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Low Complexity Blind Residual Carrier Offset Estimation in OFDM-based Wireless LAN Systems

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Abstract—In [1], a blind carrier frequency offset (CFO) estimation method was proposed for orthogonal frequency multiplexing (OFDM) systems. This method makes use of the fact that in practical OFDM systems, there are some subcarriers that do not carry any data. However, the computation complexity of the method is rather high. In this paper, based on practical considerations, we propose a low complexity blind CFO estimation method. We derive an approximate closed-form solution for the CFO estimate. We show that the proposed algorithm achieves similar performance as the method in [1] with significantly reduced computational complexity. For relatively large CFO values, we show that we could use the iterative algorithm in [2] to improve the performance of the proposed algorithm.

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has been adopted as a standard transmission technology for various wireless communication systems such as the digital audio and video broadcasting and IEEE 802.11a wireless local area networks (LAN) [3]. The advantages of OFDM include its high spectral efficiency and robustness against frequency selective fading. Figure 1 shows a typical OFDM transceiver chain. In such a system, frequency synchronization is performed in

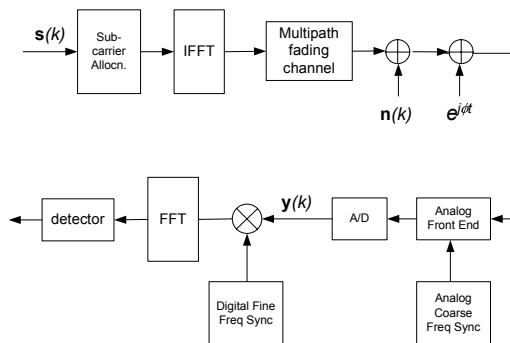


Fig. 1. Block diagram of an OFDM system with both analog and digital frequency synchronization.

both the analog and digital domains to reduce the detrimental effect of the carrier frequency offset (CFO). The CFO comes from mainly two sources: the mismatch between the local oscillators of the transmitter and receiver as well as the doppler effect of the channel due to the movement of the receiver. Such CFO must be estimated with sufficient accuracy

and correctly compensated. In the analog domain, a coarse frequency synchronization is carried out using a phase-locked loop [4]. The received signal after analog to digital conversion can be expressed as [1] ¹

$$y(k) = \mathbf{E}\mathbf{W}_P\mathbf{H}\mathbf{s}(k)e^{j\phi_o(k-1)(N+N_g)} + \mathbf{n}(k) \quad (1)$$

where $\mathbf{E} = \text{diag}(1, e^{j\phi_o}, \dots, e^{j(N-1)\phi_o})$ is a diagonal matrix containing the residual CFO $e^{j\phi_o}$ after analog coarse frequency synchronization. In a practical OFDM system, some subcarriers at both ends of the allocated spectrum are left empty to avoid aliasing to the adjacent channels. In this paper, we will refer to these non-data-carrying subcarriers as null subcarriers, and to the data-carrying subcarriers as data subcarriers. \mathbf{W}_P is a $N \times P$ submatrix that is obtained from the $N \times N$ inverse discrete Fourier transform (IDFT) matrix \mathbf{W}_N , where P is the number of data subcarriers. \mathbf{H} is a diagonal matrix containing the channel frequency response, $\mathbf{s}(k)$ is a $P \times 1$ vector containing the transmitted symbols, N_g denotes the length of the cyclic prefix, and $\mathbf{n}(k)$ is an additive white Gaussian noise (AWGN) vector.

Liu and Tureli presented a blind CFO estimation method [1] based on minimizing the received signal power on all the null subcarriers [1]:

$$\mathcal{J}(z) = \sum_{i=1}^L \sum_{k=1}^K \|\mathbf{w}_{l_i}^H \mathbf{Z}^{-1} \mathbf{y}(k)\|^2 \quad (2)$$

where $[l_1, l_2, \dots, l_L]$ are the indices of L null subcarriers, K is the total number of OFDM symbols used for CFO estimation, and $\mathbf{Z} = \text{diag}(1, z, z^2, \dots, z^{(N-1)})$. $\mathbf{w}_{l_i}^H$ is the l_i th row of the DFT matrix. Obviously, in the absence of noise, $z = e^{j\phi_o}$ is a zero of $\mathcal{J}(z)$. To find the CFO value that minimizes (2), it was shown in [1][5] that either a Music-like search algorithm or a rooting method can be used. Note that the cost function $\mathcal{J}(z)$ is a polynomial in the complex variable z of order $2(N-1)$. For a typical application, like wireless LAN (IEEE 802.11a standard), $N = 64$. Hence, $\mathcal{J}(z)$ order becomes 126, and the computational complexity required to find its zeroes is very high. To reduce this complexity, Tureli *et al.* proposed an ESPRIT-like method in [5]. However, the computational

¹This model was derived by using a cyclic prefix that is longer than or equal to the channel memory length.

complexity is still very high as a subspace computation is required.

In practical OFDM systems, the residual CFO value after analog frequency synchronization is normally very small due to the availability of high precision local oscillators. In IEEE Standard 802.11a-1999 [3], coarse and fine CFO estimation are both used, and the stability of the carrier frequency that is required is ± 20 ppm maximum. This leads to worst case CFO value of ± 0.063 . Based on this observation, we proposed a method to reduce the computational complexity of Liu *et al.* method in [6]. As compared to [5], the method in [6] was shown to achieves similar performance with much less implementation cost for both Gaussian and multipath channels.

In this paper, we propose a new factorization for the CFO matrix \mathbf{Z}^{-1} to further improve the method in [6]. By limiting the number of terms in Taylor's series approximation to 2 (first order approximation), we derive a closed-form solution for the CFO estimation. We also evaluate the computational complexity as well as the performance of the proposed method. We show that the proposed method has comparable performance as the method in [1] with significantly lower complexity for small CFO values. For relative large CFO values, the proposed method suffers some performance degradation due to the lower order Taylor's series approximation of (2). We further propose to use the iterative algorithm proposed in [2] in conjunction with the proposed method to mitigate such performance degradation. We introduce a new convergence monitoring method to ensure the convergence of the iterative algorithm. We will show that the performance of this method can converge to the performance of [1] in 2 to 3 iterations even for the worst case CFO values of ± 0.063 specified by the IEEE 802.11a standard [3]. Moreover, the complexity of the method with iterations is still significantly lower than that of [5].

II. PREVIOUS METHOD

In this section, we briefly re-visit the method in [6], which will help to understand our newly proposed method in the subsequent section. The inverse diagonal matrix \mathbf{Z}^{-1} in (2) can be re-written as follows

$$\begin{aligned} \mathbf{Z}^{-1} &= \text{diag}(1, e^{-j\phi}, e^{-j2\phi}, \dots, e^{-j(N-1)\phi}) \\ &= e^{-j\phi(N-1)/2} \text{diag}(e^{j\phi\frac{(N-1)}{2}}, e^{j\phi\frac{(N-3)}{2}}, \dots, e^{j\phi\frac{(1-N)}{2}}) \end{aligned} \quad (3)$$

where ϕ now denotes a real variable. Using Taylor's series expansion of an exponential function, we can have

$$\mathbf{Z}^{-1} \approx e^{-j\phi(N-1)/2} \times \sum_{n=0}^Q \frac{(j\phi)^n}{2^n n!} \mathbf{D}^n \quad (4)$$

where $\mathbf{D} = \text{diag}((N-1), (N-3), \dots, (1-N))$ and Q is a suitable integer ($Q \ll N$) satisfying

$$Q \geq \frac{|\phi|(N-1)}{2}, \quad (5)$$

such that the error due to the series truncation is negligible [6]. Substituting (4) into (2) and letting $a_{i,n}(k) = \mathbf{w}_i^H \mathbf{D}^n \mathbf{y}(k)$, we get the approximated cost function (see [6] for more details)

$$\mathcal{J}_{2Q}(\phi) = \sum_{l=0}^{2Q} c_l \phi^l \quad (6)$$

where the polynomial coefficients c_l are given by

$$c_l = \left(\frac{j}{2}\right)^l \sum_{m=0}^l \frac{(-1)^m}{(l-m)!m!} \sum_{i=1}^L \sum_{k=1}^K a_{i,l-m}(k) a_{i,m}^*(k) \quad (7)$$

with $a_{i,l}(k) = 0$ for $l > Q$. The new cost function (6) is a polynomial of the real variable ϕ of degree $2Q$ with real coefficients [6]. The minimization of (6) can be carried out by setting the derivative to 0 and using some standard rooting methods to search for the estimated CFO. As both ϕ and the polynomial coefficients are real, the rooting can be performed with much less complexity compared to [1], where both the argument and the coefficients of the cost function are complex.

III. PROPOSED APPROXIMATION METHOD AND THE CLOSED-FORM SOLUTION

In this section, we describe a new approximation method for the CFO matrix \mathbf{Z}^{-1} . We also derive the closed-form solution for the CFO estimates and evaluate the computational complexity of the proposed method.

A. Improving the approximation of \mathbf{Z}^{-1}

The advantage of the factorization in (3) is that it introduces a factor of 2^n at the denominators of the Taylor's series expansion in (4) such that the amplitudes of the high order terms decreases to 0 faster. In the following, we will propose a new factorization which allows to increase this number to $(2^n)^2 = 4^n$ and more if needed. Based on (3), we can write

$$\mathbf{Z}^{-1} = e^{-j\phi(N-1)/2} (\mathbf{E}_1 + \mathbf{E}_2) \quad (8)$$

where

$$\begin{aligned} \mathbf{E}_1 &= \text{diag}(e^{j\phi\frac{(N-1)}{2}}, e^{j\phi\frac{(N-3)}{2}}, \dots, e^{j\phi\frac{1}{2}}, 0, \dots, 0) \\ &= e^{j\phi(N-1)/4} \text{diag}(e^{j\phi\frac{(N-1)}{4}}, \dots, e^{j\phi\frac{(3-N)}{4}}, 0, \dots, 0) \end{aligned} \quad (9)$$

and

$$\begin{aligned} \mathbf{E}_2 &= \text{diag}(0, \dots, 0, e^{-j\phi\frac{1}{2}}, \dots, e^{j\phi\frac{(1-N)}{2}}) \\ &= e^{j\phi(1-N)/4} \text{diag}(0, \dots, 0, e^{j\phi\frac{(N-3)}{4}}, \dots, e^{j\phi\frac{(1-N)}{4}}) \end{aligned} \quad (10)$$

Now, using Taylor's series expansion in (9), we obtain

$$\begin{aligned} \mathbf{E}_1 &= \sum_{m=0}^{+\infty} \frac{(j\phi)^m}{4^m m!} (N-1)^m \sum_{n=0}^{+\infty} \frac{(j\phi)^n}{4^n n!} \mathbf{D}_1^n \\ &= \sum_{m=0}^{+\infty} \sum_{n=0}^{+\infty} \frac{(j\phi)^{n+m}}{4^{n+m} n! m!} (N-1)^m \mathbf{D}_1^n \end{aligned} \quad (11)$$

where

$$\mathbf{D}_1^n = \text{diag}((N-1)^n, (N-5)^n, \dots, (3-N)^n, 0, \dots, 0) \quad (12)$$

²In general, one complex multiplication is equal to four real multiplications. A further reduction in complexity is also achieved by limiting the number of Taylor's series to minimum.

Similarly, we can show that

$$\mathbf{E}_2 = \sum_{m=0}^{+\infty} \sum_{n=0}^{+\infty} (-1)^m \frac{(j\phi)^{n+m}}{4^{n+m} n! m!} (N-1)^m \mathbf{D}_2^n \quad (13)$$

where

$$\mathbf{D}_2^n = \text{diag}(0, \dots, 0, (N-3)^n, (N-7)^n, \dots, (1-N)^n) \quad (14)$$

Substituting (11) and (13) into (8), we get

$$\mathbf{Z}^{-1} = e^{j\phi(N-1)/2} \sum_{m=0}^{+\infty} \sum_{n=0}^{+\infty} \frac{(j\phi)^{n+m}}{4^{n+m} n! m!} (N-1)^m [\mathbf{D}_1^n + (-1)^m \mathbf{D}_2^n] \quad (15)$$

If we truncate the Taylor's series expansion of \mathbf{E}_1 and \mathbf{E}_2 to Q terms, then, (15) can be approximated by the following

$$\mathbf{Z}^{-1} \approx e^{j\phi(N-1)/2} \sum_{q=0}^{2Q} \mathbf{C}_q \phi^q \quad (16)$$

where

$$\mathbf{C}_q = \left(\frac{j}{4}\right)^q \sum_{m=0}^q \frac{(N-1)^m}{m!(q-m)!} [\mathbf{D}_1^{(q-m)} + (-1)^m \mathbf{D}_2^{(q-m)}] \quad (17)$$

Note that \mathbf{C}_q just given above is defined for $q = 0, 1, \dots, 2Q$ and it is equal to the zero-matrix for $q > 2Q$. To simplify the calculations, let us define the scalar $b_{i,q}(k)$ as follows

$$b_{i,q}(k) = \mathbf{w}_{i_i}^H \mathbf{C}_q \mathbf{y}(k). \quad (18)$$

Substituting (16) into the cost function (2), we get the new approximate cost function

$$\begin{aligned} \mathcal{J}(z = e^{j\phi}) &\approx \mathcal{J}_{4Q}(\phi) \\ &= \sum_{r=0}^{2Q} \sum_{l=0}^{2Q} \phi^{l+r} \sum_{i=1}^L \sum_{k=1}^K b_{i,l}(k) b_{i,r}^*(k) \\ &= \sum_{q=0}^{4Q} d_q \phi^q \end{aligned} \quad (19)$$

where the polynomial coefficients are given by

$$d_q = \sum_{s=0}^q \sum_{i=1}^L \sum_{k=1}^K b_{i,q-s}(k) b_{i,s}^*(k) \quad (20)$$

We can notice that $b_{i,l}(k) = 0$ for $l > 2Q$ and $d_q = d_q^*$, that is all the coefficients of the polynomial are real-valued. This method can be reiterated again if needed. However, this also leads to an increase in the polynomial degree. Since our aim is to reduce the computational complexity of the carrier offset estimation problem while minimizing the error due to the truncation of the Taylor's series and to estimate the carrier offset using a closed-form expression³, we will not proceed any further.

³Let n be a positive integer. The roots of a polynomial of degree n can be computed using an algebraic formula if and only if $n \leq 4$ [10]. If the proposed method is reiterated, then we will obtain a polynomial for the cost function whose degree can be shown to be much larger than 4 even if we set $Q = 1$.

B. Closed-form solution for the new approximation method

For $Q = 1$, the cost function polynomial is of degree four and its derivative with respect to ϕ is a cubic polynomial whose roots can be computed directly using Cardano's formula [11]. We first rewrite the derivative of the cost function as follows:

$$\phi^3 + u\phi^2 + v\phi + r = 0 \quad (21)$$

where

$$u = \frac{3d_3}{4d_4}, \quad v = \frac{d_2}{2d_4}, \quad r = \frac{d_1}{4d_4} \quad (22)$$

Now, let us compute

$$\begin{aligned} a &= (3v - u^2)/3 \\ b &= (u^3 - 9uv + 27r)/27 \\ S &= \left(-\frac{b}{2} + \sqrt{\frac{b^4}{4} + \frac{a^3}{27}}\right)^{3/2} \\ T &= \left(-\frac{b}{2} - \sqrt{\frac{b^4}{4} + \frac{a^3}{27}}\right)^{3/2} \end{aligned} \quad (23)$$

Finally, the three roots are given by [11]

$$\begin{aligned} \phi_1 &= (S + T) - \frac{u}{3} \\ \phi_2 &= -\frac{1}{2}(S + T) + j\frac{\sqrt{3}}{2}(S - T) - \frac{u}{3} \\ \phi_3 &= -\frac{1}{2}(S + T) - j\frac{\sqrt{3}}{2}(S - T) - \frac{u}{3} \end{aligned} \quad (24)$$

The optimal CFO estimate is then the root that leads to minimum of (19). As the value of the CFO estimate is real, we should test only the real roots for the minimum and discard the complex roots. It is worthwhile to mention that for a cubic polynomial with real coefficients as in our case one of the roots will always be real [10]. The summary of this algorithm is given in table I.

Initialization

Start with the received signal vector $\mathbf{y}(0)$, and set $Q = 1$.

Algorithm:

- 1) Compute the coefficients d_q of the cost function polynomial $\mathcal{J}_4(\phi)$ using (20) for $q = 1, 2, 3, 4$.
 - 2) Compute the coefficients u, v and r of $(1/4d_4) \frac{\partial \mathcal{J}_4(\phi)}{\partial \phi}$ using (22).
 - 3) Compute the three roots ϕ_1, ϕ_2 and ϕ_3 using (24) and discard those which are complex. At least one root should be real as explained.
 - 4) If more than one root is real, then compute $\mathcal{J}_4(\phi)$ for each root and choose the one that leads to the smallest value for $\mathcal{J}_4(\phi)$.
-

TABLE I

THE PROPOSED CLOSED-FORM SOLUTION CFO ESTIMATION METHOD.

C. Computational Complexity of the proposed method

In the following we will evaluate the computational complexity of the proposed algorithm. Since $Q = 1$, the computational complexity to find the roots using (22), (23) and (24) is negligible as compared to the complexity of computing the 4 polynomial coefficients d_1, d_2, d_3 , and d_4 . To find each coefficient d_q , we need to compute $b_{i,q}(k)$ in (18). Here, both $\mathbf{w}_{i_i}^H$ and

$\mathbf{y}(k)$ are vectors and \mathbf{C}_q is a diagonal matrix. Therefore, the computation required to find each polynomial coefficient can be shown to be about $O(LKN)$. The ESPRIT method in [5] is based on the estimation and computation of a certain matrix \mathbf{A} , its pseudo inverse and the construction of another matrix called Δ . These operations have at least the following computational complexities $O(K(N - M)(M + 1)^2)$, $O(P^3) + O(MP^2)$ and $O((MP)^2)$, where $M \geq P$ and P is the number of data subcarriers. In practice, both M and P are of comparable size as N . Hence, it should be obvious that the computational complexity of the proposed method is much lower than the methods in [1] and [5].

IV. ITERATIVE BLIND FREQUENCY OFFSET ESTIMATION AND COMPENSATION FOR LARGER CFO VALUES

The accuracy of the proposed Taylor's series approximation degrades for relatively large CFO values as we only use $Q = 1$. As a result, there will be some performance degradation compared to the method in [1]. In [2], we proposed an iterative CFO estimation and compensation method to improve the performance of the low complexity method in [6]. Here, we use this iterative algorithm with proposed new approximation method for the same purpose of reducing the performance degradation due to lower order approximation. We introduce a new convergence monitoring mechanism that guarantees the convergence of the iterative method.

The iterative algorithm can be summarized as follows. Let the true CFO be ϕ_0 . In the first iteration, we use the proposed low complexity method to obtain an initial estimate of CFO ϕ_1 . Then this CFO value is compensated from the received signal such that the residual CFO after the first iteration is $\phi_{r1} = \phi_0 - \phi_1$. Consider the case where the CFO estimate ϕ_1 is close enough to ϕ_0 such that the following condition is satisfied

$$|\phi_{r1}| = |\phi_0 - \phi_1| < |\phi_0|. \quad (25)$$

Then, in the second iteration, $e^{j\phi_{r1}}$ can be approximated more accurately using the proposed Taylor's series approximation as compared to $e^{j\phi_0}$. Therefore, in the second iteration, our estimation may lead to a better estimate on ϕ_{r1} . As a result, the residual CFO after the second iteration $|\phi_{r2}| = |\phi_{r1} - \phi_2| = |\phi_0 - (\phi_1 + \phi_2)|$ is very small. If at every iteration, we can have $|\phi_{r_{i+1}}| = |\phi_{r_i} - \phi_{i+1}| < |\phi_{r_i}|$ satisfied, the residual CFO gets smaller and smaller as number of iterations increases. Eventually we could reach the same solution as the method in [1]. The question now is that how can we ensure the previous condition is satisfied at each iteration so as to guarantee convergence. In the following, we will propose a convergence monitoring method to do that.

In the iterative algorithm, we could monitor the amplitude of $|\phi_i|$ from the second iteration onwards. If $|\phi_i| < |\phi_{i-1}|^4$,

⁴Ideally, we should monitor the amplitude of $|\phi_{r_i}|$ for different iterations. If $|\phi_{r_i}| < |\phi_{r_{i-1}}|$, the algorithm is converging. However, we do not have knowledge of $|\phi_{r_i}|$ as this requires the knowledge of actually CFO value ϕ_0 . Therefore, an alternative way is to monitor the amplitude of the CFO estimates $|\phi_i|$ at different iterations. If the algorithm is converging, we should expect the amplitude of the CFO estimate $|\phi_i|$ to decrease as number of iterations increases.

then the algorithm is moving towards the right direction. If $|\phi_i| > |\phi_{i-1}|$, we know that the iteration is likely to diverge. In this case, we need a better approximation to get a more accurate estimate on the CFO value. This could be achieved, for example, by increasing the order of Taylor's series approximation in the $[i - 1]$ th iteration. From our computer simulations, we have found that for all practical CFO values, the first order Taylor's series approximation, i.e. $Q = 1$, is fine. We can also use $|\phi_i|$ as the stopping criteria of the iterative algorithm, i.e. if we find that $|\phi_i|$ is too small, we shall stop the iteration as further iterations will not lead to any significant improvement in CFO estimation.

The algorithm is summarized in Table II. Here η is a small threshold value for CFO. When the estimated residual CFO gets smaller than this value, there is no point in going on with the iterations as the improvement will be marginal.

Initialization

Set iteration number $i = 1$ and the CFO threshold η .

Algorithm:

- 1) Substitute $\mathbf{y}(k)$ into cost function (2) and solve for ϕ_i that minimizes (2) using the proposed method (Table I).
 - 2) Perform carrier CFO compensation using the CFO estimate.
 - 3) If $\phi_i < \eta$, exit iteration, else go to 4)
 - 4) If $\{|\phi_i| < |\phi_{i-1}| \text{ and } i > 1\}$ or $\{i = 1\}$
 $i = i + 1$, go back to 1).
 else
 Go back to the $[i - 1]$ th iteration and increase the order of Taylor's series approximation.
-

TABLE II

SUMMARY OF THE ITERATIVE CFO ESTIMATION AND COMPENSATION ALGORITHM

The computational complexity of this method is roughly n_{itn} times the complexity of the proposed closed-form solution, where n_{itn} is the number of iterations. As we are going to show later in the simulations, the algorithm converges to the performance of the method in [1] in 2 to 3 iterations for practical CFO values. Therefore, the complexity of the successive algorithm is still much less than the ESPRIT method in [5].

V. SIMULATION RESULTS

We conducted computer simulation to study the performance of the proposed CFO estimation method. The OFDM system follows the specifications given in IEEE 802.11a [3]. There are $N = 64$ subcarriers and the null subcarriers are placed consecutively from subcarriers 27 to 37. The subcarrier spacing is given by $\omega = 2\pi/N$. The estimation MSE is defined by [5]

$$\text{MSE} = \frac{1}{N_s} \sum_{i=1}^{N_s} \left(\frac{\hat{\phi} - \phi_0}{\omega} \right)^2, \quad (26)$$

where $\hat{\phi}$ and ϕ_0 represent the estimated and true CFO's, respectively, and N_s denotes the total number of Monte Carlo

trials. The channel models we use is channel model A of the HiperLAN II channel models [9]. It is a multipath Rayleigh fading channel with exponential power delay profile and root mean square (RMS) delay spread equal to one modulation symbol interval. In all the simulations, we only use 1 OFDM symbol to perform CFO estimation, i.e $K = 1$ in all the cost functions.

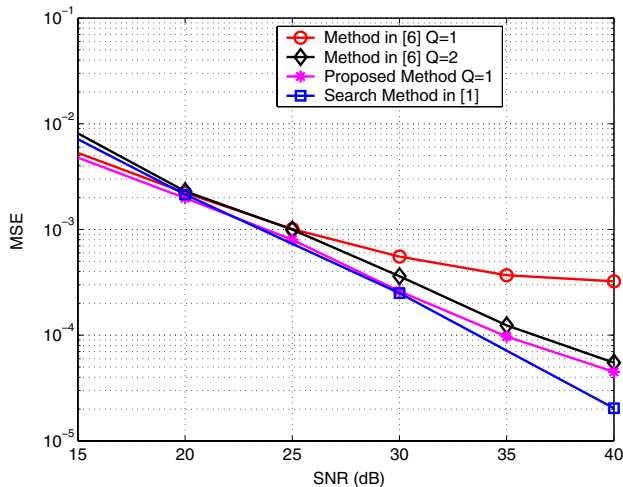


Fig. 2. CFO estimation for $-0.25\omega \leq \phi_0 \leq 0.25\omega$.

The MSE performance of the proposed low complexity method is shown in Figure 2. Here, the true CFO for each OFDM symbol is modelled as an uniformly-distributed random variable between $[-0.25\omega, 0.25\omega]$. We can see that the proposed method with $Q = 1$ shows a better MSE performance than the method in [6] with both $Q = 1$ and $Q = 2$ due to the improved approximation of the Taylor Series expansion. Comparing the performance of the proposed method and the search method in [1], we can see that the proposed method suffers small degradation only at high SNR regions.

It was shown in [8] that the optimal placement of the null subcarriers to minimize the Cramer-Rao bound is to place them with even spacing across the whole OFDM symbol. We adopted this optimal placement of null subcarriers in the subsequent simulations. We used a total of 11 subcarriers (same as the consecutive null subcarrier case) placed at the following locations $[1, 7, \dots, 55, 61]$ ⁵.

The performance of the proposed algorithm with the iterative CFO estimation and compensation method for $-0.7\omega \leq \phi_0 \leq 0.7\omega$ is shown in Figure 3. Here we purposely increased the values of the CFO such that the worst case CFO of $\pm 0.64\omega$ specified by IEEE 802.11a [3] is included. Because of the large CFO value used, we can see that using the proposed low complexity method alone (i.e. the result after the first iteration) results in a large degradation compared to the method in [1]. It has an error floor at MSE around 3×10^{-3} . However, with the iterative algorithm, the performance of

⁵If it is necessary to place some null subcarriers consecutively at both ends of the spectrum as guard band, we could still place a few extra null subcarriers evenly across the remaining subcarriers to achieve optimality as in [8].

the CFO estimation improves significantly. The error floor is removed and the performance converges to the performance of the search method in [1] after the second iteration. We have implemented the convergence monitoring mechanism shown in Table II in the algorithm. We found that $Q = 1$ is good enough to guarantee convergence for all the SNR values.

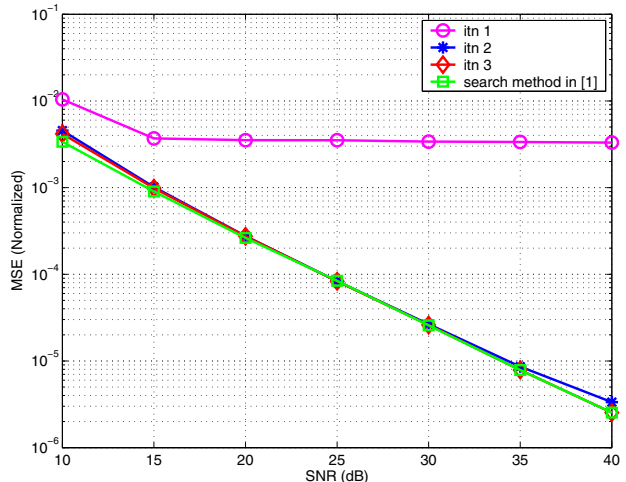


Fig. 3. CFO estimation for $-0.7\omega \leq \phi_0 \leq 0.7\omega$ using the successive method.

Figure 4 shows the SER performance of the iterative CFO estimation and compensation method. We can see that the SER performance takes 2 iterations only to achieve a performance similar to the case where we have a perfect knowledge and compensation of the CFO.

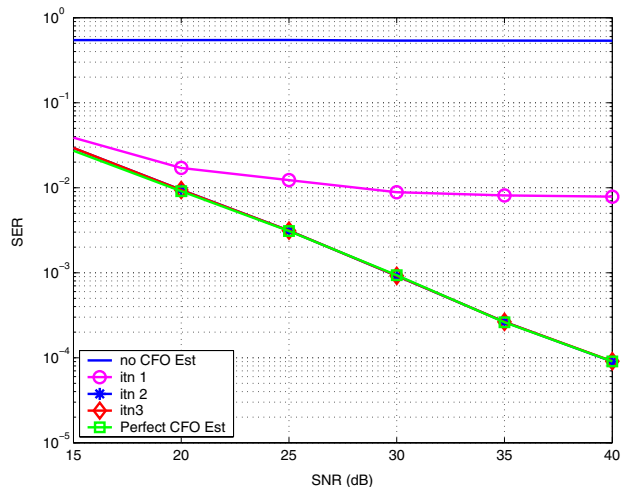


Fig. 4. SER of iterative CFO estimation and compensation using the proposed method with $Q = 1$ for $-0.7\omega \leq \phi_0 \leq 0.7\omega$.

VI. CONCLUSION

In this paper, we proposed an approximate closed-form solution for blind estimation of residual carrier offset in OFDM-based wireless LAN systems. We also proposed to use the iterative method in [2] to reduce the degradation

of the proposed method due to lower order Taylor's series approximation. We introduced a new convergence monitoring algorithm to ensure the convergence of the iterative algorithm. We have shown by using the proposed algorithms, we are able to achieve similar performance as the method in [1] yet at a much lower computational complexity.

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