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Adaptive Computed Reference Computed Torque Control of Flexible Robots

This paper presents a motion control technique for flexible robots and manipulators. It takes into account both joint and link flexibility and can be applied in adaptive form if robot parameters are unknown. It solves the main problems that are related to the fact that the number of degrees of freedom exceeds both the number of actuators and the number of output variables. The proposed method results in trajectory tracking while all state variables remain bounded. Global, asymptotic stability is ensured for all values of the stiffnesses of joints and links. To show the characteristics of the proposed control law, some simulation results are presented.

Introduction

Until now, robot control has been based on the assumption that the transmissions are stiff and that the links behave as rigid bodies. For higher operating speeds, the links can deform significantly. Besides, flexibility can be present in the transmissions between actuators and links (joint elasticity). Flexibility in links and/or joints decreases tracking accuracy and can cause instability if the controller is designed under the assumption of perfect rigidity. In order to improve the performance of the robot, a model that accounts for the flexibilities is required for control. Nowadays, real-time application of complex control algorithms is possible with advanced multiprocessor equipment.

Because of the complexity of the equations of motion for robots with flexibilities in both joints and links (e.g., Shabana, 1989), often only joint flexibility (e.g., Spong et al., 1987), only link flexibility (e.g., Fukuda, 1985), and/or special configurations such as single link examples (e.g., Marino and Spong, 1986) are considered. Among some others, Henrichfreise (1990) and Lin (1991) considered combined joint and link flexibility.

One possible approach to control flexible robots is to use feedback linearisation (e.g., Khorasani and Kokotovic, 1985). This method is computationally expensive and requires an accurate model. An alternative approach, based on singular perturbation techniques (e.g., Spong et al., 1987), is restricted to robots with small flexibilities only. Although various approaches have been presented to control flexible robots (see for a survey, e.g., Desoyer et al., 1986; Lammerts, 1993), this problem is still a challenge.

Starting from the idea of computed torque control for rigid robots (e.g., Slotine and Li, 1991), Lammerts et al. (1991) introduced a control technique for robots with rigid and elastic joints. Further development of this technique resulted in the so-called "Computed Reference Computed Torque Control" (CRCTC) technique for flexible robots with *both joint elasticity and link flexibility* (Lammerts, 1993). In this paper, the CRCTC approach is formalized. We first present the theory of CRCTC and show that global asymptotic stability is ensured, regardless of the magnitude of the flexibilities. For unknown robot parameters, an *adaptive* version can be used such that trajectory tracking is achieved while all signals in the system remain bounded. The (adaptive) control technique is illustrated with some simulation results on a flexible TR-robot with one elastic joint and one flexible link. Results on some experimental setups were reported in Lammerts (1993).

Model and Control Objective

We consider a robot (or manipulator) that can be modeled as an open chain of m links, connected by cylindrical, revolute, or prismatic joints with one-degree-of-freedom per joint. The base is fixed to the ground and the other end (with end-effector) has to follow a specified trajectory. Sensors and one actuator are located in each joint and eack link is connected to its driving actuator via a (gear) transmission.

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We neglect actuator dynamics and use the actuator torques $\mathbf{u} \in R^m$ as the input variables of the system.

If all links and joints are rigid, the number of degrees of freedom equals the number of inputs. However, because the robot has non-negligible link and/or joint flexibility, *the number of degrees of freedom, n, is larger than the number of actuators, m.* We assume that the number of output (end-effector) variables is equal to the number of actuators. These output quantities are the components of $\mathbf{y} \in R^m$.

With Lagrange's formalism, it is possible to derive a set of second order differential equations, the equations of motion, which relate the torques $\mathbf{u}$ to the chosen generalized coordinates $\mathbf{q}$. We assume that the structure of this model is correct, i.e., that the model perfectly matches the real system if all parameters are exact. Uncertainties in parameters (masses, stiffnesses, etc.) will be taken into account. The a priori unknown parameters are the entries of $\mathbf{p} \in R^p$. They are assumed to be constant or to change much slower than the generalized coordinates. Hence, it is assumed that $\dot{\mathbf{p}} = \mathbf{0}$.

A fairly general model for a flexible robot is given by (Desoyer et al., 1986; Lin, 1991; Lammerts, 1993):

$$M\ddot{\mathbf{q}} + C\dot{\mathbf{q}} + \mathbf{f} = H\mathbf{u} \tag{1}$$

$$\mathbf{y} = \mathbf{h}(\mathbf{q}) \tag{2}$$

where $M = M(\mathbf{q}, \mathbf{p})$ is symmetric and positive definite, and $C = C(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{p})$ accounts for the Coriolis and centrifugal torques, such that the matrix $\dot{M} - 2C$ is skew-symmetric (Lammerts, 1993). $H\mathbf{u} - \mathbf{f}$ represents the generalized torques. The distribution matrix $H = H(\mathbf{q})$ will have full rank, i.e., rank $(H) = m$. The vector $\mathbf{f} = \mathbf{f}(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{p}, t)$ accounts for external loading (e.g., gravity) and for all internal torques other than the actuator torques. These internal torques are due to, for instance, damping and friction and, last but not least, link and joint flexibility. Often, $\mathbf{f}$ can be written as (Henrichfreise, 1990):

$$\mathbf{f} = K\mathbf{q} + B\dot{\mathbf{q}} + \mathbf{f}_n \tag{3}$$

where K and B are the symmetric, (semi-)positive definite stiffness and damping matrix.

The functions $\mathbf{h} = \mathbf{h}(\mathbf{q})$ in the output Eq. (2) are assumed to be smooth. Furthermore, it is assumed that the Jacobian of these functions is a matrix of full rank for all relevant $\mathbf{q}$.

The desired trajectory of $\mathbf{y}$ is given by the smooth, bounded vector function $\mathbf{y}_d = \mathbf{y}_d(t)$. Without loss of generality, we may assume that the generalised coordinates are chosen such that the desired trajectory of m of these coordinates is determined by the m equations $\mathbf{y} = \mathbf{h}(\mathbf{q})$ in combination with the desired trajectory $\mathbf{y}_d = \mathbf{y}_d(t)$. These coordinates are the components of $\mathbf{q}_k \in R^m$ and their desired trajectory is denoted by $\mathbf{q}_{kd} = \mathbf{q}_{kd}(t)$. The other coordinates are the components of $\mathbf{q}_u \in R^{n-m}$. Hence:

$$\mathbf{q} = L_k\mathbf{q}_k + L_u\mathbf{q}_u \tag{4}$$

where $[L_k\ L_u]$ is a permutation matrix. In Lammerts (1993), it is assumed that $\mathbf{q}_k$ is the vector with rigid-body coordinates and that $\mathbf{q}_u$ is the vector with flexible coordinates.

The objective of the current investigation is to determine such a bounded input $\mathbf{u} = \mathbf{u}(t)$ that the coordinates in $\mathbf{q}_k$ track their desired trajectory asymptotically while the coordinates in $\mathbf{q}_u$ and their derivatives in $\dot{\mathbf{q}}_u$ remain bounded. It is assumed that both $\mathbf{q}$ and $\dot{\mathbf{q}}$ are known for all $t \geq t_0$ from measurements and/or reconstruction.

Let $\mathbf{e}_k = \mathbf{q}_{kd} - \mathbf{q}_k$ be the tracking error for the coordinates in $\mathbf{q}_k$. Then, the control objective is to determine such a bounded input $\mathbf{u}$ that $\mathbf{q}$ and $\dot{\mathbf{q}}$ remain bounded while $\mathbf{e}_k$ and $\dot{\mathbf{e}}_k$ converge to zero. According to Slotine and Li (1991), we can write this requirement in a more compact form if we introduce a reference trajectory $\mathbf{q}_{kr}$ for $\mathbf{q}_k$ and a reference error $\mathbf{e}_{kr}$, defined by:

$$\dot{\mathbf{q}}_{kr} = \dot{\mathbf{q}}_{kd} + \Lambda_k\mathbf{e}_k; \quad \mathbf{e}_{kr} = \mathbf{q}_{kr} - \mathbf{q}_k \tag{5}$$

with positive definite Λ_k. A trivial choice for the initial value $\mathbf{q}_{kr}(t_0)$ is $\mathbf{q}_{kr}(t_0) = \mathbf{q}_k(t_0)$. Since $\Lambda_k > 0$ and $\dot{\mathbf{e}}_{kr} = \dot{\mathbf{e}}_k + \Lambda_k\mathbf{e}_k$, the requirements with respect to $\mathbf{e}_k$ and $\dot{\mathbf{e}}_k$ are satisfied if $\dot{\mathbf{e}}_{kr}(t) \to \mathbf{0}$ for $t \to \infty$.

To satisfy the condition that $\mathbf{q}_u$ and $\dot{\mathbf{q}}_u$ remain bounded, we assume that it is possible to compute a smooth, bounded desired trajectory $\mathbf{q}_{ud} = \mathbf{q}_{ud}(t)$ for $\mathbf{q}_u$. The determination of such a trajectory is one of the topics of the next section. Let $\mathbf{e}_u = \mathbf{q}_{ud} - \mathbf{q}_u$ be the associated tracking error and let the reference trajectory $\mathbf{q}_{ur} = \mathbf{q}_{ur}(t)$ and the reference error $\mathbf{e}_{ur}$ be defined by:

$$\dot{\mathbf{q}}_{ur} = \dot{\mathbf{q}}_{ud} + \Lambda_u\mathbf{e};\ \mathbf{e}_{ur} = \mathbf{q}_{ur} - \mathbf{q}_u \tag{6}$$

with positive definite matrix Λ_u and any suitable initial value for $\mathbf{q}_{ur}$, for instance $\mathbf{q}_{ur}(t_0) = \mathbf{q}_u(t_0)$. Then, the total reference trajectory $\mathbf{q}_r$ and the reference error $\mathbf{e}_r$ are given by:

$$\mathbf{q}_r = L_k\mathbf{q}_{kr} + L_u\mathbf{q}_{ur}; \tag{7}$$

Nomenclature

$\mathbf{q} \in R^n$ = generalized coordinates
$\mathbf{q}_k \in R^m$ = generalized coordinates with known desired trajectory
$\mathbf{q}_U \in R^{n-m}$ = generalized coordinates with unknown desired trajectory
$\mathbf{q}_d, \mathbf{q}_{kd}, \mathbf{q}_{ud}$ = desired trajectories
$\mathbf{q}_r, \mathbf{q}_{kr}, \mathbf{q}_{ur}$ = reference trajectories
$\mathbf{e} \in R^n$ = tracking errors
$\mathbf{e}_r \in R^n$ = reference errors
$\mathbf{u} \in R^m$ = input torques/forces
$\mathbf{y} \in R^m$ = output, end-effector variables
$\mathbf{p} \in R^p$ = unknown parameters
$M, C, K, B \in R^{n\times n}$ = model matrices
$\mathbf{f}, \mathbf{f}_n, \mathbf{f}^* \in R^n$ = torque/force vectors
$H \in R^{n\times m}$ = distribution matrix
$N \in R^{n\times(n-m)}$ = matrix such that $N^TH = 0$
$W \in R^{n\times p}$ = regression matrix
$\hat{\mathbf{p}} \in R^p$ = estimated parameters
$\hat{M}, \hat{C}, \hat{K}, \hat{B}, \hat{\mathbf{f}}^*$ = model quantities based on $\hat{\mathbf{p}}$
V = Lyapunov function
$K_r \in R^{n\times n}$ = control gain
$L_k \in R^{n\times m}$, $L_u \in R^{n\times(n-m)}$ = permutation matrices
$\Lambda_k \in R^{m\times m}$, $\Lambda_u \in R^{(n-m)\times(n-m)}$ = control matrices

$$\mathbf{e}_r = \mathbf{q}_r - \mathbf{q} = L_k\mathbf{e}_{kr} + L_u\mathbf{e}_{ur} \quad (8)$$

and the control objective is achieved if such a bounded input $\mathbf{u}$ can be found that $\mathbf{e}_r$ and $\dot{\mathbf{e}}_r$ are bounded and that $\dot{\mathbf{e}}_r \to \mathbf{0}$ for $t \to \infty$. Since $\mathbf{q}_{ur}$ has to be calculated on-line, the resulting control is called computed reference computed torque control (CRCTC).

Computed Reference Computed Torque Control (CRCTC)

In this section, we extend the *nonadaptive* control law of Slotine and Li (1987) for rigid robots to arrive at a formulation that covers a broad class of flexible robots. The starting-point is given by the equations of motion (1) with $\mathbf{q} \in R^n$, $\mathbf{u} \in R^m$ and $m < n$. Similarly to Slotine and Li, it seems appropriate to determine the input $\mathbf{u}$ from:

$$H\mathbf{u} = M\ddot{\mathbf{q}}_r + C\dot{\mathbf{q}}_r + \mathbf{f} + K_r\dot{\mathbf{e}}_r \quad (9)$$

where K_r is positive definite. One problem is that only the components of $\mathbf{q}_{kr} \in R^m$ are determined by the desired output. An obvious solution is to determine the unknown components of $\mathbf{q}_{ur} \in R^{n-m}$ and the components of $\mathbf{u} \in R^m$ from the set (9). However, this is often impossible because this set is insufficient and the resulting $\mathbf{q}_{ur}$ is not bounded. This problem occurs, for instance, when a flexible joint is modeled according to Spong (1987). For more details we refer to Lammerts (1993). A possible solution is to consider the set (9) as just a set of equations for $\mathbf{u}$ and $\mathbf{q}_{ur}$ and to replace the functions $\mathbf{f} = \mathbf{f}(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{p}, t)$ in this set by suitably chosen functions $\mathbf{f}^* = \mathbf{f}^*(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{q}_r, \dot{\mathbf{q}}_r, \mathbf{p}, t)$. This "new" functions follow from $\mathbf{f} = \mathbf{f}(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{p}, t)$, for instance, by partial or complete replacement of $\mathbf{q}$ and $\dot{\mathbf{q}}$ by $\mathbf{q}_r$ and $\dot{\mathbf{q}}_r$, respectively. For the moment, it is assumed that it is possible to determine such functions $\mathbf{f}^* = \mathbf{f}^*(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{q}_r, \dot{\mathbf{q}}_r, \mathbf{p}, t)$ that the set:

$$H\mathbf{u} = M\ddot{\mathbf{q}}_r + C\dot{\mathbf{q}}_r + \mathbf{f}^* + K_r\dot{\mathbf{e}}_r \quad (10)$$

has a bounded solution for $\mathbf{q}_{ur}$ and $\mathbf{u}$. Then, the error equation for the controlled system is given by:

$$M\ddot{\mathbf{e}}_r + C\dot{\mathbf{e}}_r + \mathbf{f}^* - \mathbf{f} + K_r\dot{\mathbf{e}}_r = \mathbf{0} \quad (11)$$

A further requirement with respect to the functions $\mathbf{f}^* = \mathbf{f}^*(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{q}_r, \dot{\mathbf{q}}_r, \mathbf{p}, t)$ is that $\dot{\mathbf{e}}_r = \mathbf{0}$ must be an asymptotically stable equilibrium point of this system. This will be studied in more detail in the next section. Here, we focus on the first requirement.

The distribution matrix $H \in R^{n\times m}$ has full rank, so H^TH is regular and there exist matrices $N \in R^{n\times(n-m)}$ of full rank that satisfy $N^TH = 0$. Hence, the set (0.10) is equivalent to:

$$\mathbf{u} = (H^TH)^{-1}H^T[M\ddot{\mathbf{q}}_r + C\dot{\mathbf{q}}_r + \mathbf{f}^* + K_r\dot{\mathbf{e}}_r] \quad (12)$$

$$\mathbf{0} = N^T[M\ddot{\mathbf{q}}_r + C\dot{\mathbf{q}}_r + \mathbf{f}^* + K_r\dot{\mathbf{e}}_r] \quad (13)$$

The unknown part $\mathbf{q}_{ur}$ of $\mathbf{q}_r$ must be solved from the set (13) and the solution has to be bounded. As soon as $\mathbf{q}_{ur}$, $\dot{\mathbf{q}}_{ur}$, and $\ddot{\mathbf{q}}_{ur}$ are known, $\mathbf{q}_r$, $\dot{\mathbf{q}}_r$, and $\ddot{\mathbf{q}}_r$ can be determined and the input $\mathbf{u}$ can be computed from the set (12).

As an example, we consider the case where $\mathbf{f}$ is given by (3), i.e., where $\mathbf{f} = K\mathbf{q} + B\dot{\mathbf{q}} + \mathbf{f}_n$, with constant matrices K and B and $\mathbf{f}_n = \mathbf{f}_n(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{p}, t)$. Then, $\mathbf{f}^*$ can be chosen as:

$$\mathbf{f}^* = K\mathbf{q}_r + B\dot{\mathbf{q}}_r + \mathbf{f}_n \quad (14)$$

and Eq. (13) for $\mathbf{q}_{ur}$ becomes:

$$N^T[ML_u\ddot{\mathbf{q}}_{ur} + (C + B + K_r)L_u\dot{\mathbf{q}}_{ur} + KL_u\mathbf{q}_{ur}] =$$
$$-N^T[ML_k\ddot{\mathbf{q}}_{kr} + (C + B + K_r)L_k\dot{\mathbf{q}}_{kr} + KL_k\mathbf{q}_{kr} + +K_r\dot{\mathbf{q}} + \mathbf{f}_n]$$

The procedure to determine $\mathbf{q}_{ur}$ strongly depends on the properties of the matrices N^TML_u, $N^T(C + B + K_r)L_u$ and N^TKL_u on the left-hand side (Lammerts, 1993). It is noted that Eq. (15) allows a bounded solution for $\mathbf{q}_{ur}$, for instance, for robots with stiff links and elastic joints. The suitability of the CRCTC technique for this kind of robots was shown earlier by Lammerts et al. (1991).

Stability

Premultiplication of the error equation (11) with $\dot{\mathbf{e}}_r$ yields:

$$\dot{\mathbf{e}}_r^T(M\ddot{\mathbf{e}}_r + C\dot{\mathbf{e}}_r) = -\dot{\mathbf{e}}_r^TK_r\dot{\mathbf{e}}_r - \dot{\mathbf{e}}_r^T(\mathbf{f}^* - \mathbf{f}) \quad (15)$$

where $\dot{\mathbf{e}}_r^TC\dot{\mathbf{e}}_r = \frac{1}{2}\dot{\mathbf{e}}_r^T\dot{M}\dot{\mathbf{e}}_r$ for every $\dot{\mathbf{e}}_r$, because $C - \frac{1}{2}\dot{M}$ is skew-symmetric. Hence:

$$\frac{d}{dt}\left(\frac{1}{2}\dot{\mathbf{e}}_r^TM\dot{\mathbf{e}}_r\right) = -\dot{\mathbf{e}}_r^TK_r\dot{\mathbf{e}}_r - \dot{\mathbf{e}}_r^T(\mathbf{f}^* - \mathbf{f}) \quad (16)$$

The second method of Lyapunov guarantees that $\dot{\mathbf{e}}_r \to \mathbf{0}$ for $t \to \infty$ if the term $\dot{\mathbf{e}}_r^T(\mathbf{f}^* - \mathbf{f})$ can be written as the sum of a quantity ψ that is non-negative for each $\dot{\mathbf{e}}_r \neq \mathbf{0}$, and the time derivative of a non-negative quantity ϕ:

$$\dot{\mathbf{e}}_r^T(\mathbf{f}^* - \mathbf{f}) = \psi + \dot{\phi} \quad (17)$$

Here, ψ may depend on $\mathbf{q}$, $\dot{\mathbf{q}}$, $\mathbf{e}_r$, $\dot{\mathbf{e}}_r$, $\mathbf{p}$ and t while ϕ may be a function of $\mathbf{q}$, $\mathbf{e}_r$ and $\mathbf{p}$:

$$\psi = \psi(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{e}_r, \dot{\mathbf{e}}_r, \mathbf{p}, t); \quad \psi \geq 0 \text{ for each } \dot{\mathbf{e}}_r$$
$$\phi = \phi(\mathbf{q}, \mathbf{e}_r, \mathbf{p}); \quad \phi \geq 0 \quad (18)$$

Suppose that functions ψ and ϕ of this kind exist for the chosen function $\mathbf{f}^*$. Then, we can define a Lyapunov function V by:

$$V = \frac{1}{2}\dot{\mathbf{e}}_r^TM\dot{\mathbf{e}}_r + \phi \quad (19)$$

and it is easily seen that:

$$\dot{V} = -\dot{\mathbf{e}}_r^TK_r\dot{\mathbf{e}}_r - \psi \quad (20)$$

Hence, $\dot{V} < 0$ for each $\dot{\mathbf{e}}_r \neq \mathbf{0}$ and therefore $\dot{\mathbf{e}}_r(t) \to \mathbf{0}$ for $t \to \infty$. Since $\dot{\mathbf{e}}_r = L_k\dot{\mathbf{e}}_{kr} + L_u\dot{\mathbf{e}}_{ur}$, this implies that both $\dot{\mathbf{e}}_{kr}(t) \to \mathbf{0}$ and $\dot{\mathbf{e}}_{ur}(t) \to \mathbf{0}$ for $t \to \infty$. From $\dot{\mathbf{e}}_{kr} = \dot{\mathbf{e}}_k + \Lambda_k\mathbf{e}_k$ and $\Lambda_k > 0$, it then follows that $\mathbf{e}_k(t) \to \mathbf{0}$ for $\mathbf{t} \to \infty$, i.e., that asymptotic tracking of the desired trajectory for $\mathbf{q}_k$ and therefore also for the output $\mathbf{y}$ is guaranteed. Furthermore, it can be shown that $\mathbf{e}_{ur} = \mathbf{q}_{ur} - \mathbf{q}_u$ is bounded and hence that $\mathbf{q}_u$ is bounded since $\mathbf{q}_{ur}$ is bounded. It cannot be shown that $\mathbf{q}_u$ will converge to the reference trajectory $\mathbf{q}_{ur}$. However, $\dot{\mathbf{q}}_u$ will converge asymptotically to the reference velocity $\dot{\mathbf{q}}_{ur}$.

As an example, we again consider the case where $\mathbf{f}$ is given by $\mathbf{f} = K\mathbf{q} + B\dot{\mathbf{q}} + \mathbf{f}_n$ and where $\mathbf{f}^*$ is chosen as $\mathbf{f}^* = K\mathbf{q}_r + B\dot{\mathbf{q}}_r + \mathbf{f}_n$. Equation (17) then yields:

$$\dot{\mathbf{e}}_r^T(\mathbf{f}^* - \mathbf{f}) = \dot{\mathbf{e}}_r^TK\mathbf{e}_r + \dot{\mathbf{e}}_r^TB\dot{\mathbf{e}}_r \quad (21)$$

Because $B \leq 0$ and $K \leq 0$ and K is constant, it is easily seen that $\psi = \dot{\mathbf{e}}_r^TB\dot{\mathbf{e}}_r$ and $\phi = \frac{1}{2}\mathbf{e}_r^TK\,\mathbf{e}_r$ satisfy the conditions of

Eq. (18). Therefore, in this case, asymptotical tracking of the desired end-effector trajectory is guaranteed and all coordinates remain bounded. This was shown earlier by Lammerts et al. (1991), for the special case of a robot with rigid links and flexible joints.

It is emphasized that the control law Eq. (12) and the stability proof make sense only if we can determine such functions $\mathbf{f}^* = \mathbf{f}^*(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{q}_r, \dot{\mathbf{q}}_r, \mathbf{p}, t)$ that Eq. (13) allows a smooth, bounded solution of $\mathbf{q}_{ur}$ and that Eqs. (17) and (18) are satisfied. The determination of vector functions is the essence of this control method.

Adaptive CRCTC

The CRCTC law and the stability analysis are based on the assumption that the parameters used in the control law match the parameters $\mathbf{p}$ of the actual system. In practice, only an estimate $\hat{\mathbf{p}}$ for $\mathbf{p}$ is available. For rigid robots, Slotine and Li (1987) proposed a method to adapt the estimate $\hat{\mathbf{p}}$ such that the stability of the closed loop system is guaranteed. In this section, the method of Slotine and Li (1987) is extended to robots with flexible links and flexible joints.

It is often possible to choose such unknown parameters $\mathbf{p}$ that the equations of motion are linear in $\mathbf{p}$ (e.g., Slotine and Li, 1991). In that case, the left-hand side of Eq. (1) can be written as:

$$M(\mathbf{q}, \mathbf{p})\ddot{\mathbf{q}} + C(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{p})\dot{\mathbf{q}} + \mathbf{f}(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{p}, t) = W(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}, t)\mathbf{p} + \mathbf{w}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}, t) \quad (22)$$

where all components of $W \in R^{n\times p}$ and of $\mathbf{w} \in R^n$ are known functions of $\mathbf{q}$, $\dot{\mathbf{q}}$, $\ddot{\mathbf{q}}$ and t.

The control law for the case with unknown parameters is essentially the same as for the case with known parameters. The only difference is the replacement of $\mathbf{p}$ in Eq. (12) and Eq. (13) by the available estimate $\hat{\mathbf{p}}$. Therefore, the input $\mathbf{u}$ and the unknown reference $\mathbf{q}_{ur}$ have to be determined from:

$$\mathbf{u} = (H^T H)^{-1} H^T [\hat{M}\ddot{\mathbf{q}}_r + \hat{C}\dot{\mathbf{q}}_r + \hat{\mathbf{f}}^* + K_r \dot{\mathbf{e}}_r] \quad (23)$$

$$\mathbf{0} = N^T [\hat{M}\ddot{\mathbf{q}}_r + \hat{C}\dot{\mathbf{q}}_r + \hat{\mathbf{f}}^* + K_r \dot{\mathbf{e}}_r] \quad (24)$$

with $\hat{M}(\mathbf{q}) = M(\mathbf{q}, \hat{\mathbf{p}})$, $\hat{C}(\mathbf{q}, \dot{\mathbf{q}}) = C(\mathbf{q}, \dot{\mathbf{q}}, \hat{\mathbf{p}})$, etc. Then, the error equation for the closed-loop system becomes:

$$M\ddot{\mathbf{e}}_r + C\dot{\mathbf{e}}_r + \mathbf{f}^* - \mathbf{f} + K_r\dot{\mathbf{e}}_r = (M - \hat{M})\ddot{\mathbf{q}}_r + (C - \hat{C})\dot{\mathbf{q}}_r + (\mathbf{f}^* - \hat{\mathbf{f}}^*) \quad (25)$$

Analogously to Eq. (22), we require that the functions $\hat{\mathbf{f}}^*$ are linear in $\hat{\mathbf{p}}$. Then, the right-hand side of Eq. (25) is proportional to the error $\mathbf{e}_p = \mathbf{p} - \hat{\mathbf{p}}$ and there exists such a matrix $W_r = W_r(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{q}_r, \dot{\mathbf{q}}_r, \ddot{\mathbf{q}}_r, t)$ that:

$$M\ddot{\mathbf{e}}_r + C\dot{\mathbf{e}}_r + \mathbf{f}^* - \mathbf{f} + K_r\dot{\mathbf{e}}_r = W_r\mathbf{e}_p \quad (26)$$

To investigate the asymptotic properties of this system, we consider the same Lyapunov function V as in the previous section, but now augmented with a positive definite term in the parameter error:

$$V = \frac{1}{2}\dot{\mathbf{e}}_r^T M \dot{\mathbf{e}}_r + \phi + \frac{1}{2}\mathbf{e}_p^T \Gamma^{-1}\mathbf{e}_p; \quad \Gamma > 0 \quad (27)$$

For constant Γ, the time-derivative of V is given by:

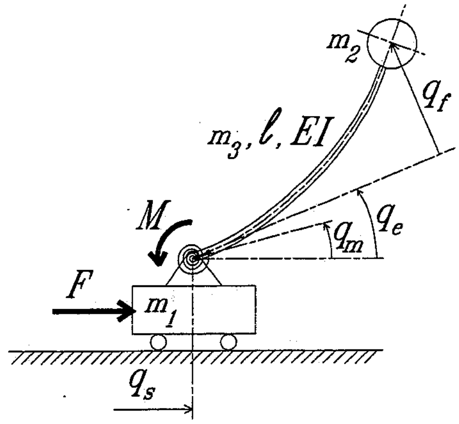

Fig. 1 Flexible TR-manipulator

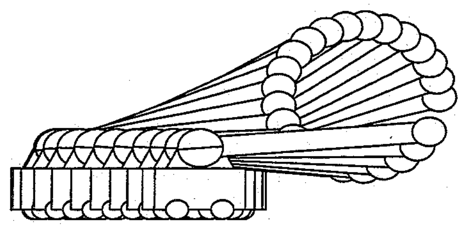

Fig. 2 Desired end-effector trajectory

$$\dot{V} = -\dot{\mathbf{e}}_r^T K_r \dot{\mathbf{e}}_r - \psi + \mathbf{e}_p^T (W_r^T \dot{\mathbf{e}}_r + \Gamma^{-1}\dot{\mathbf{e}}_p) \quad (28)$$

and it is readily seen that $\dot{V}$ is (semi-)negative if:

$$\dot{\mathbf{e}}_p = -\Gamma W_r^T \dot{\mathbf{e}}_r; \quad \dot{\mathbf{e}}_p = -\dot{\hat{\mathbf{p}}} \quad (29)$$

With the adaptation algorithm $\dot{\hat{\mathbf{p}}} = \Gamma W_r^T \dot{\mathbf{e}}_r$, the direct method of Lyapunov states that the reference velocity error $\dot{\mathbf{e}}_r$ converges to $\mathbf{0}$. However, it is not guaranteed that the parameter estimates $\hat{\mathbf{p}}$ converge to the true value $\mathbf{p}$. Nevertheless, the parameter error $\dot{\mathbf{e}}_p$ converges to $\mathbf{0}$, i.e., $\hat{\mathbf{p}}(t)$ becomes constant for $t \to \infty$. In general it is not a problem that $\mathbf{e}_p(t) \neq \mathbf{0}$ for $t \to \infty$ since it is guaranteed that $\dot{\mathbf{e}}_r(t) \to \mathbf{0}$ for $t \to \infty$.

Simulations

The simulation example (see Fig. 1) concerns a two-dimensional translation-rotation (TR) robot with four degrees of freedom ($n = 4$) and two inputs ($m = 2$). The first actuator can move the carriage (mass $m_1 = 10$ [kg]) via a stiff transmission in horizontal direction (coordinate q_s). At the revolute joint a flexible link (mass $m_3 = 3$[kg], length $l = 0.75$[m], stiffness $EI = 10$[Nm^2]) with payload (mass $m_2 = 2$[kg]) is attached. This link is driven via an elastic motor transmission, which is modeled as a linear-elastic, massless torsional spring (stiffness $k = 2$[Nm/rad]). The rotation of the motor rotor (inertia $J = 5$[kgm^2], friction $b = 0.5$[Nm]) is given by q_m, while the rotation of the link at the revolute joint is given by q_e. The distributed link flexibility is approximated with one extra degree of freedom q_f, being the extra displacement at the free end of the link perpendicular to the axis of the link. The desired end-effector trajectory is a circle that has to be followed with a constant velocity of 1[rad/s]. The desired trajectories q_{sd} and q_{ed} for q_s and q_e, respectively, are determined from this desired end-effector trajectory under the assumption that q_f is negligible (see Fig. 2).

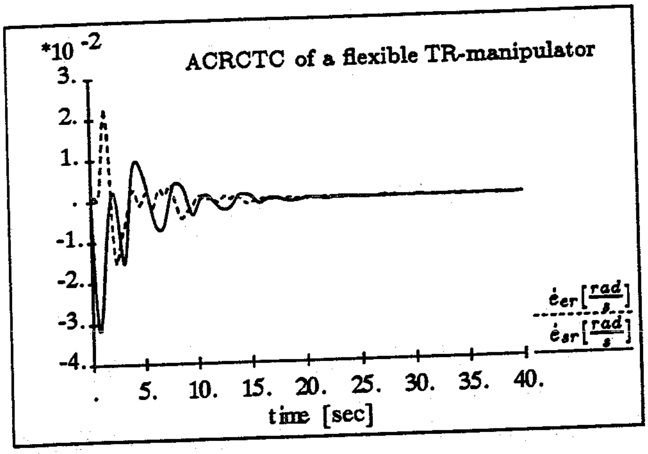

Fig. 3 Reference velocity errors

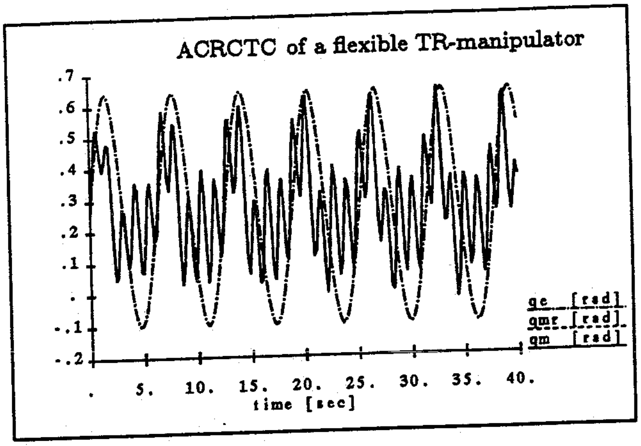

Fig. 6 Link and rotor rotation

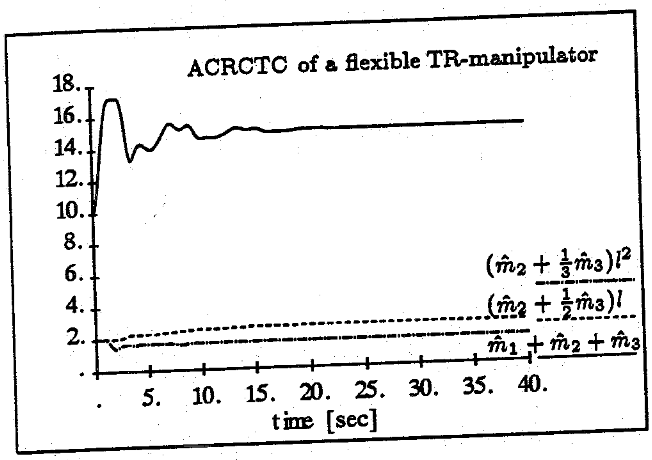

Fig. 4 Adjusted parameters

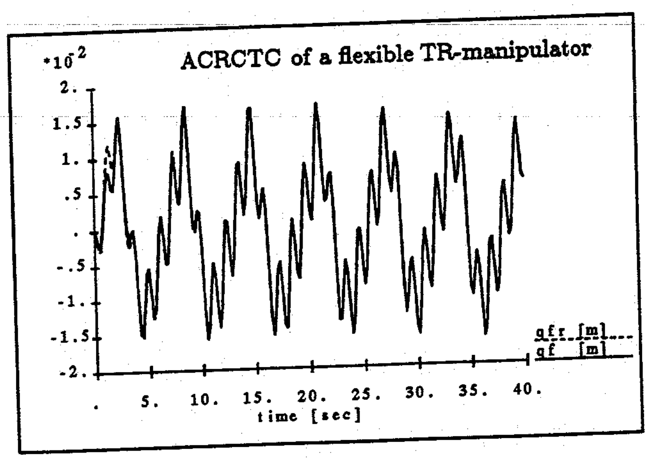

Fig. 5 Link deformation

In this example $\mathbf{q}_k = [q_s\, q_e]^T$ and $\mathbf{q}_u = [q_f\, q_m]^T$. From $\mathbf{q}_{kd} = [q_{sd}\, q_{ed}]^T$, the references $\mathbf{q}_{kr} = [q_{sr}\, q_{er}]^T$ can be determined using definition (5). With the (adaptive) CRCTC law the unknown references $\mathbf{q}_{ur} = [q_{fr}\, q_{mr}]^T$ can be computed using Eq. (13) or (24), while the input signals $\mathbf{u} = [F\, M]^T$ can be computed using Eq. (12) or (23). For details, we refer to Lammerts (1993).

Figures 3, 4, 5, and 6 show adaptive CRCTC simulation results for the case where the masses m_1, m_2, and m_3 are unknown. The initial tracking error $\mathbf{e}(0)$ and the initial parameter estimate $\hat{\mathbf{p}}(0)$ are chosen equal to $\mathbf{0}$. It turns out that the reference velocity errors $\dot{\mathbf{e}}_{sr}$, $\dot{\mathbf{e}}_{er}$, $\dot{\mathbf{e}}_{fr}$, and $\dot{\mathbf{e}}_{mr}$ become zero again after a relatively fast adaptation process, which forces the parameter estimates to their true values (Fig. 4). According to definition (5), the convergence of $\dot{\mathbf{e}}_{kr} = [\dot{\mathbf{e}}_{sr}\, \dot{\mathbf{e}}_{er}]^T$ to $\mathbf{0}$ (Fig. 3) implies that trajectory tracking is realized ($q_s \rightarrow q_{sd}$, $q_e \rightarrow q_{ed}$). Furthermore, q_s, q_e, q_f, and q_m remain bounded, which can also be seen in Figs. 5 and 6 where the elastic degrees of freedom q_f and q_m are shown. The actual deformation of the torsional spring is the difference between q_m (solid line) and q_e (dashed-point line) in Fig. 6. The required input force F and input torque M are very acceptable in all simulation tests (Lammerts et al., 1991; Lammerts, 1993). If the link is quite stiff, a high-frequency signal can occur in the input variables. This is unacceptable for practical implementation. In that case, rigidity of the link must be assumed in control design.

In Lammerts et al. (1991), some simulation results were presented for the special case of an elastic-joint TR robot with rigid links. It was shown that the magnitude of the stiffness of the revolute joint does not influence the tracking performance of the controlled system. Simulations of the TR robot with four degrees of freedom have shown similar results for different stiffnesses of the link and joint (Lammerts, 1993). Both the motor coordinate q_m and the link deformation q_f fluctuate with bounded amplitudes, and all four reference velocity errors $\dot{\mathbf{e}}_{sr}$, $\dot{\mathbf{e}}_{er}$, $\dot{\mathbf{e}}_{fr}$, and $\dot{\mathbf{e}}_{mr}$ converge to zero.

Conclusions and Future Research

A control law for flexible robots is presented that takes into account flexibility in joints and links and also parametric uncertainty. The inclusion of flexibilities in the control model improves the tracking performance in comparison with a controller that is based on a rigid model. Although the number of degrees of freedom exceeds the number of control inputs, global asymptotic stability of the closed-loop system is ensured without restrictions on the magnitudes of the stiffnesses of the links and joints. Since the full state must be available, the system must be equipped with additional sensors for the elastic vibrations and a suitable observer has to be used to reconstruct unknown state variables (e.g., Van de Molengraft, 1990). To handle unknown constant or slowly time-varying robot parameters, an adaptive version of the control law is proposed. Simulations for a TR robot with one elastic joint and one flexible link have shown promising results. If the desired trajectory $\mathbf{q}_{kd}$ depends not only on $\mathbf{y}_d$ but also on the a priori unknown desired trajectory $\mathbf{q}_{ud}$, the problem can be handled with the help of coordinate transformation. For a suitable choice of generalized coordinates to avoid this complexity, as well as for direct end-effector control of the flexible TR-robot, we refer to Lammerts (1993). Experiments on some experimental setups have shown the practical possibilities of the proposed adaptive control law (Lammerts, 1993). In the future, experiments on the adaptive end-effector control of a flexible *xy*-table will be exploited, where the end-effector position will be measured directly with the help of a camera system.

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