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# Efficient MIMO Iterative Feedback Tuning via Randomization\*

Leontine Aarnoudse<sup>1</sup> and Tom Oomen<sup>1,2</sup>

**Abstract**—Iterative feedback tuning (IFT) enables the tuning of feedback controllers based on measured data without the need for a parametric model. The aim of this paper is to develop an efficient method for MIMO IFT that reduces the required number of experiments. Using a randomization technique, an unbiased gradient estimate is obtained from a single dedicated experiment, regardless of the size of the MIMO system. This gradient estimate is employed in a stochastic gradient descent algorithm. Simulation examples illustrate that the approach reduces the number of experiments required to converge.

## I. INTRODUCTION

Feedback controllers are typically designed in one of two ways: through fully model-based or data-driven methods. In a model-based approach, a system is first modeled, often through system identification. This is an expensive and difficult process, especially for multiple-input multiple-output (MIMO) systems. Therefore, data-driven approaches that depend less on accurate modeling because they combine models with data are appealing. An overview of data-driven control methods is given in [1], which includes, a.o., frequency-domain tuning [2], virtual reference feedback tuning (VRFT) [3], [4], correlation-based tuning [5], and iterative feedback tuning (IFT) [6], [7]. In this paper the focus is on IFT, which can be interpreted as using local modeling of the dependence of closed-loop signals on the controller [8], yet which does not require an explicit model.

Iterative feedback tuning is an iterative optimization-based approach that uses a stochastic gradient-descent parameter update. Unbiased gradient estimates are obtained directly from experiments on the noisy system. IFT has been applied successfully to, e.g., process industry [9], motion systems [10]–[13], cascaded control for quadrotors [14] and robotic systems [15], and intelligent PID control [16]. Extensions of IFT involve, e.g., robustness [12], system constraints [15], convergence speed [17] and prefiltering [18]. Most of these applications and extensions consider SISO systems.

IFT is extended to MIMO systems in [6], where it is shown that an unbiased estimate of the gradient for a MIMO system with  $n_i$  inputs and  $n_o$  outputs can be obtained through  $n_i \times n_o$  experiments. Similar approaches that generate gradients for MIMO systems through  $n_i \times n_o$  system experiments include VRFT [19],  $H_\infty$ -norm estimation [20] and data-driven iterative learning control (ILC) [21]. These approaches remove

the need for modeling for MIMO systems, but they do not scale well for large MIMO systems since the number of dedicated experiments increases with the size of the system.

A randomized MIMO IFT algorithm aimed at reducing the number of dedicated experiments is introduced in [22]. This approach is based on simultaneous perturbation stochastic approximation (SPSA) [23], in which the gradient is replaced by an estimated derivative in a random combination of input directions. However, the approach in [22] is limited to systems with periodic reference signals and negligible measurement noise. In this paper efficient unbiased gradient estimates for IFT are introduced. Similar to the data-driven MIMO ILC approach in [24], it is shown that for MIMO IFT unbiased gradient estimates can be obtained from a single experiment. These unbiased gradient estimates are used in a Robbins-Monro stochastic gradient descent algorithm [25]. Since standard IFT assumes noisy system evaluations that result in unbiased gradient estimates, it also employs a stochastic gradient descent parameter update and as such, the framework enables the introduction of an SPSA-like gradient estimate. The method is reminiscent of SPSA, but it is fundamentally different because it retains the gradient expressions from MIMO IFT and does not replace them with less accurate parameter perturbations.

Although MIMO iterative feedback tuning enables fully data-driven tuning of feedback controllers for MIMO systems, it does not scale well for large MIMO systems because of the high number of dedicated experiments. The aim of this paper is to develop an efficient MIMO IFT method that employs unbiased gradient estimates obtained from a single experiment. The contribution consists of the following elements.

- 1) A MIMO IFT approach is developed that uses a single experiment per iteration to obtain an unbiased gradient estimate.
- 2) The unbiased gradient estimate is used in a stochastic gradient descent algorithm and convergence is proved.
- 3) The MIMO IFT approach is illustrated using two simulation examples.

This paper is structured as follows. In Section II, the problem is introduced. Standard MIMO IFT is introduced in Section III. In Section IV, it is shown that an unbiased gradient estimate can be obtained from a single experiment, which is applied in stochastic approximation MIMO IFT. In Section V several implementation aspects are considered. The approach is validated in simulations in Section VI and conclusions are given in Section VII.

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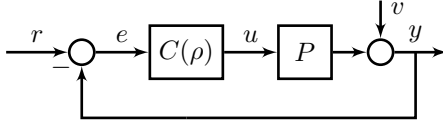


Fig. 1: Control scheme.

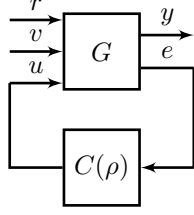


Fig. 2: General control scheme.

## II. PROBLEM FORMULATION

Consider a discrete-time linear time-invariant MIMO system with  $n_i$  inputs and  $n_o$  outputs, given by

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{n_o} \end{bmatrix} = \begin{bmatrix} P_{11} & \cdots & P_{1n_i} \\ \vdots & \ddots & \vdots \\ P_{n_o 1} & \cdots & P_{n_o n_i} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_{n_i} \end{bmatrix}. \quad (1)$$

This system is placed in closed loop with a controller  $C(\rho)$  that is parameterized by the parameter vector  $\rho \in \mathbb{R}^{n_\rho}$ , as shown in Fig. 1. The output  $y \in \mathbb{R}^{N n_o}$  is then given by

$$y = (I + PC(\rho))^{-1} PC(\rho)r + (I + PC(\rho))^{-1}v, \quad (2)$$

with reference  $r \in \mathbb{R}^{N n_o}$  and disturbance  $v \in \mathbb{R}^{N n_o}$ . The feedback error  $e \in \mathbb{R}^{N n_o}$  is given by  $e = r - y$ . The system is rewritten to the generalized plant  $G$  with feedback controller  $C(\rho)$  shown in Fig. 2, resulting in

$$\begin{pmatrix} y \\ e \end{pmatrix} = G \begin{pmatrix} r \\ v \\ u \end{pmatrix}, \quad (3)$$

$$u = C(\rho)e, \quad (4)$$

with  $u \in \mathbb{R}^{N n_i}$  denoting the controller output. The aim is to find controller parameters  $\rho$  such that the cost function

$$\mathcal{J}(\rho) = \frac{1}{2N} \sum_{t=1}^N [(y_d(t) - y(\rho, t))^T (y_d(t) - y(\rho, t))], \quad (5)$$

is minimized. Here  $y(\rho, t)$  denotes entry  $t$  of  $y(\rho)$ , and  $y_d = Mr$  is the output of a reference model  $M$ .

The aim of this paper is to develop an efficient approach, in terms of the number of experiments, to minimizing (5) for an unknown MIMO system. A gradient-based approach based on iterative feedback tuning (IFT) is used, in which the experimentally expensive gradient expressions are replaced by gradient estimates that follow from a single experiment.

## III. ITERATIVE FEEDBACK TUNING FOR MIMO SYSTEMS

In this section MIMO iterative feedback tuning is introduced. In IFT, the controller parameters  $\rho$  are updated iteratively based on estimates  $\widehat{\frac{\partial \mathcal{J}(\rho_j)}{\partial \rho_j}}$  of the gradient of cost

function (5). The gradient estimates follow from system experiments, and since they are unbiased, these directly allow a stochastic gradient descent update according to

$$\rho_{j+1} = \rho_j - \varepsilon_j \widehat{\frac{\partial \mathcal{J}(\rho_j)}{\partial \rho_j}}. \quad (6)$$

**Remark 1.** Typically, the stochastic nature of the gradient estimate results from measurement noise and disturbances during system experiments. In this paper, a method is developed to obtain unbiased gradient estimates that are stochastic by design, even if system experiments are deterministic.

The gradient  $g(\rho_j) = \frac{\partial \mathcal{J}(\rho_j)}{\partial \rho_j}$  of (5) is given by

$$g(\rho_j) = \frac{1}{N} \sum_{t=1}^N \left[ \left( \frac{\partial y(\rho_j)}{\partial \rho_j}(t) \right)^T (y_d(t) - y(\rho_j, t)) \right]. \quad (7)$$

Given  $y(\rho_j)$ , IFT aims to obtain an estimate of  $\frac{\partial y(\rho_j)}{\partial \rho_j}$ , i.e., the derivative of  $y(\rho)$  to the complete parameter vector  $\rho$ , from experiments. In the remainder of the paper, the iteration index  $j$  is omitted for ease of notation.

The experiments to obtain gradient estimates in IFT are constructed as follows. The derivative of the output  $y(\rho)$  to the  $x^{\text{th}}$  entry  $\rho(x)$ , i.e., to a single parameter, is denoted by  $y'(\rho(x)) \in \mathbb{R}^{N n_o}$ , and is given by

$$\begin{pmatrix} y' \\ e' \end{pmatrix} = G \begin{pmatrix} 0 \\ 0 \\ u' \end{pmatrix} \quad (8)$$

$$u' = C'(\rho(x))e + C(\rho)e'. \quad (9)$$

Here  $C'(\rho(x))$  is the derivative of  $C(\rho)$  to the  $x^{\text{th}}$  entry of  $\rho$ . The  $n_\rho$  derivatives  $y'(\rho(x))$  follow from  $n_i \times n_o$  experiments [6], by observing that for each  $x$ ,  $y'(\rho(x))$  is given by

$$y'(\rho(x)) = \begin{pmatrix} \sum_{l=1}^{n_o} \sum_{k=1}^{n_i} J_{1k}(\rho) C'_{kl}(\rho(x)) e_l(\rho) \\ \sum_{l=1}^{n_o} \sum_{k=1}^{n_i} J_{2k}(\rho) C'_{kl}(\rho(x)) e_l(\rho) \\ \vdots \\ \sum_{l=1}^{n_o} \sum_{k=1}^{n_i} J_{n_o k}(\rho) C'_{kl}(\rho(x)) e_l(\rho) \end{pmatrix}. \quad (10)$$

The system  $J$  is the transfer from  $u$  to  $y$ , e.g., in the closed-loop system shown in Fig. 1,  $J(\rho)$  is the process sensitivity  $(I + PC(\rho))^{-1}P$ . Because the derivatives  $C'_{kl}$  are the SISO elements of  $C'$ ,  $C'_{kl}$  and  $J_{mk}$  commute and calculating  $y'(\rho(x))$  requires only the signals  $J_{mk}(\rho) e_l(\rho)$  for  $m = 1, \dots, n_o$ ,  $k = 1, \dots, n_i$ ,  $l = 1, \dots, n_o$ . These signals follow from applying the measured error of each output direction,  $e_1, e_2, \dots, e_{n_o}$ , to each of the  $n_i$  input directions of  $J$ , requiring  $n_i \times n_o$  experiments for a MIMO system with  $n_i$  inputs and  $n_o$  outputs. With the signals  $J_{mk}(\rho) e_l(\rho)$ , the derivatives  $y'(\rho(x))$  for each  $x = 1, \dots, n_\rho$  are determined according to (10), after which the gradient estimate follows from (7).

In certain cases, for example when not all controller channels are used, fewer than  $n_i \times n_o$  experiments per iteration may suffice, but overall this approach is experimentally inefficient for large MIMO systems. Therefore, unbiased gradient estimates from a single experiment are developed in the next section.

#### IV. EFFICIENT UNBIASED GRADIENT ESTIMATES FOR MIMO IFT

The previous section shows that standard IFT requires  $n_i \times n_o$  experiments per iteration to obtain the gradient  $g(\rho_j)$  for an  $n_i \times n_o$  MIMO system. As such, it does not scale well for large MIMO systems. In this section, it is shown that an unbiased gradient estimate can be obtained from a single dedicated experiment instead, regardless of the size of the MIMO system. First, the main idea is illustrated for a  $2 \times 2$  example. Then, the approach is extended to MIMO systems of any size. Lastly, the convergence is analyzed.

The essential step for the gradient experiment is the introduction of the matrix  $A_j \in \mathbb{R}^{(Nn_i) \times (Nn_o)}$ , which is constructed as

$$A_j = \begin{bmatrix} a_j^{11} & \dots & a_j^{1n_o} \\ \vdots & \ddots & \vdots \\ a_j^{n_i 1} & \dots & a_j^{n_i n_o} \end{bmatrix} \otimes I_N. \quad (11)$$

Here,  $I_N$  is the  $N \times N$  identity matrix and the entries  $a_j^{lm}$  are samples from a symmetric Bernoulli  $\pm 1$  distribution, i.e.,  $a_j^{lm} \in \{-1, 1\}$  with probabilities  $P(a_j^{lm} = 1) = 1/2$  and  $P(a_j^{lm} = -1) = 1/2$ .

##### A. SAIFT: $2 \times 2$ example

The approach, which is referred to as stochastic approximation IFT (SAIFT), is illustrated using a  $2 \times 2$  system. SAIFT is aimed at finding an efficient way to estimate the signals  $J_{mk}(\rho)e_l(\rho)$  for  $m = 1, \dots, n_o$ ,  $k = 1, \dots, n_i$ ,  $l = 1, \dots, n_o$  in (10). In particular, a single experiment is defined from which unbiased estimates  $\hat{y}'(\rho)$  can be determined for each parameter  $\rho$ . For ease of notation, the dependency of  $J$ ,  $C'$  and  $e$  on  $\rho$  is omitted.

For a  $2 \times 2$  system, the derivative  $y'(\rho(x)) \in \mathbb{R}^{2N}$  that is required to compute the gradient (7) is given by

$$y'(\rho(x)) = \begin{bmatrix} J_{11}(C'_{11}e_1 + C'_{12}e_2) + J_{12}(C'_{21}e_1 + C'_{22}e_2) \\ J_{21}(C'_{11}e_1 + C'_{12}e_2) + J_{22}(C'_{21}e_1 + C'_{22}e_2) \end{bmatrix} \quad (12)$$

The matrix  $A_j$  is now given by

$$A_j = \begin{bmatrix} a_j^{11} & a_j^{12} \\ a_j^{21} & a_j^{22} \end{bmatrix} \otimes I_N. \quad (13)$$

This matrix is used in an experiment of the form

$$\begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} A_j \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \begin{bmatrix} J_{11}(a_j^{11}e_1 + a_j^{12}e_2) + J_{12}(a_j^{21}e_1 + a_j^{22}e_2) \\ J_{21}(a_j^{11}e_1 + a_j^{12}e_2) + J_{22}(a_j^{21}e_1 + a_j^{22}e_2) \end{bmatrix} \quad (14)$$

For each parameter  $\rho(x)$ ,  $x = 1, 2, \dots, n_\rho$ , an unbiased estimate  $\hat{y}'(\rho(x))$  is obtained by taking

$$\hat{y}'(\rho(x)) = \begin{bmatrix} (a_j^{11}C'_{11} + a_j^{12}C'_{12} + a_j^{21}C'_{21} + a_j^{22}C'_{22}) \times \\ (J_{11}(a_j^{11}e_1 + a_j^{12}e_2) + J_{12}(a_j^{21}e_1 + a_j^{22}e_2)) \\ (a_j^{11}C'_{11} + a_j^{12}C'_{12} + a_j^{21}C'_{21} + a_j^{22}C'_{22}) \times \\ (J_{21}(a_j^{11}e_1 + a_j^{12}e_2) + J_{22}(a_j^{21}e_1 + a_j^{22}e_2)) \end{bmatrix}, \quad (15)$$

where  $C'_{kl}$  are entries of the matrix  $C' = \frac{\partial C}{\partial \rho}$  for any entry of  $\rho$ . The expected value of this expression is obtained by noting that  $E\{a^{\alpha\beta}a^{\gamma\delta}\} = 1$  if  $\alpha = \gamma$ ,  $\beta = \delta$  and otherwise, if  $\alpha \neq \gamma$  and/or  $\beta \neq \delta$ ,  $E\{a^{\alpha\beta}a^{\gamma\delta}\} = 0$ . The expected value of  $\hat{y}'(\rho(x))$  is given by

$$\begin{aligned} E\{\hat{y}'(\rho(x))\} &= \begin{bmatrix} a_j^{11}C'_{11}J_{11}a_j^{11}e_1 + a_j^{12}C'_{12}J_{11}a_j^{12}e_2 + \\ a_j^{11}C'_{11}J_{21}a_j^{11}e_1 + a_j^{12}C'_{12}J_{21}a_j^{12}e_2 + \\ a_j^{21}C'_{21}J_{12}a_j^{21}e_1 + a_j^{22}C'_{22}J_{12}a_j^{22}e_2 \\ a_j^{21}C'_{21}J_{22}a_j^{21}e_1 + a_j^{22}C'_{22}J_{22}a_j^{22}e_2 \end{bmatrix} \\ &= \begin{bmatrix} J_{11}(C'_{11}e_1 + C'_{12}e_2) + J_{12}(C'_{21}e_1 + C'_{22}e_2) \\ J_{21}(C'_{11}e_1 + C'_{12}e_2) + J_{22}(C'_{21}e_1 + C'_{22}e_2) \end{bmatrix}, \quad (16) \end{aligned}$$

i.e.,  $E\{\hat{y}'(\rho(x))\} = y'(\rho(x))$ , which shows that the single experiment (14) leads to an unbiased estimate of the gradient.

##### B. SAIFT for massive MIMO systems

The approach illustrated for a  $2 \times 2$  system in the previous subsection is formalized for general  $n_i \times n_o$  MIMO systems in the following theorem.

**Theorem 1.** *Let*

$$\hat{y}'(\rho(x)) = \begin{bmatrix} \sum_{m=1}^{n_i} \sum_{q=1}^{n_o} a_j^{mq} C'_{mq} \left( \sum_{k=1}^{n_i} J_{1k} \left( \sum_{l=1}^{n_o} a_j^{kl} e_l \right) \right) \\ \vdots \\ \sum_{m=1}^{n_i} \sum_{q=1}^{n_o} a_j^{mq} C'_{mq} \left( \sum_{k=1}^{n_i} J_{n_o k} \left( \sum_{l=1}^{n_o} a_j^{kl} e_l \right) \right) \end{bmatrix}, \quad (17)$$

with  $a_j^{mq}$  and  $a_j^{kl}$  the entries of matrix  $A_j$  in (11), and define

$$\hat{g}(\rho) = \frac{1}{N} \sum_{t=1}^N \left[ \left( \widehat{\frac{\partial y(\rho)}{\partial \rho}}(t) \right)^\top (y_d(t) - y(\rho, t)) \right], \quad (18)$$

with

$$\begin{aligned} \widehat{\frac{\partial y(\rho)}{\partial \rho}}(t) &= \begin{bmatrix} \hat{y}'(\rho(1))(t) & \hat{y}'(\rho(2))(t) & \dots & \hat{y}'(\rho(n_\rho))(t) \end{bmatrix} \in \mathbb{R}^{n_o \times N n_\rho}. \end{aligned} \quad (19)$$

Then  $E \left\{ \widehat{\frac{\partial y(\rho)}{\partial \rho}}(t) \right\} = \frac{\partial y(\rho)}{\partial \rho}(t)$  and consequently,

$$E\{\hat{g}(\rho)\} = g(\rho). \quad (20)$$

*Proof.* For the entries of (17) it holds that  $E\{a_j^{\alpha\beta}a_j^{\gamma\delta}\} = 1$  if  $\alpha = \gamma$ ,  $\beta = \delta$  and  $E\{a_j^{\alpha\beta}a_j^{\gamma\delta}\} = 0$  otherwise. Therefore, (17) is equal to  $\tilde{y}'(\rho(x)) + \eta$ , with

$$\tilde{y}'(\rho(x)) = \begin{bmatrix} \sum_{k=1}^{n_i} \sum_{l=1}^{n_o} C'_{kl} J_{1k} e_l \\ \vdots \\ \sum_{k=1}^{n_i} \sum_{l=1}^{n_o} C'_{kl} J_{n_o k} e_l \end{bmatrix} \quad (21)$$

and  $\eta$  containing all the terms for which  $\alpha \neq \gamma$  or  $\beta \neq \delta$ . Since  $E\{\eta\} = 0$ , and since the SISO elements  $C'_{kl}$  and  $J_{1k}$  commute, it holds that

$$E\{\hat{y}'(\rho(x))\} = \tilde{y}'(\rho(x)) = y'(\rho(x)). \quad (22)$$

From (22) and (19) it follows that  $E \left\{ \widehat{\frac{\partial y(\rho)}{\partial \rho}}(t) \right\} = \frac{\partial y(\rho)}{\partial \rho}(t)$  and consequently, it holds that  $E\{\hat{g}(\rho)\} = g(\rho)$ .  $\square$

The unbiased estimate  $\hat{g}(\rho)$  in Theorem 1 is obtained from a single experiment on  $J$ , given by

$$JA_j e = \begin{bmatrix} J_{11} & \dots & J_{1n_i} \\ \vdots & \ddots & \vdots \\ J_{n_o 1} & \dots & J_{n_o n_i} \\ \sum_{k=1}^{n_i} J_{1k} \left( \sum_{l=1}^{n_o} a_j^{kl} e_l \right) \\ \vdots \\ \sum_{k=1}^{n_i} J_{n_o k} \left( \sum_{l=1}^{n_o} a_j^{kl} e_l \right) \end{bmatrix} A_j \begin{pmatrix} e_1 \\ \vdots \\ e_{n_o} \end{pmatrix} = \quad (23)$$

Experiment (23) gives the terms  $\sum_{k=1}^{n_i} J_{ik} \left( \sum_{l=1}^{n_o} a_j^{kl} e_l \right)$  for  $i = 1, \dots, n_o$ , which are the entries of (17) that allow for computation of (19) and finally  $\hat{g}(\rho)$ . This estimate is used in a stochastic gradient descent update given by

$$\rho_{j+1} = \rho_j - \varepsilon_j \hat{g}(\rho_j). \quad (24)$$

Using Theorem 1,  $\hat{g}(\rho_j)$  is rewritten as

$$\hat{g}(\rho_j) = g(\rho_j) + \omega_j, \quad (25)$$

where the disturbance term  $\omega_j$  results from  $\eta_j$  in (17), which is propagated when (19) and  $\hat{g}(\rho_j)$  are constructed. Similar to  $\eta_j$ , for  $\omega_j$  it holds that  $E\{\omega_j\} = 0$  by Theorem 1.

### C. Convergence

Next, convergence of the SAIFT algorithm under certain assumptions is shown. In the previous subsections, the presence of noise and disturbances in the system evaluations was omitted for brevity. In practice, there are disturbances during experiments, for which the following is assumed.

**Assumption 1.** *The disturbance signal  $v$  is a bounded discrete-time zero-mean stochastic process. The second-order statistics are the same for all experiments, and sequences from different experiments are mutually independent.*

Assumption 1 ensures that the gradient estimates  $\hat{g}$  remain unbiased when disturbances are taken into account. The SAIFT algorithm is written as a Robbins-Monro algorithm:

$$\rho_{j+1} = \rho_j - \varepsilon_j (g(\rho_j) + \omega_j), \quad (26)$$

with  $\omega_j$  now containing both the propagation of  $\eta_j$  in (17) and the terms resulting from disturbances during the different experiments. Consider the following assumptions, which are standard for these type of algorithms.

**Assumption 2.** *The iterates  $\rho_j$  remain almost surely bounded.*

**Assumption 3.** *The step size  $\varepsilon_j$  is chosen such that*

$$\sum_j \varepsilon_j = \infty, \quad \sum_j \varepsilon_j^2 < \infty.$$

**Assumption 4.** *The gradient  $g(\rho)$  is Lipschitz continuous, and the sequence  $\{\omega_j\}$  is square-integrable.*

Assumptions 2 and 3 can be satisfied by choosing  $\varepsilon_j$  appropriately. The assumptions lead to the following theorem.

**Theorem 2.** *Under Assumption 2 and 3, the sequence of iterates  $\{\rho_j\}$  in (26) converges to a stationary point  $\rho^*$  for which  $g(\rho) = 0$  almost surely.*

The disturbance term  $\omega_j$  is a Martingale difference sequence since  $E\{\omega_j\} = 0$  and it is square integrable. The proof of Theorem 2 follows from this property and the almost sure convergence of a Robbins-Monro algorithm under Assumptions 2 and 3, see, e.g., [26, Chapter 2].

**Remark 2.** *In IFT, satisfying Assumption 2 is not trivial. A bounded iterate  $\rho_j$  may result in a controller for which the closed-loop system is unstable. This leads to an unbounded error  $e(\rho_j)$  and consequently the gradient estimate and the next iterate  $\rho_{j+1}$  are unbounded. It is therefore essential that the parameter update leads to a stabilizing controller. This can be ensured by choosing  $\varepsilon_j$  small enough, but it is difficult to guarantee in practice, see also Section V-C.*

**Remark 3.** *Note that the convergence theorem 2 is similar to one in [7, Theorem 1], which considers only the disturbance terms that are due to noisy system evaluations, and in which boundedness of the iterates is ensured by requiring that the closed-loop system remains stable. Again, this stability must be ensured by the user through a suitable choice of step size.*

## V. IMPLEMENTATION ASPECTS

In this section, several implementation aspects of the SAIFT algorithm are considered. First, an overview of the implementation is given. Secondly, scaling of the gradient experiments is introduced. Thirdly, some approaches to ensure boundedness of the iterates are suggested.

### A. Implementation of the SAIFT algorithm

The implementation of SAIFT is outlined in Algorithm 1. The algorithm illustrates that each iteration requires only one additional experiment to obtain an unbiased estimate of the gradient, regardless of the size of the MIMO system.

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#### Algorithm 1 Stochastic Approximation IFT

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- 1: **for**  $j = 1 : n_{\text{iteration}}$
  - 2:   Apply controller  $C(\rho_j)$  and measure  $e_j = r - y(\rho_j)$ .
  - 3:   Apply the input  $A_j e_j$  to the system with  $C(\rho_j)$  and measure  $JA_j e_j$  according to (23).
  - 4:   Compute the entries  $\hat{y}'(\rho(x))$  of  $\widehat{\frac{\partial y(\rho)}{\partial \rho}}$  according to (17), using  $JA_j e_j$ ,  $A_j$  and the derivatives  $C'(\rho_j(x))$ .
  - 5:   Use  $\widehat{\frac{\partial y(\rho)}{\partial \rho}}$  to compute  $\hat{g}(\rho_j)$  according to (18).
  - 6:   Determine a suitable step size  $\varepsilon_j$ , see Section V-C.
  - 7:   Update  $\rho_{j+1} = \rho_j - \varepsilon_j \hat{g}_j$  according to (24).
  - 8: **end**
- 

### B. Scaling of dedicated experiments

Algorithm 1 involves a dedicated experiment, in which a measured error signal is applied as an input at the location of signal  $u$ . Since  $u = C(\rho)e$ , the magnitude of  $e$  may not be similar to that of  $u$ . For example,  $e$  may be too large to apply to  $P$  because it exceeds actuator limits. In such cases

it is necessary to scale  $e$ , which can be done using a scaling factor  $\alpha$  that is implemented as

$$JA_j e_j = \frac{1}{\alpha} J \alpha A_j e_j. \quad (27)$$

The scaling factor can be chosen separately for each iteration. It is assumed that  $\alpha$  is chosen such that the system behaves linearly. Note that while this paper considers LTI systems, in practice nonlinear effects such as saturation occur in most systems and therefore they should be taken into account.

### C. Ensuring stability

In Remark 2 it is mentioned that in order to apply iterative feedback tuning, the user must choose the step size  $\varepsilon_j$  such that the parameter update (24) results in a stabilizing controller  $C(\rho_j)$ . This holds for standard IFT, yet for stochastic approximation IFT it is even more important because the search direction given by the unbiased gradient estimates may deviate strongly from the real gradient. Some possible approaches to ensuring stability are listed below.

- An approximate parametric model of the system may be available, for example when IFT is used to improve the controllers for each copy of a series of systems. Such a model, combined with appropriate bounds to ensure robustness, can be used to assess stability.
- A non-parametric model such as a frequency response measurement can be used directly to assess stability. Alternatively, one could use the non-parametric model to determine the generalized stability margin, and choose the step size such that the Vinnicombe distance [27] between a stabilizing controller and the controller update remains below this value, see also [2].
- In order to ensure robust stability, frequency-domain constraints such as developed in [12] for SISO systems can be added to the optimization problem.

These approaches require at least a non-parametric model of the system. Although one often mentioned advantage of IFT is that no model knowledge is required, safe implementation in practice requires at least a non-parametric frequency response function. For many systems, these models are relatively inexpensive, fast and accurate to obtain.

## VI. EXAMPLE

In this section, the SAIFT algorithm is applied to two different simulation models and compared to standard IFT.

### A. Example 1

Consider the following discrete-time  $2 \times 2$  plant [5],

$$P(q^{-1}) = \begin{bmatrix} \frac{0.0951q^{-1}}{1-0.9048q^{-1}} & \frac{0.03807q^{-1}}{1-0.9048q^{-1}} \\ \frac{-0.02974q^{-1}}{1-0.9048q^{-1}} & \frac{0.04758q^{-1}}{1-0.9048q^{-1}} \end{bmatrix}, \quad (28)$$

and let the initial controller be given by

$$C_0(q^{-1}) = \begin{bmatrix} \frac{1-0.99q^{-1}}{1-q^{-1}} & \frac{0.1-0.099q^{-1}}{1-q^{-1}} \\ -\frac{1-0.99q^{-1}}{1-q^{-1}} & \frac{1-0.99q^{-1}}{1-q^{-1}} \end{bmatrix}. \quad (29)$$

The parameterized controller  $C(\rho)$  has a structure similar to  $C_0$ , i.e., all elements have an integrator and a first degree

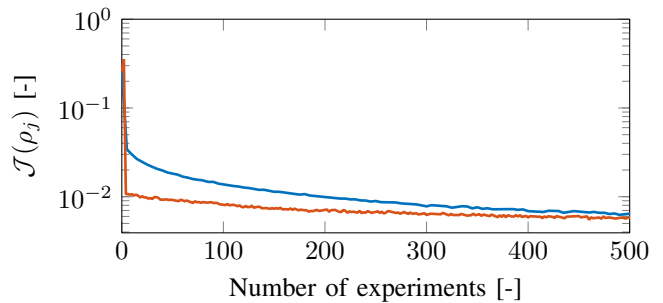


Fig. 3: For example 1, SAIFT (—) with  $\varepsilon = 0.005$  reaches the same cost as standard IFT (—) with  $\varepsilon = 0.01$  in far fewer experiments: SAIFT requires two experiments per iteration, whereas IFT requires five for the  $2 \times 2$  system.

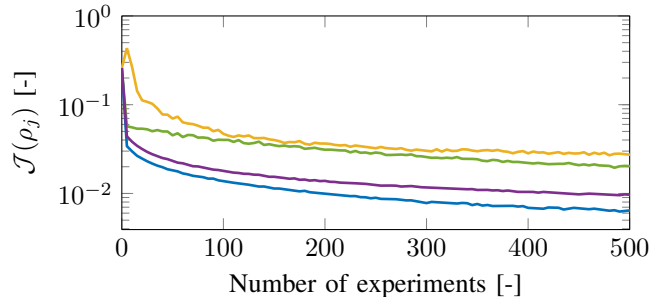


Fig. 4: Convergence for IFT (example 1) with varying step sizes 0.1 (—), 0.05 (—), 0.01 (—) and 0.005 (—). A larger step size does not necessarily result in faster convergence.

numerator with two parameters, such that  $\rho \in \mathbb{R}^{8 \times 1}$ . In addition, a reference model is defined given by

$$M(q^{-1}) = \frac{0.1148q^{-1} - 0.0942q^{-2}}{1 - 1.79q^{-1} + 0.8106q^{-2}} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \quad (30)$$

Reference  $r$  consist of a step for the first output at sample 10 and a step for the second output at sample 100, and measurement noise with a variance of 0.01 is added.

Stochastic approximation IFT and standard IFT are applied to this system with step sizes of respectively  $\varepsilon_j = 0.005 \forall j$  and  $\varepsilon_j = 0.01 \forall j$ . In Fig. 3 it is shown that SAIFT requires far fewer experiments to converge to the same cost, which is not compensated by the larger step size used for standard IFT. A smaller step size is used for SAIFT, because the larger variance of the gradient estimates causes SAIFT to become unstable faster. For IFT the step size that leads to the fastest convergence is chosen based on Fig. 4.

### B. Example 2: LV100 gas turbine engine

The second example is a model of a  $2 \times 2$  LV100 gas turbine engine, see [28]. The same example has been employed for IFT with Hessian estimates in [7], correlation-based tuning in [5], and virtual-reference feedback tuning in [4]. The simulation conditions are taken from [7]. SAIFT and IFT are applied with step sizes of respectively  $\varepsilon_j = 0.01 \forall j$  and  $\varepsilon_j = 0.02 \forall j$ . Again, for IFT the step size for which fastest convergence is observed is used. Fig. 5 shows that SAIFT requires fewer experiments to reach the same cost; especially in the beginning convergence is much faster.

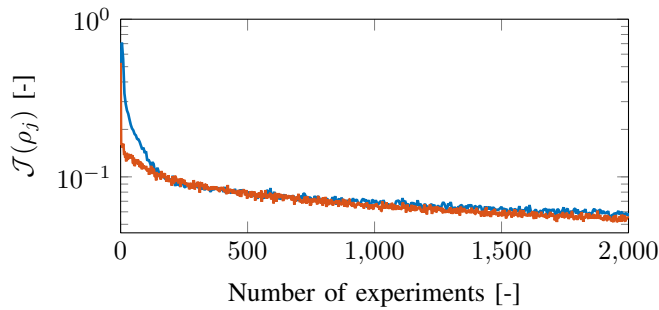


Fig. 5: For the LV100 system, SAIFT (—) with  $\varepsilon = 0.01$  reaches the same cost as standard IFT (—) with  $\varepsilon = 0.02$  in fewer experiments. SAIFT requires two experiments per iteration, whereas IFT requires five for the  $2 \times 2$  system.

## VII. CONCLUSIONS AND OPEN CHALLENGES

A new method of randomization allows IFT to converge much faster for MIMO systems, enabling a broader implementation on complex systems. The stochastic approximation IFT algorithm uses an unbiased gradient estimate that follows from a single randomized experiment on the system. This gradient estimate is then used in a stochastic gradient descent parameter update, the convergence of which is analyzed. Simulation results show that SAIFT requires fewer experiments to reach the same cost compared to the current MIMO IFT approach. Future research involves experimental implementation of SAIFT and simulation of larger systems, as well as the open challenges mentioned next.

### A. Open challenges for MIMO (SA)IFT

The simulation results illustrate that stochastic approximation IFT is promising, as it can improve upon the performance of standard IFT. However, there are two open challenges for the implementation of MIMO IFT in practice that are important to mention. First, stability of the parameter update has to be ensured. Suggestions on how to approach this are given in Section V-C. Additionally, the choice of reference model might influence the stability of SAIFT.

Secondly, gradient descent methods are known to converge slowly, as can be seen in the simulation results. In order to improve the convergence speed, one could consider extending the methods in Section V-C to not only check what step sizes lead to stability, but also what step sizes are expected to lead to an improved cost. To this end, a model-based inexact line search could be employed. Additionally, Hessian estimates could be included in the parameter update, see, e.g., [29] for unbiased Hessian estimates for SISO IFT.

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