an *interface function*, that together form the control refinement [4]. Then, we investigate the conditions under which a single controller for \hat{M} can be refined to a controller for all $M(\theta)$ independent of the parameter θ . Note that this is the central contribution of this work, since instead of synthesizing one parametrized controller $C(\theta)$ for each model $M(\theta)$ by using the respective parameter θ as would be possible based on [1], it allows us to synthesize a single controller C that works uniformly for all models $M(\theta)$ in the set of models $\{M(\theta) \text{ with } \theta \in \Theta\}$. This leads us to the concept of *sub-probability couplings* and simulation relations developed recently in [4].

4 RESULTS

We apply our control synthesis method based on sub-similarity relations to a linear and nonlinear system. In ongoing work, we combine this with data-driven parameter estimation. As expected, the satisfaction probability increases when more data is obtained from the true system. In Figure 1, the satisfaction probability is increased by obtaining more data, which shrinks the confidence set Θ by approximately a factor of three.

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