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A queuing–location–allocation model for designing a capacitated bus garage system

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ABSTRACT

The spatial distribution of bus garages determines the total vehicular dead mileage of the transit system because buses must travel between bus garages and terminals at the start or conclusion of a day. By contrast, the size of the garages determines the queuing status when buses enter or leave the garages. Thus, a bus garage system with reasonable distribution and size is required to address these problems. In this article, a queuing–location–allocation model for optimizing a bus garage system is developed. Since a nonlinear objective function is involved, a linearization technique is introduced to convert the proposed model into an equivalent linear form. Next, a Lagrangian relaxation algorithm is designed to solve the linear form model. To validate the proposed algorithm, two groups of randomly generated test instances and a real-life case, the Dalian transit system in China, are applied. The results show that the proposed Lagrangian heuristic is efficient and stable.

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Bus garages; dead mileage cost; queuing–location–allocation model; linearization; Lagrangian relaxation

1. Introduction

Public transit holds an irreplaceable position in urban transport. It is regarded as the most efficient means of moving large numbers of commuters across a city (Ebrahimi and Bridgelall 2020). However, inadequate planning and management in the urban transport sector still lead to tough challenges being faced within cities. Because of limited urban space and high land prices, most transit agencies build bus garages on the outskirts of the city. As a result, the starting/ending points of the routes, as well as the bus garages, are distributed among different locations, leading to vehicular dead (or deadhead) miles and dead costs. Under the conditions of market economics, the influence of these ineffective costs on the operation of the transit system becomes more and more noticeable (Perre and Oudheusden 1997).

Vehicle allocation is also a critical factor in reducing the dead mileage costs. Allocating buses to their nearest garage could decrease the total dead miles. However, idle waiting time and associated invalid costs may also be generated if too many vehicles are allocated to the same garage because drivers have to queue up to pull into or out of the garage. To capture this feature, this article explicitly models the bus garage as a congested system. When taken together, the appropriate number, location and size of bus garages and a reasonable assignment of buses can not only help transit agencies to save on the total operational costs but also use the allocated funding reasonably to provide a better service.

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The methodology presented in this article is a queuing–location–allocation model for designing the bus garage system to reduce the total operational costs of transit agencies (Ball *et al.* 1984). The cost components are classified into two parts: (1) garage-related cost and (2) assignment-related cost. The garage-related cost contains two parts: one is the initial investment (fixed charge) for a new garage with the necessary facilities, and the other is the variable cost that increases linearly with garage capacity. The assignment-related costs can be divided into three parts: dead mileage cost for buses, staff cost for the garage system and waiting-time cost for drivers. Considering that the level of service varies throughout the day, the vehicles in this model are divided into two parts to respond to the fluctuations in passenger demand, *i.e.* (1) the base service throughout the day and (2) an additional service used as a supplement during peak periods. Different vehicle types are also considered in this mathematical model, since different vehicle types require different facilities.

Furthermore, as discussed later in more detail, the proposed model contains a nonlinear objective. Following Sherali and Alameddine (1992), the model is converted to an equivalent linear form. Then, a Lagrangian relaxation (LR) algorithm is designed to solve it. Two randomly generated test instances and a real-life case are finally presented to demonstrate the effectiveness of the model and algorithm.

The remainder of this article proceeds as follows. In Section 2, the literature on the bus garage system's location and allocation problems is reviewed. Section 3 describes the proposed model. Section 4 details the linearization technique and the LR algorithm. Section 5 gives the computational results, and a real-life application is examined in Section 6. Finally, Section 7 provides some conclusions and suggests directions for future study.

2. Literature review

The determination of the optimal location, number and size of bus garages can be classified as the location–allocation problem. It is one of the oldest problems studied in management science (Salhi and Rand 1989; Chen, Tian, and Yao 2019; Othman *et al.* 2020). The pioneering studies on the location–allocation problem for designing bus garages were conducted by Maze, Khasnabis, and Kutsal (1982, 1983). At approximately the same time, Ball *et al.* (1984) also discussed a model for the bus garage location–allocation problem but with the significant difference that it disaggregated the demand into three parts: base requirement, morning and afternoon incremental requirements. Waters *et al.* (1986) analysed a complementary approach compared to the models proposed by Maze, Khasnabis, and Kutsal (1982, 1983) and Ball *et al.* (1984). In their article, the number of bus garages is first obtained using the basic facility location model. Then, the garages' size and location and the allocation of vehicles are determined using a traditional, discrete-space location–allocation model. Uyeno and Willoughby (1995) also formulated a mixed-integer programming (MIP) model considering an existing configuration of bus routes and a set of existing and candidate transit centre locations to minimize the cost for transit centre location–allocation decisions. However, the results showed one weakness, recognized as a managerial inconvenience by scheduling personnel, that buses for a given route are split between transit centres. Uyeno and Willoughby (2001) later further developed a heuristic procedure to make sure that all buses on a route are assigned to the same garage. To further extend the application of the MIP model, Willoughby (2002) explored how solutions change under different additional transit planning scenarios, which include no candidate facilities, forced Oakridge allocation and Greenfields approach. Unlike the assumption of the independent relationship between vehicle assignments and the location and size of bus garages, Maze and Khasnabis (1985) developed a technique that can simultaneously design vehicle scheduling and determine the location and size of bus garages.

All of the literature discussed above indicates that minimization of the total cost of the transit system is always chosen as the objective function and the dead mileage cost accounts for a large proportion of the total cost. In practice, a transit agency may operate different bus brands/types for a given route, for which different maintenance facilities are also required. Thus, vehicles with the same brand/type should be grouped in the same garage to the greatest extent possible. In recent decades, many studies have paid attention to the bus allocation problem to consider the effect of different bus brands/types, aiming to minimize dead mileage costs (Perre and Oudheusden 1997; Kepaptsoglou, Karlaftis, and Bitsikas 2009; Djiba *et al.* 2012; Nasibov *et al.* 2013; Kontou *et al.* 2014; Mahadikar, Mulangi, and Sitharam 2015; Yao *et al.* 2019).

Although many studies have focused on the bus garage location–allocation problem or bus allocation problem to reduce the costs of bus agencies, few researchers have considered the bus garage as a congestion system. It becomes difficult for buses to enter or leave the garage when too many vehicles are allocated to the same garage. This phenomenon not only causes the wastage of fuel but also increases the drivers' idle time. Some applications of the queuing–location–allocation problem in other fields have been studied (Wang, Batta, and Rump 2002; Shavandi and Mahlooji 2006; Berman and Drezner 2007; Yao *et al.* 2019; Shan *et al.* 2019; Liu *et al.* 2020). For more details about the queuing–location–allocation problem and model variations, interested readers may refer to Boffey, Galvão, and Espejo (2007) and Shavarani *et al.* (2019).

In this article, a location–allocation model for designing the bus garage system that incorporates the queuing problem is developed. This model considers different service times and vehicle types and takes the waiting time of drivers into consideration. The contributions to the current literature are as follows.

First, this work aims to design a bus garage system for a transit agency. The existing literature limits its focus to minimizing the dead mileage cost or the total costs. As a result, these methods tend to allocate buses to their nearest garage. However, the queuing phenomenon occurs if too many buses are allocated to the same garage, increasing the drivers' invalid cost and working time. Thus, this article models the bus garage as a congested facility to determine the optimal location, number and size of the bus garage system.

Secondly, different vehicle types are always operated by one transit agency, and different vehicle types need different maintenance facilities. For instance, electric vehicles can only be parked in garages with charging points. However, previous studies on the location–allocation of the bus garage system have always ignored this effect. This article aims to cope with the related problems raised by considering different types of vehicles.

3. Model development

The model developed in this study is based on the following assumptions. (1) The numbers and locations of bus routes and garages are known. Moreover, the number of buses contained in each bus route is also known. (2) Buses that pull out from a bus garage to their starting service point must return to the same garage after a full day's service. (3) Multiple servers (doors) are allowed for each bus garage. (4) Each bus garage has a service limitation. (5) Vehicles for service are divided into two parts: one part is for the base service throughout the day, while the other part is used as a supplement during peak periods. (6) The bus garage is regarded as an M/M/S system; basically, it has to accord with the queuing system's typical queuing behaviour. In this article, bus arrivals are assumed to be subject to the Poisson distribution and garage service time follows an exponential distribution. (7) Servers (doors) in a given garage have a unique service rate. In contrast, the service rates of servers (doors) belonging to different garages are different. Therefore, the waiting time, queuing length and other measures can be calculated by standard formulae (Kleinrock 1975).

For easy reference, the notation used in the model is listed as follows.

Index sets:

I	Set of bus routes indexed by i; $i = 1, 2, \ldots, M$
J	Set of sites of potential bus garages indexed by j; $j = 1, 2, \ldots, N$
K	Set of vehicle types indexed by k
S	Set of numbers of servers (doors) $S = \{\theta_1, \theta_2, \ldots, \theta_{p-1}, \theta_p\}$ indexed by s

T Set of serving times indexed by t; $t = \begin{cases} 1, & \text{for base service} \\ 2, & \text{for peak service} \end{cases}$

Decision variables:

$$y_{jk} = \begin{cases} 1, & \text{if bus garage } j \text{ is open and the } k\text{th type of bus can be parked in it} \\ 0, & \text{otherwise} \end{cases}$$

$$x_{ijkt} = \begin{cases} 1, & \text{if the } k\text{th type of bus on route } i \text{ in serving time } t \text{ is assigned to bus garage } j \\ 0, & \text{otherwise} \end{cases}$$

$$z_{js} = \begin{cases} 1, & \text{if } s \text{ servers are built at garage } j \\ 0, & \text{otherwise} \end{cases}$$

Parameters:

μ_{jt}	Service rate of a single server at bus garage j in serving time t
λ_{ikt}	Arrival rate of the kth type of bus on route i in serving time t
d_{ij}	Average travel distance from bus route i to garage j
f_{jk}	Fixed cost for building garage j with facilities that can serve the kth type of bus
Tc_{ijkt}	Average unit dead mileage cost of the kth type of bus from bus route i to garage j in serving time t
Wc_{jkt}	Average unit waiting cost of the kth type of bus at garage j in serving time t
Wt_{jkts}	Average waiting time of the kth type of bus at garage j in the serving time t when the server number is s
Sc_j	Unit staff cost at garage j
π_{js}	Number of staff members at garage j when the server number is s
τ_k	Required area per bus of type k
δ_j	Unit land price for building bus garage j
A_j	Service limitation of bus garage j; (a prespecified number for existing garages or a considerable number for garages that are to be built)
σ_i	Length of bus route i
v_{ikt}	Free-flow speed of buses of type k on route i and in serving time t
P_L	Lower limit of the number of opened garages
P_U	Upper limit of the number of opened garages
Θ	Funding limitation
ξ	Discount rate parameter

The objective of the present model is to minimize the total cost of the transit agency, including the fixed charge and variable portion of garage construction costs, staff cost, waiting-time cost in the queuing system and dead mileage cost.

The model is formulated as follows.

$$\begin{aligned} \text{Min } & \sum_{j\in J}\sum_{k\in K} f_{jk}^{\xi} \cdot y_{jk} + \sum_{j\in J}\sum_{s\in S} (\pi_{js} \cdot Sc_j) \cdot z_{js} \\ & + \sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\sum_{t\in T} (\lambda_{ikt} \cdot d_{ij} \cdot Tc_{ijkt} + \lambda_{ikt} \cdot \tau_k \cdot \delta_j) \cdot x_{ijkt} \\ & + \sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\sum_{t\in T}\sum_{s\in S} (Wt_{jkts} \cdot Wc_{jkt}) \cdot \lambda_{ikt} \cdot x_{ijkt} \cdot z_{js} \end{aligned} \tag{1}$$

s.t.

$$\sum_{j\in J} x_{ijkt} = 1, \quad \forall i \in I, k \in K, t \in T \tag{2}$$

$$x_{ijkt} \leqslant y_{jk}, \forall i \in I, j \in J, k \in K, t \in T \tag{3}$$

$$\sum_{s\in S} z_{js} \leqslant 1, \quad \forall j \in J \tag{4}$$

$$\sum_{s\in S} z_{js} \geqslant x_{ijkt}, \forall i \in I, j \in J, k \in K, t \in T \tag{5}$$

$$\sum_{i\in I}\sum_{k\in K} \lambda_{ikt} \cdot x_{ijkt} \leqslant \mu_{jt} \cdot z_{js}, \forall j \in J, t \in T, s \in S \tag{6}$$

$$\sum_{i\in I}\sum_{k\in K} \frac{2\sigma_i/v_{ikt}}{60/\lambda_{ikt}} \cdot x_{ijkt} \cdot \tau_k \leqslant A_j, t = 2, \forall j \in J \tag{7}$$

$$x_{ijkt_1} = x_{ijkt_2}, t_1 \neq t_2, \forall i \in I, j \in J, k \in K, t_1, t_2 \in T \tag{8}$$

$$\sum_{j\in J}\sum_{k\in K} f_{jk}^{\xi} \cdot y_{jk} + \sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\sum_{t\in T} (\lambda_{ikt} \cdot \tau_k \cdot \delta_j) \cdot x_{ijkt} \leqslant \Theta^{\xi} \tag{9}$$

$$P_L \leqslant \sum_{j\in J}\sum_{k\in K} y_{jk} \leqslant P_U \tag{10}$$

$$y_{jk}, x_{ijkt}, z_{js} = 1 \text{ or } 0, \forall i \in I, j \in J, k \in K, t \in T, s \in S \tag{11}$$

Objective (1) is to minimize the total cost of the bus garage system. Constraint (2) states that for all vehicles of type k on route i in serving time t, they must be assigned to precisely one garage. Constraint (3) indicates that the assignment of bus routes can only be made if the bus garage is opened ($= 1$). Constraints (4) and (5) guarantee that any opened bus garage j must provide at least one server (door). Constraint (6) requires that the service rate must be larger than the arrival rate of vehicles. Constraint (7) ensures that the number of vehicles assigned to garage j does not exceed its capacity during the peak period. Constraint (8) ensures that buses of type k on route i are assigned to the same garage, regardless of the serving time. Constraint (9) indicates the funding limitation. Constraint (10) is a surrogate constraint for improving the bounds obtained in the solution algorithm. Furthermore, the bounds P_L and P_U will be updated iteratively to provide tighter limits on the number of opened garages.

4. Solution algorithm

4.1. Linearization of the model

The model developed in Section 3.2 is nonlinear as a result of the product of the binary variables x_{ijkt} and z_{js} in the objective function. Therefore, a linearization technique introduced by Sherali and Alameddine (1992) is used in this article to convert the model into an equivalent linear form by introducing a new variable ψ_{ijkts} to replace $x_{ijkt} \cdot z_{js}$. As a result, for any $i \in I, j \in J, k \in K, t \in t, s \in S$, a set of constraints is added, as follows:

$$\psi_{ijkts} \leqslant x_{ijkt}, \forall i \in I, j \in J, k \in K, t \in T, s \in S \tag{12}$$

$$\psi_{ijkts} \leqslant z_{js}, \forall i \in I, j \in J, k \in K, t \in T, s \in S \tag{13}$$

$$\psi_{ijkts} \geqslant x_{ijkt} + z_{js} - 1, \quad \forall i \in I, j \in J, k \in K, t \in T, s \in S \tag{14}$$

$$\psi_{ijkts} = 1 \text{ or } 0, \forall i \in I, j \in J, k \in K, t \in T, s \in S \tag{15}$$

The equivalent linear model is then converted as follows:

$$\sum_{j\in J}\sum_{k\in K} f_{jk}^{\xi}\cdot y_{jk} + \sum_{j\in J}\sum_{s\in S}(\pi_{js}\cdot Sc_j)\cdot z_{js}$$
$$+\sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\sum_{t\in T}(\lambda_{ikt}\cdot d_{ij}\cdot Tc_{ijkt}+\lambda_{ikt}\cdot\tau_k\cdot\delta_j)\cdot x_{ijkt}$$
$$+\sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\sum_{t\in T}\sum_{s\in S}(Wt_{jkts}\cdot Wc_{jkt})\cdot\lambda_{ikt}\cdot\psi_{ijkt} \tag{16}$$

subject to Constraints (2)–(15).

4.2. Lagrangian relaxation algorithm

The above 0–1 integer programming model is an instance of an NP-complete problem. The problem difficulty increases rapidly with the increase in problem size. The LR heuristic is an algorithm that can efficiently solve integer or mixed-integer optimization models and has been proven to be an effective technique in a variety of applications (Beasley 1993; Liu, Li, and Liu 2017; Wang, Liu, and Zheng 2020; Tolouei *et al.* 2020; Hamdan and Diabat 2020; Zhang *et al.* 2020). By relaxing the complicating constraints and adding them to the objective function, this method provides a lower bound to the original optimization model (for a minimization problem) (Fisher 1985).

In this article, Constraints (3), (5), (6), (9) and (12)–(14) are relaxed by adding them to the objective function. Relaxing these constraints is conducive to the model decomposition as more than one variable is involved in each of the constraints. The details of the LR decomposition method are provided in the next subsection.

4.2.1. Lagrangian dual problem

The Lagrangian multipliers $\alpha_{ijkt}, \beta_{ijkt}, \gamma_{jts}, \chi, \varepsilon_{ijkts}, \nu_{ijkts}, w_{ijkt}$ associated with these 'hard constraints' are introduced. The LR model after mathematical manipulation is shown as follows:

$$Z\lfloor LR(\alpha_{ijkt},\beta_{ijkt},\gamma_{jts},\varepsilon_{ijkts},\nu_{ijkts},\omega_{ijkts})\rfloor$$
$$= \text{Min}\left\{\begin{array}{l} PO+\sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\sum_{t\in T}\left(\alpha_{ijkt}\cdot(x_{ijkt}-y_{jk})+\beta_{ijkt}\cdot\left(x_{ijkt}-\sum_{s\in S}z_{js}\right)\right)\\ +\sum_{j\in J}\sum_{t\in T}\sum_{s\in S}\gamma_{jts}\cdot\left(\sum_{i\in I}\sum_{k\in K}\lambda_{ikt}\cdot x_{ijkt}-\mu_{jt}\cdot z_{js}\right)\\ +\sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\sum_{t\in T}\sum_{s\in S}(\varepsilon_{ijkts}\cdot(\psi_{ijkts}-x_{ijkt})+\nu_{ijkts}\cdot(\psi_{ijkts}-z_{js})\\ +\omega_{ijkts}\cdot(x_{ijkt}+z_{js}-1-\psi_{ijkts}))\\ +\chi\cdot\left(\sum_{j\in J}\sum_{k\in K}f_{jk}^{\xi}\cdot y_{jk}+\sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\sum_{t\in T}(\lambda_{ikt}\cdot\tau_k\cdot\delta_j)\cdot x_{ijkt}-\Theta^{\xi}\right)\end{array}\right\} \tag{17}$$

subject to Constraints (2), (4), (7), (8) and (10):

$$y_{jk}, x_{ijkt}, z_{js}, \psi_{ijkts} = 1 \text{ or } 0, \forall i\in I, j\in J, k\in K, t\in T, s\in S \tag{18}$$

Here, PO denotes the objective function of the primal model. The objective function of the LR model can be rewritten in another form:

$$Z\lfloor LR(\alpha_{ijkt}, \beta_{ijkt}, \gamma_{jts}, \varepsilon_{ijkts}, \nu_{ijkts}, \omega_{ijkts})\rfloor$$

$$= ZMin \left\{ \begin{array}{l} \sum_{j\in J}\sum_{t\in T}\sum_{s\in S}\left\{\pi_{js}\cdot Sc_j - \gamma_{jts}\mu_{jt} + \sum_{i\in I}\sum_{k\in K}(\omega_{ijkts}-\beta_{ijkt}-\nu_{ijkts})\right\}\cdot z_{js} \\ \quad +\sum_{j\in J}\sum_{k\in K}\left\{(1+\chi)\cdot f_{jk}^{\xi} - \sum_{i\in I}\sum_{t\in T}\alpha_{ijkt}\right\}\cdot y_{jk} \\ \quad +\sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\sum_{t\in T}\left\{\lambda_{ikt}\cdot\left[d_{ij}\cdot Tc_{ijkt} + (1+\chi)\cdot\tau_k\cdot\delta_j + \left(\sum_{s\in S}\gamma_{jts}\right)\right]\right. \\ \quad\quad \left. +\,\alpha_{ijkt} + \beta_{ijkt} + \sum_{s\in S}(\omega_{ijkts}-\varepsilon_{ijkts})\right\}\cdot x_{ijkt} \\ \quad +\sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\sum_{t\in T}\sum_{s\in S}(Wt_{jkts}\cdot Wc_{jkt}\cdot\lambda_{ikt} + \varepsilon_{ijkts} + \gamma_{ijkts} - \omega_{ijkts})\cdot\psi_{ijkts} \\ \quad -\sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\sum_{t\in T}\sum_{s\in S}\omega_{ijkts} - \chi\cdot\Theta^{\xi} \end{array} \right\} \tag{19}$$

Let LD be the Lagrangian dual of $LR(\alpha_{ijkt}, \beta_{ijkt}, \gamma_{jts}, \varepsilon_{ijkts}, \nu_{ijkts}, \omega_{ijkts})$, then the Lagrangian dual problem is to maximize the Lagrangian objective (lower bound) by continually adjusting the values of Lagrangian multipliers.

$$(LD) \quad Z(LD) = \text{Max}\, Z\lfloor LR(\alpha_{ijkt}, \beta_{ijkt}, \gamma_{jts}, \varepsilon_{ijkts}, \nu_{ijkts}, \omega_{ijkts})\rfloor \tag{20}$$

4.2.2. Lower bounds

The lower bound of the original problem can be obtained by solving the LR model described in Section 4.2.1. It may be observed that the LR model can be further decomposed by variables. In addition, the terms $\sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\sum_{t\in T}\sum_{s\in S}\omega_{ijkts}$ and $\chi\cdot\Theta^{\xi}$ are both constant. Thus, the four submodels of the LR model are presented as follows.

Submodel I

$$\text{Min} \sum_{j\in J}\sum_{k\in K}\left\{(1+\chi)\cdot f_{jk}^{\xi} - \sum_{i\in I}\sum_{t\in T}\alpha_{ijkt}\right\}\cdot y_{jk} \tag{21}$$

subject to Constraint (10)

$$y_{jk} = 0 \text{ or } 1, \forall j\in J, k\in K \tag{22}$$

Submodel II

$$\begin{aligned} \text{Min} \sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\sum_{t\in T}&\left\{\lambda_{ikt}\cdot[d_{ij}\cdot Tc_{ijkt} + (1+\chi)\cdot\tau_k\cdot\delta_j] + \alpha_{ijkt} + \beta_{ijkt}\right. \\ &\left. +\left(\sum_{s\in S}\gamma_{ikt}\right)\cdot\lambda_{ikt} + \sum_{s\in S}(\omega_{ijkts}-\varepsilon_{ijkts})\right\}\cdot x_{ijkt} \end{aligned} \tag{23}$$

subject to Constraints (2), (7) and (8)

$$x_{ijkt} = 0 \text{ or } 1, \forall i \in I, j \in J, k \in K, t \in T \tag{24}$$

Submodel III

$$\text{Min} \sum_{j \in J} \sum_{t \in T} \sum_{s \in S} \left\{ \pi_{js} \cdot Sc_j - \gamma_{jts} \cdot \mu_{jt} + \sum_{i \in I} \sum_{k \in K} (\omega_{ijkts} - \beta_{ijkt} - \nu_{ijkts}) \right\} \cdot z_{js} \tag{25}$$

subject to Constraint (4)

$$z_{js} = 0 \text{ or } 1, \quad \forall j \in J, s \in S \tag{26}$$

Submodel IV

$$\text{Min} \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} \sum_{t \in T} \sum_{s \in S} (Wt_{jkts} \cdot Wc_{jkt} \cdot \lambda_{ikt} + \varepsilon_{ijkts} + \gamma_{ijkts} - \omega_{ijkts}) \cdot \psi_{ijkts} \tag{27}$$

$$s.t. \quad \psi_{ijkts} = 0 \text{ or } 1, \forall i \in I, j \in J, k \in K, t \in T, s \in S \tag{28}$$

Thus, for given multipliers, the LR model can be solved separately, corresponding to the four submodels. The lower bound of the original problem is obtained as follows.

Step 1. Initialization

Step 1.1. Initialize the lower bound of the original problem, LB_{inc}, to a low value.
Step 1.2. Calculate the lower bound at each iteration and store the value in LB_{iter}. The details of the procedure are shown in Steps 2–5.

Step 2. Find the best solution of Submodel I

Step 2.1. Sort the coefficient of y_{jk},$(1 + \chi) \cdot f_{jk}^{\xi} - \sum_{i \in I} \sum_{t \in T} \alpha_{ijkt}$, with given multipliers in ascending order.
Step 2.2. Set $y_{jk} = 1$ for the first P_L garages; otherwise, $y_{jk} = 0$.
Step 2.3. If $P_L = P_U$, go to Step 2.5.
Step 2.4. Keep setting $y_{jk} = 1$ until either (1) the coefficients of y_{jk} have become positive or (2) P_U garages have been opened.
Step 2.5. Let the set of y_{jk} be chosen as y_{jk}^* and save y_{jk}^* to LB_{iter}^1.

Step 3. In Submodel II

Step 3.1. For each bus garage j during peak periods (*i.e.* serving time equals 2), form the coefficient $\lambda_{ikt} \cdot [d_{ij} \cdot Tc_{ijkt} + (1 + \chi) \cdot \tau_k \cdot \delta_j] + \alpha_{ijkt} + \beta_{ijkt} + \left(\sum_{s \in S} \gamma_{jts}\right) \cdot \lambda_{ikt} + \sum_{s \in S} (\omega_{ijkts} - \varepsilon_{ijkts})$ with the combination of route i and vehicle type k.
Step 3.2. Record all potential garage numbers if Constraint (7) is not violated. Then, identify j^* corresponding to $Min \lambda_{ikt} \cdot [d_{ij} \cdot Tc_{ijkt} + (1 + \chi) \cdot \tau_k \cdot \delta_j] + \alpha_{ijkt} + \beta_{ijkt} + \left(\sum_{s \in S} \gamma_{jts}\right) \cdot \lambda_{ikt} + \sum_{s \in S} (\omega_{ijkts} - \varepsilon_{ijkts})$.
Step 3.3. Set $x_{ijkt_2} = 1$ for j^* and $x_{ijkt_2} = 0$ for other garages. Finally, set $x_{ijkt_1} = 1$ for the same combination of route i and vehicle type k but in another serving time according to Constraint (8).
Step 3.4. Let the set of x_{ijkt} be chosen as x_{ijkt}^* and save x_{ijkt}^* to LB_{iter}^2.

Step 4. For Submodel III

Step 4.1. Calculate the coefficient of z_{js}, which means the cost for garage j with s servers (doors).
Step 4.2. For given multipliers and each garage j, record s corresponding to $Min\pi_{js} \cdot (Sc_{jt} - \gamma_{jts} \cdot \mu_{jt}) + \sum_{i \in I} \sum_{k \in K} (\omega_{ijkts} - \beta_{ijkt} - \nu_{ijkts})$.
Step 4.3. Check whether the value of the coefficient is negative. If so, save $s = s^*$ and set $z_{js} = 1$ for $s = s^*$. Otherwise, set $z_{js} = 0$.
Step 4.4. Let the set of z_{jts} be chosen as z^*_{jts} and save z^*_{jts} to LB^3_{iter}.

Step 5. To solve Submodel IV

Step 5.1. Calculate the coefficient $(Wt_{jkts} \cdot Wc_{jkt} \cdot \lambda_{ikt} + \varepsilon_{ijkts} + \gamma_{ijkts} - \omega_{ijkts})$ for each combination of route i, garage j, vehicle type k, serving time t and server number s.
Step 5.2. Set $\psi_{ijkts} = 1$ for the coefficient less than zero; otherwise, set $\psi_{ijkts} = 0$.
Step 5.3. Let the set of ψ_{ijkts} be chosen as ψ^*_{ijkts} and save ψ^*_{ijkts} to LB^4_{iter}.

Step 6. At the end of the iteration, the solutions to the LR model are y^*_{jk}, x^*_{ijkt}, z^*_{js} and ψ^*_{ijkts}. By summing up the LB^i_{iter}, a lower bound of the original problem, LB_{iter}, is also obtained. After comparing with the original LB_{inc}, the incumbent lower bound is then updated by letting $LB_{inc} = LB_{iter}$ if $LB_{inc} \geq LB_{iter}$.

The determination of the initial and improved upper bounds is described in the next subsection.

4.2.3. Upper bounds

The lower bound solution calculated by the Lagrangian iteration is generally not a feasible solution to the primal model. However, finding a feasible solution is likely to accelerate the convergence of the Lagrangian heuristic. In this subsection, a heuristic used to find the initial feasible solution (*i.e.* the initial upper bound), and the improved feasible solution from the lower bound, is given. The procedure is described below.

Step 1. Initialization

Step 1.1. Initialize the upper bound of the original problem, UB_{inc}, to a large value.
Step 1.2. The upper bound calculated at each iteration is stored in UB_{iter}. Set $UB_{iter} = M$ (M is a large number) in the first iteration. The details of the later procedure are shown as follows.

Step 2. Determine the decision variable x^*_{ijkt}

Step 2.1. Let Y be the set of opened garages obtained in the lower bound, *i.e.* $Y = \{(j,k) : y_{jk} = 1\}$. Then, the decision variable x_{ijkt} of the improved upper bound solution can be solved by an allocation subproblem, with the set of opened garages being given by the lower bound.
Step 2.2. For each combination of route i, vehicle type k and serving time t_2, solve the following allocation subproblem when the set of opened garages is fixed:

$$\text{(GAP)} \qquad \text{Min} \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} \sum_{t \in T} (\lambda_{ikt} \cdot d_{ij} \cdot Tc_{ijkt} + \lambda_{ikt} \cdot \tau_k \cdot \delta_j) \cdot x_{ijkt} \tag{29}$$

$$\text{s.t.} \qquad \sum_{j \in J} x_{ijkt_2} = 1, \quad \forall i \in I, k \in K \tag{30}$$

$$\sum_{i \in I} \sum_{k \in K} \frac{2\sigma_i / v_{ikt}}{60/\lambda_{ikt}} \cdot x_{ijkt_2} \cdot \tau_k \leqslant A_j, \forall j \in J \tag{31}$$

$$x_{ijkt_2} = 1 \text{ or } 0, \forall i \in I, j \in J, k \in K \tag{32}$$

Step 2.3. Set $x_{ijkt_1} = 1$ for the same combination of route i and vehicle type k but in another serving time, according to Constraint (8). Check whether Constraint (9) is violated or not. If not, go to Step 2.4; otherwise, go to Step 2.2 and find another suboptimal solution.

Step 2.4. Save x^*_{ijkt} to UB_{iter}.

Step 3. Solve z^*_{js}

Step 3.1. According to Constraints (4) and (5), exactly one server can be provided at each opened garage j.

Step 3.2. For each $x_{ijkt} = 1$ obtained in Step 2, identify all possible combinations of z_{js} without violating Constraint (6). Traverse all eligible z_{js} and choose those gaining the least staff cost to the objective value.

Step 3.3. Save z^*_{js} to UB_{iter}.

Step 4. Find the best ψ^*_{ijkts}

Step 4.1. For each combination of route i, garage j, vehicle type k, serving time t and server number s, set $\psi_{ijkts} = 1$ if z_{jts} and x_{ijkt} are both equal to 1 according to Steps 2 and 3.

Step 4.2. Save ψ^*_{ijkts} to UB_{iter}.

Step 5. All portions of the objective function in the primal problem can be calculated and constitute the UB_{iter}. The incumbent upper bound UB_{inc} is also updated by letting $UB_{inc} = UB_{iter}$ if $UB_{inc} \geq UB_{iter}$.

4.2.4. The subgradient procedure

Finding good Lagrangian multipliers is crucial in designing an LR algorithm (Fisher 1985). In the algorithm, the dual multipliers are updated at each iteration using the subgradient method. The subgradients of the problem are computed as follows:

$$\begin{aligned}
&NU(\alpha_{ijkt}) = x_{ijkt} - y_{jk}, \quad NU(\beta_{ijkt}) = x_{ijkt} - \textstyle\sum_{s\in S} z_{js}, NU(\gamma_{jts}) = \sum_{i\in I}\sum_{k\in K}\lambda_{ikt}\cdot x_{ijkt} - \mu_{jt}\cdot z_{js}\\
&NU(\varepsilon_{ijkts}) = \psi_{ijkts} - x_{ijkt}, NU(\nu_{ijkts}) = \psi_{ijkts} - z_{js}, NU(\omega_{ijks}) = x_{ijkt} + z_{js} - 1 - \psi_{ijkts}\\
&NU(\chi) = \textstyle\sum_{j\in J}\sum_{k\in K} f^{\xi}_{jk}\cdot y_{jk} + \sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\sum_{t\in T}(\lambda_{ikt}\cdot\tau_k\cdot\delta_j)\cdot x_{ijkt} - \Theta^{\xi}
\end{aligned} \tag{33}$$

Then, the Lagrangian multipliers α_{ijkt} are updated; for example, as

$$\alpha^{n+1}_{ijkt} = \max\{\alpha^n_{ijkt} + \Phi_n \cdot NU(\alpha_{ijkt}), 0\} \tag{34}$$

where the step size $\Phi_n = \vartheta_n \frac{(UB_n - LB_n)}{||s^n||^2}$, s^n is the vector of subgradients at iteration n, and ϑ_n is a scalar chosen between 0 and 2. UB_n and LB_n are the incumbent upper and lower bounds of the problem, respectively. In this article, the norm of s^n is taken as the Euclidean norm. The procedure starts with an initial $\vartheta_0 = 2$ and keeps the value unchanged if the solution is improved, while ϑ_n will be set to $\vartheta_n = {}^1\!/_2\vartheta_{n-1}$ if the solution is not improved in two consecutive iterations. The other multipliers are updated in the same manner. Then, the lower bound problem in the next iteration is solved by applying the updated dual multipliers, and the procedure is terminated after 20 iterations. Finally, the general algorithm procedure for solving the problem proposed in this paper is summarized and illustrated in Figure 1.

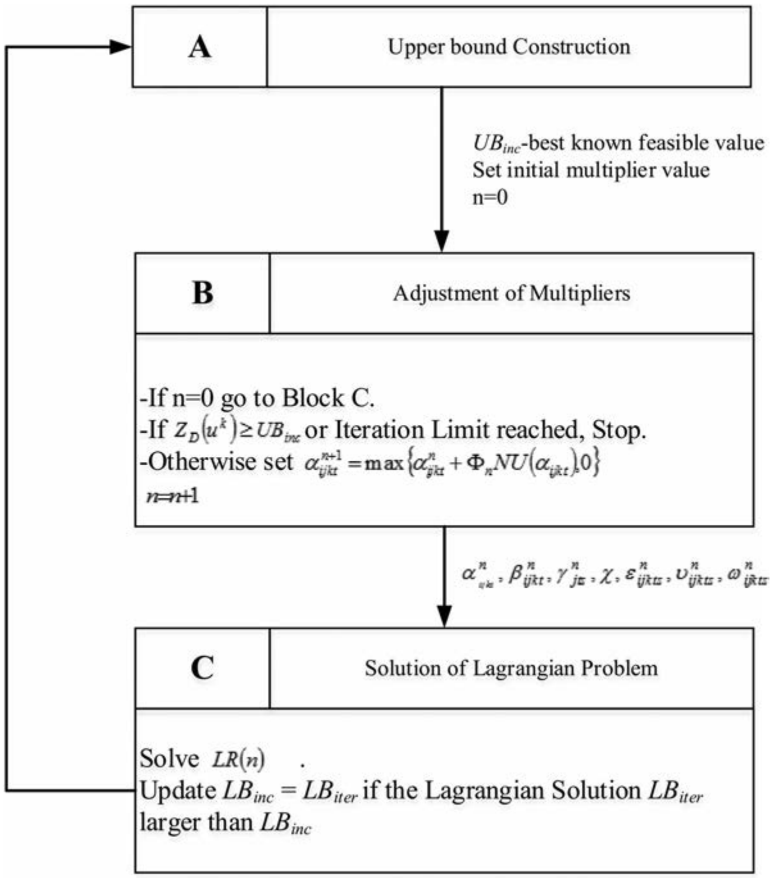

Figure 1. Procedure of the Lagrangian relaxation algorithm.

5. Computational experiments

In this section, a set of numerical experiments is employed to validate the proposed LR algorithm. The data used in these experiments are obtained by simulation. The algorithm is coded in Java and run on a personal desktop computer with Intel Core E5-2603, 1.7 GHz CPU, 48 GB of RAM and Windows 7 64-bit operating system. Two test problems are generated by varying the number of bus routes and potential garage locations. In the first group, the number of bus routes is set as 80, 100, 150 or 200, while the number of potential locations is fixed at 60. In the second group, the number of possible garage locations is 40, 60, 80 or 100, while the number of bus routes is fixed at 100. For simplicity, the arrival rates (operation schedules) of different bus routes are assumed to be the same in peak and off-peak periods. The number of required garages is set as 30 in all experiments (*i.e.* $P_L = P_U = 30$).

5.1. Data generation

- Problem parameters
  - Number of servers (doors) for each garage j is fixed at 2.
  - Number of vehicle types is fixed at 2.
  - Service rate for each combination of garage j and serving time t: $U(15, 20)$.
  - Arrival rate of each combination of bus route i, vehicle type k and serving time t: $U(6, 12)$.
  - Average distance between bus route i and garage j: $U(15, 30)$.

Table 1. Impact of the number of bus routes.

Instance no.	No. of bus routes	No. of required garages	No. of potential locations	Average gap % between lower and upper bounds	Average computation time (s)
a-1	80	30	60	0.879	18.53
a-2	100	30	60	1.046	51.33
a-3	150	30	60	1.114	80.15
a-4	200	30	60	1.251	120.94
	Average			1.072	67.74

Table 2. Impact of the number of potential bus garages.

Instance no.	No. of bus routes	No. of required garages	No. of potential locations	Average gap % between lower and upper bounds	Average computation time (s)
b-1	100	30	40	0.814	36.14
b-2	100	30	60	1.046	51.33
b-3	100	30	80	0.872	70.75
b-4	100	30	100	0.988	86.45
	Average			0.894	61.17

- The freeflow speed for each combination of bus route i, vehicle type k and serving time t: $U(10, 25)$ for peak period, whereas $U(15, 30)$ for off-peak period.
- Fixed cost for building garage j with facilities that can serve vehicle type k: $U(20,000\ \text{CNY}, 40,000\ \text{CNY})$.
- Average dead mileage cost per bus for each combination of bus route i, garage j, vehicle type k and serving time t: $U(2\ \text{CNY},\ 8\ \text{CNY})$.
- Waiting cost per bus for each combination of garage j, vehicle type k and serving time t: $U(0.3\ CNY,\ 0.8\ CNY)$.
- Staff cost per server (door): $U(200\ \text{CNY},\ 300\ \text{CNY})$.
- The required area per bus of type k is fixed at 180 m^2.
- Unit land price is fixed at 500 CNY.
- Discount rate parameter is fixed at 0.0325.

- Parameters in Langrangian heuristic
  - Maximum number of iterations is set to 20.
  - Parameter ϑ_0 in the step size equation is initially set to 2.0.
  - Lower limit of ϑ_0 is set to 0.05.

5.2. Results analysis

The results in Tables 1 and 2 report a positive correlation between the computation time and precision and the size of the instance. For more details, Table 1 shows that the computation time increases rapidly with the increased number of bus routes. In contrast, Table 2 reports that the computation time only increases linearly with the number of potential garage locations. The results suggest that the number of bus routes has a more significant impact on computation time than the number of potential garage locations. Besides, the average computation time shows that this algorithm can solve the problem within a reasonable time, even for a large version of the transit system. Regarding the precision, the average gaps between the upper and lower bounds are also reasonable. For the instances in group 1, the average gap is closed to 1.07, while in group 2, the average gap is 0.894. Even for the largest instance investigated in this article, a-4 or b-4, the average gap can be limited to 1.5%.

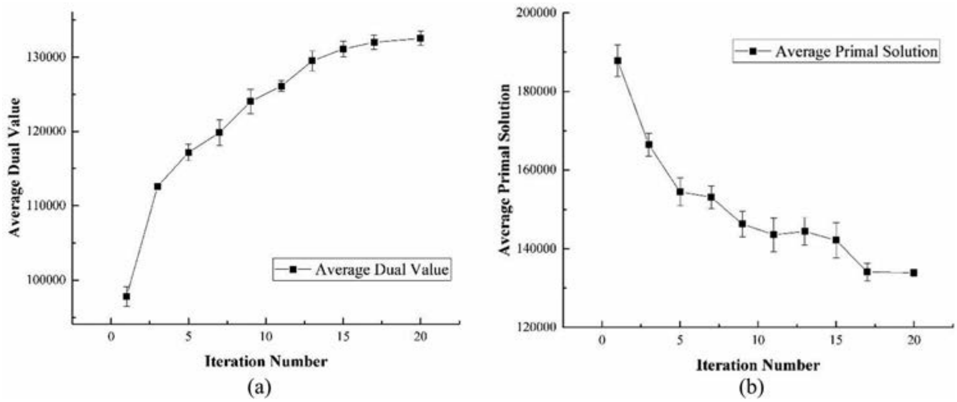

Figure 2. Iterative process of instance a-2/b-2: (a) dual value; (b) primal value.

As one instance appears in both test groups (*i.e.* a-2/b-2) and to show the solutions in detail, the detailed output of instance a-2/b-2 in five repeated computations is provided in Figure 2. The error bars in Figure 2 indicate the standard deviation of the five repeated calculations.

Figure 2 also demonstrates the reliability of the proposed method. Compared with the iterative process of primal solutions, the dual value seems to have a more stable performance. Although the primal solutions are unstable in different calculations, the method could still provide a favourable solution to the dual value and the algorithm eventually converged to a stable condition. Consequently, all observations from the above analysis suggest that the proposed Lagrangian heuristic is efficient and is suitable for solving large and sophisticated problems.

6. A real-life case study

According to the Financial Report of the Dalian Government, the losses of the Dalian Passenger Transport Group (DPTG) system in 2016 reached 1.4 billion CNY (http://www.czj.dl.gov.cn/zwgk/czgg.htm), of which invalid operation costs made up a large proportion. Therefore, it is necessary to reoptimize the largest city public transport system in Dalian, China, to reduce these invalid operational costs. Since most vehicles in this system belong to conventional public transit, the analysis in this article only focuses on these conventional buses.

6.1. Data preparation

6.1.1. Existing and candidate bus garage locations

In the DPTG system, all vehicles are parked, maintained and repaired in 26 bus garages located around Dalian city. Figure 3 illustrates the locations of all bus routes and garages. Ten additional candidate sites used to design a more efficient bus garage system are also selected and visualized in Figure 3. Appendix A provides data on the capacities of the bus garages and the vehicle allocations of the existing bus garage system. Some information about the candidate garages is also provided.

In Figure 3, the dots at the ends of all bold lines represent the terminal stations of bus routes, boxes with symbol P the current garages, and circles with symbol P the candidate garage sites; the bold lines are the present bus routes and fine lines are the Dalian city road network.

6.1.2. Estimation of related cost and parameters

According to the investigations on the DPTG, the average operational cost of a diesel bus is 9.45 CNY/mile, while for a bus running on liquefied natural gas (LNG) the cost is 1.32 CNY/mile and for a hybrid bus 5.72 CNY/mile. The actual distances between bus garages and terminal stations were

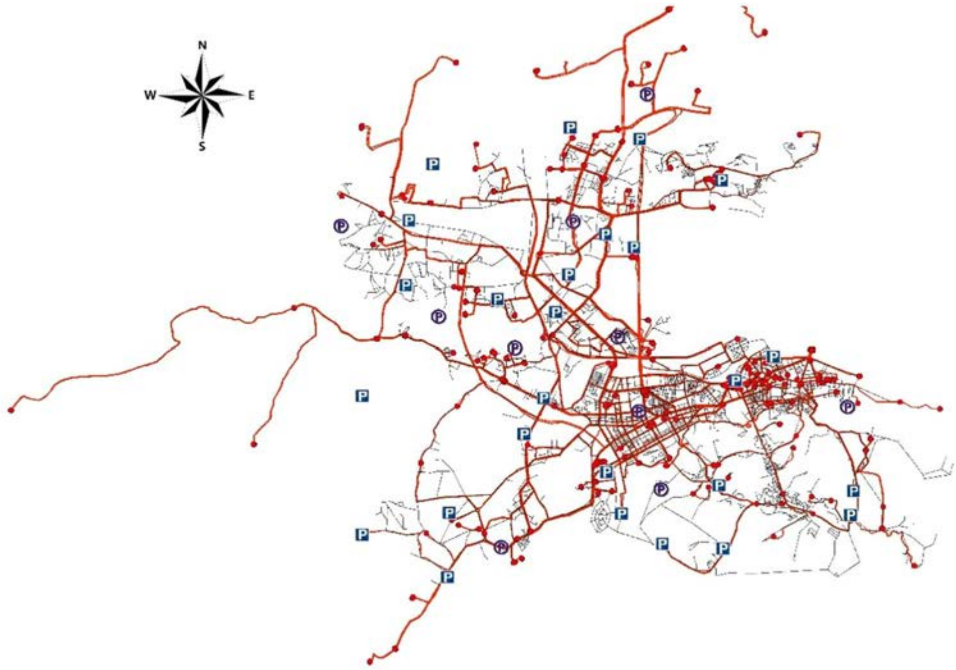

Figure 3. Current transit network in Dalian, China.

obtained using the GIS software MapInfo 11.0. By assuming that 60% of vehicles on each bus route are assigned to the departure station, while the remainder are allocated to the terminal station, the average travel cost per bus can be calculated. Besides, the average monthly wage for drivers and other staff is 3500 CNY.

For the garage construction cost, this article assumes that the unit land price is determined by reference to the China Real Estate Index System (CREIS). Note that the existing garages cannot be expanded because available land resources are scarce. Furthermore, because different vehicle types have different fuelling and maintenance requirements, some bus garages can only accommodate certain vehicle types. In this case, the initial construction costs of existing garages are set to 0. In contrast, the initial construction costs of new candidate garages are determined by which vehicle type can be parked there. In this article, vehicles are grouped into three categories: diesel, LNG and hybrid. Therefore, garages with diesel-related equipment can only park diesel buses and those with LNG-related facilities can only hold LNG buses, while hybrid buses can only be served in garages that have both diesel and LNG facilities. The area needed per bus for all vehicle types is 180 m^2, and the discount rate is set to 0.0325. Other related parameters are based on the real operational situation of the DPTG.

6.2. Application and results

Based on the data presented in Section 6.1, the proposed model and algorithm are finally applied to the real-life case. Figure 4 illustrates the objective values and gaps between the upper and lower bounds in five repeated calculations. The detailed results, including the best location, size and vehicle allocation, of the best calculation are presented in Table 3. A comparison of the associated cost components before and after optimization is provided in Table 4.

From Table 3, it can be seen that all existing garages are still retained, and only four of the 10 candidate garages are opened. This is because opening a new garage involves a costly initial investment. However, many vehicles are reallocated as a result of the constraints of capacities of the garages and

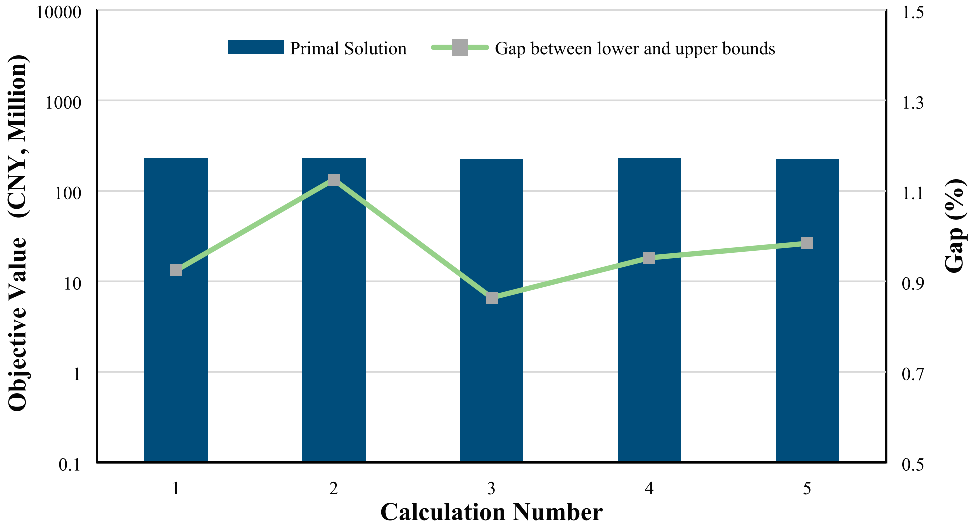

Figure 4. Objective values and gaps in the five repeated calculations.

Table 3. Configuration of the bus garage system after optimization.

No.	Current allocation	Optimal allocation	Optimal size	No.	Current allocation	Optimal allocation	Optimal size
1	20	19	3,420	19	82	65	11,700
2	81	166	29,880	20	34	26	4,680
3	90	116	20,880	21	156	108	19,440
4	62	33	5,940	22	39	32	5,760
5	103	125	22,500	23	89	52	9,360
6	176	74	13,320	24	166	150	27,000
7	96	65	11,700	25	35	15	2,700
8	155	174	31,320	26	63	105	18,900
9	96	55	9,900	27	–	–	
10	80	52	9,360	28	–	74	28,440
11	120	119	21,420	29	–	88	17,640
12	103	86	15,480	30	–	–	
13	206	102	18,360	31	–	–	
14	70	45	8,100	32	–	120	20,520
15	171	94	16,290	33	–	–	
16	119	83	14,940	34	–	94	23,040
17	103	41	7,380	35	–	–	
18	102	117	21,060	36	–	–	

Table 4. Comparison of associated cost components before and after optimization.

Cost category (CNY)	Current system	Optimal system	Gap (%)
Capital cost	–	34,173,720	–
Dead mileage cost	228,162,143	200,597,445	−12.08
Waiting-time cost	5,328,161	4,936,682	−7.34
Server staff cost	182,000	225,000	+23.6
Discounted value	–	18,073,306	–
Total cost	233,672,304	221,859,541	−5.06

the required area per bus. For example, Yunong Road Station (no. 15) initially has 171 vehicles before optimization, with only 99.5 m^2 for each bus. As a result, drivers waste a lot of time pulling into and out of the garage. The new vehicle allocation of Yunong Road Station is reduced to 94, but the garage area is nearly unchanged after optimization. It may cost a lot to build additional spaces to park these extra buses, but in the long run, this is a considerable reduction in the amount of waiting time and fuel costs.

Table 4 shows a comparison of the associated cost components of the bus garage system before and after optimization. After optimization, about 1.12 million CNY is saved, accounting for about 5.06% of the initial total cost of the bus garage system. Concerning the dead mileage cost, there is a reduction of almost 2.76 million CNY, reflecting a decrease of around 12.08%. However, with the increase in the number of bus garages, the staff cost increases by 23.6%. In short, although extra investment costs are required for these new garages, the total cost of the whole garage system can be decreased through the reduction in dead mileage and waiting-time costs.

7. Conclusions

A nonlinear integer programming model for determining the best location, number and size of bus garages, and a heuristic algorithm based on Lagrangian relaxation have been developed in this article. Although previous literature has focused on the bus garage location–allocation problem, few researchers have considered the bus garage as a congestion system. Drivers need to queue up to drive out of and into the garage before the beginning or after the end of daily service if too many buses are allocated to the same garage. Thus, the proposed model incorporates a wide range of costs, such as garage-related cost, dead mileage cost, staff cost and waiting-time cost. To further conform to the practical situation, different vehicle types and server numbers are also considered. Two groups of randomly generated test instances and a real-life case, the Dalian transit system in China, are applied to validate the algorithm. The results indicate that the proposed Lagrangian heuristic is efficient and stable.

Based on the modelling discussed in this article, several directions to further elaborate the modelling remain. One direction is that the queuing behaviour of the bus garage system may not act as an M/M/C queue. Examining complicated scheduling policies in future studies could provide a more elaborate model to explain the impacts of the queuing phenomenon on the optimization of the bus garage system. Furthermore, in this study, two randomly generated test instances and a real-life case were developed to validate the proposed LR algorithm. Future research may use other techniques to improve the performance of the proposed algorithm further.

Disclosure statement

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Appendix A. Information about the current bus garage system

No.	Location	Current area (m^2)	Current allocation	No.	Location	Current area (m^2)	Current allocation
1	MiniBus Station	3,500	20	19	Changchun Road Station	11,770	82
2	Fuguo Station	30,000	81	20	Fujia Manor Station	4,800	34
3	AnjiaWang Station	21,000	90	21	Lvbo Station	19,582	156
4	Baihe Village Station	6,000	62	22	Jinxiu Station	5,900	39
5	Tongde Road Station	22,527	103	23	Taishan Road	9,538	89
6	Jinliu Road Station	20,000	176	24	Zhangjia Station	30,000	166
7	Haikou Road Station	11,866	96	25	Zhongxia Road Station	3,000	35
8	Huadong Road Station	31,333	155	26	Lvcheng Station	18,925	63
9	Huanan Road Station	10,500	96	27	Donggang Station	–	–
10	Huanan Station	9,500	80	28	Miaoling Station	–	–
11	Jinjia Road Station	21,530	120	29	Beihai industrial zone Station	–	–
12	Youyi Road Station	15,481	103	30	Xiajia River Station	–	–
13	Yaojia Station	21,095	206	31	Xiaopingdao Station	–	–
14	Ganjingzi Station	8,190	70	32	Xianglujiao Station	–	–
15	Yunong Road Station	17,000	171	33	Suoyuwan Station	–	–
16	PaoYazi Station	15,000	119	34	Mingzhu Road Station	–	–
17	Zhongnan Road Station	8,900	103	35	Huanan Square Station	–	–
18	Zhongnan Station	23,825	102	36	Novy Valleys Station	–	–