

For the record, a Construction of an Optimal Non-linear Binary Error-Correcting Code of Length~10 and Minimum Distance~6

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For the record, a Construction of an Optimal Non-linear Binary Error-Correcting Code of Length 10 and Minimum Distance 6

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According to [1], there exists a (not necessarily linear) binary error-correcting code of length 10 and minimum distance 6, consisting of 6 codewords. I could not immediately find a concrete example. In [2], this is given as Exercise 114 in Chapter 2 “Bounds on the size of codes”:

By Theorem 2.1.2, $A_2(9, 5) = A_2(10, 6)$.

- (a) Compute upper bounds on both $A_2(9, 5)$ and $A_2(10, 6)$ using the Sphere Packing Bound, the Plotkin Bound, and the Elias Bound. When computing the Elias Bound make sure all possible values of w have been checked.
- (b) What is the best upper bound for $A_2(9, 5) = A_2(10, 6)$?
- (c) Find a binary code of length 10 and minimum distance 6 meeting the bound in part (b). Hint: This can be constructed using the zero vector with the remaining codewords having weight 6. (Note: This verifies the entry in Table 2.1.)

Note that Table 2.1 in [2] is improved by [1]. In both, the entry for $A_2(10, 6) = 6$.

Construction

The hint suggests that one finds 5 codewords of weight 6 that have minimum distance 6. By also including the all-0 codeword, one then gets a code of 6 codewords and minimum distance 6.

Imagine one could find 5 codewords of length 5, weight 3, and minimum distance 3. Then doubling each bit would yield 5 codewords of length 10 and weight 6 at distance 6. However, [1] and [2] list $A_2(5, 3) = A_2(6, 4) = 4$. So, that is not going to work.

Consider the 5 cyclic shifts of 11000. It is easy to verify that this yields 5 codewords of length 5, weight 2, and minimum distance 2. In fact, because of the cyclic nature, there are only 2 pairs to consider for the distance:

- $\begin{matrix} 11000 \\ 01100 \end{matrix}$, at distance 2
- $\begin{matrix} 11000 \\ 00110 \end{matrix}$, at distance 4

Now double each 1-bit and replace the 0-bits by respectively 00, 01, and 10. This yields 5 codewords of length 10 and weight 6. The 2 pairs for the distance distribution are then:

- $\begin{matrix} 11 & 11 & 00 & 01 & 10 \\ 10 & 11 & 11 & 00 & 01 \end{matrix}$, at distance $1 + 0 + 2 + 1 + 2 = 6$
- $\begin{matrix} 11 & 11 & 00 & 01 & 10 \\ 01 & 10 & 11 & 11 & 00 \end{matrix}$, at distance $1 + 1 + 2 + 1 + 1 = 6$

Thus, we found as 6 codewords of length 10:

```
0000000000
1111000110
1100011011
0001101111
0110111100
1011110001
```

All pairwise distances equal 6. Note that this code is not linear, since its size is not a power of 2. In fact, $1111000110 + 1100011011 = 0011011101$, which does not occur in the code.

References

- [1] Andries Brouwer. *Table of general binary codes*. 2018-11-27. <https://www.win.tue.nl/~aeb/codes/binary.html>
- [2] Huffman, W. Carry and Pless, Vera. *Fundamentals of Error-Correcting Codes*. Cambridge University Press, 2003. <http://www.cambridge.org/9780521782807>