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Dependence of the transition from Townsend to glow discharge on secondary emission

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In a recent paper, Šijačić and Ebert [Phys. Rev. E. **66**, 006410 (2002)] systematically studied the transition from Townsend to glow discharge, referring to older work by von Engel and M. Steenbeck [*Elektrische Gasentladungen. Ihre Physik und Technik* (Springer, Berlin 1934), Vol. II] up to Raizer [*Gas Discharge Physics* (Springer, Berlin, 1991)]. Šijačić and Ebert stated that this transition strongly depends on secondary emission γ from the cathode. We show here that the earlier results of von Engel and Raizer on the small current expansion about the Townsend limit actually are the limit of small γ of the Šijačić and Ebert expression, and that for larger γ the old and the Šijačić and Ebert new results vary by no more than a factor of 2. We discuss the γ dependence of the transition, which is rather strong for short gaps.

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In a recent article [1], the transition from Townsend to glow discharge was reinvestigated with analytical and numerical means. On the analytical side, a systematic, small current expansion about the Townsend limit was performed and it was stated:

“The result agrees qualitatively with the one given by Raizer [2] and Engel and Steenbeck [3]. In particular, the leading order correction is also of order $\alpha''(j/\mu)^2$. However, the explicit coefficient of j^2 differs: while the coefficient in [2,3] does not depend on γ at all, we find that the dependence on γ is essential, as the plot of F in Fig. 1 of [1] clearly indicates. In fact, within the relevant range of $10^{-6} \leq \gamma \leq 10^0$, this coefficient varies by almost four orders of magnitude. We remark that it indeed would be quite a surprising mathematical result if the Townsend limit itself would depend on γ , but the small current expansion about it would not.”

Here, we remark that while the systematic calculation in Ref. [1] was correct, the interpretation and comparison to earlier work requires some correction.

To be precise, the model treated in [1–3] and by many other authors is a one-dimensional time independent Townsend or glow discharge characterized by the classical equations for electron and ion particle current $J_{e,+}$ and electric field E , given by

$$\partial_x J_e = |J_e| \bar{\alpha}(|E|), \quad \partial_x J_+ = |J_+| \bar{\alpha}(|E|), \quad (1)$$

$$\partial_x E = \frac{e}{\epsilon_0} (n_+ - n_e), \quad (2)$$

$$J_e = -n_e \mu_e E, \quad J_+ = n_+ \mu_+ E. \quad (3)$$

Impact ionization in the bulk of the discharge is given by the Townsend approximation

$$\bar{\alpha}(|E|) = \alpha_0 e^{-E_0/|E|}. \quad (4)$$

(In [1], the generalized case $\bar{\alpha}(|E|) = \alpha_0 \exp(-E_0/|E|)^s$ was treated.) Boundary conditions at the anode ($x=0$) and for secondary emission at the cathode ($x=d$) are

$$J_+(0) = 0, \quad |J_e(d)| = \gamma |J_+(d)|. \quad (5)$$

The discharge is characterized by the potential U and total electric current J , as

$$U = \int_0^d dx E(x), \quad J = e(n_+ \mu_+ + n_e \mu_e) E. \quad (6)$$

It is useful to introduce dimensionless voltage and current, as

$$u = \frac{U}{E_0/\alpha_0}, \quad \bar{j} = \frac{J}{\epsilon_0 \alpha_0 E_0 \mu_+ E_0}, \quad (7)$$

where $\bar{j} = j/\mu$ with the definition of j from [1]. It should be noted that only bulk gas parameters have been used as units; therefore, the dimensionless u and \bar{j} are independent of γ .

Further dimensional analysis yields that the current-voltage characteristics $u = u(\bar{j})$ can depend on three parameters only; namely, on the dimensionless gap length $L = \alpha_0 d$, on the coefficient γ of secondary emission, and on the mobility ratio $\mu = \mu_+/\mu_e$. In practice, the dependence on the small parameter μ is almost negligibly weak [1]; therefore, $u = u(\bar{j}, L, \gamma)$. Here, the dimensionless gap length L is related to pd through $L = Apd$ as long as the coefficient α_0 is related to pressure as $\alpha_0 = Ap$.

How strongly does the characteristics $u = u(\bar{j}, L, \gamma)$ depend on γ ? In [1], Šijačić and Ebert (SE) calculated the whole Townsend-to-glow regime numerically and derived, by expanding systematically in powers of current \bar{j} about the Townsend limit, that

$$u = u_T - A_{SE} \bar{j}^2 + O(\bar{j}^3), \quad (8)$$

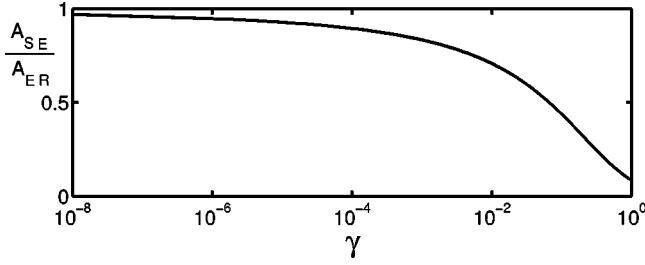


FIG. 1. The ratio A_{SE}/A_{ER} of the small current expansions by Šijačić and Ebert and by von Engel and Raizer as a function of γ .

$$A_{SE} = \frac{\mathcal{E}_T \alpha'' F(\gamma, \mu)}{2 \alpha' (\alpha \mathcal{E}_T)^3}, \quad (9)$$

$$\alpha(\mathcal{E}_T) = e^{-1/|\mathcal{E}_T|}, \quad (10)$$

which gave an excellent fit to the numerical solutions. Here,

$$F(\gamma, \mu) = \frac{L_\gamma^3}{12} + (1 + \mu)(2 - L_\gamma - 2e^{-L_\gamma} - L_\gamma e^{-L_\gamma}) + (1 + \mu)^2 \left(\frac{1 - e^{-2L_\gamma}}{2} - \frac{(1 - e^{-L_\gamma})^2}{L_\gamma} \right),$$

$$L_\gamma = \ln \frac{1 + \gamma}{\gamma}, \quad (11)$$

and \mathcal{E}_T and u_T are field and potential in the Townsend limit of “vanishing” current, i.e., with breakdown values

$$\mathcal{E}_T = \frac{1}{\ln(L/L_\gamma)}, \quad u_T = \frac{L}{\ln(L/L_\gamma)}. \quad (12)$$

The minimal potential u_T is $L_\gamma e^1$, it is attained for gap length $L = L_\gamma e^1$ on the Paschen curve $u_T = u_T(L)$ [1–3].

In [1], it was argued that the coefficient A_{SE} in (8) strongly depends on γ due to the factor $F(\gamma, \mu)$ in (9). This factor $F(\gamma, \mu)$ indeed strongly depends on γ , for small γ actually in leading order like $L_\gamma^3/12$. (Note that there is a discrepancy between equation (50) in [1] for $F(\gamma, \mu)$ which is reproduced as Eq. (11) in the present paper, and the plot in Fig. 1 of [1] for $10^{-1} < \gamma < 10^0$. Equation (50) in [1] is correct and the figure erroneous. $F(\gamma, \mu)$ actually varies by five orders of magnitude on $10^{-6} < \gamma < 10^0$, not only by four.)

At this point, the question of how the remaining factors in A_{SE} depend on γ was omitted. In fact, the denominator $(\alpha \mathcal{E}_T)^3$ in (8) has in leading order the same strong dependence on γ , since

$$\frac{1}{(\alpha \mathcal{E}_T)^3} = \left[\frac{L}{L_\gamma} \ln(L/L_\gamma) \right]^3, \quad (13)$$

according to the Townsend breakdown criterion $\alpha L = L_\gamma$; cf. (10)–(12). Therefore, the leading order dependence on L_γ^3 of the coefficient of \bar{j}^2 in (8) is cancelled and replaced by a dependence on L^3 , while the term with α'' has the classical explicit form

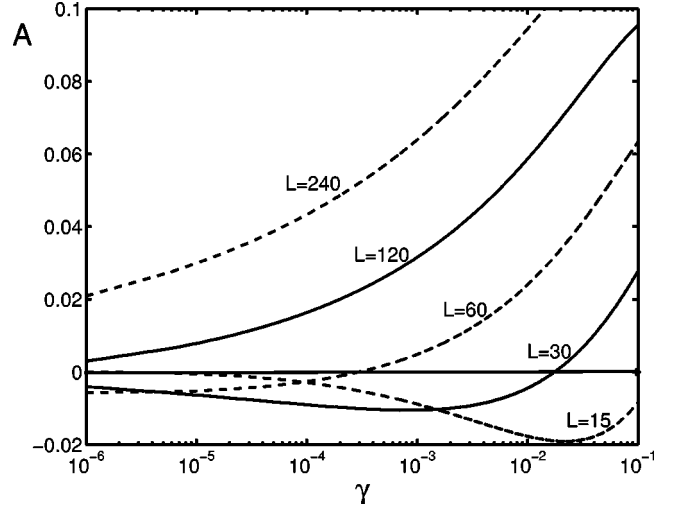


FIG. 2. The normalized coefficient $A = 24 A_{SE}/(L^3 \ln^4 L)$ as a function of γ for gap lengths $L = \text{Apd} = 15, 30, 60, 120, 240$ (dashed and solid lines with labels).

$$\frac{\mathcal{E}_T \alpha''}{2 \alpha'} = \frac{1 - 2\mathcal{E}_T}{2\mathcal{E}_T} = \frac{\ln(L/L_\gamma) - 2}{2}. \quad (14)$$

In [2,3], another small current expansion was derived from (1)–(3), assuming $n_+ \gg n_e$ and $n_+(x) \approx \text{const}$. This approximation was criticized in [1], since it is in contradiction with the boundary condition (5); however, for very small γ , it is a good approximation in a large part of the gap. The resulting equations (8.8) and (8.10) from [2] read in the notation of the present paper

$$U = U_T - \frac{U_T}{48} \frac{1 - 2\mathcal{E}_T}{2\mathcal{E}_T} \left(\frac{J}{J_L} \right)^2, \quad (15)$$

$$J_L = \frac{\epsilon_0 \mu_+ U_T^2}{2d^3}. \quad (16)$$

(Here, a misprint in [2] was corrected, namely, the missing factor U_T in the coefficient of J^2 in (15), is now included. Furthermore, the factor $1/(8\pi)$ in (8.8) is substituted by $\epsilon_0/2$ in (16), since we here write the Poisson equation (2) in MKS units rather than in Gaussian units; cf. (8.6) in [2].)

In (15), the physical current density J is compared to J_L . J_L is the current density at which deviations from the Townsend limit through space charges start to occur; it explicitly depends on γ through U_T (12).

Comparison of the results of Šijačić and Ebert (8) and of von Engel and Raizer (ER) (15) show that the coefficients $A_{SE,ER}$ in the expansion (8) are related as

$$A_{SE} = A_{ER} \frac{12 F(\gamma, \mu)}{L_\gamma^3}, \quad A_{ER} = \frac{1 - 2\mathcal{E}_T}{2\mathcal{E}_T} \frac{L^3}{12 \mathcal{E}_T^3}. \quad (17)$$

The coefficients A_{SE} and A_{ER} depend in the same way on L , and they are essentially independent of μ for realistic values of μ . Therefore, the ratio A_{SE}/A_{ER} depends only on γ as shown in Fig. 1. For $\gamma \rightarrow 0$, the ratio tends to unity. For a

large range of γ values, the deviation is not too large, approaching a factor 0.44 for $\gamma=10^{-1}$.

Figure 2 shows that the factor A_{SE} indeed strongly depends on γ for the given L .

The strong dependence of A_{SE} or A_{ER} on γ for a given short gap length L means that we can obtain both negative and positive differential resistance dU/dJ close to the Townsend limit for the same gap length. Therefore, the choice of γ is important since it can change the differential

conductivity and therefore the stability of a Townsend discharge in a short gap.

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[1] D. D. Šijačić, U. Ebert, Phys. Rev. E **66**, 066410 (2002).

[2] Yu. P. Raizer, *Gas Discharge Physics* (Springer, Berlin, 1991).

[3] A. von Engel and M. Steenbeck, *Elektrische Gasentladungen. Ihre Physik und Technik* (Springer, Berlin 1934), Vol. II.