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Application potential of combining strain hardening cementitious composites and helical reinforcement for 3D concrete printed structures: Case study of a spiral staircase

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ABSTRACT

Despite 3D Concrete Printing having the potential to advance the construction industry through material savings, design flexibility, and a more efficient workflow from design to construction, the challenge of reinforcing the layers of created structures remains unresolved. Recent developments within reinforcement techniques for 3D concrete printing, such as helical reinforcement and strain-hardening cementitious composites, have shown promise to work as potential solutions. To demonstrate the effectiveness of these techniques on a load-bearing structure, an automated production of spiral staircase elements is presented combining automated placement of helical reinforcement and strain-hardening cementitious composites. A design study is presented and different staircase elements were produced, experimentally tested in bending and compared to analytical results. The findings of the study reveal that the strengths and weaknesses of each reinforcement concept are evident when examined individually. However, when integrated together, these concepts complement each other, resulting in a successful and efficient reinforcement method for printed concrete.

1. Introduction

Compared to other sectors, the construction industry has historically been slower in adopting innovation and novel technologies [1–3]. The slow adoption of novel technologies and diffusion of innovation is not solely caused by a lack of technological developments but is also hindered by the industry’s fragmentation [4]. Fortunately, new techniques of Digital Fabrication with Concrete (DFC) have shown the potential to provide solutions, combining automated manufacturing technologies and digital design. Among these methods, 3D Concrete Printing (3DCP) is the most promising and thoroughly studied DFC technology presently in operation [5].

Despite numerous large-scale 3DCP demonstrations, incorporating reinforcement is still a major challenge that hinders the use of 3DCP at a larger scale [6]. Due to the nature of the printing process, conventional reinforcement techniques cannot be used [7], resulting in most structures currently utilizing printed concrete for non-load-bearing walls, facades, or lost formwork for cast reinforced concrete [8]. Solutions where printed concrete itself is used for load-bearing purposes typically rely on compression structures, drawing inspiration from either unreinforced masonry or post-tensioning techniques [9–11]. To date, the authors are not aware of any
load-bearing 3D-printed concrete structures with reinforced layers themselves designed to withstand global tensile and shear forces.

Researchers globally are currently exploring various approaches to tackle this issue, however none of the reinforcement techniques have been employed in practice yet. The main challenges can be attributed to factors such as poor bond strength [6], low concrete ductility [12], lack of automation [13], directional dependency [14] and an insufficient understanding on the interdependency between the design methodology and production process [9]. Consequently, the utilization of 3DCP is currently restricted to structures where the printed layers are not designed to uphold global tensile and shear forces. Fundamental research on innovative solutions such as helical reinforcement [15] and strain-hardening cementitious composites (SHCC) [12,16,17] have shown to overcome specific challenges mentioned before, however, the techniques have not yet been applied in a load-bearing printed structure to reinforce the printed concrete itself.

This paper seeks to reduce this gap by presenting a design using a fully automated production process for structural spiral staircase elements. To withstand main bending and shear forces, and to provide reinforcement confinement, a combination of helical reinforcement inserted by the Automated Screwing Device (ASD) [18] and SHCC is utilized. The combination of these two techniques within the 3DCP process has not been researched together and is where the novelty of this study lies.

The paper has been divided into six sections. Section 2 gives background information on the reinforcement techniques used. Section 3 covers the spiral staircase design approach, used materials and structural considerations. Section 4 details the production, specimen preparation and experimental testing used in this study, while Section 5 explains the analytical analysis applied. Section 6 provides a discussion through comparing experimental and analytical results. Finally, Section 7 summarizes the conclusions of the study.

2. Background

2.1. Helical reinforcement

One of the methods currently under development for reinforcing printed layers is the helical reinforcement technique [15]. The method involves inserting a discrete reinforcement element with a helical surface into fresh mortar within a specified timeframe following deposition, through a screwing motion. During screwing, the translational and rotational motion need to be synchronized in such a way that during one 360° rotation, a translation equal to the lead of the helical reinforcement is performed. It has been demonstrated that this technique creates a high mechanical interlocking with the printed matrix, as shown through the implementation of the Automated Screwing Device (ASD) for placement [18] and confirmed by pull-out tests [15]. Helical reinforcement can be theoretically applied in all directions, however its utilization is constrained by the length of the reinforcement and the inability to be curved naturally, not taking advantage of the available design freedom of 3DCP process. The previous requires careful design considerations and implementation of splicing techniques.

2.2. Strain hardening cementitious composites (SHCC)

A wide range of SHCCs have been developed [19–22] and applied in a variety of different applications, ranging from bridge decks [23] to earthquake resistance building nodes [24], with recent demonstrations showcasing their utilization in the 3D printing process [16,17]. These advancements illustrate the potential of a material-level reinforcement technique that enables the reinforcement of more complex shapes, thereby opening up possibilities to optimize load-bearing structures. Considering the influence of processes like pumping and extrusion, the alignment of fibers with the printing direction varies to some extent, depending on the specific printing method applied. Experimental and analytical investigations have shown the mechanical behavior of the material in different orientations, highlighting the significant impact of the printing process and demonstrating a directionality dependent strain hardening behavior. Although the ductility of SHCC surpasses that of plain mortars, its tensile strength remains within a comparable range. As a result, SHCC may not be suitable for applications in larger load-bearing structures where tensile or bending forces are predominant factors. Acknowledging that there have been recent advancements in high-strength SHCCs that also exhibit increased levels of tensile strength [20,21], however, their applicability in 3D printing process still has to be evaluated. Nonetheless, due to its high ductility, the material holds great potential for applications as local reinforcement, such as addressing restrained deformation or providing confinement when combined with other reinforcement techniques.

2.3. General background

Automated placement of reinforcement is certainly desirable from the perspective of safety and productivity. However, it is also necessary from the process perspective, as for example the time of placement of helical reinforcement has to be controlled depending on the setting rapidity of the print mortar in use, while also taking into consideration structural design and directionality aspects. Current developments in reinforcement techniques for 3DCP are significantly influenced by directionality, leading to variations in structural properties and making the integration of these techniques highly design and process specific. As defined in the RILEM classification framework for integrating reinforcement in DFC [6], the penetration of helical reinforcement is categorized as a contiguous single step process during concrete shaping, while SHCC is considered as a pre-process reinforcement method applied during mixing.

The applicability of various design strategies for 3DCP, including yield line theory [25], topology optimization [26], shape optimization, principal stress approach [27], and stringer panel method [28], has been significantly enhanced by recent advancements in printable materials and mortar deposition processes [29]. However, these design approaches are, generally, not compatible with the integration of reinforcement. To fulfill structural requirements multiple reinforcement techniques might have to be combined, which will undoubtedly affect potential integration scenarios and process characteristics. Without achieving an understanding on the interdependence between design methodology and the production process, a reinforcement technique (or their combination) cannot
be effectively applied for structural functionality. This paper employs a design approach to characterize the structural performance, functionality, and printability of the spiral staircase, while emphasizing the need for full automation across all stages of the production process, including reinforcement.

Both helical reinforcement and SHCC have the potential to be applied in the 3D Concrete printing process. However, each reinforcement technique has distinct constraints in terms of directionality, strength and ductility. Combining these two reinforcements strategies might help to overcome the limitations of one another. For example, to increase structural behaviour of helical reinforcement with small cover sizes, it is expected that a ductile material such as SHCC would spread peak stresses around the reinforcement, avoid a brittle failure of the structure and work as transverse reinforcement to combat splitting tensile forces. This research paper presents a 3D-printed spiral staircase design utilizing automated reinforcement techniques, integrating helical reinforcement and SHCC, to achieve structural performance.

3. Design and process

A spiral staircase design will be developed by incorporating helical reinforcement and SHCC using 3DCP. The design and production processes will be interdependent, considering various boundary conditions such as automated production, structural integrity and sustainability. An analytical model will be applied to evaluate these factors and make a well-informed final design decision.

3.1. Sustainability considerations

Concrete has become the most used construction material worldwide due to its cost efficiency, global accessibility and high durability [30]. It features a comparatively lower CO₂ output per volume in contrast to other construction materials [31]; nevertheless, the significant quantity of concrete used in the built environment has resulted in a colossal carbon footprint [32]. While developments are ongoing on cement alternatives [33] and decarbonization of cement production [34], it remains equally important to prioritize optimization and long structural lifespan for (reusable) concrete structures. Designing with reusability as a fundamental principle becomes crucial in addressing the disparity between the functional and structural lifespan of buildings and infrastructure, as the built environment constantly evolves. From this perspective, reusability principles such as durability, modularity, adaptability and repairability will be considered within the design process of the spiral staircase. The previous will be done while utilizing the optimization possibilities of 3DCP, in combination with the process conditions of used materials and reinforcement techniques.

For reinforced concrete to withstand long term use, resistance to degradation and coping with conditions such as fire and corrosion are essential factors to consider. Concrete presents itself as an excellent option, offering exceptional durability and fire resistance characteristics (among other reasons); nonetheless, the use of steel reinforcement demands careful consideration for long term use [35].

The spiral staircase will be constructed by arranging multiple staircase elements on top of each other, at 15° intervals to form a spiral configuration. With an emphasis on modularity, the staircase will incorporate standardized element dimensions and dry

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Fig. 1. Three different static systems for the spiral staircase: (i) cantilevered, (ii) counter-balanced, and (iii) suspended showing bending moments and shear forces.
connections within the middle column, allowing for simple de- and reassembly.

To ensure stability and prevent tensile forces in the central column, a post-tension tendon is applied through the longitudinal channel formed by stacked staircase elements, placing the column entirely under compression.

3.2. Static system

As illustrated on Fig. 1 three different structural designs for the spiral staircase were considered: (i) cantilevered, (ii) counter-balanced, and (iii) suspended. Within the cantilevered (i) approach a free standing spiral staircase can be formed, resulting in relatively high bending moment near the base of the element. Similar bending moments can be seen for the counter-balanced (ii) variant, however they require a significantly lower level of prestress. The suspended system allows for the thinnest elements, as the bending moment are relatively low compared to the two other systems. While all concepts are viable from a structural perspective, the cantilevering design approach was chosen for its practicality and compatibility with the 3D concrete printing process, e.g., automated production of staircase elements combining different reinforcement strategies to resist main bending and shear forces.

3.3. Design study

A preliminary variant study was conducted to explore various design- and material options for spiral staircase elements. As shown in Fig. 2 a categorization of elements types has been made: (a) concrete elements, (b) hybrid elements (e.g. structural timber-concrete, SHCC-concrete), and (c) custom element (e.g. glass-concrete, timber-concrete). The concrete element category (Fig. 2a) involved creating a reinforced concrete element that was entirely composed of concrete, with optimization techniques applied specifically to the compression zone. To modify the texture of the exposed surface of the staircase elements, the roughness of the print bed can be utilized. Furthermore, for the hybrid elements (Fig. 2b) a timber-concrete element was created by incorporating a timber plate that served a structural role in tension. The two materials were joined together using screws strategically placed by the ASD (Automated Screw Driving), ensuring resistance against shear forces and enabling the structure to function as a timber-concrete composite. If required, the timber plate can be detached from the concrete by unscrewing it, allowing for easy replacement, whether it’s for maintenance purposes or to achieve desired aesthetic modifications. To optimize the staircase element even further, there may be gaps intentionally left in the exposed surface, rendering it unsuitable for direct stepping. In such cases, structural glass could be employed to bear the live loads, while the primary bending forces continue to be supported by the reinforced concrete. This element is an example of the custom element category (Fig. 2c). Several elements from the preliminary variant study were produced, as shown on Fig. 3, and assembled into an exemplary spiral staircase, as can be seen in Fig. 4. Based on the initial design study the cantilevering spiral staircase in combination with complete concrete elements was selected for further mechanical assessment as it would best demonstrate the automated reinforcement techniques in bending and be more representative for other structures in the field of 3DCP. A video illustrating the automated production process of the staircase elements is embedded to the online version of this paper, at the following Youtube link: https://youtu.be/18HOe7LmMvg.

3.4. Materials

The study utilized two mortars: 1) Weber 3D 145-2, a commercially available printing mortar (referred to as plain mortar henceforth), and 2) a printable SHCC developed in a previous study, referred to as mix D in Ref. [36], containing 2 VOL% of structural PVA fibres. The composition of the selected SHCC is presented in Table 1, while the exact composition of plain mortar is undisclosed. Table 2 provides an overview of the relevant mechanical properties for both materials used for printing in this study.

Three different types of discreet reinforcement, as shown on Fig. 5, were used: 1) a helical bar made out of stainless steel Grade 316, with the product name Helibar by Helifix; 2) a carbon steel screw made out of cold-rolled wire, from SchraubenXL and; 3) a conventional ribbed reinforcement bar, hot rolled from steel type B500B. It should be noted that the reinforcement geometry and materials used in this study are not optimized specifically for 3DCP, but were chosen based on their length-to-diameter ratio and availability. Tensile tests for the reinforcement types were conducted on five specimens following ASTM A370 standards, in order to get the respective mechanical properties. These tests were performed on an Instron 250 kN test rig equipped with a non-contact video extensometer, applying a loading rate of 0.5 mm/min.

For the steel screws the force-displacement diagrams obtained exhibited approximately bilinear behavior (as expected from cold-rolled steel) with an average ultimate load of 50.4 kN. The cross-section of the screw was measured to be 40 mm², which corresponds to

![Fig. 2. Design study categories for the cantilevered staircase elements: (a) concrete elements, (b) hybrid elements, and (c) custom elements. For each category two possible variants are shown, with plain mortar (light grey), SHCC (dark grey), timber (brwon), and structural glass (light grey in custom elements).](image-url)
The stainless steel helical bar had an average ultimate load of 10.0 kN with a cross sectional area of 10.15 mm$^2$, corresponding to the ultimate strength of 982 N/mm$^2$. The average axial strain at the yield load was 0.43%, which increased to 2.5% at the maximum load.

Conventional rebar using steel grade B500B had an average ultimate load of 34.2 kN with a cross sectional area of 50.26 mm$^2$, corresponding to the ultimate strength of 681.1 N/mm$^2$. The average axial strain at the yield load was 0.29%, which increased to 12.5% at the maximum load. Table 2 provides an overview of the mechanical properties.

### 3.5. Structural design

To analyze the structural behaviour of the design variants a moment-curvature analysis can be performed. The analysis is an established analytical method for calculating reinforced concrete structures in bending, in which the effect of both materials (i.e., steel and concrete) are taken into account [37]. The method is based on the Euler-Bernoulli beam theory and its main assumptions that plane sections remain plane and strains are linearly distributed over the beam height. Often four distinct phases are described, in which a certain curvature is applied to which the element will be in one of the following states; the moment at crack initiation (Fig. 6a), the moment of yielding of the reinforcement (Fig. 6b), the moment at which compressive forces start to redistribute (Fig. 6c), the moment when concrete fails in compression (Fig. 6d). A four-linear moment-curvature diagram can be formed based on these four points and the origin. Furthermore, failure of the rebar itself might take place (Fig. 6e) before concrete compressive failure is reached.

### 3.6. Final design

To choose a final design for the staircase element the moment curvature analysis was leveraged, comparing the bending capacity to the governing design load acting on the structure. This analysis and subsequent comparison is applied on a staircase element comprising of plain mortar and conventional reinforcement bars (PM-CR). To calculate the bending capacity (Eurocode NEN-1992-1-1) the material factors of $\gamma_c = 1.5$ for plain mortar and $\gamma_s = 1.15$ for reinforcement were used. The governing design load (Eurocode NEN-8700) for this type of structure within safety class CC2, is a 4.50 kN point load ($Q_k = 3.00$ kN and $\psi_0 = 1.5$) when applied at the end of the element, corresponding to a moment when multiplying with the distance to the respective cross-section. For the cross-
section near the base of the element this means bending capacity of at least 3.65 kNm should be achieved. Through this comparison of bending capacity to design load, optimization by reduction in cross-section along the length of the element took place. After this process to achieve the final design, the cross-sectional area of the concrete was gradually increasing from the loading point towards the base of the element, increasing the bending capacity closer to the base (i.e. the clamping location). This resulted in significant material savings without significant reduction of maximum bending capacity, when compared to that of a full cross-section with the same overall height and width along the element. Specifically, the maximum design bending moment capacities were approximately 5.44 kNm for the gradually increasing cross-section and 6.05 kNm for the full cross-section. It should be noted that making the most optimal design is outside the scope of this paper, rather the focus is on validation of the effectiveness of incorporating newly developed

<table>
<thead>
<tr>
<th>Property</th>
<th>Dir.</th>
<th>Symbol</th>
<th>Value (±STD)</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plain mortar (Weber 145-2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>45</td>
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<td>N/mm²</td>
</tr>
<tr>
<td>SHCC (mix D [36])</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tensile strength</td>
<td>( u )</td>
<td>( f_{tu,shcc-u} )</td>
<td>3.19 (±0.53)</td>
<td>N/mm²</td>
</tr>
<tr>
<td></td>
<td>( v )</td>
<td>( f_{tu,shcc-v} )</td>
<td>1.84 (±0.16)</td>
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<tr>
<td>Compressive strength</td>
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<tr>
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<td>Screw</td>
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<td></td>
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<tr>
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<tr>
<td>Inner area</td>
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<td>40.00</td>
<td>mm²</td>
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<tr>
<td>Conventional rebar B500B</td>
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<tr>
<td>Inner area</td>
<td>–</td>
<td>( A_{rebar} )</td>
<td>50.26</td>
<td>mm²</td>
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</tbody>
</table>

Fig. 5. Reinforcement variants from left to right: Helibar (1), screw (2) and conventional rebar (3).
reinforcement strategies with the 3DCP process. The geometry of the final design is not altered by integrating the different reinforcement strategies used, instead process was adjusted as possible to fit the design. However, it is acknowledged that as a result this may not necessarily represent the best configurations for each specific reinforcement strategy. The print path and dimensions of the

Fig. 6. Moment-curvature analysis including four stages: (a) crack initiation, and (b) yielding of the reinforcement, and (c) redistribution of compressive forces, and (d) concrete compressive failure or (e) reinforcement tensile failure. The analytical method was used to create the final design for spiral staircase elements in this study.

Fig. 7. On the left each layer of the print path of the final design is seen, in which the line indicates the centre point of the nozzle over time. The print path indicated with an asterisk is for the first three layers of the SHCC staircase element with a perpendicular print path (SHCC-perp). On the right, an isometric visualization of the printed structure can be seen with dimensions: the entire print path (bottom), visualization after production (middle) and after the element is flipped over before testing (top).
Based on this final design a series of variations with different reinforcement strategies and materials is made. An overview of the reinforcement schemes can be seen in Fig. 8 and configurations and their abbreviations are listed below:

1. Plain mortar conventional reinforcement (PM-CR)
2. Plain mortar unreinforced (PM-UR)
3. SHCC plain (SHCC-plain)
4. SHCC perpendicular (SHCC-perp)
5. Plain mortar helibar (PM-helibar)
6. SHCC helibar (SHCC-helibar)
7. Plain mortar screw (PM-screw)
8. SHCC screw (SHCC-screw)

4. Experimental work

4.1. Production process

The technology for 3D concrete printing available at Eindhoven University of Technology (TU/e) [14] was used to produce the spiral staircase elements. The production process involved the use of a 4-degree-of-freedom gantry robot for concrete printing, complemented by a ABB robotic arm equipped with the ASD [18] for simultaneous helical reinforcement placement, shown in Fig. 10. Two different type of pumps were applied for the deposition of two different types of mortar. To facilitate the production and transportation of plain mortar, the M-Tec Duo Mix Connect mixing pump was utilized alongside an overhead silo designed to store the dry material. Once mixed, the wet mortar was transported using a rotor stator pump through a 10-m long hose with a diameter of 25.4 mm, ultimately reaching the gantry robot with a nozzle opening of 50 × 10 mm².

The deposition process for the SHCC involved the utilization of the M-Tec P20 Connect pump. The material were manually batch mixed in distinct phases, employing two Hobart A200N mixers. The production of SHCC involves several distinct phases. First, the dry materials (as listed in Table 1) are mixed together with fibers, superplasticizer, and one-third of the VMA for a duration of 2 min. Next, water is continuously added for 1 min, followed by an additional 2 min of mixing. Afterward, two-thirds of the VMA is added, and the mixture is mixed for another 2 min. Finally, the mixed material is subsequently loaded into the material chamber of the M-Tec P20 Connect, transported through a 5 m long hose with a diameter of 25.4 mm to the same nozzle and gantry system employed for printing plain mortar. The pump frequency is regulated by a dedicated computer and maintained at a constant rate of 0.93 Hz (56 revolutions per minute). A visual representation of these printing systems can be seen in Fig. 9.

During the printing process, the ASD picks up the helical reinforcement bars from a nearby rack and inserts them inside the object. The insertion speed for the screw and helibar reinforcement is 150 revolutions per minute, resulting in a translational movement of 10 mm/s and 95 mm/s, respectively, based on the lead of the reinforcement.

In order to preserve bond properties and minimize unnecessary influence during insertion, the tip of the reinforcement was milled into a conical shape. The helical reinforcements were inserted 10 min after the plain mortar was deposited, resulting in sufficient bond strength according to Ref. [15]. Because of the slower printing speed for SHCC, the helical reinforcement was inserted 45 min after SHCC deposition. The conventional reinforcement was placed manually during the printing process. All reinforcements were placed at a height of 9.25 mm from the printbed, between the first and second layer. Due to the difference in cross-sectional area of the reinforcement bars (illustrated on Fig. 5), different amount of bars were used. In total the cross-sectional area within all reinforced staircase

![Fig. 8. Reinforcement schemes for all manufactured (and experimentally tested) staircase elements, with discrete reinforcement bars indicated in red. (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)](image-url)
elements using screws, helibars and conventional rebars were 120.0 mm$^2$, 50.8 mm$^2$ and 150.8 mm$^2$ respectively. Fig. 8 illustrates the reinforcement layout and printpath used for the different staircase elements.

The print paths were programmed in G-Code using the Salad Slicer [38] plugin in Rhinoceros Grasshopper, with a layer offset of 9.25 mm. To generate the RAPID code that controlled the ABB robotic hand, the Robot Components plugin [39] within Rhinoceros Grasshopper and ABB Robotstudio was used. Additionally, a custom script using RAPID code was utilized to issue control commands specifically for the ASD [18]. The overall print path of the element can be seen in Fig. 7. After production, the staircase elements were covered with a plastic foil for 24 h and subsequently placed underwater for 27 days, before testing, in order to have a controlled and consistent curing environment.

4.2. Mechanical testing

A custom testing apparatus was used to determine the bending resistance of staircase elements, as can be seen in Fig. 11. As described in Section 3 the middle column of the spiral staircase was post-tensioned. For this reasons, and to ensure minimal rotation during testing, the base of the staircase elements were post tensioned within the testing apparatus. Post-tensioning is applied through a bolted steel thread, by using concrete blocks with rubber interlayers and a load cell for data acquisition.

The structure was subjected to a vertical force until failure, at the edge of the staircase element, using a manually operated hydraulic jack, at an average loading rate of approximately 20 N/s ($\pm$5 N/s), yielding an initial stress rate of 0.1 MPa/s. The load was applied using a roller and two steel plates with Teflon in between, to reduce horizontal friction. The deformation of the element was measured using a Panasonic HG-C1400-P laser during the application of the load.

5. Analytical analysis

The moment-curvature method, as presented in Section 3, can be used to determine the bending capacity of the selected staircase
elements. To accurately assess the maximum bending capacity a valid comparison can only be made when bending failure occurs during mechanical testing. As illustrated in Fig. 12, an analytical prediction of the force-displacement behavior of the elements can be achieved by: 1) determining the effective beam stiffness during each loading step in each section of the element; 2) utilizing traditional beam theory to derive deflections for each loading step. The effective beam stiffness at each loading step can be based on the slope in the moment-curvature graph for each section of the element. While the 4-linear moment-curvature graphs presented in Section 3 is appropriate for design consideration, it is not accurate enough to analyze actual (experimental) beam behaviour as it only considers a single section rather than the cumulative behaviour of sections along the length of a beam. To obtain a prediction of the full force-displacement behaviour of a beam, an iterative procedure are illustrated in Fig. 18 can be followed.

To initiate the calculation, four predetermined parameters must be known; the cross-sectional geometry, positioning of discrete reinforcement bars, the stress-strain relationship of the reinforcement, and the stress-strain relationship of the concrete. All input values that were used in the analytical calculation can be seen in Table 2.

The analyses of a specific element can start by dividing the specific cross-section into a large number of imaginary layers (50 layers for concrete and 1 layer for reinforcement are used in this study). After which an initial curvature of the element is assumed and the neutral line of the element is calculated based on the assumption of a plain concrete element. From this point the calculation is divided into two parts: 1) the addition to the overall bending resistance due to the reinforcement layer and; 2) the addition to overall bending resistance due to the concrete layers. Subsequently, the distance from all layers to the neutral line are determined. Based on the applied curvature and the distance to the neutral line the strain can be calculated from the respective stress-strain relations. Based on the cross
sectional area of and the stress within each of the defined layers the normal force can be calculated.

In the case of the sum of all normal forces remains below a pre-set threshold of 0.02 kN, the moment resistance of a section can be calculated based on the force and distance from the defined layers to the neutral line. If the threshold is exceeded, the neutral line will be shifted, before the cross-section is re-calculated. After each successful iteration, the curvature is increased. The iterative process is stopped if all reinforcement bars have failed or compressive failure in the concrete is reached.

The moment-curvature analysis is applied to all cross sections within the 8 selected designs. The results for the plain mortar and SHCC elements respectively can be seen in Figs. 13 and 14. The sections are named ‘AB’ till ‘FG’, according to Fig. 12, in which ‘AB’ is the section closest to the base of the element and thus is assumed to have the highest bending moment. The elements are designed to increase bending resistance closer to the base of the elements, as can be seen in all M-κ diagrams. However, it should be noted that the bending moment also increases towards the base of the element. For reinforced plain mortars (see Fig. 13b till 13d), it should be noted that a drop in moment capacity can be observed at low curvatures. This effect happens after the initial cracking of the plain mortar in each of the reinforced elements, due to the curvature-controlled manner of obtaining the moment resistance in each of the sections, as can be seen in the flowchart of Fig. 18. By increasing curvature, the neutral line for the sections shifts down, however, due to the absence of ductility in the material, as can be seen in Fig. 13a, and varying effective width of the cross-section, the loss of tensile resistance in the concrete is larger compared to the increase in tensile force in the respective reinforcement bars. Therefore, a reduction of moment capacity can be observed because the total amount of forces in the cross-section are reduced at these specific induced curvatures.

This effect does not occur in the analysis of the SHCC elements, because a significant amount of material ductility is present, which avoids the loss of tensile resistance in the concrete cross-section, even after the initial cracking of the concrete. Furthermore, this ductility results in a slightly higher maximum moment capacity for the SHCC elements compared to plain mortar elements.

From the M-κ diagrams, it can be seen that the elements with screw reinforcement (i.e. PM-screw and SHCC-screw) should achieve a higher bending resistance compared to the PM-CR element during mechanical testing. The element reinforced with helibar in plain mortar (i.e. PM-helibar) show a significant lower bending resistance, while the maximum bending capacity difference with the SHCC-helibar element is limited. The PM-UR, SHCC-perp and SHCC-plain have an increasing bending resistance respectively, however, due to the absence of discrete reinforcement are not comparable to the PM-CR element in terms of bending resistance.

6. Results and discussion

The following discussion examines the 8 different reinforcement and material configurations of the final design using both experimental and analytical approaches, as previously illustrated in Fig. 8. In order to present the structural behaviour of staircase elements in bending, the elements are presented in pairs for comparison reasons. A force-displacement graph is presented to compare the analytical moment-curvature analysis to the experimental bending test results, evaluating the bending capacity and failure mode of the elements. It is important to note that the analytical model does not take into consideration the imperfections resulting from the manufacturing process (e.g. concrete and reinforcement placement) and assumes a bending failure for the elements. The other failure modes, where applicable, are therefore solely described through the experimental work. Rotation of the clamped staircase elements during experimental testing was recognized and measured through images taken before testing and before failure. The measured rotational stiffness was taken into account when plotting the force displacement graphs by subtracting from the recorded deformation.

![Fig. 13. M-κ diagrams for each cross-section for all plain mortar elements.](image-url)
As explained in Section 3, the final design has been based on the PM-CR element. The failure load of each experimentally tested element is therefore assessed against the analytical failure load of this element, to show the application potential of the newly developed reinforcement strategies for 3DCP.

In the following selected elements are discussed in groups, presented together with force displacement graphs and visual representation of the elements post-testing. An overview of the experimental and analytical maximum loads and displacement for all tested elements are presented in Table 3.

6.1. Plain mortar conventionally reinforced (PM-CR) and plain mortar unreinforced (PM-UR)

The experimentally tested PM-CR element shown on Fig. 15a, where the use of conventional reinforcement, manually placed between the layers during printing, was expected to result in a failure load of 8.51 kN, shown on Table 3. However, the experimental findings present a different behaviour, revealing a significantly lower bending capacity at failure, as shown on the force displacement graph on Fig. 15a and by the difference between the maximum loads at failure of PM-CR-ana and PM-UR-exp, shown in Table 3.

This observation is further detailed by the brittle failure mode, indicated by the failure pattern near the loading point. This pattern near the loading point, where the concrete suddenly separated during testing, can be seen in Fig. 15a. Although the precise reasons behind this outcome lie beyond the scope of this paper, assumptions can be made regarding the potential causes: 1) insufficient bond between the conventional reinforcement and concrete; 2) inadequate reinforcement confinement.

In this case, reinforcement confinement is defined as the capacity to resist and distribute the tensile stresses developed in the normal plane to the reinforcement due to the pulling force applied on the rebar (also known as splitting tensile forces).

The PM-CR element was produced by manually placing reinforcement between the layers, posing a significant complication to the production process, involving human interaction during printing and resulting in questionable bond properties. As a result this type of production is not suitable without significant improvement to safety and quality, and as such is not considered as a viable solution for staircase element production.

As anticipated, the plain mortar unreinforced element (PM-UR) exhibits a relatively low bending capacity with an abrupt failure occurring at a low deformation due to the absence of ductility, shown on Fig. 15a and indicated by the PM-UR-exp plotted line. Both the analytical and experimental results align in terms of this capacity (PM-UR-ana and PM-UR-exp); however, the experimental results demonstrate a lower stiffness. It is not surprising that the unreinforced element falls short of the analytical result of the PM-CR element indicating a reinforced solution is needed.

6.2. SHCC plain and SHCC perpendicular (SHCC-perp)

Considering the utilization of the printable material itself as reinforcement, SHCC emerges as a viable option. The SHCC-plain element, shown on Fig. 15b demonstrates a notable increase in both load capacity and ductility compared to the PM-UR element on Fig. 15a, indicated by the respective plots of SHCC-plain-exp and PM-UR-exp. Furthermore, the analytical model accurately captured the force-displacement behaviour of the mechanically tested element, shown by SHCC-plain-ana. However, it is evident that the analytical strength of the PM-CR is not achieved.

Nevertheless, the failure mode transitions towards a more ductile behavior, allowing for greater deformation prior to failure, shown by the load displacement values in Table 3. This enhanced ductility can be advantageous when combined with different materials, in addition to potentially addressing the earlier challenge of inadequate confinement. Moreover, the application of SHCC is feasible.
6.3. Plain mortar helibar (PM-helibar) and SHCC helibar

To further enhance the bending moment capacity and automated the production, helibar was chosen as a reinforcement strategy in combination with both plain mortar (PM) and SHCC. While the analytical calculation of SHCC-helibar predicted a notably higher failure load than PM-helibar, the difference between their respective experimental counterparts was small, as shown on Fig. 16a plotted lines SHCC-helibar-exp and PM-helibar-exp. However, it should be noted that the experimental tests exhibited a lower bending moment capacity than showed in analytical work.

The analytical calculations predicted that neither the SHCC-helibar nor PM-helibar variants would meet the analytical result of the PM-CR element. This can be attributed to the fact that only approximately $\frac{1}{3}$ of the steel (150.78 mm$^2$ compared to 50.75 mm$^2$) is present, while the helibar has less than $1\frac{1}{2}$ times the strength of the conventional reinforcement (982 MPa compared to 681 MPa, respectively).

The automated integration of reinforcement appears to be successful for both plain mortar and SHCC. Nevertheless, there is still a need to increase the bending capacity, suggesting the inclusion of additional reinforcement in the cross-section. However, practical limitations arise when considering the addition of more helibar. Therefore, alternative helical reinforcement types are explored to

### Table 3

Maximum force and the accompanying displacement of all staircase elements.

<table>
<thead>
<tr>
<th></th>
<th>PM</th>
<th>PM</th>
<th>SHCC</th>
<th>SHCC</th>
<th>PM</th>
<th>SHCC</th>
<th>PM</th>
<th>SHCC</th>
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<td></td>
<td>CR</td>
<td>UR</td>
<td>plain</td>
<td>perp</td>
<td>helibar</td>
<td>helibar</td>
<td>screw</td>
<td>screw</td>
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<td><strong>Experimental</strong></td>
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<td></td>
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<td></td>
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</tr>
<tr>
<td>$F_{\text{max}}$ [kN]</td>
<td>3.25</td>
<td>0.95</td>
<td>2.65</td>
<td>1.49</td>
<td>3.69</td>
<td>4.94</td>
<td>7.88</td>
<td>9.04</td>
</tr>
<tr>
<td>$\delta_F$ [mm]</td>
<td>14.55</td>
<td>2.27</td>
<td>25.60</td>
<td>3.59</td>
<td>31.91</td>
<td>44.28</td>
<td>32.43</td>
<td>58.93</td>
</tr>
<tr>
<td><strong>Analytical</strong></td>
<td></td>
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<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>$F_{\text{max}}$ [kN]</td>
<td>8.51</td>
<td>1.15</td>
<td>2.44</td>
<td>1.43</td>
<td>4.97</td>
<td>6.85</td>
<td>9.28</td>
<td>9.83</td>
</tr>
<tr>
<td>$\delta_F$ [mm]</td>
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<td>9.57</td>
<td>50.61</td>
<td>34.03</td>
<td>35.32</td>
<td>31.79</td>
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</table>

Fig. 15. Experimental and analytical force displacement graphs and images post testing of respective Plain Mortar and SHCC staircase elements. Brittle failure of the elements has been indicated with a red cross at the end of the graphs, furthermore, the loading location has been indicated with a red dashed rectangle on the images. An overview of the failure loads and deformations for each element is given in Table 3. (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)

Within the printing process.

Additionally, it is important to acknowledge that the printing direction strongly influences the behavior of SHCC, as evident in the SHCC-perp element. Thus, this aspect should be taken into consideration during the design phase. Regardless, the incorporation of discrete bars is necessary to further increase the bending capacity.

### 6.3. Plain mortar helibar (PM-helibar) and SHCC helibar

To further enhance the bending moment capacity and automated the production, helibar was chosen as a reinforcement strategy in combination with both plain mortar (PM) and SHCC. While the analytical calculation of SHCC-helibar predicted a notably higher failure load than PM-helibar, the difference between their respective experimental counterparts was small, as shown on Fig. 16a plotted lines SHCC-helibar-exp and PM-helibar-exp. However, it should be noted that the experimental tests exhibited a lower bending moment capacity than showed in analytical work.

The analytical calculations predicted that neither the SHCC-helibar nor PM-helibar variants would meet the analytical result of the PM-CR element. This can be attributed to the fact that only approximately $\frac{1}{3}$ of the steel (150.78 mm$^2$ compared to 50.75 mm$^2$) is present, while the helibar has less than $1\frac{1}{2}$ times the strength of the conventional reinforcement (982 MPa compared to 681 MPa, respectively).

The automated integration of reinforcement appears to be successful for both plain mortar and SHCC. Nevertheless, there is still a need to increase the bending capacity, suggesting the inclusion of additional reinforcement in the cross-section. However, practical limitations arise when considering the addition of more helibar. Therefore, alternative helical reinforcement types are explored to
overcome these limitations.

6.4. Plain mortar screw (PM-screw) and SHCC screw

The PM-screw element shows an increased bending capacity in the analytical analysis compared to the PM-CR analytical result (9.28 kN–8.51 kN, respectively), shown on Fig. 16b and in Table 3. This can be attributed to the difference in steel strength and total cross-sectional area of screw and conventional reinforcement, as indicated in Table 2. During the experimental testing of PM-screw an abrupt failure took place close to it’s potential bending moment capacity during loading, illustrated by Figs. 16b and 17a. Similarly to the PM-CR experimental test shown on Fig. 15a, the premature failure might have happened due to inadequate confinement, as the

Fig. 16. Experimental and analytical force displacement graphs and images post testing of respective Plain Mortar and SHCC staircase elements. Brittle failure of the elements has been indicated with a red cross at the end of the graphs, furthermore, the loading location has been indicated with a red dashed rectangle on the images. (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)

(a) Plain mortar helibar and SHCC helibar elements (b) Plain mortar screw and SHCC screw elements

Fig. 17. Staircase element during testing (left) and after fracture (right), for (a) PM-screw and (b) SHCC-screw.
crack pattern seen on Fig. 16b does not indicate a typical bending behaviour and the bond properties between screw and plain mortar has been shown to be sufficient in previous studies [15].

The SHCC-screw element shows an increased bending capacity in the analytical analysis compared to the PM-CR analytical results (9.83 kN–8.51 kN, respectively). This behaviour was further confirmed in the experimental results, where a bending failure at 9.04 kN was recorded, as can be seen in Figs. 16b and 17b. This positive change in failure mode could be attributed to the confinement provided around the reinforcement by the SHCC.

The incorporation of the screw in the fibrous material did not result in any complications for the production of the elements, while the desired bending failure and sufficient capacity were achieved by combining SHCC with the screw reinforcement. Using Automated Screw Device (ASD) for placement, the production process for the staircase element became fully automated.

7. Conclusions

The main conclusions from the study are:

● Integration of helical reinforcement for automated production of a 3D printed structure using the Automated Screwing Device has been successfully demonstrated. However, the combination of helical (screw) reinforcement with plain mortar was shown to lead to an abrupt premature failure by splitting of concrete during loading, suggesting inadequate confinement, and therefore not reaching its full bending moment capacity.

● The application of SHCC as the main reinforcement in a 3D printed structure was shown to provide a significant increase in bending resistance and ductility compared to plain mortar. However, the tensile strength of SHCC seems to be insufficient for bending structures of this scale without additional reinforcement.

● The combination of helical reinforcement and SHCC was shown to be an effective reinforcement strategy for load-bearing printed structures to reinforce the layers themselves. To achieve high bending capacity, the favourable characteristics of both methods were effectively used, in which the high tensile strength of discrete helical (screw) reinforcement was combined with the ductility of SHCC to provide confinement.

● The comparisons revealed differences in distinct failure mechanisms (other than failure in bending) in printed staircase elements subjected to bending loads, in which the need for confinement seems apparent. Furthermore, due to typical printed structures consisting of narrow layers of concrete, there is an evident challenge of ensuring adequate confinement for discrete reinforcement in tension.

● The integration of SHCC with the 3D printing process was successfully shown by producing a staircase element. The printing direction has significant influence on the macro-scale properties of 3D printed SHCC, shown through the reduction in strength and ductility of the staircase element printed in the perpendicular direction.

● The moment-curvature analytical method shows good agreement with reinforced 3D concrete printed specimens that failed in bending.

Author contribution

Lauri Hass: Investigation, Methodology, Writing – original draft.
Karsten Nefs: Investigation, Methodology, Writing – original draft.
F.P. Bos: Conceptualization, Supervision, Writing – review & editing.
T.A.M. Salet: Conceptualization, Supervision.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: Lauri Hass has patent #N2030291 pending to Eindhoven University of Technology.

Data availability

Data will be made available on request.

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A. Appendix

For each section of the staircase element, the deflection can be derived, as shown below for section-AB:
\[ u_{AB} = \frac{PL - Px}{EI_{AB}} \]
\[ u'_{AB} = \frac{P}{2EI_{AB}} (2Lx - x^2) + C_1 \]
\[ u_{AB} = \frac{P}{6EI_{AB}} (3Lx^2 - x^3) + C_1x + C_2, \]

in which \(x\) is the distance along the element starting from the base of the element, and \(P\) the force applied to the element as illustrated in Fig. 12. The deflection for section-FG is therefore given by:
\[ u_{FG} = \frac{P}{6EI_{FG}} (3Lx^2 - x^3) + C_{11}x + C_{12} \]

For each of the sections two boundary conditions (bc) can be formulated:
\[ bc_1 : u_{AB}(0) = 0 \]
\[ bc_2 : u'_{AB}(0) = 0 \]
\[ bc_3 : u_{AB}(B) = u_{BC}(B) \]
\[ bc_4 : u'_{AB}(B) = u_{BC}(B) \]
\[ bc_5 : u_{BC}(C) = u_{CD}(C) \]
\[ bc_6 : u'_{BC}(C) = u_{CD}(C) \]
\[ bc_7 : u_{CD}(D) = u_{DE}(D) \]
\[ bc_8 : u'_{CD}(D) = u_{DE}(D) \]
\[ bc_9 : u_{DE}(E) = u_{EF}(E) \]
\[ bc_10 : u'_{DE}(E) = u'_{EF}(E) \]
\[ bc_11 : u_{EF}(F) = u_{FG}(F) \]
\[ bc_12 : u'_{EF}(F) = u'_{FG}(F). \]

By solving for the previous mentioned boundary conditions the 12 unknown variables can be solved:
\[ C_1 = 0 \]
\[ C_2 = 0 \]
\[ C_3 = \frac{P}{2EI_{AB}} (2LB - B^2) + C_1 - \frac{P}{2EI_{BC}} (2LB - B^2) \]
\[ C_4 = \frac{P}{6EI_{AB}} (3LB^2 - B^3) + C_1B + C_2 - \frac{P}{6EI_{BC}} (3LB^2 - B^3) - C_3B \]
\[ C_5 = \frac{P}{2EI_{BC}} (2LC - C^2) + C_3 - \frac{P}{2EI_{CD}} (2LC - C^2) \]
\[ C_6 = \frac{P}{6EI_{BC}} (3LC^2 - C^3) + C_4C + C_4 - \frac{P}{6EI_{CD}} (3LC^2 - C^3) - C_5C \]
\[ C_7 = \frac{P}{2EI_{CD}} (2LD - D^2) + C_5 - \frac{P}{2EI_{DE}} (2LD - D^2) \]
\[ C_8 = \frac{P}{6EI_{CD}} (3LD^2 - D^3) + C_5D + C_6 - \frac{P}{6EI_{DE}} (3LD^2 - D^3) - C_6D \]
\[
C_9 = \frac{P}{2EI_{DE}} (2LE - E^3) + C_7 - \frac{P}{2EI_{EF}} (2LE - E^3)
\]

\[
C_{10} = \frac{P}{6EI_{DE}} (3LE^2 - E^3) + C_7E + C_8 - \frac{P}{6EI_{EF}} (3LE^2 - E^3) - C_8E
\]

\[
C_{11} = \frac{P}{2EI_{EF}} (2LF - F^3) + C_9 - \frac{P}{2EI_{FG}} (2LF - F^3)
\]

\[
C_{12} = \frac{P}{6EI_{FG}} (3LF^2 - F^3) + C_9F + C_{10} - \frac{P}{6EI_{FG}} (3LF^2 - F^3) - C_{11}F.
\]

In which \(B\) to \(F\) are the \(x\) coordinates from which the cross-section changes to the respective next cross section. So, the vertical deflection along any point \(x\) on the section-FG can be calculated with the following formula:

\[
u_{FG} = \frac{P}{6EI_{FG}} (3Lx^2 - x^3) + C_{13}x + C_{12}
\]

---

References

